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Econ 899
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1 Solution to questions (using Julia)

Note: Questions 1-3 are answered using the output of our Julia program. However, the answers are obviously the same for any of the languages. We include the output for Matlab and Fortran in the next section. All code, figures, and auxiliary files can be found here.

1. The agent's dynamic programming problem is as follows. Given a current level of capital k and realization of productivity level z, the agent's value is given by the following:

$$V(k, z) = \max_{(k', c) \in \Gamma(k, z)} u(c) + \beta \sum_{z' \in Z} V(k', z') \Pi(z', z)$$

where $\Pi(z', z)$ is the probability of transitioning to productivity z' given current state z, $Z = \{1.25, 0.2\}$ is the set of productivity levels, and

$$\Gamma(k, z) = \{(k', c) \in \mathbb{R}_+^2 \mid k' + c \leq zk^\theta + (1 - \delta)k\}$$

is the set of feasible choices of consumption and future capital given current capital k and productivity z.

2. Below, we plot the value function for K over each state Z. It is evident that $V(k, z)$ is increasing in z since the high productivity function lies above the low productivity function. Furthermore, it is 'concave' in K, as the difference between the two shrinks as K grows. To more clearly see these relationships, we have plotted $V(\cdot, Z^g) - V(\cdot, Z^b)$ for all K. It is clear that the difference is positive and decreasing in K, verifying the previous claims.

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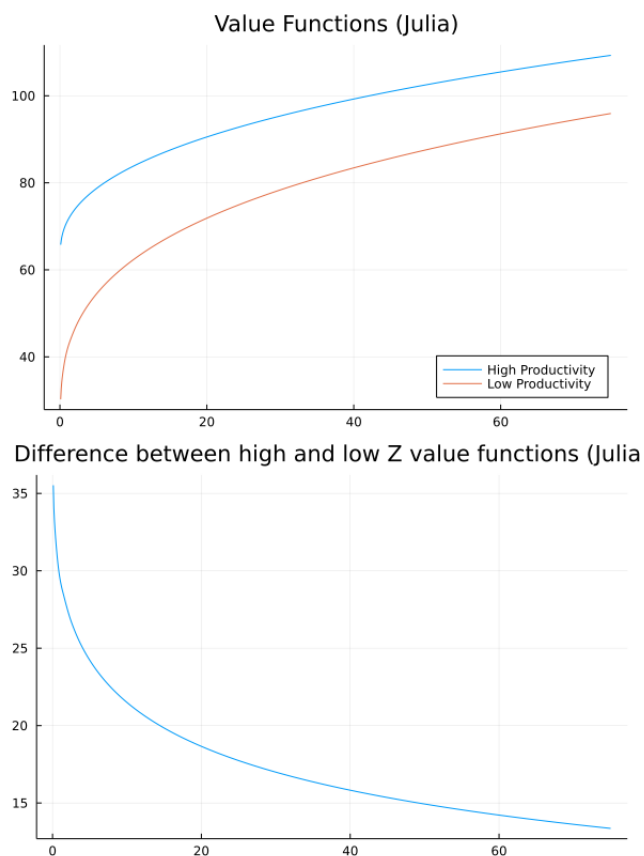
Figures generated using Julia

3. The decision rule is increasing in both K and Z. To see this, we plot the policy function as a function of K for both high and low productivity levels below. For both types, $K'(k, z)$ is increasing in k. Furthermore, it is evident that the policy function for high productivity agents is higher than that of low productivity agents.
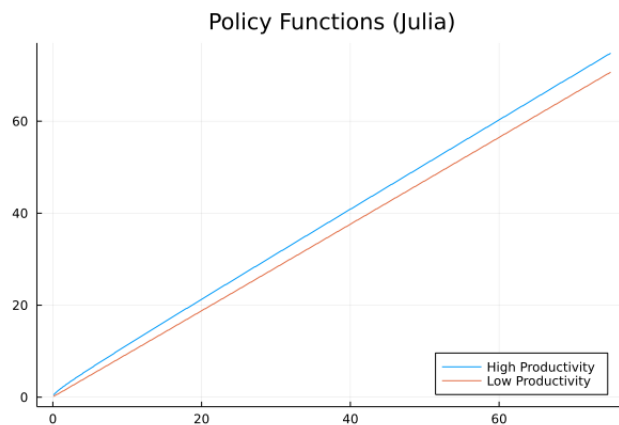
However, when we plot $K'(K, Z) - K$ across K in the following figure to ascertain agents' savings as a function of K and Z, we can see that the relationship is somewhat more complicated.

