ECE 6380: Problem Set #6

1. Quadratic interpolation functions can be defined in one dimension as the three functions

$$B_1(x) = \frac{x(x-\Delta)}{2\Delta^2}$$

$$B_2(x) = 1 - \left(\frac{x}{\Delta}\right)^2$$

$$B_3(x) = \frac{x(x+\Delta)}{2\Delta^2}$$

overlapping the interval $-\Delta < x < \Delta$. Show that the element matrices associated with the 1D scalar Helmholtz equation can be evaluated to produce:

$$\int_{-\Delta}^{\Delta} \frac{dB_m}{dx}\frac{dB_n}{dx}\,dx = \begin{bmatrix} \frac{7}{6\Delta} & \frac{-8}{6\Delta} & \frac{1}{6\Delta} \\ \frac{-8}{6\Delta} & \frac{16}{6\Delta} & \frac{-8}{6\Delta} \\ \frac{1}{6\Delta} & \frac{-8}{6\Delta} & \frac{7}{6\Delta} \end{bmatrix}$$

$$\int_{-\Delta}^{\Delta} B_m B_n\,dx = \begin{bmatrix} \frac{4\Delta}{15} & \frac{2\Delta}{15} & -\frac{\Delta}{15} \\ \frac{2\Delta}{15} & \frac{16\Delta}{15} & \frac{2\Delta}{15} \\ -\frac{\Delta}{15} & \frac{2\Delta}{15} & \frac{4\Delta}{15} \end{bmatrix}$$

2. Extend the dispersion analysis introduced in Note #13 to the quadratic representation used in Problem 1 (above).

(a) Show that the phase constant obtained is

$$\beta = \frac{1}{2\Delta}\cos^{-1}\left\{\frac{15 - 26(k\Delta)^2 + 3(k\Delta)^4}{15 + 4(k\Delta)^2 + (k\Delta)^4}\right\}$$

(b) Identify the region of $(k\Delta)$ where β is real valued.