CS 6505 - Homework 12

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We first show that DENSE SUBGRAPH is in NP. Namely, we wish to show that when the answer is YES, there exists a certificate that can be verified in polynomial time. Such a certificate would be the set of k vertices. To verify it, we check all entries in the upper triangle (so that we don't double count) of the adjacency matrix corresponding to this subset and if there are at least l 1's we have verified.

To show that DENSE SUBGRAPH is NP-complete, we reduce CLIQUE to it. Given a graph $G = (V, E)$ we wish to find a CLIQUE in G if one exists or return NO if none exists. To do so, we do the following:

- For i = 2 to $|V|$:

    - Run DENSE SUBGRAPH on G, k=i, l=k choose 2.
    - If DENSE SUBGRAPH returns NO, pass.
    - Otherwise, return the output of DENSE SUBGRAPH.

All that remains to argue is that this reduction is polynomial. Why? Say DENSE SUBGRAPH has running time T(G,k,l) which is bounded (in the worst case over all, k, l) by T'(G). Then the runtime of our implementation of CLIQUE would be $O(|V|T'(G))$.