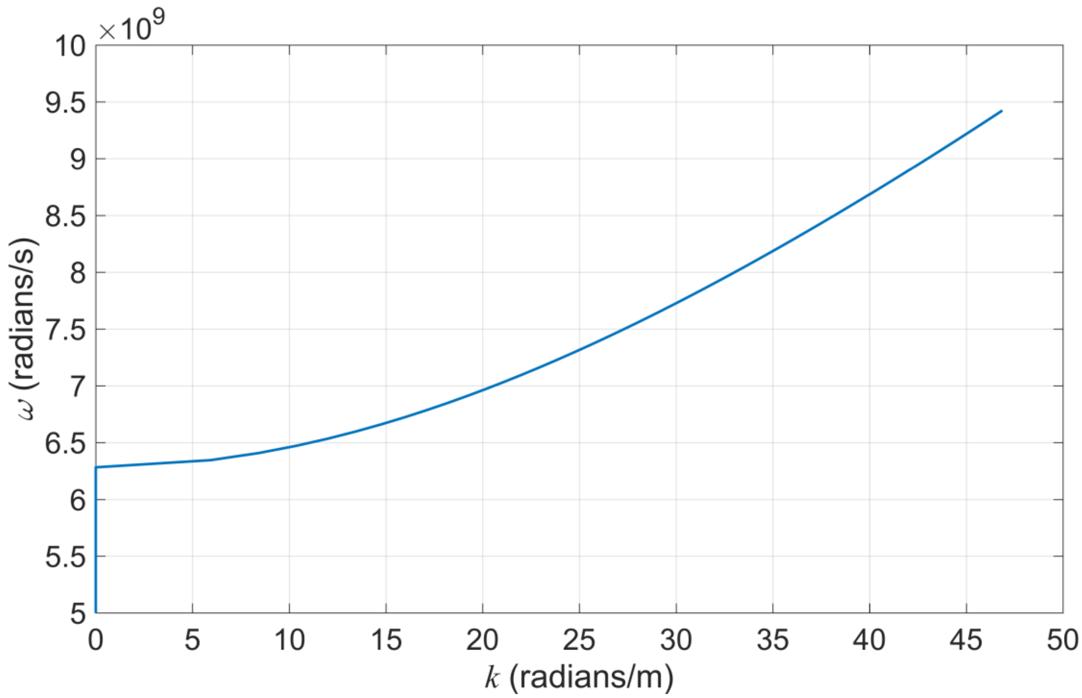
Homework 1 – due by start of class on Monday, February 4
ECE 8823a Spring 2019
Prof Morris Cohen
100 points

1. **(20 points)** You are developing a communication system which will work through a dispersive material. By injecting single-frequency broadcasts into the system, and measuring the wavenumber at each angular frequency, you construct the below curve of ω vs k.



   a. Your communication system is to be operated at 1275 MHz. Estimate (approximately) the group velocity at this frequency, and explain how you did it.

   b. Estimate the phase velocity at this frequency and explain how you did it.

   c. At 1100 MHz, is the group velocity higher or lower than what you estimated in part (a)? Why?

   d. At 1100 MHz, is the phase velocity higher or lower than what you estimated in part (b)? Why?

   e. At which frequency will dispersion be a bigger problem? 1275 MHz or 1100 MHz? Explain your answer.

2. **(30 points)** The dielectric constant of raw potato at 2.45 GHz is $\epsilon = \varepsilon_0(70 - j20)$. Assume $\mu = \mu_0$. 2.45 GHz is the frequency that your microwave oven uses. Consider the potato to be an infinitely long slab, and the microwave emits a uniform plane wave at normal incidence with power density 83 kW/m^2. (A typical microwave emits 700 W over an area 29 cm x 29 cm). Assume that any energy reflected from the potato is then re-absorbed at the source, rather than resonate back and forth.

   a. What is the microwave power density just on the potato side of the air-potato boundary?

   b. At what rate is energy being dissipated (i.e. converted to heat) on the other side of the boundary, as the plane wave penetrates into the potato, per unit volume? Note: power flow and energy are proportional to the square of the electric field.

   c. How long will it take to heat the very edge of the potato from room temperature (20°C) to the temperature where the water boils (100°C)?

      I'll save you a step: The specific heat of a raw potato is 3430 Joule/kg/°C, and the density is about 1090 kg/m^3. So multiplying those two numbers together and then multiplying by 80 degrees, implies that 2.99 x 10^8 Joules/m^3 is needed to heat the potato by 80 degrees (and it's just a coincidence that number is almost identical to the speed of light).

   d. How long will it take the heat the potato at 1 cm depth? Does this seem reasonable given how long potatoes usually take? Speculate as to why it is or is not roughly correct?

3. **(25 points)** The relative permittivity (ε_r) of sandy soil at 3 GHz is about 2.5, and the conductivity, σ, is about 0.005 S/m. You are building a communication beacon that is mounted on top of a tall tower along the coast at 3 GHz. Multipath effect due to ground reflections needs to be taken into account.

   a. Plot the magnitude of the reflection coefficient of 3 GHz wave incident from air, as a function of incidence angle from 0 to 90 degrees. Do this separately for parallel and perpendicular polarization.

   b. What if the beacon is at a coastal tower, and you are pointing it over the water? Repeat part a for sea water, for which you can assume σ=4 A/m and ε_r=81.

   c. If you are trying to minimize the effect of ground reflections, which polarization would be better to use, parallel or perpendicular? Assume that the receiver is at a long distance which means that the ground reflections are at a low grazing angle.

4. **(25 points)** A weather system passing over the ocean bring dry air at cloud height but moist air remains close to the surface. As such, the refractive index falls over quickly with height. The refractivity N is decreasing by 300 units per km.

    a. What is the effective Earth radius?

    b. A highly directed radio wave "ray" is launched at an angle of 0.3 degrees above horizontal, from a transmitter on the surface. How far away should a receiver, also on the ground, be positioned in order to detect the signal?