I apologize for the terrible formatting but I was not expecting to show these to anyone. However, I think it may be useful to see notes I took from the Graph Theory conference I attended.

Also, one of the speakers yesterday gave a talk about embedding complete graphs into various surfaces of different genus (he was looking at those with positive euler genus which so like everything but the plane and the sphere--euler genus is some simple formula in terms of the number of holes and crosscaps of a surface). Also, I was so happy because I already had enough background knowledge from my Graph Theory topics class to understand the keynote speaker's talk on the Erdos Hajnal conjecture which basically says that if G is the family of H-free graphs (all graphs containing no induced copy of H) then there exists a number epsilon which depends on H such that there is a clique of size at least n^epsilon or an independent set of size at least n^epsilon. So, this is like Ramsey theory except we're restricting to class of H-free graphs instead of just all graphs. Cool things to remember: we probably want our class to be closed under taking complements, also want it to be closed under containment. Key idea: look for intermediate graph between whole graph and desired graph we wish to find--so perfect graphs in this case because those have chromatic number equal to clique number for ever induced subgraph.

Things talked about in this conference:

Covering graphs with complete graphs size 2 to k in minimum cost where cost of a clique is the number of vertices in it. Speaker just restricted to covering with K_2 and K_3 so edges and triangles. Obviously you want to use only triangles when possible but with bipartite graphs for example there are no triangles. Also, you can change the costs so that you don't want to always maximize number of triangles.

Random Graphs from metric space and isomorphisms between them:

A really weird result is that if we look at the uniform random graph on countably infinitely many vertices, so G_1=G(\inf,p) and another G_2 = G(\inf,p), then the probability they are isomorphic tends to ONE in the limit. Would think it would be zero. But, with the back and forth technique and property such that for any finite subgraphs A,B in G, we can find a vertex v in G such that v is complete to A and anticomplete to B. I don't remember the name of this property and I don't know if all infinite graphs satisfy it but when they do its easy to construct an isomorphism. Namely, take any v_0 in G_1, map it to any u_0 in G_2. Then, pick any u_1 in G_2 (BACK AND FORTH technique because we're at step one picking the new vert from G_1 and finding a corresponding vertex in G_2, then at the next step we pick our new vertex from G_2 and find its corresponding vert in G_1, then we pick the new vertex in G_1... so on). Now, let A' be the set of vertices in G_2 which weve already seen (mapped onto as some u_k with k < i) that are adjacent to u_i in G_2, let B’ be the set of vertices already seen which are not adjacent to u_i in G_2. Let A and B be the inverse image of these sets under f (f is the isomorphism we are constructing). Then, there exists some vertex w in G_1 which is complete to A and anticomplete to B, pick that as f^{-1}(u_i)… Then keep going. Like I said, I’m not sure if all infinite graphs satisfy this property (I’ll find the name) but maybe he proved they do or he’s just looking at the cases where they do.

IDEA: Probably every graph appears as a subgraph of a fixed infinite graph, use some probabilistic argument about subgraph isomorphism and expected number of times a graph on n vertices appears as part of a random erdos renyi graph (just G(\inf, p) where an edge exists with probability p, usually p is 0.5). So, I could look at the expected number over all randomly generated graphs. Finite? Infinite? Countably infinite?

Continuing on: he also talked about making infinite graphs from countable dense subsets of metric spaces where we add an edge between to vertices with probability 1/2 only if they are within distance one from each other in the metric space (so, we're not adding edges between every pair within distance 1, we’re just saying we only CONSIDER adding them, which we do with probability 1, whenever they are within distance 1 apart). Then, he looked at the graph arising from the metric space with is the set of points around some circle of a given circumference. I guess this is not a normed space but it is a metric space. He said that different things happen when the circumference L is rational versus irrational. In one case we get that any 2 graphs arising from using countable dense subset S and countable dense subset S’ are strongly isomorphic. Other case almost never isomorphic. I need to add these definitions. Basically they are almost always isomorphic almost never and some middle ground. I also need to fill in how we generate graphs from S and S’.

