- What I did since the last meeting

▶ Contents:

① considering seasonality (main part)

② forecasting demand by ARIMA (underway)

→ I also made some minor modifications, I describe main parts.

◉ Considering seasonality

- I'm searching for the method to consider seasonality; there are various methods.

→ Among them, I particularly focus on 2 methods:

① multiply $h(t)$ by the corresponding coefficient for each month
[マルチプライ] [コエフィシェント]

② multiply parameter λ by the corresponding coefficient for each month

① multiply $h(t)$ by the corresponding coefficient for each month

wind turbines (in 2024)

- $h(t)$: the rate that an failure occurs at the moment $t+dt$, where $dt \to 0$

instantaneous failure occurence rate at time
[インスタンテイニアス] [オカーレンス]

$$h(t) = W_s(t) \cdot h_0(t)$$

$h_0(t)$: baseline failure model

$W_s(t)$: the corresponding coefficient for each month.

→ the function of time t

→ take the distinct value in each month

$$W_s(t) = [W_{s1}, W_{s2}, \ldots, W_{s12}]$$

Jan. Feb. Dec.

- The month when demand increases,

→ $W_s(t)$ = "large"

→ increase $h(t)$

- The month when demand is usual,

→ $W_s(t)$ = "1"

② multiply parameter λ by the corresponding coefficient for each month

- λ : parameter that determines the average and median failure time
- AFT model
- The idea is very similar to first method:

- specific literatures
- I applied it since I think it is applicable

$$\lambda = \lambda_0 \exp(\beta(t))$$

λ_0 : baseline parameter

$\beta(t)$: the corresponding coefficient for each month.

→ the function of time t

→ take the distinct value in each month

$$\begin{array}{c} \beta(t) = [\beta_{s1}, \beta_{s2}, \ldots, \beta_{s12}] \\ \text{Jan.} \quad \text{Feb.} \qquad\qquad \text{Dec.} \end{array}$$

- The month when demand increases,
  - → $\beta(t)$ = "large"
    giving it a value larger than 0
  - → increase λ $\left(T \propto \frac{1}{\lambda}\right)$
    inversely proportional of life time T
  - → life time gets shorter
- The month when demand is usual,
  - → $W_s(t)$ = "1"
  - → life time does not change

- How to design the coefficient for each month
  - 3 types of combination
    - — summer - winter : considering that demand increases in both , ~~give~~ large coef. in both season
    - — summer : considering that demand increases only in summer
    - — winter : considering that demand increases only in winter

  ※ transitional period : period when the demand changes from high to usual / from usual to high

- Results : with 2 dealers, 30 trucks and 30 parts , simulation period is set to 2 years.
  - sum of failures of each part type (in each dealer) in each month
  - the number of daily failure of each holding part type in each deler

◉ Forecasting demand by ARIMA

- started coding last night, I don't fully understand ARIMA.
  → $\boxed{\text{I need to study it further}}$

- I just used python library as it is, without adding any particular modifications.

- How I trained the model:
  - I build the model and forecast the demand, for each part type (in each dealer)
  - Provide "one year of demand data" as train data, predict "the demand for the following year"

    Part-type A (dealer 1)

time	sum(failure)
1	0
2	0
3	2
⋮	⋮

  - Evaluate "the prediction accuracy" by comparing with the synthetic demand

- To conclude, it hasn't worked well.

- MTG 2024/10/22

- Questions of them:
  1. What's the difference between the seasonaly method.
  2. Why does it take exponential.

- Directions of Slowomir:
  - Should use the following method.

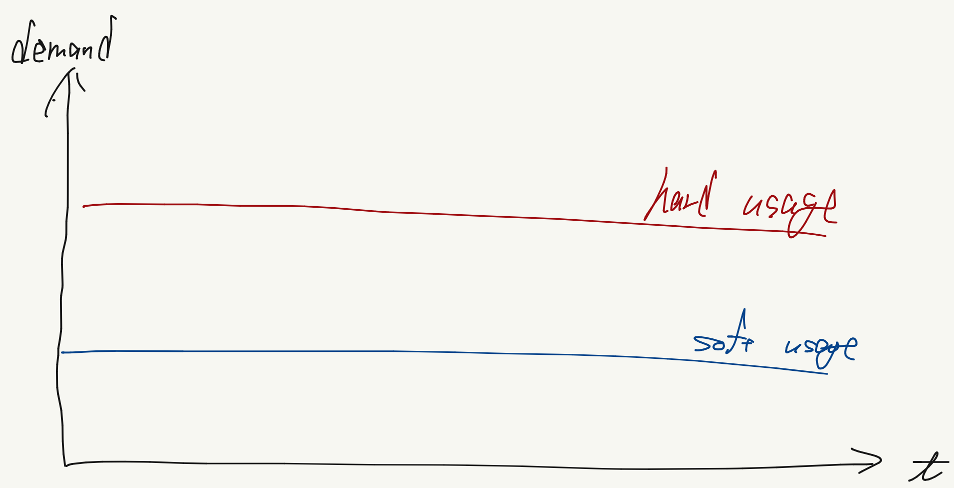
$$\lambda(t) = W_s(t)\cdot \lambda_0(t)$$

