ECE4560 - Homework #5

Due: Oct. 2 2017

Problem 1. [10 pts] Consider the setup depicted in Figure 1. The configurations associated to the different frames are

$$g_{\mathcal{A}}^{\mathcal{O}} = \left(\left\{ \begin{array}{c} 9 \\ 4 \end{array} \right\}, R(\pi/2) \right), \quad g_{\mathcal{B}}^{\mathcal{O}} = \left(\left\{ \begin{array}{c} 3 \\ 6 \end{array} \right\}, R(\pi/3) \right), \quad g_{\mathcal{C}}^{\mathcal{B}} = \left(\left\{ \begin{array}{c} 4 \\ 0 \end{array} \right\}, R(\pi/12) \right).$$

(a) Suppose that from frame $\mathcal{A}$ the 2D vector v_1 is $\mathrm{v}_1^{\mathcal{A}} = (1, 1)^{\mathrm{T}}$. Then what is the same vector written in $\mathcal{O}$'s frame? in $\mathcal{B}$'s frame?

(b) Now, let frame $\mathcal{C}$ see the vector $\mathrm{v}_2^{\mathcal{C}} = (\frac{\sqrt{3}}{2}, -\frac{1}{2})^{\mathrm{T}}$. What are the coordinates of the same vector in $\mathcal{O}$'s frame? in $\mathcal{B}$'s frame?

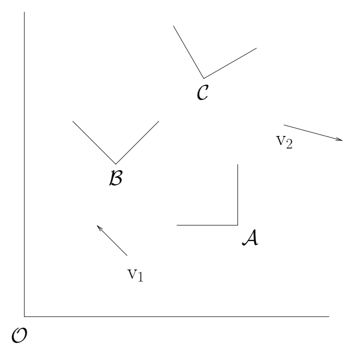

Figure 1: Problem #1 vector setup.

Problem 2. [10 pts] Suppose that we have the following frames

$$g_{\mathcal{A}}^{\mathcal{O}} = \left(\left\{ \begin{array}{c} 5 \\ 2 \end{array} \right\}, R(\pi/3) \right), \quad g_{\mathcal{B}}^{\mathcal{O}} = \left(\left\{ \begin{array}{c} 2 \\ 7 \end{array} \right\}, R(\pi/2) \right), \quad \text{and} \quad g_{\mathcal{C}}^{\mathcal{B}} = \left(\left\{ \begin{array}{c} 0 \\ 3 \end{array} \right\}, R(-\pi/6) \right).$$

We have seen how these frames function with regards to Euclidean 2-vectors. Now it is time to better understand special Euclidean 2-vectors, e.g., vectors associated to planar rigid body motion. You observe motions associated to $SE(2)$ as per Figure 2. Notice that in Figure 2 the vectors also have this semi-circle arrow by them, this is to denote the rotational component that is now a part of vectors in $SE(2)$.

(a) If frame $\mathcal{A}$ sees the vector $\xi_1^{\mathcal{A}} = (2, 5, -\pi/12)^T$, then what is this same vector with respect to frames $\mathcal{B}$ and $\mathcal{O}$?

(b) If frame $\mathcal{C}$ sees the vector $\xi_2^{\mathcal{C}} = (1, -4, \pi/15)^T$, then what is the same vector with respect to frames $\mathcal{B}$ and $\mathcal{O}$?

Problem 3 [10 pts] Continuing with the setup from the previous Problem, let's suppose that frame $\mathcal{B}$ and frame $\mathcal{C}$ were actually attached to the same rigid body and that you watched the body move from its original configuration to some new configuration ($\mathcal{B}'$ and $\mathcal{C}'$). As the rigid body was moving, you measured that the body velocity of the frame $\mathcal{B}$ was $\xi_{\mathcal{B}}^b = (2, -3, \pi/9)^T$. What is the body velocity associated to the frame $\mathcal{C}$, i.e., what is $\xi_{\mathcal{C}}^b$? The setup is depicted in Figure 3.

Problem 4. [15 pts] The main part of this exercise is to make sure that we are all comfortable with the mathematical notation thus far.

(a) Given that $\xi \in \mathfrak{se}(2)$ and in particular, $\xi = (4, 6, \pi/12)^T$, what is $\hat{\xi}$?

(b) Given

$$\hat{\xi} = \begin{bmatrix} 0 & -\frac{\pi}{8} & 7 \\ \frac{\pi}{8} & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix},$$

what is $\xi \in \mathfrak{se}(2)$?

(c) Given that $\xi \in \mathfrak{se}(3)$ and in particular, $\xi = (1, -9, 3, 1, -0.8, -0.5)^T$, what is $\hat{\xi}$?

(d) Given that

$$\hat{\xi} = \begin{bmatrix} 0 & -0.9 & -0.1 & 3 \\ 0.9 & 0 & -0.2 & -2 \\ 0.1 & 0.2 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

what is $\xi \in \mathfrak{se}(3)$?

Problem 5. (10 pts) The differential equations represented by:

$$\dot{g} = g * \xi?$$

for some constant $\xi = (\xi_1, \xi_2, \xi_3)$ is given more explicitly by

$$\left\{ \begin{matrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{matrix} \right\} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{matrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{matrix} \right\}. \tag{1}$$

Integrating this for some time τ and some initial condition g_i should mimic the effect of moving along a trajectory with a constant body velocity (starting from g_i). Let's see what that would look like.

