ECE4560 - Solution #8

Problem 1. (20 pts) Consider the three-link planar manipulator discussed in class and depicted in Figure 1. Modify the `scara_r2.m` code so that it can plot the reachable workspace and the complete workspace for the system. Also plot in the plane the dextrous workspace limits (inner and outer rings); it can be the same plot, or on another one. Call it `planar_r3.m`. Let the link lengths be 1.5, 1.0, and 0.3 inches from base to end-effector, respectively.

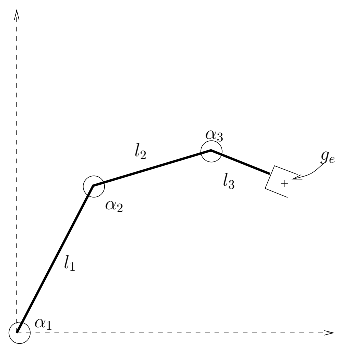

Figure 1: 3R Planar manipulator.

Solution 1. The rewritten code is

```
function planar_r3(l1, l2, l3, method)

if (nargin < 4)
  method = 'plot';
end

alpha1 = 2*(-90:10:90) * pi()/180;
alpha2 = 2*(-90:10:90) * pi()/180;
alpha3 = 2*(-90:10:90) * pi()/180;

alpha1Cnt = numel(alpha1);
alpha2Cnt = numel(alpha2);
alpha3Cnt = numel(alpha3);
total = alpha1Cnt*alpha2Cnt*alpha3Cnt;
x     = zeros(1,total);
y     = zeros(1,total);
theta = zeros(1,total);

ll = 1;
for ii=1:alpha1Cnt
  for jj=1:alpha2Cnt
    for kk=1:alpha3Cnt
      x(ll) = l1*cos(alpha1(ii))+l2*cos(alpha1(ii)+alpha2(jj)) ...
                                +l3*cos(alpha1(ii)+alpha2(jj)+alpha3(kk));
      y(ll) = l1*sin(alpha1(ii))+l2*sin(alpha1(ii)+alpha2(jj)) ...
                                +l3*sin(alpha1(ii)+alpha2(jj)+alpha3(kk));
      theta(ll) = alpha1(ii)+alpha2(jj)+alpha3(kk);
      ll = ll+1;
```

```
    end
  end
end

switch method
  case 'mesh',
    x     = reshape(x, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    y     = reshape(y, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    theta = reshape(theta, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    figure(1);
    patch(x, y,'g*');
  case 'plot'
    plot3(x, y, theta,'r*');
    hold on;
    plot(x, y, 'g*');
    axis equal
    hold off;
    %surf(x, y, theta);
end
```

Sample plots for different viewpoints are in Figure 2. The first plot depicts the entire range of configurations (it was generated with a `2*` in the equation for `alpha1`, `alpha2`, and `alpha3`. The second and third plots show a cut-away using only `1*` factor in the equation (as per the code above), allowing one to see the structure inside of the surface/volume. The inner radius of the dextrous workspace annulus is 1.8 and the outer one is 2.2. The reachable workspace is the disc of radius 2.8 (since the sum of the last two link lengths equals the first link length).

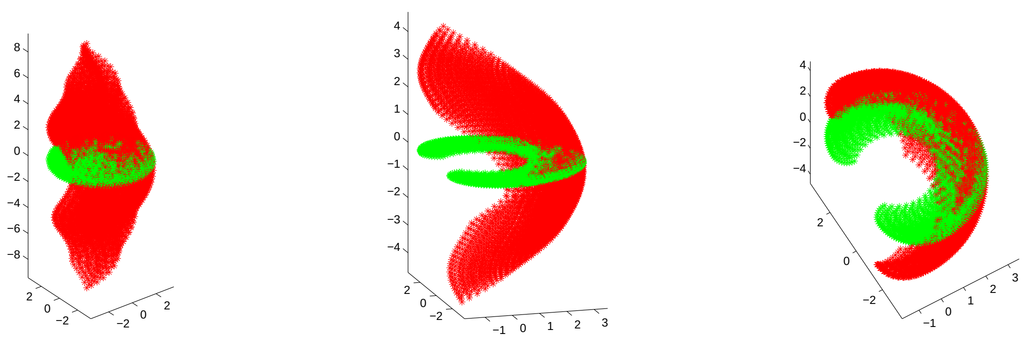

Figure 2: Sample plots of reachable (red) and complete (green) workspace.

Problem 2. (20 pts) Consider the wristless Armatron manipulator of Figure 3. It is not possible to achieve an arbitrary configuration without the wrist, so let's consider the reachable workspace, e.g., the set of reachable positions of the end-effector.

Supposing that the link lengths were $l_0 = 1$, $l_1 = 3/4$, $l_2 = 1/2$:

(a) What are the forward kinematics for position? Use any technique you'd like.