$\lambda_0(t)$: baseline failure model

$W_s(t)$: the coef. to each month

$W_s(t) = [W_{s1}, W_{s2}, \dots, W_{s12}]$

  - Consider "the bias of usage".

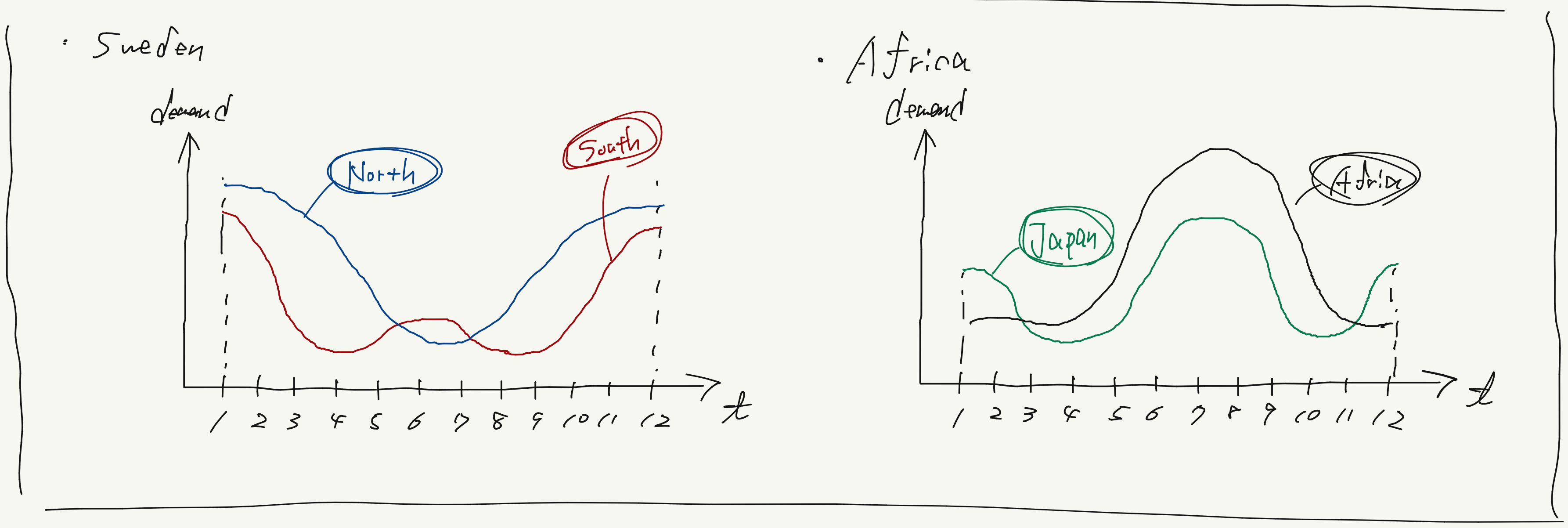


- Directions of Slawomir :
  - Consider the location of dealer.

→ As the location differs, the seasonality changes.

- the number of the demand peaks
- the location of the demand peaks
- the height of the demand peaks
- the width of the demand peaks

ex:



→ How to consider the differences :

Adjust the coefficient for each location.