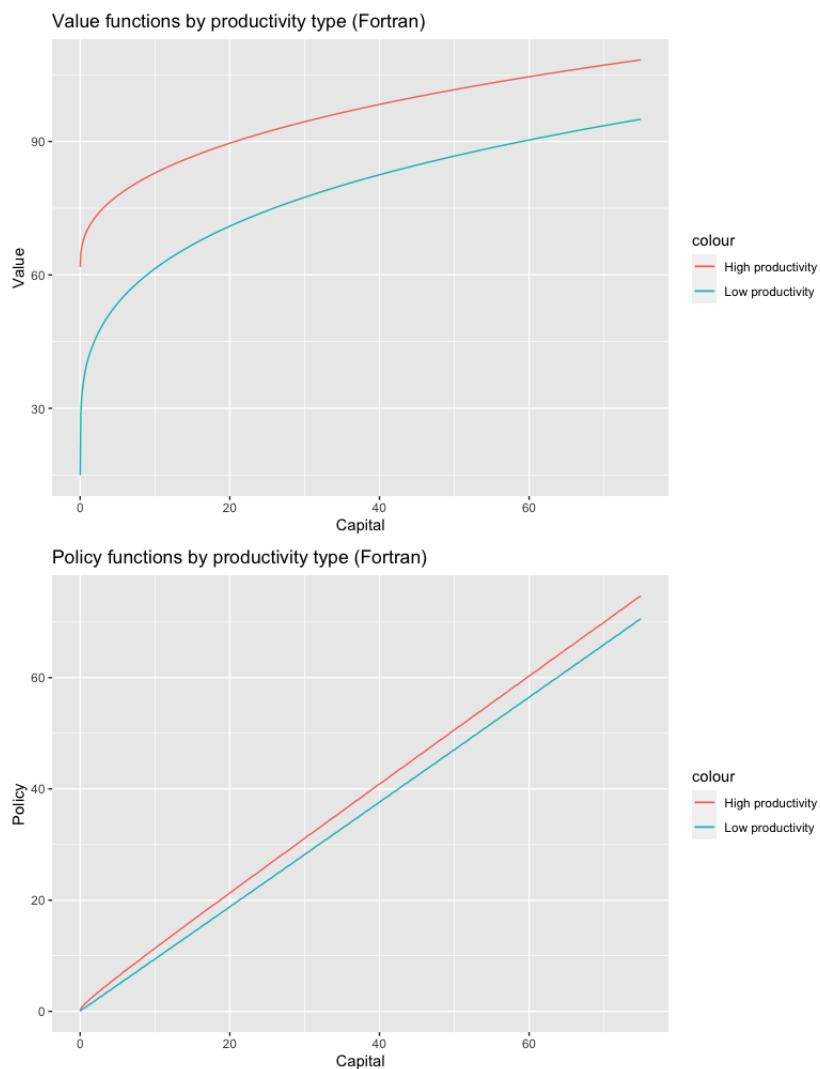Change in decision rule, by K (Julia)
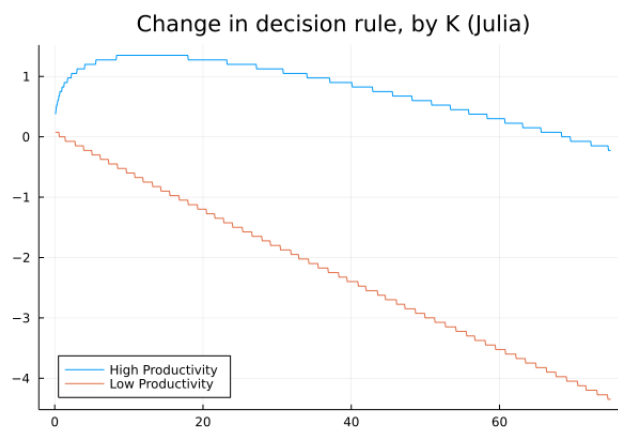
Upon inspection, we note that high productivity agents have