Another talk: Increasing paths in countable hypergraphs.

There are different definitions of increasing paths for hypergraphs, using the idea of a loose path, since now edges can have more than two vertices. So, we say a loose path is increasing if we have a set of edges where the highest index vertex in one edge is the lowest index vertex of the next edge in the path.

He talked about how our “enemy” labels the graph in the worst case way, and we want to know what’s the guaranteed length of a longest increasing path we will be able to find. (So, even before all this the graph itself is fixed).

IDEA: the only idea I got from this was related to graph isomorphism. Like what if I looked at certain permutations of labelings of the vertices of G, like those with some cycle type and then examined the length of the longest increasing path. So, this is some number that even among the same isomorphism class will change with the given labeling. OH! Really important idea: maybe this will give me some probabilistic way or sneaky counting (yes) way of estimating or predicting the number of automorphism of G. It would be nice to compare the number of bijections of G compare with the number of relabelings that preserve this number (length of longest increasing path) and I feel like there should be some kind of inverse relation, like a quotient thing. Also, he gave some useful results about infinite graphs and paths along vertices 0 mod 2 or 0 mod p. I can fill in these details later.

The local version of Sidorenko's conjecture: Sadly, I had very little background knowledge for this talk and didn't particularly absorb anything. The one thing I took away was that one can associate a piecewise function on the unit square made from the adjacency matrix of a graph by making a grid on the unit square n by n and then wherever that entry is a 1 make that box on the grid take value 1, otherwise take value 0. Then there is some useful quantity which is an integral of that function. I can look at her talk or email her for details if I ever need them. Fan Wei Stanford soon to go to Princeton.

Circular Coloring of Planar Graphs:

Want to generalize 3 colorability. So, 3 colorability is the same as whether G has a homomorphism to the triangle, One can also ask whether G has a homomorphism into C_5 or C_7 or C_{2t+1}. If it has a homomorphism into larger odd C, for instance C_137, then it has a homomorphism into any lower odd C, so C_135, C_133, … C_7,C_5,C_3 because there is a homomorphism from C_9 to C_7 and C_7 to C_5 and so on. So, we want to find the largest t such that there is a homomorphism from G into C_{2t+1}. For planar graphs, it has been proved that girth at least 6t is enough to get a homomorphism into C_{2t+1}. It is conjectured that even girth at least 4t suffices. You can also look just at odd girth since even cycles "don't cause problems". Intuitively this makes sense since bipartite graphs are those with no odd cycles and are two colorable.

Idea: Direct product coloring. Remember this! Say I have G partitioned into subgraphs H_1 ,,, H_k, each are c(a) colorable, where H_a is that part. Then, say max over a of c(a) is A, I color each part with A colors. So, I have a coloring function d:V to [A] that is a proper coloring just usng edges WITHIN each H_i, will not be proper overall Then say that the subgraph of the multipartite graph between all these parts H_1, … H_k is B colorable. So, for every vertex in this multipartite graph I have a coloring function (NOT PROPER IN GENERAL, only proper in this multipartite subgraph where we are ignoring edges within the parts) k:V(verts that show up in this multipartite graph) but can extend to all of V by coloring other ones not seen color 1, and it'll still work cause well take the direct product youll see \to [B]. NOW to get a proper coloring using AB colors. For any vertex w in G we assign r(w) = (d(v),k(v)) and r is our new coloring function, so actually our colors are now ordered pairs (rather than just single numbers) so now we use at most A times B colors to color our whole graph. This is really useful! Remember this! I can lower some shitty bounds using this technique (That's right)