- $W_s = [W_{s1}, W_{s2}, \dots, W_{s12}]$
  - scale → number, height, width
  - shift → location

How to

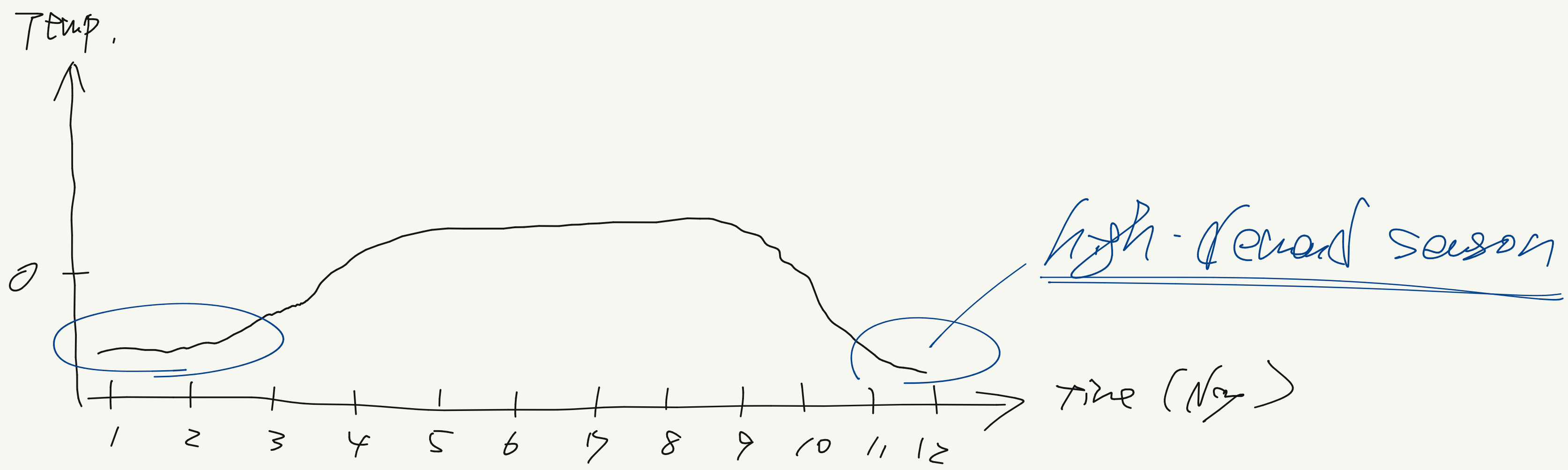
- Abdallah : Cosider "daily coefficients" instead of monthly ones

idea

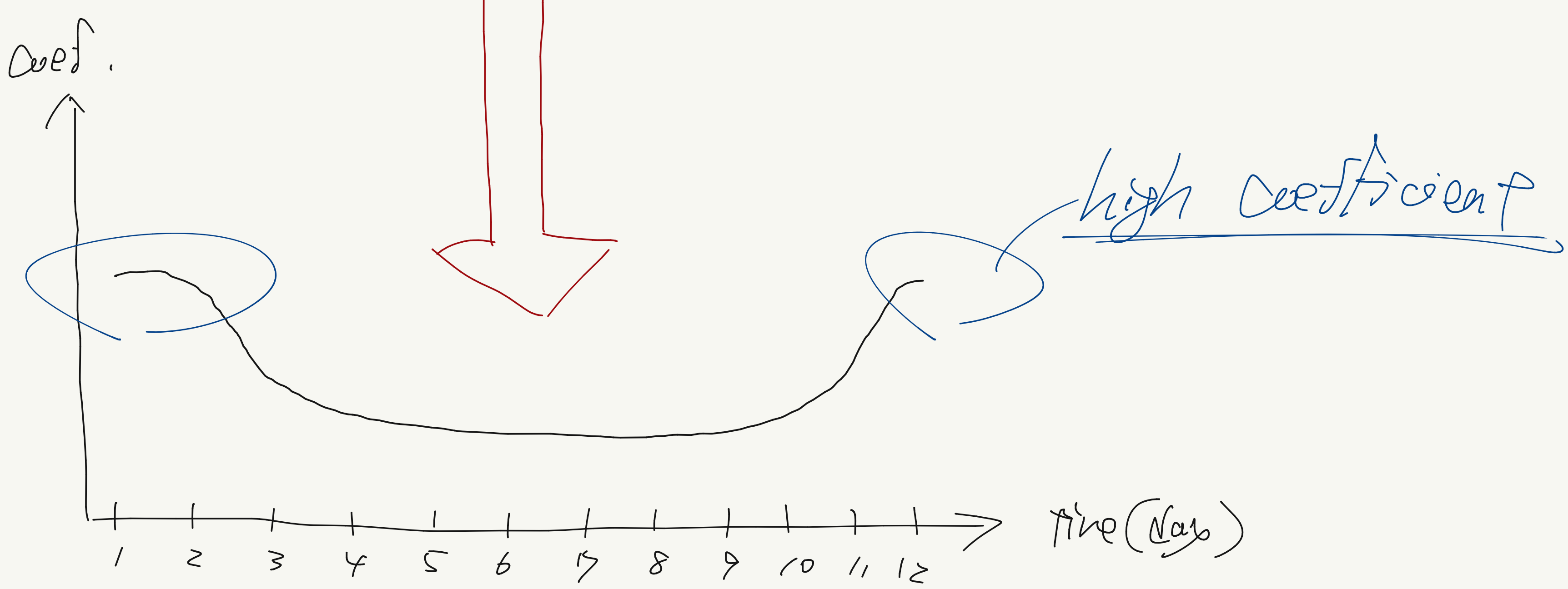
- Reproduce the temperature tendency by seasonal model (coefficient)

