ECE4560 - Homework #1

Due: Aug. 21, 2017

The homework has two different problem categories. The first two problems are little primer or refresher problems, one on coordinates and one on Matlab. The second also serves to make sure that everyone has Matlab up and running. The rest is the beginning material of the course. You'll note that the way the frames are described differs a bit. Look at the notes to see what the different representations are (in short, there is vector form, translation + rotation form, and homogeneous form). They all describe the same thing, a coordinate frame or a displacement, depending on the problem.

I recommend doing as much of it as possible in Matlab, then using Matlab's publish option to generate the homework solutions. This should lead to a pdf file with both the code and the solution, for a clean homework submission.

Problem 1. [10 pts] The purpose of this problem is to cover some basic trigonometry. In this problem, we are examining displacements of an object. Recall that the rotation of a point $p = (p_1, p_2)^T$ through the rotation angle θ (in radians) is given by:

$$p_1' = \cos(\theta)p_1 - \sin(\theta)p_2$$
$$p_2' = \sin(\theta)p_1 + \cos(\theta)p_2$$

Consider Figure 1 below, where a hammer is depicted in two different configurations. The reference point and vectors of each hammer configuration is depicted in the figure. Suppose that in its unrotated state, and with the base of the hammer at the origin, the hammer head is located at $(6, 0)$. The figure depicts the hammer translated and also rotated relative to the origin at $\mathcal{O}$. What are the new hammer head points for the following rotation and translation information?

(a) The base is translated by $(10, 25)$ to $\mathcal{A}$, and the hammer undergoes a rotation of $\pi/6$ radians?

(b) The base is translated by $(21, 7)$ to $\mathcal{B}$, and the hammer undergoes a rotation of π radians?

Note that it makes a difference when you do the rotation and when you do the translation. One will give sensible results that agree with the figure. The other will give non-sensical results. What is the proper order?

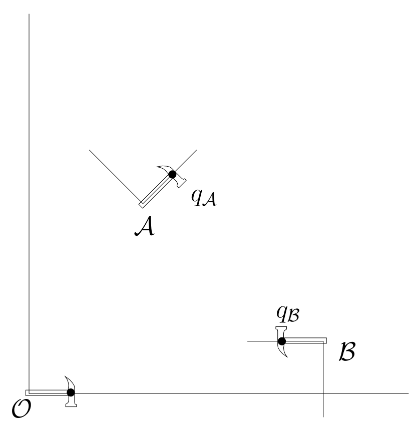


Figure 1: Rigid body displacements and point displacements.

Problem 2. [10 pts]

a. Numerically integrate the following differential equation using the code stub found on t-square:

$$\dot{x} = -2.5x + 0.75u, \qquad \text{where } x(0) = 1,$$

for $u(t) = \sin(t)$ and $u(t) = 8e^{-t}$. The current code works as though there were no input. Your job is to add the input part to the differential equation and numerically integrate. Hand in (i) a print-out of the modified code, and (ii) a plot of the system response versus time. Try to describe what happens with the response of the system to each of the two signals.

b. Integrate the following differential equation using the code stub found on t-square:

$$\begin{aligned} \dot{x} &= \cos(\theta)u(t), \\ \dot{y} &= \sin(\theta)u(t), \\ \dot{\theta} &= v(t). \end{aligned}$$

where $x(0) = 0, y(0) = 0, \theta(0) = 0$. Simulate for both **(i)** $u(t) = 0.2\sin(t)$ and $v(t) = 0.2$ and **(ii)** $u(t) = 0.5\cos(t)$ and $v(t) = 0.3\sin(t)$. Hand in (i) a print-out of the code, and (ii) a plot of the system response versus time. Try to describe what happens with the response of the system to each of the two signals.

To integrate this, you will have to vectorize the equations and define $x_1 = x$, $x_2 = y$, and $x_3 = \theta$ in Matlab. The stub sort of hints at how to do that.

File for Problem 2: odeProb.zip

Preface: The next set of problems are all related. Consider Figure 2(a) below, where a hammer is depicted in two different configurations. The reference frame of each hammer configuration specifies the position and orientation of the tip of the handle. Suppose that the configurations associated with the two reference frames depicted were $g_{\mathcal{A}}^{\mathcal{O}} = (5, 12, \pi/3)$ and $g_{\mathcal{B}}^{\mathcal{O}} = (2, -1, \pi)$, with $\mathcal{O}$ being the observer frame. We are interested in knowing where the location of the head is, supposing that the head is at $q = (1, 0)$ relative to the reference frame of the hammer.

