CS 6505 - Homework 7

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1. Label all vertices from 1 to n. Construct a directed graph, G, from our undirected graph by orienting each edge both ways and allowing the capacity each direction to be the capacity of the underlying undirected edge.
2. For all pairs of vertices, (i,j), with $i < j$:
   (a) Let the current residual graph be G.
   (b) While there exists an augmenting path in the residual graph:
      i. Find an augmenting path.
      ii. Augment it and update the current residual graph.
   (c) Breadth First Search from i in the residual graph to mark the component containing i.
   (d) Identify the set, $S_{i,j}$, of all edges from the component containing i to the component containing j (everything which is not marked).
   (e) Store the pair $(S_{i,j}, \sum_{e \in S_{i,j}} c_e)$.
3. Identify a pair $(S_{i,j}, \sum_{e \in S_{i,j}} c_e)$ such that $\sum_{e \in S_{i,j}} c_e$ is minimized.
4. Return $S_{i,j}$.

The runtime of this algorithm is $O(n^2((nm) + (n + m)) + n^2)$ because Ford Fulkerson runs in O(nm) time and BFS runs in O(n+m) time (we do these $n^2/2$ times) then finding the running min of $n^2/2$ elements takes $O(n^2)$ time.