▶ Get real-world temperature data

- region: several areas in the world, which have "different climate types"
  - e.g. Köppen climate classification
- periods: one-year



▶ Build seasonal model (coefficient model) by referring real data



▶ Just idea

- I can account for other types of data, e.g. (rain data
  snow data

Seasonality function $W_s(t)$

- $W_s(t) = G(t) \cdot F(t)$

$G(t)$

- Generate "gaussian curves" that repeats with a certain period.

$$\boxed{G(t) = \exp\left(A \cdot \sin\left(2\pi \cdot \frac{t-\phi}{T}\right)\right)}$$

$F(t)$

- Filter function

$$\boxed{F(t) = \begin{cases} 1, & \text{if } t \in S_{active} = \{\underbrace{12, 1, 2}_{\text{high-demand month}}\} \\ 0, & \text{otherwise} \end{cases}}$$

Characteristics

- Gaussian curves are "generated" according to "a certain frequency".
- Sigma in each Gaussian curve takes "same value".
- the peak positions are "determined" based on "ϕ".

→ calculation of ϕ to match peak with specific month m.

$$2\pi \cdot \frac{m-\phi}{T} = \frac{\pi}{2}$$

$$4m - 4\phi = T$$

$$\boxed{\phi = m - \frac{T}{4}}$$

$$\phi = 1 - \frac{90}{4} = \frac{60-90}{4} = \frac{-30}{4} = -\frac{15}{2}$$

- Seasonality function $W_s(t)$

- requirements
  - To shift the peak
  - To scale the height of peak
  - To adjust the width
  - To generate multiple peak.

Seasonality function $W_s(t)$ method ①

- $W_s(t) = \sum_{p=1} G_p(t) \cdot F_p(t)$

▶ $G(t)$

- Generate "gaussian curves" that repeats with a certain period.

$$\boxed{G_p(t) = \exp\left(A \cdot \sin\left(2\pi \cdot \frac{t-\phi}{T}\right)\right)}$$

▶ $F(t)$

- Filter function

$$F_p(t) = \begin{cases} 1, & \text{if } t \in S_{active_p} = \{ \underbrace{12, 1, 2}_{\text{high-demand}} \} \text{ month} \\ 0, & \text{otherwise} \end{cases}$$

▶ Characteristics

- difficulty : ☆ cut out "the bell curve" clearly using filter function

→ bell curve becomes intermittent if the range of the filter function is too narrow

→ How to.

① Match the peak (phase ϕ) with the center of the range

② ?

- Seasonality function $W_s(t)$ (method②)
  - Sum of Radial Basis Function.
    (レイディアル)

$$\boxed{W_s(t) = \sum_p \exp\left(-\frac{\|\mu_p - t\|^2}{\sigma^2}\right)}$$

① Place RBFs at the peak month

② Sum up each RBF

- change the height at the peak for each RBF
- define high-demand range, where the function takes values larger than 0 or 1, by inversely calculating σ
  → so that the function takes specific value at certain time t

- Seasonality function $W_s(t)$ method ③

- $R(t)$: RBFN (Radial Basis Function Network)
  - Regression model → accordig to the sum of RBF — an extended version
  - We can draw the seasonal curve automatically by using RBF, by setting demand degree for each month
  - Calculation cost is relative low : Only the calculation of inverse matrix is required.
  - one ~~example~~ example:
    - Train the model so that : give two peaks - demand
      - → output (takes large value at high-demand months / takes 0 at no-demand months)
      - → give the number of days to sigma of each RBF in each month
    - Use circular coordinate to connect Jan. to Dec.

$$\boxed{\sigma = \min(|c_j - t|, 12 - |c_j - t|)}$$

  - weighted sum of gaussian functions.