Useful result came out of this circular coloring talk: if I pick desired flow excesses on vertices such that all those excesses sum to 0 mod 5 over the whole graph, then if I can orient edges of G (may be a multigraph with parallel edges) so that the desired flow excesses hold (all addition is done mod 5, but since we're just orienting edges we have flow values on edges 1 or -1, which just amount to reversing an edge but remember there are multiple edges (or parallel edges), so we can have flow greater than one between two adjacent vertices), then we call G strongly Z_5 connected (notice for the def of strongly Z_5 connected we only required that when the desired excesses sum to 0 mod 5 over the whole graph we can find the desired orientation, so if they don't sum to 0 mod 5 we don't even look at those, note flow excess is flow in minus flow out). Rough note: there are certain troublesome graphs, K_2 with 2 copies of that edge, K_2 with 3 copies of that edge (so to be clear, two vertices with 3 edges between them), a triangle where there are 3 edges, 3 edges, and 1 edge between the pairs respectively and a triangle where there are 3 edges, 2 edges, and 2 edges between the pairs respectively.

A result of nash showed that if for all partitious P_i of the vertices of G we have W = sum_i =1 to t (d(P_i)-11t+19) (cant remember if that was plus or minus 11t can lookup later if needed) or something then we get k edge disjoint spanning trees (the paper of nash is referenced in this paper: https://www.math.wvu.edu/~hjlai/Pdf/Catlin_Pdf/Catlin49a.pdf
) my statement above is probably wrong. That's either what the speaker prove about something if W equals that or he was referencing tha result from nash, anyway the idea is if we have enough edges going around in some uniform way such that for any partition of G we have lots of edges between parts then we will get K edge disjoint spanning trees, so that brings in the idea of edge connectivity because if d(P_i) = num edges leaving P_i (our graph is undirected at this point so when I say leaving I just mean edges across from P_i to its complement) < some number then G is not that number edge connected (give or take 1) (cause theres a cut of that size)

NEW IDEA presented here: or maybe it was another talk, what if just look at min cut among only cuts of ODD size, what if I generalized to I mod p where p is not 2. Probably not a super great or useful idea, but its there...

Younghoos talk (hes here working with robin)

basically special cases of Hadwiger conjecture, there aren't really any notes I feel worth mentioning I can look up his results also very similar results were given in a (more comprehensive advanced talk by chun hung liu

https://math.gatech.edu/news/2018-aco-prize-goes-chun-hung-liu-0

I can look at his webpage and papers or email him to get those results if I ever need them)

Oh wait! useful definitions graph linklessly embeddable in 3 space iff exists embedding such that no 2 disjoint cycles are linked like two circles linked like this:

https://www.bing.com/images/search?view=detailV2&ccid=Kaak5KPX&id=2BE55714247A5DD563D4CD7BDF9FB5D957B798BA&thid=OIP.Kaak5KPX4UCheJ2RU3OtXAHaFw&mediaurl=https%3a%2f%2fupload.wikimedia.org%2fwikipedia%2fcommons%2fthumb%2fc%2fce%2fHopf_Link.png%2f1200px-Hopf_Link.png&exph=933&expw=1200&q=linked+circles&simid=607987181097257646&selectedIndex=0&ajaxhist=0
he gave equivalent characterizations, these are actually really useful write this down and cite where they came from:

TFAE G linklessly embeddable into R^3 iff:

1. admits linkless embedding (definition)

2. admits a flat embedding (embedding in R^n such that each cycles forms boundary of disk and that disk does not intersect and part of the graph (as it is embedded in R^n obviously))

3. has no minor in the Peterson family (robin Thomas showed that the Peterson family is exactly the class of graphs closed under map f and map g

f replaces a triangle in G with a star on 3 edges, g is the reverse, where we start with K_6 and then take closure under those operations, I can check but I think that right shouldn't be hard to find online if google robin Peterson family)

4. de verdiere number mu(G) <= 4

de verdiere number Is maximum multiplicity of the 2nd largest eigenvalue of a weighted adjacency matrix of G (where we impose some conditions on what a valid weight function is google colin de verdiere)

