Problem Set #6, Problem 2:

Hint: Using the element matrices from problem 1, show that the m-th FEM equation for the scalar Helmholtz equation (away from any boundaries)

$$\sum_i E_i \left\{ \int \frac{\partial B_m}{\partial x} \frac{\partial B_i}{\partial x} dx - k^2 \int B_m B_i \, dx \right\} = 0$$

is

$$\left(\frac{1}{6\Delta} E_{m-2} - \frac{8}{6\Delta} E_{m-1} + \frac{14}{6\Delta} E_m - \frac{8}{6\Delta} E_{m+1} + \frac{1}{6\Delta} E_{m+2} \right)$$
$$-k^2 \left(-\frac{\Delta}{15} E_{m-2} + \frac{2\Delta}{15} E_{m-1} + \frac{8\Delta}{15} E_m + \frac{2\Delta}{15} E_{m+1} - \frac{\Delta}{15} E_{m+2} \right) = 0$$

Then show that the (m–1)st equation (mid-cell) is

$$\left(-\frac{8}{6\Delta} E_{m-2} + \frac{16}{6\Delta} E_{m-1} - \frac{8}{6\Delta} E_m \right)$$
$$-k^2 \left(\frac{2\Delta}{15} E_{m-2} + \frac{16\Delta}{15} E_{m-1} + \frac{2\Delta}{15} E_m \right) = 0$$

You must substitute the second equation into the first to eliminate E_{m-1} (and by symmetry E_{m+1}).

Then, the idea is to seek a solution to this equation of the form $E(x) = e^{-j\beta x}$, or equivalently

$$E_{m-2} = e^{+j\beta 2\Delta} \qquad E_m = 1 \qquad E_{m+2} = e^{-j\beta 2\Delta}$$

and solve the system for β.