CS 6505 - Homework 4

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(We assume the input is given in the proper form). Our Turing Machine D works as follows: Store $\langle M \rangle$, x on the left of the tape. Afterwards, to the right leave a # (assume that wasn't in our alphabet before) keep our counter to the right of the pound sign starting with 1 digit with "#0". Then, we leave a pound sign to the right of this counter. We do so at the start in at most $|\langle M \rangle| + |x| + 3$ steps (since the position of our head starts at the beginning of the tape). Then move to the start of x and begin the simulation in at most $|x| + 3$ steps. This startup cost is at most $|\langle M \rangle| + 2|x| + 6$ steps.
Now, say we have taken k steps of our simulation so far (at the start k = 0). We do as follows.

1. Starting at the current head position, read the input, then go back to the description of the Turing Machine at the very left in at most $T(n) + |x| + |\langle M \rangle|$ steps.

2. Then, go to the counter in at most $|\langle M \rangle| + |x|$ steps and increment it. This will take $log(T(n)) + 3T(n) + 3$ steps if we need to increase the size of the counter, in which case we shift everything to the right of the counter to the right by $[currentsizeofcounter]$ squares by going to the right end of the tape (in at most log(T(n)) + T(n) steps), then we copy each entry, paste it one to the right, then move two to the left T(n) + 1 times to move over the rightmost pound sign and everything to the right of it (in 3(T(n)+1) steps). Then, we increment the counter and pad the side with 0's (in at most log(T(n)) steps). Overall, this step takes at most $|\langle M \rangle| + |x| + log(T(n)) + 3T(n) + 3$ steps.

3. Then, move back to the head position we had before we went to read the machine description (in at most $log(T(n)) + |x|$ steps) and do what it says in 1 step. This step take at most $log(T(n)) + |x| + 1$ steps.

4. Repeat steps 1 through 3 until M halts which will happen after T(n) loops through 1,2, and 3.

Therefore, this algorithm will terminate in at most $R(n) = |\langle M \rangle| + 2|x| + 6 + \Big(T(n)\Big)\Big((T(n) + |x| + |\langle M \rangle|) + (|\langle M \rangle| + |x| + log(T(n)) + 3T(n) + 3) + (log(T(n)) + |x| + 1)\Big) = O((T(n))^2)$ steps.