ECE4560 - Homework #6

Due: Oct. 11 2017

Problem 1. (20 pts) Now that you know how to numerically integrate the Lie algebra element $\xi \in \mathfrak{se}(2)$, given an initial configuration $g_i \in SE(2)$ and a time duration τ, to get a Lie group element $g_f \in SE(2)$, it is time to compare against the closed-form solution. Given the numerical integration from a previous homework problem, let's compare against the closed-form solution determined by the exponential.

For the case that $\xi = (1, -1, \frac{1}{4})$, $g_i = g(0) = (0, 0.5, \pi/6)$ and final time is $\tau = \pi$, explicitly solve for $e^{\xi\tau}$ using ξ and τ. Show that $g_f = g_i e^{\xi\tau}$ agrees with what you got from the previous problem.

You also had to do the case where the body velocity was $\xi = (1, -1, 0)$. Repeat the same procedure for this body velocity.

Problem 2. (15 pts) If the initial configuration of an object in $SE(2)$ is $g_i = (-3, 2, \pi/4)^T$ and the final configuration is $g_f = (3, -3, -\pi/3)^T$, what is the Lie group displacement, g, associated to the object. What is the Lie algebra element, $\xi \in \mathfrak{se}(2)$, associated with this rigid body motion (presuming that $\tau = 1$)?

Problem 3. (15 pts) Compute the exponential of the following $\mathfrak{so}(3)$ rotation matrix vector,

$$\hat{\omega} = \begin{bmatrix} 0 & -1 & -0.25 \\ 1 & 0 & 0.5 \\ 0.25 & -0.5 & 0 \end{bmatrix}$$

for $\tau = 2$.

Problem 4. (25 pts) If the initial and final configurations of an object in $SE(3)$ are

$$g_i = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 1 & 0 & 0 & 2 \\ \hline & 0 & & 1 \end{array}\right] \qquad \text{and} \qquad g_f = \left[\begin{array}{ccc|c} \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 6 \\ \frac{1}{2} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & -3 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 9 \\ \hline & 0 & & 1 \end{array}\right].$$

What is the Lie algebra element, $\xi \in \mathfrak{se}(3)$ associated with this transformation? Verify your solution by computing the exponential of it and showing that $g_f = g_i e^{\hat{\xi}\tau}$.

Problem 5. (10 pts)

(a) Let's get the actions (what I call products) of group elements with vectors (or velocities) coded up so we don't have to worry about it. In particular, we will have to extend the left action to apply to either a point or a vector representation for the velocity. The good thing is that a point is a 2x1 vector and a vector representation of the velocity is a 3x1 vector (translational + rotational velocity).

The code had an `if` statement with two versions. Maybe you coded up both, or just one of them. Add in the missing code. Verify that your code gives the same answers as in the solutions, both for the group times homogeneous translation vector and the group times vector-form of velocity for the planar case.

(b) Also amend the adjoint to operate on vectors as well. Modify the adjoint code reposted and reproduce both the vector solution and the homogeneous solution. Hopefully you see that this code can be verified by reproducing the solution to the Adjoint times vector-form of velocity problem from the last homework. Demonstrate that you have verified the code.

(c) To make life easier, create a new section at the bottom of the SE2 class that performs the hat and unhat operations. At the end of the class definition add the following:

```
methods(Static)

  %------------------------- hat -------------------------
  %
  %  Perform the hat operation with a vector form of se(2).
  %  Output is the homogeneous matrix form of se(2).
  %
  %
  function xiHat = hat(xiVec)

  CODE HERE;

  end

  %------------------------ unhat ------------------------
  %
  %  Perform the unhat operation with a matrix form of se(2).
  %  Output is the vector form of se(2).
  %
  %
  function xiHat = hat(xiVec)

  CODE HERE;

  end
end
```

Use your new code to then show that $(\mathrm{Ad}_g \xi)^\wedge = \mathrm{Ad}_g \hat{\xi}$ and vice-versa $\left(\mathrm{Ad}_g \hat{\xi}\right)^\vee = \mathrm{Ad}_g \hat{\xi}$. You can make up the g and ξ elements. The important thing is to have the right code and to also show that the outputs agree, in vector form for the first version and homogeneous form for the second version.

The `hat` and `unhat` functions are accessed as follows:

```
xiHat = SE2.hat(xiVec);
xiVec = SE2.unhat(xiHat);
```

Static functions don't need an actual group element, but they do need to be referred to by the class they belong to. It's kind of neat. Pretty soon we'll redo this coding practice for $SE(3)$.

Problem 6. [10 pts] It's time to start moving into the 3D world. Get the Lie group product, the left action, and the inverse functions written. The Lie group element should be stored as a homogeneous matrix. Also fill out the functions that get the translation and rotation components individually, plus the helper functions for whatever has been covered already. Code stubs are on t-square.

Verify that it works by reproducing the following:

```
>> g1 = SE3([1;2;3],[1 0 0;0 0 -1; 0 1 0]);
>> g2 = SE3([2;-1;-1], [sqrt(2)/2 0 -sqrt(2)/2;0 1 0;sqrt(2)/2 0 sqrt(2)/2]);
>> p1 = [4;5;6];
>> p2 = [4;5;6;1];
>> g3 = g1*g2

g3 =
```

```
    0.7071         0   -0.7071    3.0000
   -0.7071         0   -0.7071    3.0000
         0    1.0000         0    2.0000
         0         0         0    1.0000

>> g3.*p1

ans =

    1.5858
   -4.0711
    7.0000

>> g3.*p1

ans =

    1.5858
   -4.0711
    7.0000

>> g3.*p2

ans =

    1.5858
   -4.0711
    7.0000
    1.0000

>> inv(g3)

ans =

    0.7071   -0.7071         0         0
         0         0    1.0000   -2.0000
   -0.7071   -0.7071         0    4.2426
         0         0         0    1.0000

>>
```

Problem 7. (30 pts) This week, the objective is to work out the forward kinematics for the Piktul manipulators. Work out Module Set 0, part 2 from the class wiki.

Problem 7. [30 pts] For the turtlebot track, work out the first two parts of Module Set 1, "Basic ROS Commands" and also "Moving the turtlebot."

Problem 7. (30 pts) For the biped track, work out Module Set 1, Part 1. It explores the kinematics for different frames, and plotting of the kinematic chain relative to them.

Problem 7. (30 pts). If you are working on a group project, then your group contact will review the submissions associated to the prior tasks and provide a new set of tasks. You should work out whatever baby steps this group contact assigns the group as opposed to the lab problem. It is your responsibility to turn in the results to the group contact. There should be sufficient output for them to assess progress.