higher savings than low productivity agents.
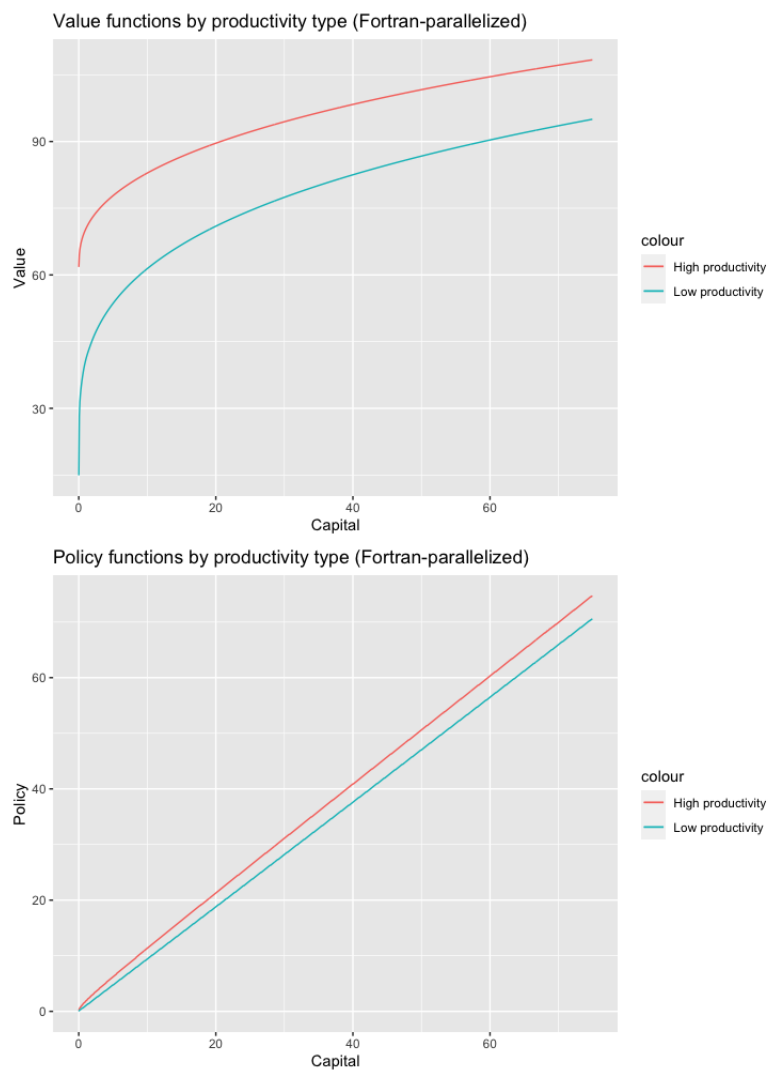
Furthermore, low productivity agents' savings is decreasing in their current level of capital K.