Conjecture which speaker thinks false, I also think false

mu(G) <= k-1 implies that G k colorable

eulers formula gives an upper bound on number of edges In a planar graph can we also also get an upper bound on num edges here? num edges in a linklessly embeddable graph with no K_p minor and V>=p-1, yeah younghood yo and robin got bound for p = 2,3,4,...9 so can look up their paper

bound is (for specifically K_5 minor free graphs which are also triangle free) E<= (p-2)V - (p-2)^2 and that bound is tight by looking at the example K_{p-2},{V-(p-2)}

note to self: don't work on linklessly embeddable graph not a lot of progress waste of time, even if I thnk I want to come back and work on this later don't someone will figure whatever I want to conjecture out first and I should just wait and use their result otherwise they'll publish before me 778 bhjnkml

problems I could work on:

fill in nora Hartsfields outline on embedding complete graphs into surfaces of whatever euler genus

prove C5 satisfies the erdos hajnal property speaker says probably true and probably not that hard just no one has done it, I should try (could definitely do over the summer, not too hard)

do AND FINISH this summer find counterexample to

Conjecture which speaker thinks false, I also think false

mu(G) <= k-1 implies that G k colorable

think about k_5,5 recursive constructions and K_4 minors (topological minors from contractions) (these ideas popped into my head think about it in late august this is def not a priority but if ever sick of comp studying can think about this and shouldn't be too hard or take too long so don't stress few days)

improve some erdos haj bounds? definitely not likely for me to be able to solve at all, remember that, but if a think about it for a few hours one weekend might understand something that will help me solve my other project july june august, don't think about this too long

also don't use szemeredis reg lemma not useful for my research it will seem like it but the bound that comes out of it will be worse than the current record holder

youre good you can do this, keep taking lamotrigine (cramp) so no ok or at least ween off very slowly be careful ween off over course of year or two and be fine

nora Hartsfield unwell just published outline go back fill in

4 techniques

current graphs

voltage graphs

diamond sums

diagonalization technique

see prev email for more notes on that

cool ideas:

rainbow tree embedding so embedding where we specify we want a certain vertex in this part of some partition (related to erdos haj conjecture from alex scotts talk)

idea from another talk: have planar graph, contract even cycles and "fold" associate some pairs of vertices in odd facial cycles until we get a planar graph with all facial cycles have the odd degree same as the min odd degree of a face

related idea: delete stuff or take cmopletent perform whatever operations to make graph more regular in whatever sense that could be

oh I am also now convinced by stronger graceful labeling conjecture is true (yes it is, take a year at most) some notion of almost planar graphs which are graphs whoare planar except for one apex vertex yeah I can google that

flecke algebras

people.math.gatech.edu/~mwigal3/abstracts.pdf

graphs with forbidden induced subgraphs alex scott
basic idea if chrom num G large might expect to find any certain graph as induced subgraph, not true, but have proven for suff large chrom num as function of desired subgraph can find that induced subgraph if that subgraph is a path, star, tree of radius 2, or some certain set of radius 2 trees
Chi boundedness family of graphs is chi bounded iff (def) there exists some function f:N to N such that for any graph G in that family the chrom num(G) <= f(\omega(G)).
perfect graphs chi bounded with binding function the identity
line graphs chi bounded with binding function f(x) = x+1
which famiies have linear binding functions? speaker doesnt know.

flip around prev idea and say, if X(G) (<-- that's chi X the chrom number) >= M (where M is some appropriately calculated bound), then it has a large clique of it contains some "forbidden subgraph" (see weve flipped around the idea of saying that if we define a family of graphs to be those without any of these forbidden subgraphs, then its chi bounded), just kind of turning this idea around

the above theorem has been shown when the forbidden subgraph is well, that was a bad way to put it , we are not claiming it has a large clique or it has one of the graphs in some list of forbidden subgraphs, namely we are actually saying something stronger that, pick a specific graph H, then for appropriately calculated M and A, then if Chi(G) >= M, then G has clique of size at least A or it has H as an induced subgraph. that claim has been shown when H is a
path
star
any subdivision of a star
any tree of radius 2
broom/mop
2 sided broom/mop (this is more than what XXY taught us)
certain trees of radius 3

the speaker was saying one might conjecture and hope to prove that statement where H is any induced tree (note that any tree is a subtree of the tree Tb,d which is a b-ary tree

of depth d). That hasnt necessariliy been shown to be false (i think not?) but at the very least he couldn't prove it, he made a weaker conj:
if X(G) >= M (where M some appropriate number), then G has a clique of size A or it contains a subdivision of the desired tree T,

he proved it and even proved a slightly stronger statement (though weaker than the original one), he prove if Chi G suff large either get large clique or get subdivision of desired tree T with each edge of T subdivided at most t times.

