ECE 4560
Homework 2
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① $g_A^o = (7, 4, 45°)$ $\quad g_B^o = (2, 7, 90°)$ $\quad$ homogeneous: $\left[\begin{array}{c|c} R & T \\ \hline 0 & 1 \end{array}\right]$

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \rightarrow R_A = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \qquad R_B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$T_A = \begin{bmatrix} 7 \\ 4 \end{bmatrix} \qquad T_B = \begin{bmatrix} 2 \\ 7 \end{bmatrix} \qquad g_A^o = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 7 \\ \sqrt{2}/2 & \sqrt{2}/2 & 4 \\ 0 & 0 & 1 \end{bmatrix} \quad g_B^o = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

② $q_1^o = (2, 2)$ $\quad q_1^A = g_o^A q_1^o$ $\quad q_1^B = g_o^B q_1^o$ $\quad g_o^A = (g_A^o)^{-1}$ $\quad g_o^B = (g_B^o)^{-1}$

inverse matrices computed with MATLAB - see file HW2prob2.m

$$q_1^A = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & -7.7782 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 2.1213 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4.9497 \\ 2.1213 \\ 1 \end{bmatrix} \rightarrow \boxed{q_1^A = (-4.9497, 2.1213)}$$

$$q_1^B = \begin{bmatrix} 0 & 1 & -7 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \longrightarrow \boxed{q_1^B = (-5, 0)}$$

← calculated in (1) $\quad$ ← calculated in (2) $\quad$ ← calculated in (3a)

③ $q_2^B = (1, 0)$ $\quad q_2^o = g_B^o q_2^B$ $\quad q_2^A = g_o^A q_2^o$

$$q_2^o = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 1 \end{bmatrix} \rightarrow \boxed{q_2^o = (2, 8)}$$

$$q_2^A = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & -7.7782 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 2.1213 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -.7071 \\ 6.3640 \\ 1 \end{bmatrix} \rightarrow \boxed{q_2^A = (-0.7071, 6.3640)}$$

again computed with MATLAB - see HW2prob3.m

④ $g_B^A = g_\theta^A * g_B^\theta = \begin{bmatrix} .7071 & .7071 & -7.7782 \\ -.7071 & .7071 & 2.1213 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .7071 & -.7071 & -1.414 \\ .7071 & .7071 & 5.656 \\ 0 & 0 & 1 \end{bmatrix}$

matrix calculations in MATLAB HW2prob4.m

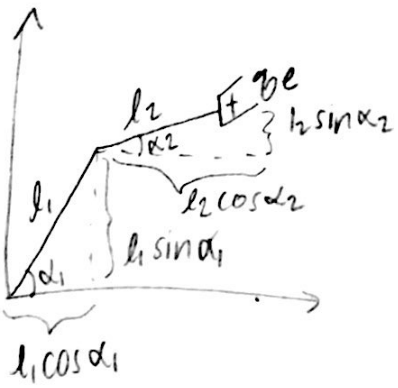
⑤ $g_C^A = (1, 4, -15°) = \begin{bmatrix} \cos(-15°) & -\sin(-15°) & 1 \\ \sin(-15°) & \cos(-15°) & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .9659 & +.2588 & 1 \\ -.2588 & .9659 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

matrix calculations in MATLAB HW2prob5.m

$g_C^\theta = \underset{\text{from (1)}}{g_C^A * g_A^\theta} = \begin{bmatrix} 0.8660 & -0.500 & 8.7968 \\ 0.5000 & 0.8660 & 6.052 \\ 0 & 0 & 1 \end{bmatrix}$

$g_C^B = g_C^\theta * \underset{\text{from (2)}}{g_\theta^B} = \begin{bmatrix} 0.500 & 0.866 & 1.7346 \\ -0.866 & 0.500 & 4.2840 \\ 0 & 0 & 1 \end{bmatrix}$

⑥ a) $(x_e, y_e) = (l_1\cos\alpha_1 + l_2\cos\alpha_2,\ l_1\sin\alpha_1 + l_2\sin\alpha_2) = g_e$



$g_e = \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} l_1\cos\alpha_1 + l_2\cos\alpha_2 \\ l_1\sin\alpha_1 + l_2\sin\alpha_2 \end{bmatrix}$

$g_e = \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} (1)\cos(\pi/2) + (\frac{1}{2})\cos(-\pi/3) \\ (1)\sin(\pi/2) + (\frac{1}{2})\sin(-\pi/3) \end{bmatrix} = \boxed{\begin{bmatrix} .25 \\ .567 \end{bmatrix}}$

6 cont.

b) $$D q_e = \begin{bmatrix} \frac{dx}{d\alpha_1} & \frac{dx}{d\alpha_2} \\ \frac{dy}{d\alpha_1} & \frac{dy}{d\alpha_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\alpha_1 & -l_2 \sin\alpha_2 \\ l_1 \cos\alpha_1 & l_2 \cos\alpha_2 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 0.433 \\ 0 & 0.25 \end{bmatrix}}$$

c) end effector velocity $= \dot{q}_e = Dq_e \cdot \dot{\alpha}$

$$\dot{q}_e = \begin{bmatrix} -1 & 0.433 \\ 0 & 0.25 \end{bmatrix} \begin{bmatrix} -1/5 \\ 1/2 \end{bmatrix} = \boxed{\begin{bmatrix} 0.41650 \\ 0.1250 \end{bmatrix}}$$

d) To be invertable, the determinant must not be 0.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \det = ad - bc = 0 = (-l_1 l_2 \sin\alpha_1 \cos\alpha_2) - (-l_1 l_2 \cos\alpha_1 \sin\alpha_2)$$

$$\Rightarrow \cancel{l_1 l_2} \sin\alpha_1 \cos\alpha_2 = \cancel{l_1 l_2} \cos\alpha_1 \sin\alpha_2$$

let $\alpha_1 = \pi/4 \rightarrow \alpha_2 = \pi/4$ $\quad \boxed{\alpha = \begin{bmatrix} \pi/4 \\ \pi/4 \end{bmatrix}}$

7 code in zip file

8 code in zip file

9 My preference is for option 1, traditional manipulator

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