Homework 2 – due by start of class on Monday, March 11
ECE 8873a Spring 2019
Prof Morris Cohen
100 points

This homework is about getting experience with realistic numerical calculations. You may want to implement the below in Matlab. You are welcome to, but not required to, turn in your actual Matlab code printed out.

1. This problem is devoted to practicing that, and I've chosen for you to implement a raytracer to simulate HF wave (3-30 MHz) propagation. We will do this in pieces. HF waves are used for a lot of things, for example amateur (ham) radio, aviation communication, long range communications. For now, make these assumptions:

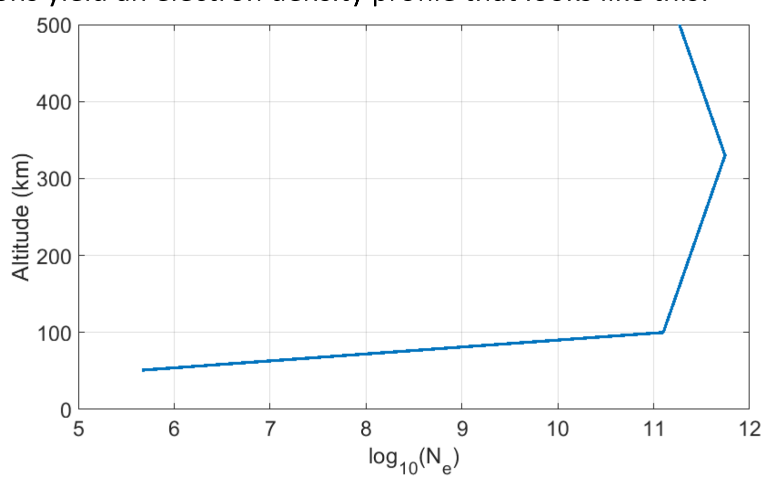
   (1) The Earth is flat, not round
   (2) There are no collisions
   (3) Propagation is in the left hand circularly polarized mode

   What is most important is propagation through the ionosphere. Let's treat the ionosphere as "horizontally stratified", meaning that there are no variations in the horizontal direction, but only in the vertical direction. Ignore atmospheric attenuation and refraction, which are negligible. Ionospheric refraction is what dominates.

   Assume the electron density profile with altitude follows this distribution:

$$N_e = \begin{cases} 10^{0.111h} & 100 \geq h \\ 10^{(0.0028(h-100)+11.1} & 330 \geq h > 100 \\ 10^{(-0.0028(h-330)+11.744} & h > 330 \end{cases}$$

   Where h is altitude (km), and N_e is electron density (m^{-3}). This is a very rough approximation of a typical profile not too far from the equator during the daytime. The above equations yield an electron density profile that looks like this:



Assume that you are propagating signals at the (geomagnetic) equator, where the Earth's magnetic field is parallel to the ground and pointed northward, with a magnetic flux density of 40 μT.

Let's get the upward propagation part working. Here are the steps to build a raytracer:

1. Assume a step size. Make sure the step size is small compared to the length scale of variations in the index of refraction.
2. Begin with an angle of incidence (or angle with respect to vertical) at the ground, and assume a starting horizontal position is $z = 0$.
3. Then calculate what its horizontal position is one step length away, from simple geometry.
4. Using Snell's Law, calculate what the angle of incidence will be at this new altitude, based on the on the other side of the boundary. You'll need to calculate the index of refraction using equations from lecture 10.
5. Return to step 3

You can ignore the effect of collisions entirely, for now. Assume for now that the signal propagates along the (geomagnetic) equator, in the east-west direction. Note that this means the propagation direction is perpendicular to the magnetic field, where LHCP propagation is known as the ordinary mode.

Implement a raytracer. For now, just worry about tracing the wave upward. Be sure you use a small enough step size. To make sure, you should test out your raytracer after doubling the step size. If your results don't change, that's a good sign. On the other hand, don't make it way too small, otherwise your simulation may take too long.

As you trace the wave, keep track of the horizontal and vertical position.

a. (15 points) Assuming that the frequency is 3 MHz, simulate the path of the HF wave. Assume the wave is launched 10 degrees off from vertical. You will notice that the ray eventually bends sideways. What is the highest altitude that the HF wave will reach?

b. (10 points) Does a 10 MHz wave launched from the ground at a 45 degree angle make it through the ionosphere? Plot the path.