IDEA USE METHOD OF TEMPLATES. recursively find some structure then within that find a similar structure, repeat until some desired condition is reached (kind of like, while the condition is not meant I can find one of these templates within my graph and eventually I wont be able to find another template who knows even just because number of vertices too small so then that means that my desired condition is met) XXY did something like this in class where we said either this graph a tree is induced or its quasi ind and contains some level one vertex with some property, then we let our new graph be made from that level one vert with that propery, and apply it again, either thats induced or it has some level one vert with that property, if its induced were happy and done otherwise we make a new tree using that new level one problem vert and keep going...)

MOP OF certain length speaker guaranteed find two end mop of certain length with means he gave conditions under which one can find two vertices of degree a,b distance m apart, THAT COULD ACTUALLY BE USEFUL
^ middle ground for checking for isomorphism (heuristic) if graph isom then both have verts of deg a,b dist m apart, maybe could check this condition

conjecture by Gyarfas:
large Chi chrom num ==> large clique (omega) or odd hole (not saying equality but now forget about antiholes) (related to strong perfect graph theorem, i think he was saying we get less result but with fewer conditions, so its a trade off)
large Chi ==> large omega or long hole (not even or odd just cycle of long length)
large Chi ==> large omega or large odd hole

he showed no odd hole ==> (Chi(G) <= 2^2^omega) so chi bounded but shitty chi binding function, very loose upper bounded using ramsey theory like arguments speaker says its quote un quote "pathetic" lolol
also have lower bound (omega(G))^(1.14) <= Chi(G)

lang (? couldnt read my handwriting i think thats right) mcdermott conj:
odd hole free graph can cut into 2 pieces decrese clique number says Chi(G) <= 2^omega(G)
speaker thinks that above conjecture is true
we have no real reason to think need exponential chi binding function could be polynomial, this is just whats known right now

recent result: there is an O(n^9) algorithm to check for odd holes in a graph so can check for odd holes in G and odd holes in complement of G and know if a graph is perfect in O(n^9) time
apparently there already was a polynomial time algorithm for checking graph perfection before according to tetali (surprising, maybe look up later)

we were talking about forbidding even, odd holes, now we forbid holes which are 0 mod 3 (has to do with betti number which is an algebraic topology thing)
"more generalized conjecture proved as well"
generalized this thm:
showed if Chi(G) >= M (some function of n, other parameters, can look up), then G contains a triangle (K_3) or it contains holes of 10^10 consecutive lengths (so both even and odd)
so think about a family of graphs A, infinite with bounded graphs

finding odd holes parity mod m
Conj or Thm not sure which one:
for every integer m, large Chi(G) implies large clique or has holes of all lengths mod m (i guess they're saying every parity)

set of lengths sparse or dense? can you get a set of density 0? I have no clue what that means....

more complicated graphs: look at graphs where can get subdivion of some graph

DEF: H called pervasive if large Chi(G) implies large omega(G) or induced subdivision of H (and have each edge subdivided at least some number of times)
Example:
-trees are pervasive bc can subdivide and look for subdivisions of these
-cycles pervasive too
so speaker thought maybe all graphs are pervasive! can see why, but thats not true ("really wrong" as he said)
the counterexample was something where there exists a triangle free "intersection graph"
(https://en.wikipedia.org/wiki/Intersection_graph
where the vertices correspond to line segments and theyre adjacent if they intersect) and arbitrarily large chromatic number
"take any planar graph subdivde it and it will still be planar" basically something about looking at classes of graphs closed under taking subdivisions

