

# CS 6505 - Homework 8

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Our certificate is the graph and our verification algorithm is Edmonds Blossom algorithm, which runs in  $O(mn^2)$  time. (I know this question asked us to show that this problem was in CO-NP (the question being is there a matching of size greater than  $k$ ), but a much stronger statement holds: namely that the problem is in P, solvable in polynomial time). By the theorem shown in class, Edmonds algorithm finds a matching of maximum size (by showing that if Edmonds doesn't find any augmenting path then there is none. Namely we used the fact that at the end of Edmonds the number of unmatched vertices is exactly equal to outer - inner (specifically it is the set of vertices we started out with in our first unmatched outer vertex level who still remain unmatched at the end of Edmonds) and if we remove the inner vertices we are left with exactly outer isolated vertices (since there are no edges between outer vertices, if there were we would augment or find a blossom and swap edges) these outer vertices must be matched to inner vertices in any matching, so at most inner of these outer vertices can be matched. In any matching at least outer - inner will remain unmatched (which is the number unmatched at the end of Edmonds) so this matching at the termination of Edmonds is maximum. If the matching found at the end of Edmonds (polynomial time) is of size  $\leq k$  our certificate is verified.