Problem Set #4:

1. Starting with the MATLAB code provided on Canvas for the 2D TE cavity problem, modify the code in order to handle the TM-to-z case for a cavity with PEC walls. This requires that (1) the primary unknown field be E_z instead of H_z and (2) that a zero Dirichlet boundary condition of the form

$$E_z\big|_{\partial\Gamma} = 0$$

be imposed around the entire boundary of the cavity. Thus fields at boundary nodes will be specified by the boundary condition and not be treated as unknowns in the eigenvalue problem. The differential equation is the same, but the code will need to be modified to read in the number of boundary nodes with the mesh (you may assume that the boundary nodes are all at the end of the node list) and not include boundary nodes when filling the system of equations.

Generate numerical results for a coaxial cavity with inner dimension $a = 1.0$ and outer dimension $b = 4.0$, using the same coax mesh software from Problem Set #3 (coaxmesh.m). Produce a table showing at least the 3 lowest resonant nonzero wavenumbers obtained with three different meshes.

Solution:

The code can be modified to truncate the global system at the number of interior nodes. Alternatively, the matrix fill loops can be modified as illustrated below:

```
function cavityTM

% compute propagation constants for waveguide cross section modeled with
% triangular cells
%
% September 23, 2018   A. F. Peterson

  global pcetond xy;

% read mesh from file 'cylfil.txt'

  nnodes = dlmread('cylfil.txt','', [0,0,0,0]);
  ncells = dlmread('cylfil.txt','', [0,1,0,1]);
  nb1 = dlmread('cylfil.txt','', [0,2,0,2]);
  nb2 = dlmread('cylfil.txt','', [0,3,0,3]);

  xy=dlmread('cylfil.txt','', [1,1,nnodes,2]);

  nstart=nnodes + 1;
  nend=nstart + ncells - 1;
```

```
  pcetond=dlmread('cylfil.txt','', [nstart,1,nend,3]);

% initialize variables

  Ninterior = nnodes - nb1 - nb2;

  W=zeros(Ninterior);
  Y=zeros(Ninterior);

% fill global matrix one cell at a time

   for icell=1:ncells

     [eleS,eleT]=elemat(icell);

%    add contributions from cell 'icell' to global matrix

     for jj=1:3
        jp=pcetond(icell,jj);
        if(jp <= Ninterior)
        for kk=1:3
           kp=pcetond(icell,kk);
           if(kp <= Ninterior)
           W(jp,kp)=W(jp,kp)+eleS(jj,kk);
           Y(jp,kp)=Y(jp,kp)+eleT(jj,kk);
           end
        end
        end
     end
   end

   fid = fopen('eigfil.txt', 'wt');

   E = eig(W,Y); % use [V,E] = eig(W,Y) to get eigenvectors as well
```

For the coax with $b = 4$ and $a = 1$, we obtain smallest resonant wavenumbers:

5 layers, 16 nodes on inner 104 interior nodes avg. edge = 0.6583	10 layers, 30 nodes on inner 432 interior nodes avg. edge = 0.3428	15 layers, 40 nodes on inner 980 interior nodes avg. edge = 0.2323
1.0476110	1.030373	1.027115
1.1390128	1.118919	1.115071
1.1390128	1.118922	1.115071
1.3645864	1.339147	1.334125
1.3751356	1.341852	1.335403
1.670636	1.623806	1.614484
1.670636	1.623821	1.614484