ECE 4560
Homework 1
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1) a) $p = (6,0)^T \quad \theta = \pi/6 \qquad p' = (p_1', p_2') = (15.2, 28.0)$

$p_1' = \cos(\pi/6)(6) - \sin(\pi/6)(0) + 10 \quad p_2' = \sin(\pi/6)(6) + \cos(\pi/6)(0) + 25$

b) $p = (6,0)^T \quad \theta = \pi \qquad p' = (p_1', p_2') = (15.0, 7.0)$

$p_1' = \cos(\pi)(6) - \sin(\pi)(0) + 21 \quad p_2' = \sin(\pi)(6) + \cos(\pi/6)(0) + 7$

2) a) As t approaches infinity, the response x resembles the driving function u(t).

b) i) $\dot{x}$ vs $\dot{y}$ oscillates as u(t) is sinusoidal
t vs θ does not oscillate as v(t) is constant

ii) $\dot{x}$ vs $\dot{y}$ and t vs θ both oscillate as both u(t) and v(t) are sinusoidal

3) $\theta_A = \pi/3$
$\theta_B = \pi$

$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad R_A = \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \quad R_B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$d_A = \sqrt{5^2 + 12^2} = 13$

$d_B = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$g_A^0 : (d_A, R_A), (z_{TA}, z_{RA})$
$g_B^0 : (d_B, R_B), (z_{TB}, z_{RB})$

$z_{TA} = 5 + 12j$

$z_{TB} = 2 - j$

$z_{RA} = e^{j\pi/3} = \frac{1}{2} + \frac{\sqrt{3}}{2}j$

$z_{RB} = e^{j\pi} = -1$

(4) $x = 1$ $\quad x' = 5 - \cos(\pi/3) + x$

$y = 0$ $\quad y' = 12 + \sin(\pi/3) + y$

$\boxed{(x', y', \theta) = (5.5,\ 12.87,\ \pi/3)}$

(5) $(1, 0, 0) \rightarrow (-1, 0, \pi) \rightarrow (1, -1, \pi)$

start $\quad$ rotate $\quad$ translate

$\boxed{(x', y', \theta) = (1, -1, \pi)}$

(6) $g_C^O = g_A^O g_C^A = (\vec{d}, R) = (\vec{d}_A^O + R(\theta_A^O)\vec{d}_A^C,\ R(\theta_A^O)R(\theta_A^C))$

$= \left(\begin{bmatrix} 5 \\ 12 \end{bmatrix} + R(\pi/3)\begin{bmatrix} -2 \\ -2 \end{bmatrix},\ R(\pi/3)R(\pi/6)\right)$

$\theta_C^O = \frac{\pi}{2}$

$R(\pi/3) = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ $\quad R(\pi/6) = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$ $\quad R(\pi/3)R(\pi/6) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = R(\pi/2)$

$g_C^O = \left(\begin{bmatrix} 5 \\ 12 \end{bmatrix} + \begin{bmatrix} -1+\sqrt{3} \\ \sqrt{3}+1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right) = \left(\begin{bmatrix} 4+\sqrt{3} \\ 13+\sqrt{3} \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right) \rightarrow g_C^O = (5.73, 14.73, \pi/2)$

$g_C^O = d_A^O + R(\overset{\pi/3}{\theta_A^O})d_C^A + R(\overset{\pi/3}{\theta_A^O})R(\overset{\pi/6}{\theta_C^A})\vec{P}$ $\qquad g_C^O = (5.73, 15.73, \pi/2)$

$= \begin{bmatrix} 5 \\ 12 \end{bmatrix} + \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5-1+\sqrt{3} \\ 12+\sqrt{3}+1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+\sqrt{3} \\ 13+\sqrt{3}+1 \end{bmatrix}$

$g_C^O(\underbrace{5-2,\ 12-2,}_{\text{translate}}\ \underbrace{\pi/3 + \pi/6}_{\text{rotate}}) \rightarrow \boxed{g_C^O(3, 10, \pi/2)}$

$g_C^O(3 + \cos(\pi/2),\ 10 + \sin(\pi/2),\ \pi/2) \rightarrow \boxed{g_C^O(3, 11, \pi/2)}$

(7) $(0.6428, -0.7660) \rightarrow (\cos 50°, \sin 50°)$

$g_A^O = (\cos 50° + 5,\ \sin 50° + 12,\ 50°) = \boxed{(5.96,\ 11.74,\ 50°)}$