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Econ 899
6 December, 2022

Note: the transition matrix $\Pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{bmatrix}$ given in the problem set differs from the transition matrix used to generate the supplied transition matrices as well as the simulated data (the transition matrix used for these appears to be $\Pi = \begin{bmatrix} 0.75 & 0.25 \\ 0.9 & 0.1 \end{bmatrix}$). We have proceeded using $\Pi = \begin{bmatrix} 0.75 & 0.25 \\ 0.9 & 0.1 \end{bmatrix}$ in our analysis for consistency).

Problem 1.

Solution: The implicit equation which defines $\bar{V}(s) = E_\varepsilon[V(s,\varepsilon)]$ is given by the following:

$$\bar{V}(s) = \log(\exp(U_0(s)) + \exp(U_1(s))) + \gamma$$

where γ is Euler's constant and

$$U_0(s) = (\alpha c)\mathbb{1}_{i>0} + \lambda \mathbb{1}(i = 0, c > 0)$$

$$U_1(s) = \alpha c - p$$

with $s = (i, c, p)$ indicating the state.

We solve for $\bar{V}(s)$ and tabulate the results in Table I for each state variable s. □

Problem 2.

Solution: We estimate the conditional choice probability $\hat{P}(s)$ using the provided simulated sequence of choices. Using this estimated $\hat{P}(s)$, we can compute the implied expected value function $\bar{V}^{\hat{P}}(s)$ using the CCP mapping.

Under the assumption that $\varepsilon \sim$ T1EV, it follows that $e(a, s) = \gamma - \ln(\hat{P}(s))$, where $e(a, s) = E[\varepsilon(a) \mid a^* = a, s]$ is the conditional expectation of $\varepsilon(a)$. The value function

Inventory	Cons. shock	Price	$\bar{V}(s)$
0.0	0.0	4.0	61.128
1.0	0.0	4.0	65.01
2.0	0.0	4.0	68.482
3.0	0.0	4.0	71.669
4.0	0.0	4.0	74.63
5.0	0.0	4.0	77.394
6.0	0.0	4.0	79.959
7.0	0.0	4.0	82.263
8.0	0.0	4.0	84.073
0.0	1.0	4.0	58.491
1.0	1.0	4.0	63.128
2.0	1.0	4.0	67.01
3.0	1.0	4.0	70.482
4.0	1.0	4.0	73.669
5.0	1.0	4.0	76.63
6.0	1.0	4.0	79.394
7.0	1.0	4.0	81.959
8.0	1.0	4.0	84.263
0.0	0.0	1.0	63.244
1.0	0.0	1.0	66.895
2.0	0.0	1.0	70.203
3.0	0.0	1.0	73.26
4.0	0.0	1.0	76.11
5.0	0.0	1.0	78.766
6.0	0.0	1.0	81.201
7.0	0.0	1.0	83.282
8.0	0.0	1.0	84.278
0.0	1.0	1.0	61.025
1.0	1.0	1.0	65.244
2.0	1.0	1.0	68.895
3.0	1.0	1.0	72.203
4.0	1.0	1.0	75.26
5.0	1.0	1.0	78.11
6.0	1.0	1.0	80.766
7.0	1.0	1.0	83.201
8.0	1.0	1.0	85.282

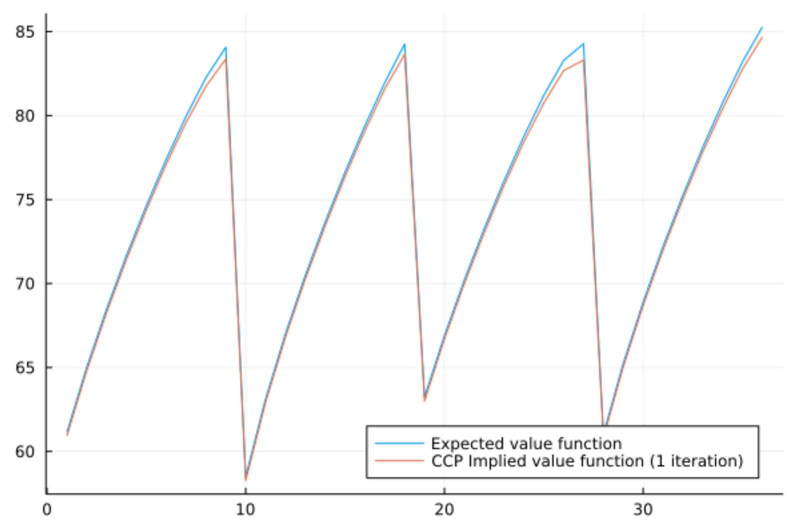
Table 1: Values of $\bar{V}(s)$

implied by the CCP vector is the following:

$$\bar{V}^P = (I - \beta F^P)^{-1}[(1 - P(s)) \odot (U_0(s) + e(0)) + P \odot (U_1(s) + e(1))]$$

where $F^P = (1 - P) \odot F(0) + P \odot F(1)$ and $\odot$ is the Hadamard element-wise product operator.

We first do one iteration of this operator using the estimated probabilities $\hat{P}$ to estimate $\bar{V}^{\hat{P}}(s)$ (i.e. one iteration of the above operator) and include our results in Table II. We see that they do differ from $\bar{V}(s)$, but only slightly (and not by very much on average). This could certainly be due to simulation error: some states are very unlikely to be realized in the sample, and we consequently will have fewer observations for those. This will adversely affect the accuracy of our implied value function estimate. We plot $\bar{V}(s)$ and $\bar{V}^{\hat{P}}(s)$ alongside each other in the following figure:



We see that the only major differences occur for the states with very high i. These are extremely unlikely to be observed (as it is costly to build up that much inventory + consumption shocks will make it even harder to get that much). These are states for which the estimated $\hat{P} = 0.001$ (our imposed non-zero constraint). As a result, we shouldn't be surprised that the implied value function is a bit off.

