CS 6505 - Homework 6

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$$\begin{bmatrix} 4 & 2 & 0 & | & 1 \\ 0 & 2 & 1 & | & 2 \\ 1 & 0 & 1 & | & 0 \end{bmatrix} \tag{1}$$

Reduce modulo 3.

$$\begin{bmatrix} 1 & 2 & 0 & | & 1 \\ 0 & 2 & 1 & | & 2 \\ 1 & 0 & 1 & | & 0 \end{bmatrix}$$
 (2)

 $R_3 = R_3 - R_1.$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & -2 & 1 & -1 \end{bmatrix}$$
 (3)

Reduce mod 3.

$$\begin{bmatrix}
1 & 2 & 1 & | & 0 \\
0 & 2 & 1 & | & 2 \\
0 & 1 & 1 & | & 2
\end{bmatrix}$$
(4)

 $R_2 = R_2 - 2R_3.$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$
 (5)

Reduce mod 3 and switch rows 2 and 3.

$$\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 0 & 1 & 1 & | & 2 \\ 0 & 0 & 2 & | & 1 \end{bmatrix}$$
 (6)

 $R_1 = R_1 - 2R_2.$

$$\begin{bmatrix} 1 & 0 & -1 & | & -4 \\ 0 & 1 & 1 & | & 2 \\ 0 & 0 & 2 & | & 1 \end{bmatrix}$$
 (7)

Reduce mod 3.

$$\begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$
 (8)

 $R_1 = R_1 - R_3.$

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 1 & | & 2 \\ 0 & 0 & 2 & | & 1 \end{bmatrix}$$
 (9)

 $R_2 = R_2 - \frac{1}{2}R_3$. $R_3 = \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$
 (10)

Reduce mod 3 using the fact that the multiplicative inverse of 2, namely $\frac{1}{2}$ is 2 mod 3.

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$$
 (11)

So, $x_1 = 1$, $x_2 = 0$, and $x_3 = 2$.