Integrate the differential equation (1) for the case that $\xi = (1, -1, \frac{1}{4})$, $g_i = g(0) = (0, 0.5, \pi/6)$ and final time is $\tau = \pi$. Now do the same for the case that the body velocity is now $\xi = (1, -1, 0)$. Does the output make sense? What is the big difference between the two velocities?

To perform the integration in Matlab, use the function **ode45** or an equivalent (you should have some familiarity with that by now). In Mathematica, you can use the **NDSolve** function. In both programs, you can access the help and online help to figure out how the integrators work if you still have questions. Plot the planar trajectory of the system with a few snapshots of the $SE(2)$ configuration overlaid on top of the trajectory (use the `hold` command). The snapshots help you visualize the orientation.

If you do it in Matlab, then you should have a script m-file to command Matlab to do the integration and process the output, plus a function m-file to specify the differential equation.

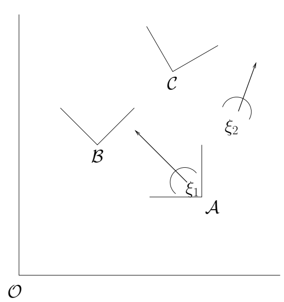

Figure 2: Vectors setup.

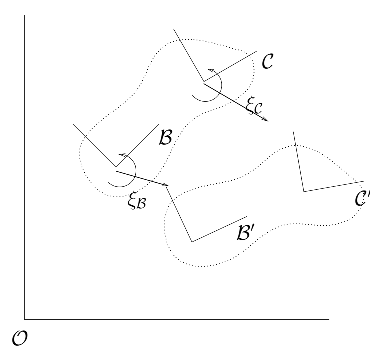

Figure 3: Rigid body vectors setup.

Problem 6. [Manipulator Option] (30 pts) This problem deals with calibration of the SCARA-type planar manipulator to be used in the lab portion of the class. Calibration usually requires measuring the link lengths associated to your manipulator, the servo command limits for each of the joints, and the angular limits of the manipulator. The limits imposed may actually be less than the full range of motion of the servo motors.

Since the link lengths have already been established based on the actual design, what is needed is to mapping between joint coordinate units and servo command units.

a. Determine what are the proper servo command limits for your particular manipulator. The manipulator code for interfacing should be uploaded to the course wiki as will a page describing in more detail what this problem is about. Please be careful, since they can break.

b. The second phase is to associate angles to the servo commands. Using the straight-out reference configuration as the zero configuration for the joint-angles, determine the angular limits associated to the joints with respect to that reference configuration as best as you can. You WILL need a protractor.

Although the robot may be able to go to bigger angles, try to round the joint-angle workspace to the nearest 30, 45, 90, or 180 degrees. For example, if you measure that it can go -37 degrees to 48 degrees, then make the joint angle limits $[-30, 45]$ in order to have nice clean limits. You'll need to figure out what servo commands correspond to those particular angle limits.

If you can hit the 90 degrees mark, then you might be able to use these little cardboard pieces on the shelf to identify that the right angle has been hit properly.

The course wiki is located at `pvela.gatech.edu/classes` and the proper link to follow is labelled *Manipulator Interface*. The manipulator is called **piktul** and you can find one on the shelving in the lab room. They are small white, RC servo manipulators. The manipulator circuit board should be plugged into the serial cable attached to the computers.

Problem 6. [Turtlebot Option] (30 pts) Following the instructions at the wiki site for Turtlebot learning `pvela.gatech.edu/classes` to get the Turtlebot up and working. Demonstrate the following:

a) The ability to teleoperate the Turtlebot using the provided laptop (or using your own if you prefer and you have some modern linux installation on it that can run ROS);

b) The ability to remotely connect to the Turtlebot (from a workstation in the lab, or from your laptop).

c) The ability to remotely teleoperate by remotely connecting to the Turtlebot.

d) Answer the questions of the *First Run* page labeled "**3. Understanding**."

You should demo this during inquiry hours on Tuesday or on Thursday. If need be, you can try to negoatiate a demo time outside of those hours with the TA.

Problem 6. [Biped Option] (30 pts) Following the instructions at the wiki site for biped learning `pvela.gatech.edu/classes` to get the motor interface understood and working for a single motor, as well as to start to model and visualize kinematically the biped robot. In particular, work out Module 0, bullet 1. Demo things working, answer any associated que:stions, and plot whatever is requested.

Problem 6. [Group Option] (30 pts)**.** If you are working on a group project, then you will be assigned a group contact. The group contact will either be me or a graduate student. You should work out whatever baby steps this group contact assigns the group as opposed to the lab problem. Most likely the info/e-mail will go out on Friday. It is your responsibility to turn in the results to the group contact. There should be sufficient output for them to assess progress.

You are welcome to negotiate a different schedule for this option with the group contact. It could be Wednesday to Wednesday, Friday to Friday, etc. whatever works, so long as it roughly cleaves to the homework schedule. At the end no matter what option is selected, there should have been n deliverables for all teams.