We have also iterated the CCP mapping (i.e. policy function iteration) to estimate $\bar{V}^P$. We iterate by updating $P^k = 1/(1 + \exp(-\tilde{v}(s)^{k-1}))$, where $\tilde{v}(s)^{k-1} = (U_1 + \beta F_1 \bar{V}^{k-1}) - (U_0 + \beta F(0)\bar{V}^{k-1})$. We stop when $\|P^k - P^{k-1}\| < \eta$ (we use $\eta = 1 \times 10^{-12}$). When we iterate this until convergence, we get an estimate for $\bar{V}^{\hat{P}}(s)$ which is (not surprisingly) numerically identical to $\bar{V}(s)$ estimated in problem 1. We don't include the plot for this one because they're literally on top of each other and no information is gained that isn't conveyed in the table. □

Problem 3.

Solution: The log-likelihood function is given by the following:

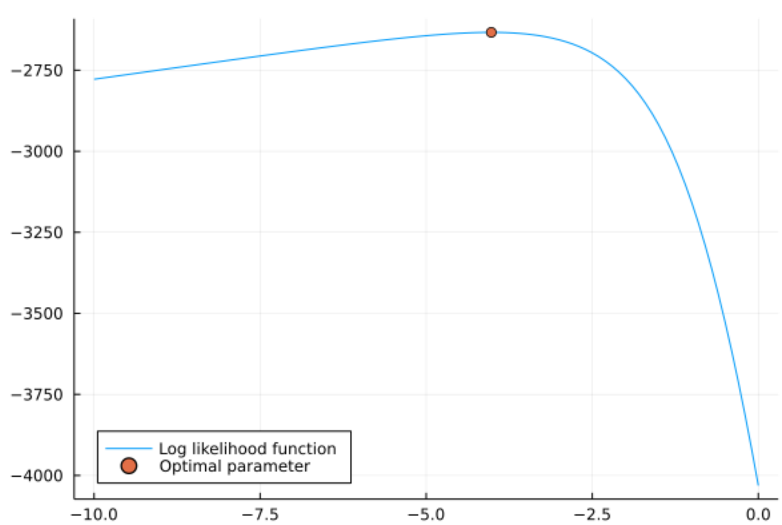
$$L(\lambda) = \sum_i a_i \ln(P(s_i)) + (1 - a_i)\ln(1 - P(s_i))$$

such that $P(s_i) = \Psi(s_i) \equiv (1 + \exp(-\tilde{v}(s_i)^{k-1}))^{-1}$, where $\tilde{v}(s_i)^{k-1}$ is as defined previously.

□

Problem 4.

Solution: We minimize the log-likelihood using the Nested-Fixed Point algorithm and find that the minimizer is $\hat{\lambda} = -4.024$, which corresponds to a log likelihood of -2633.152. We include a plot of the log-likelihood from $\lambda = -10$ to $\lambda = 0$ below:



Inventory	Cons. shock	Price	$\bar{V}(s)$	$\bar{V}^{\hat{P}}(s)$ (1 iter)	$\bar{V}^{P}(s)$ (actual)
0.0	0.0	4.0	61.128	60.914	61.128
1.0	0.0	4.0	65.01	64.8	65.01
2.0	0.0	4.0	68.482	68.267	68.482
3.0	0.0	4.0	71.669	71.44	71.669
4.0	0.0	4.0	74.63	74.38	74.63
5.0	0.0	4.0	77.394	77.104	77.394
6.0	0.0	4.0	79.959	79.594	79.959
7.0	0.0	4.0	82.263	81.744	82.263
8.0	0.0	4.0	84.073	83.376	84.073
0.0	1.0	4.0	58.491	58.273	58.491
1.0	1.0	4.0	63.128	62.915	63.128
2.0	1.0	4.0	67.01	66.8	67.01
3.0	1.0	4.0	70.482	70.267	70.482
4.0	1.0	4.0	73.669	73.434	73.669
5.0	1.0	4.0	76.63	76.377	76.63
6.0	1.0	4.0	79.394	79.103	79.394
7.0	1.0	4.0	81.959	81.595	81.959
8.0	1.0	4.0	84.263	83.645	84.263
0.0	0.0	1.0	63.244	62.999	63.244
1.0	0.0	1.0	66.895	66.684	66.895
2.0	0.0	1.0	70.203	69.982	70.203
3.0	0.0	1.0	73.26	73.014	73.26
4.0	0.0	1.0	76.11	75.842	76.11
5.0	0.0	1.0	78.766	78.437	78.766
6.0	0.0	1.0	81.201	80.744	81.201
7.0	0.0	1.0	83.282	82.68	83.282
8.0	0.0	1.0	84.278	83.308	84.278
0.0	1.0	1.0	61.025	60.807	61.025
1.0	1.0	1.0	65.244	65.034	65.244
2.0	1.0	1.0	68.895	68.681	68.895
3.0	1.0	1.0	72.203	71.983	72.203
4.0	1.0	1.0	75.26	75.019	75.26
5.0	1.0	1.0	78.11	77.838	78.11
6.0	1.0	1.0	80.766	80.405	80.766
7.0	1.0	1.0	83.201	82.771	83.201
8.0	1.0	1.0	85.282	84.682	85.282

Table 2: Values of $\bar{V}^{\hat{P}}(s)$ and true $\bar{V}^{P}(s)$

$\square$