IDEA: look at families of graphs closed under taking subdivions,
closed under containment,
closed under relabeling in certain (prespecified) ways (idk permutations of a certain cycle type) if looking at labeled graphs GOOD IDEA

if pervasive ==> then contains forest of chandeliers,
chandelier means take tree connect every trees leaf to some common vertex

True: forest chandeliers are pervasive in the class of string graphs (not sure what string graphs are and not sure why pervasive depends on what class we're in, think about later)

outline of how to prove things about Chi boundedness
grow a hole (not including the triangle, thats by def not a hole)
if graph has large Chi then we will hit a layer H with (chrom number of layer H) >= (1/2)(chrom number of G)

something about "switches" 0 mod 3 to 1 mod 3

his conjecture was false, but maybe a modified statment holds:
conj: large Chi ==> (2 ball with large Chi) OR (induced subdiv of H)
(so instead of large clique and H, subdiv H, notind H where we were modifying the part about finding H, now we're modifying the part about finding a large clique
because the idea of finding a large clique is finding a subgraph with locally large chromatic number CHi)

IDEA: look at average chromatic number, could mean lots of things,
average of chrom number over all subgraphs of a certain size
I LIKE THIS ONE: chrom number of G/|V(G)|
so scale by size of graph, like instead of looking an num edges looking at average degree that kind of thing

conj: "upper bounds are horrible"
lower bounds are all polynomial
could it be that every Chi bounded family of graphs has a polynomial binding function? has not been shown to be false, could be true, very well might be

note: forbidding small subgraphs gives us stronger results (intuitvely makes sense more bang for our buck yeah if i cant remember why just like think about it)

Does speaker Alex Scott Oxford think Gyarfas Sumner conjecture holds? he says on odd days yes. if someone can prove it for trees of radius 3, then hes pretty sure its true. my impression (me Caitlin) is that theres a more than 50 percent chance this is true.

map color theorem mark ellingham vanderbilt university
like 4 color theorem, about embedding graphs on surfaces other than plane or sphere, namely surfaces of certain euler genus
euler genus has to do with number or holes and crosscaps (simple formula i think)

hes looking at surfaces of positive euler genus he has some upper bounds on Chi(G) for graphs G embeddable in these surfaces
he breaks into looking at orientable surfaces
also nonorientable surfaces
heeway bound, hutchinson bound (i am SURE i spelled the first one wrong)
lookup CELLULAR EMBEDDING
n-m+r >= 2-gamma
(gamma instead of number of faces for planar formula)
dual version look at average face degree denoted d^* (face is defined in some weird way I could tell for surfaces of different genus, the speaker said someting abot faces being hamiltonian cycles whereas for the plane they are just cycles, so I should read about this what a face is when graph is embedded in some weird surface)
2m = r d^*
using that you get a quadratic inequality amove average degree on vertices in the graph you can solve that quadratic equation and round down to integer numbers get h(gamma)-1

vert of degree <= d remove by ind color then put back can color it with the d-1st color so have analogue to Brooks theorem
d >= h(gamma)-1
need h(gamma) colors for sure

when is the equality sharp?
quadratic equation round down
get sharp result without rounding eulers equal
2m >= 3r
if n= d+1 (d is average degree)
==> G is the complete graph
shapr if have triangular cellular(? couldnt read my writing) embedding of complete graph inot the surface (can we find those embeddings?)

hard to find sharp upper bound
easy to prove sufficiency

(opposite of 4 color them where it was easy to prove sharpness of upper bound hard to prove sufficiency)

something something ringel wrote 200 page book SUMMARIZING the proof so its a really long proof (not sure what exactly i was referring to with that note)

rest of talk is about trying to find these examples:
4 major techniques:
-CURRENT GRAPHS
-VOLTAGE GRAPHS/GRAPHICAL SURFACES
-DIAGONAL TECHNIQUES
-DIAMOND SUMS (this one is the one that works really well in all the cases)