$$\boxed{\hat{y}(t) = \sum_{i}^{m} w_i h_i(t), \quad h_i(t) = \exp\left(-\frac{\|c_j - t\|^2}{\sigma^2}\right)}$$

● seasonality function $W_s(t)$ method ③

- $R(t)$: RBFN (Radial Basis Function Network)

· How to build $\hat{y}(t)$:

$$\boxed{\begin{array}{ll} \text{find} & W \\ \min & \|y - \hat{y}(t)\|^2 + \|\lambda W\|^2 \end{array}}$$

$$\rightarrow \boxed{W = (H^T H + \Lambda)^{-1} H^T y}$$

where

$$H = \begin{bmatrix} h_1(t_1) & h_2(t_1) & \cdots & h_m(t_1) \\ h_1(t_2) & h_2(t_2) & \cdots & h_m(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ h_1(t_m) & h_2(t_m) & \cdots & h_m(t_m) \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & O \\ & \lambda_2 & \\ O & & \ddots \\ & & & \lambda_m \end{bmatrix}$$

$$y = [y_1, y_2, \ldots, y_m]^T$$

Calculation

- Monthly

$$A \cdot \sin\left(2\pi \cdot \frac{m-\phi}{T}\right)$$

- curve at m

$$2\pi \cdot \frac{m-\phi}{T} = \frac{\pi}{2} \to m-\phi = \frac{T}{4} \to \boxed{\phi = m - \frac{T}{4}}$$

- curve at m=1

$$2\pi \cdot \frac{1-\phi}{12} = \frac{\pi}{2}$$

$$\frac{1-\phi}{3} = 1$$

$$1-\phi = 3$$

$$\boxed{\phi = -2}$$

- curve at m=2

$$2\pi \cdot \frac{2-\phi}{12} = \frac{\pi}{2}$$

$$\frac{2-\phi}{3} = 1$$

$$2-\phi = 3$$

$$\boxed{\phi = -1}$$

- curve at m=12

$$2\pi \cdot \frac{12-\phi}{12} = \frac{\pi}{2}$$

$$\frac{12-\phi}{3} = 1$$

$$12-\phi = 3$$

$$\boxed{\phi = 9}$$

- curve at m=6

$$2\pi \cdot \frac{6-\phi}{12} = \frac{\pi}{2}$$

$$\frac{6-\phi}{3} = 1$$

$$6-\phi = 3$$

$$\boxed{\phi = 3}$$

- curve at m=7

$$2\pi \cdot \frac{7-\phi}{12} = \frac{\pi}{2}$$

$$\frac{7-\phi}{3} = 1$$

$$7-\phi = 3$$

$$\boxed{\phi = 4}$$

- curve at m=8

$$2\pi \cdot \frac{8-\phi}{12} = \frac{\pi}{2}$$

$$\frac{8-\phi}{3} = 1$$

$$8-\phi = 3$$

$$\boxed{\phi = 5}$$

Calculation

31 28 31 30

$$\lambda(t) = \exp\left(-\frac{\|\mu - t\|^2}{\sigma^2}\right)$$

- when $\mu = \mu_0$, range $[t^{(1)}, t^{(2)}]$

$\rightarrow$ $\boxed{\text{design } \sigma \text{ so that } \lambda(t^{(2)}) = \varepsilon \coloneqq 0}$

$$\varepsilon = \exp\left(-\frac{(\mu - t^{(2)})^2}{\sigma^2}\right)$$

$$\log(\varepsilon) = -\frac{(\mu - t^{(2)})^2}{\sigma^2}$$

$$\sigma^2 = \frac{-(\mu - t^{(2)})^2}{\log(\varepsilon)}$$

$$\sigma = \pm(\mu - t^{(2)})\sqrt{-\frac{1}{\log(\varepsilon)}}$$

Seasonality difference

- Northen area VS Southen Area
  - Northen area (Tough winter)

→ $\boxed{\text{Demand tends to be higher in winter.}}$

- winter peak

  $\mu = 1/15$, $\sigma = 25 \in [11/1 , 3/31]$

  wide range

- summer peak

  $\mu = 8/1$ $\sigma = 15 \in [6/15 , 9/15]$

  short range

- Southern area (Tough summer)

→ $\boxed{\text{Demand tends to be higher in summer.}}$

- winter peak

  $\mu = 1/15$, $\sigma = 15 \in [12/1 , 2/28]$

  short range

- summer peak

  $\mu = 8/1$ $\sigma = 25 \in [5/15 , 10/15]$

  wide range