Problem Set #2

1. Modify the MATLAB code "femtot.m" from Note #1 to incorporate variable cell sizes, using the expressions you derived in Problem Set #1, problem 3. Once you get this working (try different meshes created by the "mesh1.m" program), execute the code using the input meshes "inputfil_1.txt" and "inputfil_2.txt" which represent 2 different meshes for dielectric slabs of thickness 0.4 wavelengths and relative permittivity $\varepsilon_r = 5$. Turn in a program listing and the results you obtain for reflection and transmission coefficients for the two test cases.

Solution: The expressions for the matrix entries and RHS were given in the solutions to Problem Set #1. The portion of the code needing modification is below:

Section of code that was modified:

```
% fill global matrix

 for irow=1:n_unknowns

    if (irow == 1)
      deltaR = x(irow+1) - x(irow);
      Z(irow,irow) = 1./deltaR - k0^2 * deltaR * epsilon(irow)/3 + 1j*k0;
      Z(irow,irow+1) = -1./deltaR - k0^2 * deltaR * epsilon(irow)/6;

    elseif(irow == n_unknowns)
      deltaL = x(irow) - x(irow-1);
      Z(irow,irow-1) = -1./deltaL - k0^2 * deltaL * epsilon(irow-1)/6;
      Z(irow,irow) = 1./deltaL - k0^2*deltaL*epsilon(irow-1)/3 + 1j*k0;

    else
      deltaL = x(irow) - x(irow-1);
      deltaR = x(irow+1) - x(irow);
      Z(irow,irow-1) = -1./deltaL - k0^2 * deltaL * epsilon(irow)/6;
      Z(irow,irow) = 1./deltaL + 1./deltaR - ...
          k0^2 * (epsilon(irow-1)* deltaL + epsilon(irow)* deltaR)/3;
      Z(irow,irow+1) = -1./deltaR - k0^2 * deltaR * epsilon(irow)/6;
    end
 end

% fill excitation vector (right hand side)

 RHS(1) = 1j*2*k0;  % assumes that incident Ey(a)=1
```

The solutions for the two input files are:

Input file #1:
    reflection coeff = 0.49811 -5.402
    transmission coeff = 0.86711 -95.402

Input file #2:
    reflection coeff = 0.49454 -5.9568
    transmission coeff = 0.86916 -95.9568

2. In Note #3, a representation for the potential function $\Phi(x,y)$ was given in terms of linear "pyramid" basis functions in Equation (10). Substitute this expansion into the energy functional (as in equation 42) and use the result to generate a system of equations by setting the derivative of the functional with respect to each of the unknown coefficients $\{\phi_n\}$ to zero. How does this system differ from the system obtained in equation (14) of Note #3?

Solution: Since $V_0 = 1$, we can write the functional for Capacitance as

$$F = \frac{C}{\varepsilon_0} = \sum_{m=1}^{N} \phi_m \left\{ \sum_{n=1}^{N} \phi_n W_{mn} + \sum_{n=N+1}^{N+N_b} \tilde{\phi}_n W_{mn} \right\}$$
$$+ \sum_{m=N+1}^{N+N_b} \tilde{\phi}_m \left\{ \sum_{n=1}^{N} \phi_n W_{mn} + \sum_{n=N+1}^{N+N_b} \tilde{\phi}_n W_{mn} \right\}$$

where the tildas over some coefficients denote the fact that they are given by the boundary conditions. We differentiate with respect to the non-tilda coefficients to obtain equations such as

$$\begin{aligned}\frac{\partial F}{\partial \phi_1} &= 2\phi_1 W_{11} + \phi_2 W_{12} + \phi_3 W_{13} + \ldots + \phi_N W_{1N} \\ &\quad + \phi_2 W_{21} + \phi_3 W_{31} + \ldots + \phi_N W_{N1} \\ &\quad + \sum_{n=N+1}^{N+N_b} \tilde{\phi}_n W_{1n} + \sum_{m=N+1}^{N+N_b} \tilde{\phi}_m W_{m1} \\ &= 0\end{aligned}$$

Using the fact that $W_{mn} = W_{nm}$, we divide by 2 to obtain an equation with $m = 1$ that has the more general form

$$\sum_{n=1}^{N} \phi_n W_{mn} = -\sum_{n=N+1}^{N+N_b} \tilde{\phi}_n W_{mn} \qquad m = 1, 2, \ldots, N$$