(b) Assuming that the joint limits were $\alpha_1 \in [-\pi/2, \pi/2]$, $\alpha_2 \in [-\pi/3, \pi/3]$, and $\alpha_3 \in [-\pi/2, \pi/2]$, show a plot of the reachable workspace. You should be able to modify the `scara_r2.m` or thr `planar_r3.m` code for this.

(c) What is the end-effector configuration for $\alpha = (\pi/3, \pi/3, -\pi/4)^T$?

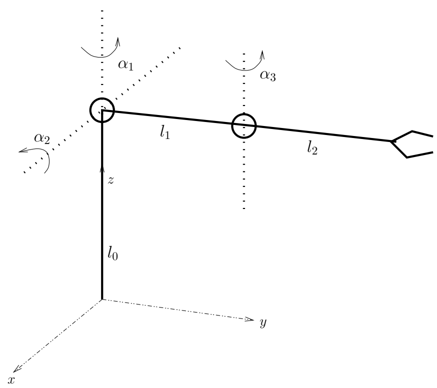

Figure 3: Wristless Armatron.

Solution 2.

(a) The forward kinematics will be done using the product of Lie groups. The four Lie group elements are:

$$g_1(\alpha_1) = \left[\begin{array}{ccc|c} \cos(\alpha_1) & -\sin(\alpha_1) & 0 & 0 \\ \sin(\alpha_1) & \cos(\alpha_1) & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ \hline & 0 & & 1 \end{array}\right] \qquad g_2(\alpha_2) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_1) & -\sin(\alpha_1) & 0 \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & 0 \\ \hline & 0 & & 1 \end{array}\right]$$

$$g_3(\alpha_3) = \left[\begin{array}{ccc|c} \cos(\alpha_3) & -\sin(\alpha_3) & 0 & 0 \\ \sin(\alpha_3) & \cos(\alpha_3) & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ \hline & 0 & & 1 \end{array}\right] \qquad g_4(\alpha_2) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ \hline & 0 & & 1 \end{array}\right].$$

The end-effector configuration is

$$g_e(\vec{\alpha}) = g_1(\alpha_1)\, g_2(\alpha_2)\, g_3(\alpha_3)\, g_4(\alpha_4).$$

The rotation matrix is:

$$R_e(\vec{\alpha}) = \left[\begin{matrix} \cos(\alpha_1)\cos(\alpha_3) - \sin(\alpha_1)\cos(\alpha_2)\sin(\alpha_3) & \dots \\ \sin(\alpha_1)\cos(\alpha_3) + \cos(\alpha_1)\cos(\alpha_2)\sin(\alpha_3) & \dots \\ \sin(\alpha_2)\sin(\alpha_3) & \dots \end{matrix}\right.$$

$$\left.\begin{matrix} -\cos(\alpha_1)\sin(\alpha_3) - \sin(\alpha_1)\cos(\alpha_2)\cos(\alpha_3) & \sin(\alpha_1)\sin(\alpha_2) \\ -\sin(\alpha_1)\sin(\alpha_3) + \cos(\alpha_1)\cos(\alpha_2)\cos(\alpha_3) & -\cos(\alpha_1)\sin(\alpha_2) \\ \sin(\alpha_2)\cos(\alpha_3) & \cos(\alpha_2) \end{matrix}\right]$$

The displacement vector component is

$$d_e(\vec{\alpha}) = \left\{ \begin{matrix} -\sin(\alpha_1)\cos(\alpha_2)\,(l_1 + l_2\cos(\alpha_3))) - l_2\cos(\alpha_2)\sin(\alpha_3) \\ \cos(\alpha_1)\cos(\alpha_2)\,(l_1 + l_2\cos(\alpha_3)) - l_2\sin(\alpha_1)\sin(\alpha_3) \\ l_0 + \sin(\alpha_2)(l_1 + l_2\cos(\alpha_3)) \end{matrix} \right\}$$

(b) The Matlab code to do the visualization is

```
function armitron(l0, l1, l2, method)

if (nargin < 4)
  method = 'plot';
end
```

```
alpha1 = (-90:5:90) * pi()/180;
alpha2 = (-45:5:60) * pi()/180;
alpha3 = (-90:5:90) * pi()/180;

alpha1Cnt = numel(alpha1);
alpha2Cnt = numel(alpha2);
alpha3Cnt = numel(alpha3);
total = alpha1Cnt*alpha2Cnt*alpha3Cnt;
x     = zeros(1,total);
y     = zeros(1,total);
z     = zeros(1,total);

ll = 1;
for ii=1:alpha1Cnt
  for jj=1:alpha2Cnt
    for kk=1:alpha3Cnt
      x(ll) = (l1+l2*cos(alpha3(kk)))*sin(alpha1(ii))*cos(alpha2(jj)) ...
                                  -l2*cos(alpha1(ii))*sin(alpha3(kk));
      y(ll) = (l1+l2*cos(alpha3(ii)))*cos(alpha1(ii))*cos(alpha2(jj)) ...
                                  -l2*sin(alpha1(ii))*sin(alpha3(kk));
      z(ll) = l0 + (l1+l2*cos(alpha3(kk)))*sin(alpha2(jj));
      ll = ll+1;
    end
  end
end

switch method
  case 'mesh',
    x = reshape(x, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    y = reshape(y, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    z = reshape(z, [alpha2Cnt*alpha3Cnt, alpha1Cnt]);
    %patch(x, y,'g*');
    surf(x,y,z);
  case 'plot'
    plot3(x, y, z,'r*');
    axis equal
end
```

