## CS 6505 - Homework 10 - Worked with Qiaomei Li

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We first establish variable names:  $x_1$  = number chairs fresh,  $x_2$  = number chairs polished regular,  $x_3$  = number chairs polished overtime,  $x_4$  = number tables fresh,  $x_5$  = number tables polished regular,  $x_6$  = number tables polished overtime,  $x_7$  = number cabinets fresh,  $x_8$  = number cabinets polished regular,  $x_9$  = number cabinets polished overtime.

Our LP is to maximize  $8x_1 + 14x_2 + 11x_3 + 4x_4 + 12x_5 + 7x_6 + 4x_7 + 13x_8 + 9x_9$  subject to  $x_1 + x_2 + x_3 \le 480$ ,  $x_4 + x_5 + x_6 \le 400$ ,  $x_7 + x_8 + x_9 \le 230$ ,  $x_2 + x_5 + x_8 \le 420$ ,  $x_3 + x_6 + x_9 \le 250$ , and  $x_i \ge 0$  for all i. We add slack variables to get,  $x_1 + x_2 + x_3 + s_1 = 480$ ,  $x_4 + x_5 + x_6 + s_2 = 400$ ,  $x_7 + x_8 + x_9 + s_3 = 230$ ,  $x_2 + x_5 + x_8 + s_4 = 420$ ,  $x_3 + x_6 + x_9 + s_5 = 250$  and  $x_i, s_j \ge 0$  for all i.j. Using the simplex method, the optimal solution is  $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (440, 0, 40, 0, 400, 0, 0, 20, 210)$ . The dual program is to minimize  $480y_1 + 400y_2 + 230y_3 + 420y_4 + 250y_5$  subject to  $y_1 \ge 8$ ,  $y_1 + y_4 \ge 14$ ,  $y_1 + y_5 \ge 11$ ,  $y_2 \ge 4$ ,  $y_2 + y_4 \ge 12$ ,  $y_2 + y_5 \ge 7$ ,  $y_3 \ge 4$ ,  $y_3 + y_4 \ge 13$ ,  $y_3 + y_5 \ge 9$ , and  $y_1 \ge 0$ . Since the optimal value for the dual problem is also 10910, we know that our solution was optimal.