ECE 6380: Problem Set #3

1. Complete the FEM MATLAB code from Note #3, available on Canvas, to incorporate the capacitance calculation in Equation 42:

$$\begin{aligned}\frac{C}{\varepsilon_0} &= \frac{1}{V_0^2}\iint_\Gamma \varepsilon_r \nabla\Phi \bullet \nabla\Phi \, dx\, dy \\ &= \frac{1}{V_0^2}\iint_\Gamma \varepsilon_r \left(\sum_{m=1}^{N+N_b} \phi_m \nabla B_m\right) \bullet \left(\sum_{n=1}^{N+N_b} \phi_n \nabla B_n\right) dx\, dy \\ &= \frac{1}{V_0^2}\sum_{m=1}^{N+N_b} \phi_m \sum_{n=1}^{N+N_b} \phi_n \iint_\Gamma \varepsilon_r \nabla B_m \bullet \nabla B_n \, dx\, dy \\ &= \frac{1}{V_0^2}\sum_{m=1}^{N+N_b} \phi_m \sum_{n=1}^{N+N_b} \phi_n \; W_{mn} \\ &= \frac{1}{V_0^2}\tilde{\phi}^{\mathrm{T}}\tilde{\mathbf{W}}\tilde{\phi}\end{aligned}$$

   Use the provided coax mesh generator to produce results for several meshes for a free-space filled coax with $b/a = 2$ and compare your results with the results in Table 1 and/or the exact solution in Equation 44. Turn in a printout of the section of the MATLAB code that you added.

2. Use the code you developed in Problem 1 to generate some capacitance results for an air-filled coax with $b/a = 4$. Determine the approximate convergence rate (Δ, Δ^2, etc) by comparing to the exact result in Equation 44 of Note #3.

3. Find the condition number of the matrix

$$\begin{bmatrix} 1 & 10{,}000 \\ 0 & 2 \end{bmatrix}$$

   using the definition in Equation (4.11) of Chapter 4 (CEM Note #6). How does the condition number compare with the ratio of largest to smallest to eigenvalues?

4. By following a procedure similar to that outlined in Equations (4.15) to (4.18) of Chapter 4 (CEM Note #6), derive Equation (4.19).