Sample plots for different viewpoints are in Figure 4.

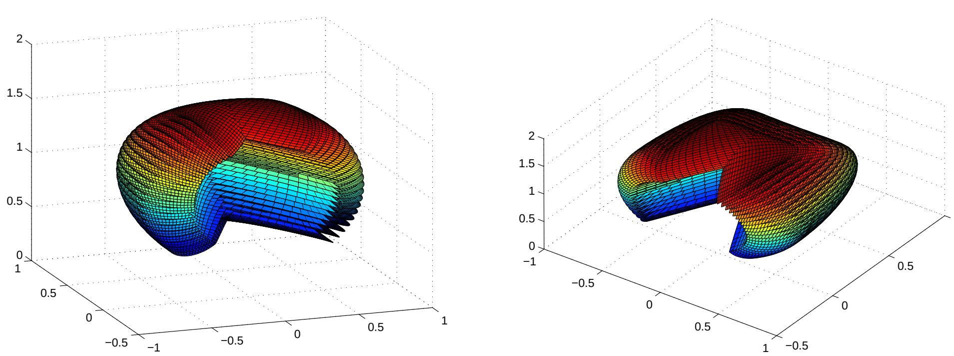

Figure 4: Armitron reachable workspace mesh plot.

(c) The forward kinematics for the joint angle $\alpha = (\pi/3, \pi/3, -\pi/4)^T$ are

$$g_e(\alpha) = \left[\begin{array}{ccc|c} 0.884 & -0.433 & 0.177 & -0.541 \\ 0.306 & 0.250 & -0.919 & 0.313 \\ 0.354 & 0.866 & 0.354 & 2.083 \\ \hline & 0 & & 1 \end{array}\right].$$

Problem 3. (20 pts) The Lynxmotion L6 is a small manipulator meant to replicate commonly found industrial manipulators in terms of its joint degrees of freedom. Unlike most industrial manipulators, which have six degrees of freedom, this one has only 5 degrees of freedom. A sketch of it is depicted below in Figure 5 with a straight-up reference configuration. Use the product of Lie groups method. You can leave the solution in symbolic form, meaning that it is sufficient to define the individual $g_i(\alpha_i)$ matrices and the final transformation to the end-effector. I do want, however, for you to give me the translation part $d_e(\alpha)$. The example from class should be a good guide as to how to proceed. Suppose for now that the link lengths are: $l_0 = 4.35$, $l_1 = 4.72$, $l_2 = 4.72$, $l_3 = 5.12$, and $l_4 = 0.79$ given in inches. Whereas l_3 go to roughly the hand area of the end-effector, the extra length l_4 goes a bit further to what would be the fingertips of the gripper (the length is not depicted in the figure). Its inclusion in the forward kinematics is optional.

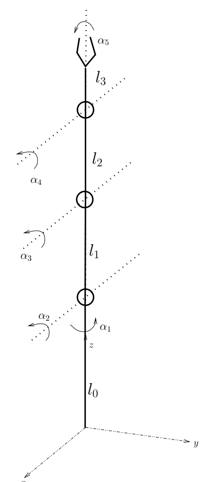

Figure 5: Lynxmotion L6 manipulator.

Solution 3. The forward kinematics using the product of Lie groups consists of six Lie group elements, the last of which is constant,

$$g_e(\alpha) = g_1(\alpha_1)g_2(\alpha_2)g_3(\alpha_3)g_4(\alpha_4)g_5(\alpha_5)g_6.$$

Of course, there are variations on this theme, meaning that the last couple of Lie group elements don't have to agree with what's below so long as they agree in the end regarding the configuration of the end-effector given a joint configuration.

