Homework #1: Conversions and Stellar Population Synthesis

AST386c: Prof. Caitlin Casey

This assignment is due by 3 pm, Tuesday, September 25^{th} (i.e. no missing colloquium for this!). Send me a pdf (in a format lastname_hw1.pdf) in email to cmcasey'at'utexas.edu. It should be formatted using LaTeX[1], showing your work, and embedding plots as needed. You can generate the plots in any code you prefer, but please include a full description of your methods so I could reproduce your plot exactly following your instructions.

1. Download all of the goodies available at the following link:
   `www.as.utexas.edu/∼cmcasey/ast386c/hw1tools/`

2. Converting between flux, flux density, luminosity and magnitudes are critical skills to professional astronomers, and often prove confusing. Here are a set of exercises to make you more adept at these conversions. In the various downloaded tools for this problem set you will find the following files:

Template Stellar A0V Spectrum	spectrum_A0V.txt
Subaru g-band filter profile	subaru_g.txt
Subaru r-band filter profile	subaru_r.txt
Subaru i-band filter profile	subaru_i.txt
Subaru z-band filter profile	subaru_z.txt
Subaru Y-band filter profile	subaru_y.txt

   The first column of every file is the wavelength (in Å). The second column of the template stellar spectrum is given in S_ν units ($\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$), and the second column of the filter profile is representative of the total system response T_λ, or the product of the filter response, detector quantum efficiency, instrument and telescope throughput and atomsphereic transmission. In other words, T_λ is the fraction of light that gets through at a given wavelength.

   (a) Plot the spectrum of the A0V star, along with a flat-spectrum source of flux density 3631 Jy, overplotting and labeling each of these filters. Make sure the plot is legible, and add labels/legends sufficient for me to fully understand what is plotted. You will need to rescale the filter curves to be clearly visible on the plot.

   (b) the AB magnitude system is defined such that a flat-spectrum source with flux density of 3631 Jy has a measured flux density of (...drumroll...) 3631 Jy in all filters, regardless of filter bandwidth/shape. In the *AB* magnitude system, such a source would have a magnitude of 0 in all bands. If the template spectrum for the A0V star were to represent Vega, what is the magnitude of Vega in the AB system across these five filters? (This is the offset between Vega magnitudes and AB magnitudes for these filters.)

   (c) What do you notice about the values of these offsets and the shape of the spectrum of Vega?

   (d) Plot the A0V spectrum in units of νL_ν, assuming it is Vega and Vega is 7.68 pc from us (note that $1\,pc = 3.086 \times 10^{16}\,m$). Overplot λL_λ on the same plot for comparison. From

[1] If you don't know LaTeX yet, don't sweat it! Come let me know and I'd be happy to show you the basics. Also talk to your peers! There is a lot of collective knowledge in this building that be really helpful to tap into.

this plot, what can you surmise about how intrinsically bright Vega is compared to the Sun? (this should give you some feeling of why people sometimes plot in units of νL_ν or λL_λ.)

3. This problem will introduce the concept of stellar population synthesis (SPS) models by building up a basic understanding of stellar populations. SPS models are critical to how we understand the stellar emission of galaxies as integrated light sources (because in the vast majority of cases we cannot see individual stars in other galaxies!), and so it is very important to understand how they are built. This problem deals with bolometric quantities, and the next problem will introduce some of the mechanics of building SPS models with real templates.

   (a) The Salpeter IMF (Salpeter 1955) is parameterized:
   $$\xi(\log m) = \frac{d(N/V)}{d\log m} = \frac{dn}{d\log m} \propto m^{-x} \tag{1}$$
   where $x = 1.35$. Using this distribution of stellar masses, plot the cumulative stellar mass fraction from high masses to low, in other words $f(>m)$ vs. m. You can stop at the brown dwarf/hydrogen burning limit, $\sim$80 $\mathrm{M_{jup}}$ and at 100 $\mathrm{M_\odot}$ at the high-mass end. What is the average mass of a star drawn at random from this Salpeter distribution (i.e. the expectation value)?

