Problem Set #7

Implement a second-order FEM solution in terms of quadratic Lagrangian basis and testing functions for the formulation described in equation (1.57) of CEM Note #1 (total *E*-field scalar Helmholtz equation for a 1D general dielectric region). Use the mesh generator (mesh2.m) and the program shell (femtot2.m) provided as a starting point to develop numerical solutions for waves scattered by a dielectric slab. Incorporate the quadratic basis functions used in Problem Set #6 (the element matrix subroutine is given in the code).

(1) Work out the expressions for the matrix entries Z_{mn} corresponding to equation (1.57), without evaluating the integrals over the cells (for instance, use S_{mn} and T_{mn} to denote the appropriate integral expressions in terms of the global m and n indices). Provide expressions for a general Z_{mn} away from the boundaries, and expressions for Z_{11} and Z_{NN} that incorporate the RBC type of boundary condition in (1.57). In addition, give the expression for RHS_1 in order to incorporate the excitation.

(2) Modify the program shell (femtot2.m) to implement the matrix entries you derived in Problem 1. Be sure that you include the relative permittivity values from the input files in your matrix entries. Provide a printout of the part of the code that you modified.

(3) Generate at least 3 results each for the two dielectric slab examples depicted in Tables 5 and 6 in CEM Note #1. The idea is to compare the results from your quadratic basis functions with those in Tables 5 and 6. Thus you should use the same air buffer on each side of the dielectric slab as in those examples to obtain the same phase values.

(4) Determine the convergence rates (such as $O(\Delta^p)$) of the numerical results you obtained in Problem 3. It may be helpful to work with exact results for the example in Table 6 with more digits:

reflection coefficient = 0.4824168 100.35461

transmission coefficient = 0.8759418 10.35461

Solution: The following pages give the code and some results. The convergence rate is p=4.

```
% fill global matrix one cell at a time using element matrices

  for icell=1:n_cells
      [eleS,eleT] = elemat(icell);

      for irow=1:3
         nrow=cell_to_node(icell,irow);
         for icol=1:3
            ncol=cell_to_node(icell,icol);
            Z(nrow,ncol) = Z(nrow,ncol) + eleS(irow,icol)...
                - k0^2 * epsilon(icell)*eleT(irow,icol);
         end
      end
  end

%  add boundary conditions for RBC terminations

Z(1,1) = Z(1,1) + 1j*k0;
Z(n_nodes,n_nodes) = Z(n_nodes,n_nodes) + 1j*k0;

% disp(Z);

% fill excitation vector (right hand side)

 RHS(1) = 1j*2*k0;  % assumes that incident Ey(a)=1
```

```
>>>>>>>>  slab with er = 4 and dt = 0.2 wavelengths:

mesh 121

                reflection coeff = 0.40628 60.5275

       transmission coeff = 0.91375 150.5275

==================================================
mesh 242

                reflection coeff = 0.40356 60.2634

       transmission coeff = 0.91495 150.2634

==================================================
mesh 484

                reflection coeff = 0.40339 60.2462

       transmission coeff = 0.91503 150.2462

==================================================
==================================================


>>>>>>>>  slab with er = 5 and dt = 0.4 wavelengths:

mesh 1,8,1

               reflection coeff = 0.48295 100.4129

        transmission coeff = 0.87565 10.4129

==================================================
mesh 2,16,2

              reflection coeff = 0.48245 100.3583

        transmission coeff = 0.87592 10.3583

==================================================
mesh 4,32,4

               reflection coeff = 0.48242 100.3548

        transmission coeff = 0.87594 10.3548

==================================================
```