The reference configuration $\alpha = 0$, is depicted in Figure 5. Analysis of the configuration indicates that the link displacements are

$$d_1 = \left\{ \begin{array}{c} 0 \\ 0 \\ l_0 \end{array} \right\}, \ d_2 = \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right\}, \ d_3 = \left\{ \begin{array}{c} 0 \\ 0 \\ l_1 \end{array} \right\}, \ d_4 = \left\{ \begin{array}{c} 0 \\ 0 \\ l_2 \end{array} \right\}, \ d_5 = \left\{ \begin{array}{c} 0 \\ 0 \\ l_3 \end{array} \right\}, \text{ and } d_6 = \left\{ \begin{array}{c} 0 \\ 0 \\ l_4 \end{array} \right\},$$

where d_1 and d_2 can actually be switched based on how the manipulator operates (or mathematicaly, how the rotation matrices multiply). The rotation matrices associated to the individual Lie group elements are

$$R_1 = R_z(\alpha_1), \ R_2 = R_x(\alpha_2), \ R_3 = R_x(\alpha_3), \ R_4 = R_x(\alpha_4), \ R_5 = R_z(\alpha_5), \text{ and } R_6 = \mathbb{1},$$

where the subscript identifies about which axis the rotation is occurring. Putting these two observations together gives the individual elements,

$$g_1(\alpha_1) = \left[\begin{array}{ccc|c} \cos(\alpha_1) & -\sin(\alpha_1) & 0 & 0 \\ \sin(\alpha_1) & \cos(\alpha_1) & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ \hline & 0 & & 1 \end{array}\right] \qquad g_2(\alpha_2) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_2) & -\sin(\alpha_2) & 0 \\ 0 & \sin(\alpha_2) & \cos(\alpha_2) & 0 \\ \hline & 0 & & 1 \end{array}\right]$$

$$g_3(\alpha_3) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_3) & -\sin(\alpha_3) & 0 \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) & l_1 \\ \hline & 0 & & 1 \end{array}\right] \qquad g_4(\alpha_4) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_4) & -\sin(\alpha_4) & 0 \\ 0 & \sin(\alpha_4) & \cos(\alpha_4) & l_1 \\ \hline & 0 & & 1 \end{array}\right]$$

$$g_5(\alpha_5) = \left[\begin{array}{ccc|c} \cos(\alpha_5) & -\sin(\alpha_5) & 0 & 0 \\ \sin(\alpha_5) & \cos(\alpha_5) & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ \hline & 0 & & 1 \end{array}\right] \qquad g_6 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_4 \\ \hline & 0 & & 1 \end{array}\right]$$

Multiplying the matrices out gives the rotation matrix

$$R_e(\alpha) = \begin{bmatrix} \cos(\alpha_1)\cos(\alpha_5) - \sin(\alpha_1)\sin(\alpha_2+\alpha_3+\alpha_4)\sin(\alpha_5) & \dots \\ \sin(\alpha_1)\cos(\alpha_5) + \cos(\alpha_1)\cos(\alpha_2+\alpha_3+\alpha_4)\sin(\alpha_5) & \dots \\ \sin(\alpha_2+\alpha_3+\alpha_4)\cos(\alpha_5) & \dots \end{bmatrix}$$
$$\begin{matrix} -\cos(\alpha_1)\sin(\alpha_5) - \sin(\alpha_1)\cos(\alpha_2+\alpha_3+\alpha_4)\cos(\alpha_5) & \sin(\alpha_1)\sin(\alpha_2+\alpha_3+\alpha_4) \\ -\sin(\alpha_1)\sin(\alpha_5) + \cos(\alpha_1)\cos(\alpha_2+\alpha_3+\alpha_4)\cos(\alpha_5) & \cos(\alpha_1)\sin(\alpha_2+\alpha_3+\alpha_4) \\ \sin(\alpha_2+\alpha_3+\alpha_4)\sin(\alpha_5) & \cos(\alpha_2+\alpha_3+\alpha_4) \end{matrix}\Bigg]$$

and the displacement matrix

$$d_e(\alpha) = \left\{ \begin{matrix} \sin(\alpha_1)\big(l_1\sin(\alpha_2) + l_2\sin(\alpha_2+\alpha_3) + (l_3+l_4)\sin(\alpha_2+\alpha_3+\alpha_4)\big) \\ -\cos(\alpha_1)\big(l_1\sin(\alpha_2) + l_2\sin(\alpha_2+\alpha_3) + (l_3+l_4)\sin(\alpha_2+\alpha_3+\alpha_4)\big) \\ l_0 + l_1\cos(\alpha_2) + l_2\cos(\alpha_2+\alpha_3) + (l_3+l_4)\cos(\alpha_2+\alpha_3+\alpha_4) \end{matrix} \right\},$$

where I wrote it in this form, as opposed to the complete homogeneous matrix form, in order to be able to display the two components.

It is OK to have only the l_3 part instead of $(l_3 + l_4)$ if you thought the l_4 seemed extra from the figure. The l_4 is for the older Lynxmotion manipulators, where the gripper would move forward as it closed. The structure of the forward kinematics should still be very similar to this solution.