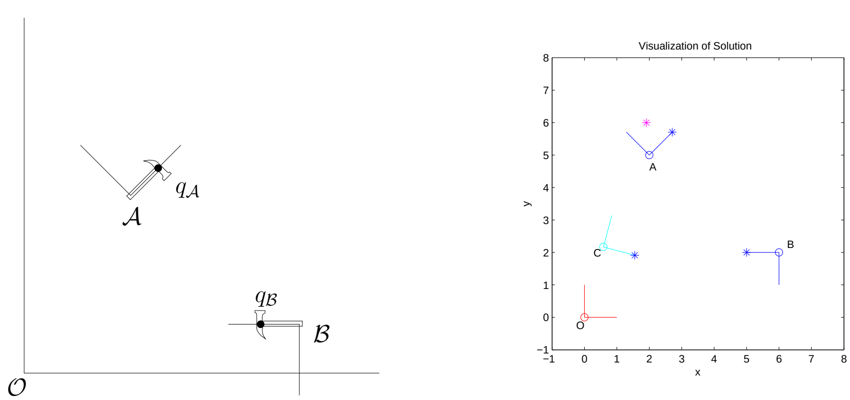
Problem 3. [5 pts] What are $g_{\mathcal{A}}^{\mathcal{O}}$ and $g_{\mathcal{B}}^{\mathcal{O}}$ in (d, R) representation? What about in complex $(z_T, z_R)^T$ representation?

Problem 4. [5 pts] Given the hammer is at configuration $\mathcal{A}$, where is the head located at? (e.g. what is $q_{\mathcal{A}}^{\mathcal{O}}$?)

Problem 5. [5 pts] Given the hammer is at configuration $\mathcal{B}$, where is the head located at? (e.g. what is $q_{\mathcal{B}}^{\mathcal{O}}$?)

Problem 6. [10 pts] Suppose that the hammer was moved from its configuration at $\mathcal{A}$ to yet another configuration, denoted by $\mathcal{C}$. Further, you are told that the new location, relative to frame $\mathcal{A}$, is $(-2, -2, \pi/6)$. What is $g_{\mathcal{C}}^{\mathcal{O}}$ and where is the head now located with respect to the observer frame (e.g., what is $q_{\mathcal{C}}^{\mathcal{O}}$)?

Problem 7. [10 pts] Suppose that someone accidentally nudges the hammer associated to frame $\mathcal{A}$. In particular, the head of the hammer rotates a bit in the frame. If the hammer head is now located in frame $\mathcal{A}$ at $(0.6428, -0.7660)$ and the base is at the origin as before, where is the hammer head located in frame $\mathcal{O}$?



(a) Coordinates: Frames and Points. (b) Sample Solution.

Numerical Integration in Matlab. Matlab provides several methods for numerically integrating a differential equation. The one I typically use is `ode45` (others include `ode23`, `ode113`, etc.). You can find out more about them by using help (type "help ode45") or through the online documentation.

Their most basic usage is as follows:

```
>> [t, x] = ode45( @mydiffeq , tspan, x0 );
```

where tspan is a 2-vector containing the initial and final times and x0 is an n-vector describing the initial condition of the system. The system is integrated from the initial to final times using the desired initial condition, where `mydiffeq` is the function giving the differential equation. It's format is usually as follows:

```
function xdot = mydiffeq(t, x)

[ some code here ]

xdot = [ your math here ]

end
```

and is given in a separate file (known as an m-function).

Numerical Integration of Vector Systems. Integration of vector systems is similar to the above, however the state x is a vector, so $\dot{x}$ should also be a vector. For each component of the vector $x(1)\ldots x(n)$ where x is an n-vector, you will have to define a time derivative. Please note that Matlab expects the variable to be a column vector (same with the initial condition) and will crap out if the vector is a row vector.

Numerical Integration of Higher-Order Systems. For higher order systems like the second order system of problem, the differential equation needs to be vectorized. For the problem, that means defining the vector $\vec{x}$ to be $\vec{x} = (x_1, x_2)^T$ where $x_1 = x$ and $x_2 = \dot{x}$. The differential equation then becomes two differential equations:

$$\dot{x}_1 = \dot{x} = x_2, \qquad \text{and} \qquad \dot{x}_2 = \ddot{x} = -2\dot{x} - 10x + u(t) = -2x_2 - 10x_1 + u(t). \tag{1}$$

These equations need to be entered into an m-function for integration. The m-function will return a 2-vector, and the initial condition is also a 2-vector. In fact, the initial condition is $\vec{x}(0) = (x_1(0), x_2(0))^T = (x(0), \dot{x}(0))^T$.

If you have any further questions, feel free to come by and discuss the problem with me.