   (b) The relationship between a star's luminosity and mass can be parameterized roughly as:
   $\frac{L}{L_\odot} \approx 0.23\left(\frac{M}{M_\odot}\right)^{2.3}$ $(M < 0.43M_\odot)$ $\quad$ $\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{4}$ $(0.43M_\odot < M < 2M_\odot)$ $\quad$ $\frac{L}{L_\odot} \approx 1.5\left(\frac{M}{M_\odot}\right)^{3.5}$ $(2M_\odot < M < 20M_\odot)$ $\quad$ $\frac{L}{L_\odot} \approx 3200\left(\frac{M}{M_\odot}\right)$ $(M > 20M_\odot)$
   Remember that the luminosity is proportional to the rate of fuel consumption and the mass of the star is proportional to the total fuel supply, and that the sun's lifetime is $\sim$10 Gyr. What is the highest mass main sequence star you expect to live past 100 Myr? 500 Myr? 1 Gyr?

   (c) If you assume that a stellar population is aged 500 Myr, what would be the average mass of a star drawn at random? How does this differ from your answer to part (a), and how would it differ for a stellar population that is aged 1 Gyr? (You can ignore evolved stars for the purposes of this problem even though we know they're... important.)

   (d) What is the fractional contribution for stars of a given mass m to the total light emitted by a given stellar population? Plot this as a cumulative distribution, $f_{\mathrm{L}}(> m)$ as a function of m. Hint: it would be wise to convert $\xi(\log m)$ to dn/dl for this step. What can you say about what types of stars dominate the light of any given stellar population?

   (e) Generate the same plot in part (d) but adjust it to represent a 500 Myr-old and 1 Gyr-old stellar population. Mark the three curves (including the 0-age curve) clearly.

4. This problem builds on the previous problem, but now you are asked to build up a stellar population model spectrally. To do this you will need to check out / unpack the following files from the directory linked above: kurucz93.tar.gz and EEM_dwarf_UBVIJHK_colors_Teff.txt. The first is a directory of stellar atmosphere model spectra from Kurucz (1993) for a range of metallicities, effective temperatures and surface gravities. The txt file should be used to map effective temperature and luminosity back to mass[2]. You can read the readme files in

[2]The table quotes $\mathrm{T_{eff}}$ for stars between $0.1 < M/M_\odot < 19.6$ with a rough gridding, so you'll have to interpolate between these points to come up with good $\mathrm{T_{eff}}$ estimates for all mass points on the scale you used in problem 1. At high masses you should interpolate towards a 100 $\mathrm{M_\odot}$ star having a 45,000 K temperature.

the Kurucz directories for more information, in addition to the annotations of the txt file.

(a) Using the same mass range as in problem 1, plot $T_{\rm eff}$ against stellar mass and stellar luminosity L vs stellar mass (by interpolating the values given in the reference txt file).

(b) Now that you know the effective temperature and luminosity across our entire mass range, you can make a composite spectrum for the stellar population as a whole. For each value of your stellar mass grid, you should read in the appropriate Kurucz model, choosing the closest one in effective temperature. For simplicity, just assume solar metallicity for all stars (only the files in the kp00 directory). For the gravity you can adopt: $T_{\rm eff} \geq 41000\,K$ (column g50), $36000 \geq T_{\rm eff} < 41000\,K$ (column g45), $9000 \geq T_{\rm eff} < 36000\,K$ (column g40), and $T_{\rm eff} < 9000\,K$ (column g45). You'll want to add all of the spectra of the stars together, proportional to how many stars of each type are in the stellar population. Plot the resulting stellar population spectrum in νL_ν units against wavelength. Be sure to make sensible choices for your axes and think about whether or not it would be best to use a linear or a log scale to present your results.

(c) Now go back and split up this spectrum into the contribution from stars in different mass ranges. You choose 3-4 mass ranges that you think convey the most interesting results. State what mass ranges you assume clearly and label them on your plot.

(d) Using your results from problem 1 now generate a spectrum of this stellar population after it has aged 500 Myr, and dilineate the contributions from each of the mass ranges chosen in part (c). Then, do the same for a 1 Gyr age.

(e) What differences and similarities do you notice across these stellar population models? What types of stars dominate the spectrum over what wavelength range?

Great! You made it to the end of Homework #1. Congrats! Now keep your stellar population models handy... we will use them in the beginning of Homework #2!