Finally, high productivity agents' savings are increasing in K when their current level of capital is below a certain threshold (about 15 or so), and are decreasing in K when their current level of capital is higher than this threshold.
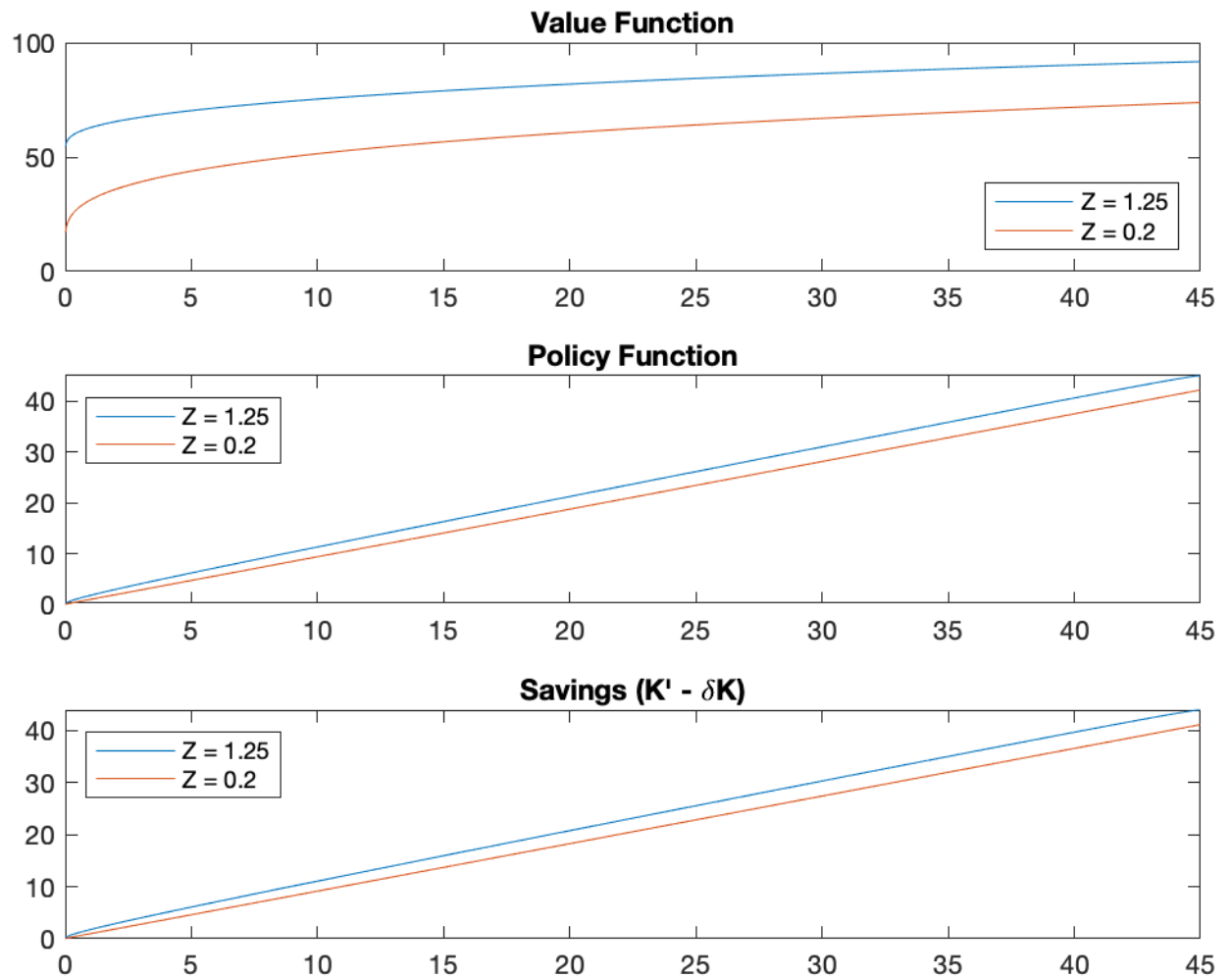
2   Repeating the analysis with Matlab and Fortran

Value functions by productivity type (Fortran)

Policy functions by productivity type (Fortran)

Results using Fortran

Results using Fortran (parallelized)

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Results using Matlab

Time comparison:

- Julia: 3.297 seconds

- Julia (parallelized): 2.173 seconds

- Matlab: 60 seconds

- Fortran (non-parallelized): 3.067 seconds

- Fortran (parallelized): 0.188 seconds

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Note: I set the max capital level at 75 for the Julia and Fortran versions. Originally, the code we were given had different capital grids (Fortran's only went up to 45, whereas the Julia one went up to 75). To enable a fair comparison, I made them all 75.