

Cut-elimination output

$$\begin{array}{c}
\frac{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta, A \swarrow \Sigma \Rightarrow \Pi, \Box A \quad \mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \swarrow \Sigma \Rightarrow \Pi, \Box A \nearrow \epsilon \Rightarrow A}{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \swarrow \Sigma \Rightarrow \Pi, \Box A} \Box_R^1 \\
\frac{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta, A \nearrow \Sigma \Rightarrow \Pi, \blacksquare A \quad \mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \nearrow \Sigma \Rightarrow \Pi, \blacksquare A \swarrow \epsilon \Rightarrow A}{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \nearrow \Sigma \Rightarrow \Pi, \blacksquare A} \blacksquare_R^1 \\
\frac{\mathcal{G} \nearrow \Gamma \Rightarrow \Delta, \Box A \nearrow \epsilon \Rightarrow A}{\mathcal{G} \nearrow \Gamma \Rightarrow \Delta, \Box A} \Box_R^2 \quad \frac{\mathcal{G} \swarrow \Gamma \Rightarrow \Delta, \blacksquare A \swarrow \epsilon \Rightarrow A}{\mathcal{G} \swarrow \Gamma \Rightarrow \Delta, \blacksquare A} \blacksquare_R^2 \\
\frac{\mathcal{G} \Downarrow \Gamma, \Box A \Rightarrow \Delta \nearrow \Sigma, A \Rightarrow \Pi}{\mathcal{G} \Downarrow \Gamma, \Box A \Rightarrow \Delta \nearrow \Sigma \Rightarrow \Pi} \Box_L^1 \quad \frac{\mathcal{G} \Downarrow \Gamma, \blacksquare A \Rightarrow \Delta \swarrow \Sigma, A \Rightarrow \Pi}{\mathcal{G} \Downarrow \Gamma, \blacksquare A \Rightarrow \Delta \swarrow \Sigma \Rightarrow \Pi} \blacksquare_L^1 \\
\frac{\mathcal{G} \Downarrow \Gamma, A \Rightarrow \Delta}{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \swarrow \Sigma, \Box A \Rightarrow \Pi} \Box_L^2 \quad \frac{\mathcal{G} \Downarrow \Gamma, A \Rightarrow \Delta}{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta \nearrow \Sigma, \blacksquare A \Rightarrow \Pi} \blacksquare_L^2 \\
\frac{\overline{\mathcal{G} \Downarrow \Gamma, p \Rightarrow p, \Delta}}{\mathcal{G} \Downarrow \Gamma, A \Rightarrow \Delta, A \rightarrow B, B} (id) \rightarrow_R \quad \frac{\overline{\mathcal{G} \Downarrow \Gamma, \perp \Rightarrow \Delta} \perp_L \quad \overline{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta} \text{EW}}{\mathcal{G} \Downarrow \Gamma, A \rightarrow B, B \Rightarrow \Delta \quad \mathcal{G} \Downarrow \Gamma, A \rightarrow B \Rightarrow \Delta, A} \rightarrow_L \\
\frac{\overline{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta, A \rightarrow B}}{\mathcal{G} \Downarrow \Gamma \Rightarrow \Delta, A \rightarrow B} \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{\overline{r, \neg r \Rightarrow r, \Box p, \Box q, \blacksquare \neg \Box \neg r}}{\overline{r, \neg r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r}} id \\
\frac{\overline{r, \neg r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r}}{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r \swarrow \Box \neg r \Rightarrow \neg \Box \neg r}} \neg_L \\
\frac{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r \swarrow \Box \neg r \Rightarrow \neg \Box \neg r}}{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r \swarrow \epsilon \Rightarrow \neg \Box \neg r}} \Box_L^2 \\
\frac{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r \swarrow \epsilon \Rightarrow \neg \Box \neg r}}{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r}} \neg_R \\
\frac{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r}}{\overline{r \Rightarrow \Box p, \Box q, \blacksquare \neg \Box \neg r}} \blacksquare_R^2
\end{array}$$