Ok, so now you've got the basics, so let's make it a little more complicated. The wave that bends to the point where it's sideways won't just stop there, but it will reflect via total internal reflection. To take this into account, we need to modify step 4 and 5 above. With each use of Snell's law, you should check whether total internal reflection has taken place. If it has, then the propagation direction should be flipped around, and downward propagation should then take place. Be sure to take care of the negative signs when you apply Snell's law, remembering that arcsin(0.99) has two answers: one slightly smaller than 90 degrees, and another slightly larger than 90 degrees.

c. (15 points) Plot the path of a 15 MHz wave launched at a 80 degree angle from vertical (or 10 degrees above the ground). How far away from the transmitter will the signal be when it returns to the ground?

Ok, so now you're getting somewhere, let's drop assumption #1 and make this work for a curved Earth. Go back to lecture 6 notes. Since atmospheric refraction is being ignored, you can use the true Earth radius (6371 km), but you'll need to use a "modified" form of Snell's law.

d. (10 points) Repeat part c with a curved Earth. How far along the Earth's surface will the signal land? (Sanity check: Is this distance higher or lower than in part (C)? Does that make sense?)

Assume now that the collision frequency can be written as

$$\nu = \begin{cases} 10^{(170-h)/15} & h < 125 \\ 10^{(170-125)/15} * 10^{(125-h)/87.5} & h \geq 125 \end{cases}$$

Then let's add the effect of collisions attenuating the wave. When you calculate the permittivity, the imaginary part will allow you to calculate the attenuation rate, in np/meter. Use the real part of the permittivity in Snell's Law. With each step, calculate the attenuation of the wave, and keep track along the ray path.

e. (15 points) For a 5 MHz wave launched at a 45 degree angle, what is the total attenuation (or loss factor), integrated along the path, due to ionospheric propagation?

And having dropped two of the three initial assumptions, let's finish it off.

f. (10 points) Overlay on the same plot the ray path for both the LHCP and RHCP wave. How different are their trajectories?

Your model is getting more sophisticated. It would take only a modest amount of work to extend it to more general situations (non-perpendicular propagation, geomagnetic field that varies along the path).

Congratulations – you are well on your way to being able to predict HF wave propagation around the world!

2. **(10 points)** The "International Reference Ionosphere" is the most prominent climatological model of the densities of atmospheric and ionospheric components as a function of altitude, time and location. It's built off of a huge number of GPS TEC observations and some other stuff, all stitched together. You can look up model results here: https://ccmc.gsfc.nasa.gov/modelweb/models/iri2016_vitmo.php

   Let's consider the ionospheric conditions in Atlanta (Geographic coordinates 34N, 276E), On March 1, 2018 (one year ago), at 20 UT (Universal time), which is 3 PM Eastern Standard Time. Enter these values in to the IRI model generator. Don't change the height profile and range stuff, and ignore all the "optional inputs". When you press submit, you'll get a table with electron density, in m^{-3}, as a function of height.

   You can also calculate the Earth's magnetic field value over Atlanta using the international geomagnetic reference field (IGRF). For Atlanta, the magnetic field is $\bar{B} = -1.9129\hat{x} + 21.5895\hat{y} - 41.3768\hat{z}$ in units of μT, where $\hat{x}$ is the eastward direction, $\hat{y}$ is the northward direction, and $\hat{z}$ is the upward direction.

   Calculate the Faraday rotation angle of a GPS L1 signal (1575 MHz) passing through the ionosphere directly vertically.

3. **(15 points)** Let's do some back of the envelope calculations on meteor scatter as a communication system. A few assumptions are necessary:

   According to slide 6 of lecture 13, roughly 5 billion overdense meteors impact the Earth's the atmosphere each year, or ~158 per second. In reality this is not constant during the year, but let's say it is. Each meteor leaves a trail that is long and thin, and preferentially scatters waves in a certain direction. But let's ignore that, and say that each meteor trail scatters signals in all directions equally, like a point sphere. Ignore refraction, diffraction, absorption, and consider only reflection from the meteor trail. However, do take into account Earth's curvature. Assume a reflection from the meteor trail at 100 km altitude. Assume a transmitter and receiver both at sea level. Assume the meteor trail lasts 0.5 s, during which time a data rate of 1 kbps is possible.

   a. How far away can a receiver be from a transmitter and still potentially communicate using meteor trail bursts?

   b. Assume the receiver is 500 km away from the transmitter. How many meteors enter the atmosphere in a location that supports communication between the transmitter and receiver?

   c. What kind of data rate, on average, can the transmitter and receiver expect between them?