Homework 3 – due by start of class on Wednesday, April 10
ECE 8873a Spring 2019
Prof Morris Cohen
100 points

1. (30 points) You are transmitting a signal in the Earth-ionosphere waveguide, with the ground on one boundary and the 80 km high D-region ionosphere on the other boundary. Treat this as a parallel plate flat waveguide with perfectly conducting boundaries. A signal is propagating down the waveguide at 2812.5 Hz. What is the maximum power per unit area that the waveguide can possibly carry? Assume that the limit is 15 kV/m, which is half the electric field it takes to make a spark at sea level. Repeat this for the following scenarios:
    a. The signal is in the TEM mode
    b. The signal is in the TE1 mode
    c. The signal is in the TM1 mode
2. (20 points) Consider a particle of mass m and charge q moving in the presence of constant and uniform electromagnetic fields given by $\bar{E} = E_0\hat{y}$ and $\bar{B} = B_0\hat{z}$, where E_0 and B_0 are constants. Assume the particle starts from rest at the origin. Find an expression for the trajectory, $x(t)$ and $y(t)$, and plot it.

3. (20 points) You live in Fairbanks, Alaska, which has geographic latitude 64.8 degrees N, geographic longitude 147.72 E. The day comes when a strong geomagnetic storm comes in, and you decide to capture the aurora. You point your camera toward the sky, but along the direction parallel to the Earth's magnetic field line. What is the zenith angle that your camera is pointed at? Zenith angle is the degrees downward from vertical, so if your camera is pointed directly outward, that would be 0 degrees.
    a. Calculate this, assuming that the Earth's geomagnetic field is a centered dipole.
    b. Now calculate and compare to the real geomagnetic field of the Earth. You can use the World Magnetic Model, that can be calculated here: https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm

4. (30 points) Consider electrons at L-shell L=4 that are stably trapped by the Earth's geomagnetic field. Assume the electron has energy level E_k. Assume the Earth's field to be a centered dipole. The equatorial loss cone angle, as defined in class, is a pitch angle at the equator that separates particles that mirror from particles that won't mirror. "Mirroring" is defines as reaching a pitch angle of 90 degrees. So consider a particle that starts out at the equator with pitch angle right at the loss cone angle. Its pitch angle will increase as it propagates down the field line, and it will just barely reach the top of atmosphere just as its pitch angle hits 90 degrees, where it then mirrors. Let's define "the top of the atmosphere" as 100 km above the Earth's surface.
    a. Assume no electric fields. What is the value of the equatorial loss cone angle?
    b. Now assume there is an electric field pointed upward from the Earth, and the voltage drop is 1 kV. What is the value of the equatorial loss cone angle? This expression may be left in terms of E_k if necessary.