1976 someone showed can improve the upper bound in some cases if have embedding into surface such that all faces have even degree (what is the degree of a face?)
same argument except require that the average face degree d^* >= 4 (rather than d^* >= 3 like before) --> then can bound ChiG

have sqrt24gamma over 2 (heewards bound)
sqrt 16 gamma (hutchinson bound)
check order of operations on those, i wrote them as they were said

surfaces with positive euler genus (euler genus is denoted gamma)
have results on sphere and plane, on sphere or plane if faces even degree implies G bipartite implies graph 2 colorable
results probably not sharp
probably sharp for proj plane and torus
not sharp for klein bottle though
heewards bound sharp for everything but klein bottle something i couldnt read my handwriting

how hutchinsons argument modified?
mimic heewards proof
d^* >= 4 and even face degree

why care about even face embeddings? why not just average face degree at least 4? well cause that doesnt work
reason is we want whatever this condition is to be closed under removing vertices
if i take a graph with averge face degree at least 4 and remove a vert the result does have ave face degree at least 4 i guess? double check but i think that was the point, i guess even face makes the condition preserved under vertex deletion

degeneracy in graphs:
a graph is k degenerate if .... dont remember can look up def online something about every subgraph or MAYBE just the graph itself maybe check having a vertex of degree at least k or something double check
planar graphs 5 degenerate
gives one line inductive proof of 6 color thm
can prove one line inductive proof that k degen implies k+1 colorable. how?
use ind on number of verts, delete vert of deg k, result still k-1 degen i guess, so k colorable, then add that vertex back in and it has at most k nbors use the k+1st color on it
(really should check that, not sure if my def is right and i just now wrote down what I thought the one line inductive proof he mentioned was, that was not notes from the conference that was me thinking about it so could be wrong)

conjecture: simple graphs with even face embeddings ==> h_even(gamma)-1 degenerate

specifically: triangle free graphs embeddable in surface h_even(gamma)-1 generate so Chi(G) bounded by h_even(gamma)
these h's are the functions hes proving exist to bound the chrom number, he definitely had them actually written down i just didnt write them down should be able to google nad find them
those bounds arent used though because theres already a better bound in those cases from ramsey theory using R(3,m) to get large ind set since G triangle free
NOTE: these color theorems DEPEND on the embedding

want to show the hutchinson bound is sharp

in heeward bounds these extreme examples proving sharpness came from triangular embeddings (doesnt say anything either way about cellular)

so we will look for faces that are squares (quadrilaterals) for hutchinson in some cases can find desired examples

hutchinson gives 4 for projective plane using K_4
using K_5 for torus? cant read my writing i think thats what that says

can we find other examples?
sometimes cant get quadrilateral embeddings of complete graph
need some kind of i mod m condition on the euler genus also condition depends on orientable vs non orientable

what we do is find something almost quadrilateral
can find sharpness except for klein bottle

this talk will give these examples
4 ways construct embeddings all work for some of the cases
1989***** current graphs
K_n [something] embeddings
orientable:
m = 5 mod 8

nonorientable:
n>=9
n = 1 mod 4
that n could have been 0 or 1 mod 4, they just looked at half the cases there also in the orientable cases there were two choices 5, or something else mod 8, they could have looked at more

nora hartsfield: non orientable graphs quadrilateral embeddings multipartite graphs
maybe someone could FLESH out her ideas into full proofs
she used the diagonalization technique to go from K_8 to K_{16} embeddings
described techniques did not include formal proof
complete graphs-- orientable or non orientable
not necessarily quadrilateral
face degree >= 4
1998
what are the min genus orientable embeddings face degree >= 4 some other group of people proved this and some of their proofs were not correct

"CURRENT GRAPH"

2019 speaker got started thinking about this stuff because wanted to construct embeddings for lexicographic products of graphs
embeddings of graphs K_{8m}

4 techniques DIAMOND SUM technique works in all cases orientable non orientable modulus of genus
[application: non orientable genus of complete tripartite graph]
diagonalization technique

complete multipartite graphs
-we have already existing quadrilateral embeddings and we'll add new vertices in pairs
add pairs along diagonal of existing embedding
problem though is new vertex joined to only 2 original verts
so need to join to all (or more? couldnt read my writing) old verts and all (or more/some?) new verts
idea: add handles between 2 quadrilateral faces
topological term? cylinder/annulus, in graph theory amounts to adding edges to a graph
so thats how to go from K_8 to K_{16} using type 1 type 2 type 3 of something nora hartsfields paper

I should go back to fill in the details and explain it
Fun!^

current graphs: used to prove 4 color theorem
algebraic construction/dual construction

directed edges get some group associated to it which is Z_8
(speaker wont explain details)
trace faces in some directed way in some picture he showed on a slide
face A gets verts $a_0,a_1,a_2,...a_7$
face B gets verts $b_0,b_1,...,b_7$
direction of traversal decides "rotation around verts"

apparently constructing embeddings is the same problem as construction rotations" around vertices topological-graph theoretic equivalence of some kind
look into later if have time^^^

so in index two current graph get 2 copies of desired graph so just keep one
so we get even better than K_8, we get K_8 and some multiple edges even (parallel) edges which we just delete
speaker wanted to cover david croft's conjecture "graphical surfaces"

something about blowing up vertices into spheres and edges into tubes then can embed into that surfaces $G[\bar{K_2}]$ <--- called LEXICOGRAPHIC (something... lexicographic something, lexicographic blowup i think? maybe? not sure)
so that gave a new proof of an already existing result
so we get quadrilateral embeddings of lexicogarphic product oooh product thats what its called using this approach

2 different techniques
voltage graph
alg constru
verts --> verts
faces --> faces

Current graph
DUAL alg construction
verts --> faces
faces --> verts

we replace all edges in crofts construction by digons (a set of 2 parallel eges so just an edge with another edge added along that edge)
and add eges between southa dn north pole and add loops some places
if assume the original graph had a perfect matching then.... i didn't finish writing that

so that gives us a quadrilateral embedding of $G[\bar{K_8}]$ lexicographic product can google for any graph G with a perfect matching i think if i wrote that down right
if take any even graph and take $G[\bar{K_4}]$ get any K_{8m}

speaker got result:
then looked at non orientable situtation
most results transfer
0 mod 8 works
0 mod 4 does NOT work

so came up with Diamond sum
1978 ---> new proof for num genus form for complete bipartite graphs

bouchet: simple inductive construction

have 2 embeddings
both have vertices of degree 5 taken them out, cut hole in both surfaces, then glue those 2 holes together
^works nicely with quadrangular embeddings

previous techniques did not work well with quadrangular embeddings

speaker generalized: to almost quadrilateral, same but then only glue the faces that are of degree 4 (almost means they aren't all degree4 , so just use the ones that are)

3 ingredients:
start with embedding of some graph K_n
put 2 graphs together:
first: a certain graph on 7 vertices (that they came up with)
second: some complete bipartite graph
then you put them together("get complete graph", not sure what that means)

can we use the diamond sum technique for also nearly quadrangular embeddings?
yeah!
orientable: starter graph size 11
nonorientable: starter graph size 7 (thats the one mentioned in the above paragraph)

that only covers surfaces of certain genus
but can add extra vertices and cmopltee graph to obtain results for surfaces/graphs of all genus

speaker prove this using a computer program

m gen surface:
take complete graph
can find embedding w all face deg even
or all (all not average)face deg >= 4
get sharpness result for map coloring bound
have sharpness results for most

minimarl triangulations
Take care of health!!!!!
map color theorem:
what is minimal triangulation on given surface? simple graph as few verts as posible
will show exists minmal triangulation that dont come from complete graphs
use hartsfield diag technique

embedding of complete graph w small faces
what is largest cycle face can use
embeddings of complete graph wher each face bounded by hamiltonian cycle

orientable: need n = 2,3 mod 4
non orientable: ... didnt hear

KIND OF LIKE BROOKS THM i should try this :L)