ECE4560 - Solution #6

Problem 1. (20 pts) Now that you know how to numerically integrate the Lie algebra element $\xi \in \mathfrak{se}(2)$, given an initial configuration $g_i \in SE(2)$ and a time duration τ, to get a Lie group element $g_f \in SE(2)$, it is time to compare against the closed-form solution. Given the numerical integration from a previous homework problem, let's compare against the closed-form solution determined by the exponential.

For the case that $\xi = (1, -1, \frac{1}{4})$, $g_i = g(0) = (0, 0.5, \pi/6)$ and final time is $\tau = \pi$, explicitly solve for $e^{\xi\tau}$ using ξ and τ. Show that $g_f = g_i e^{\xi\tau}$ agrees with what you got from the previous problem.

You also had to do the case where the body velocity was $\xi = (1, -1, 0)$. Repeat the same procedure for this body velocity.

Solution 1. As discussed in class, the closed form solution can be found through the equation for the exponential. The exponent will quantify the effect of the integration and is:

$$e^{\hat{\xi}\tau} = \left[\begin{array}{c|c} R(\xi_3\,\tau) & -\frac{1}{\xi_3}(\mathbb{1} - R(\xi_3\tau))\mathbb{J}\left\{\begin{array}{c}\xi_1\\ \xi_2\end{array}\right\} \\ \hline 0 & 1 \end{array}\right]$$

Substituting in the real values results in,

$$e^{\hat{\xi}\tau} = \left[\begin{array}{c|c} R(\pi/4) & -4(\mathbb{1} - R(\pi/4))\mathbb{J}\left\{\begin{array}{c}1\\ -1\end{array}\right\} \\ \hline 0 & 1 \end{array}\right]$$

Continuing,

$$\begin{aligned} e^{\hat{\xi}\tau} &= \left[\begin{array}{c|c} R(\pi/4) & -4(\mathbb{1} - R(\pi/4))\left\{\begin{array}{c}-1\\ -1\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{c|c} R(\pi/4) & \left\{\begin{array}{c}4\\ 4-4\sqrt{2}\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{cc|c} 0.7071 & -0.7071 & 4 \\ 0.7071 & 0.7071 & -1.6569 \\ \hline \multicolumn{2}{c|}{0} & 1 \end{array}\right] \end{aligned}$$

Perfect, now given that, we just need to determine the product

$$\begin{aligned} g_f &= g_i e^{\xi\tau} \\ &= \left[\begin{array}{c|c} R(\pi/6) & \left\{\begin{array}{c}0\\ \frac{1}{2}\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} R(\pi/4) & \left\{\begin{array}{c}4\\ 4-4\sqrt{2}\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{c|c} R(5\pi/12) & \left\{\begin{array}{c}0\\ \frac{1}{2}\end{array}\right\} + R(\pi/6)\left\{\begin{array}{c}4\\ 4-4\sqrt{2}\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{c|c} R(5\pi/12) & \left\{\begin{array}{c}0+2\sqrt{3}-2+2\sqrt{2}\\ \frac{1}{2}+2+2\sqrt{3}-2\sqrt{6}\end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{cc|c} 0.2588 & -0.9659 & 4.2925 \\ 0.9659 & 0.2588 & 1.0651 \\ \hline \multicolumn{2}{c|}{0} & 1 \end{array}\right] \end{aligned}$$

Now, for the closed form solution associated to the second velocity vector, we know that there is no rotation. The exponent simplifies:

$$e^{\hat{\xi}\tau} = \left[\begin{array}{c|c} \mathbb{1} & \left\{\begin{array}{c} 1 \\ -1 \end{array}\right\}\tau \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} \mathbb{1} & \left\{\begin{array}{c} \pi \\ -\pi \end{array}\right\} \\ \hline 0 & 1 \end{array}\right]. \tag{1}$$

The exponent then naturally leads to the final configuration,

$$\begin{aligned} g_f &= g_i e^{\xi\tau} \\ &= \left[\begin{array}{c|c} R(\pi/6) & \left\{\begin{array}{c} 0 \\ \frac{1}{2} \end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} \mathbb{1} & \left\{\begin{array}{c} \pi \\ -\pi \end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{c|c} R(\pi/6) & \left\{\begin{array}{c} 0 \\ \frac{1}{2} \end{array}\right\} + R(\pi/6)\left\{\begin{array}{c} \pi \\ -\pi \end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{c|c} R(\pi/6) & \left\{\begin{array}{c} \frac{\pi\sqrt{3}}{2} + \frac{\pi}{2} \\ \frac{1}{2} + \frac{\pi}{2} - \frac{\pi\sqrt{3}}{2} \end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \\ &= \left[\begin{array}{cc|c} 0.8660 & -0.5000 & 4.2915 \\ 0.5000 & 0.8660 & -0.6499 \\ \hline \multicolumn{2}{c|}{0} & 1 \end{array}\right] \end{aligned}$$

Problem 2. (15 pts) If the initial configuration of an object in $SE(2)$ is $g_i = (-3, 2, \pi/4)^T$ and the final configuration is $g_f = (3, -3, -\pi/3)^T$, what is the Lie group displacement, g, associated to the object. What is the Lie algebra element, $\xi \in \mathfrak{se}(2)$, associated with this rigid body motion (presuming that $\tau = 1$)?

Solution 2. There is one solution that I was looking for, but there are actually two solutions to this problem. Note that I gave you the initial and final configurations, then asked for a Lie algreba element associated with the rigid body motion. Well, the question is: what rigid body motion? There are two completely valid options, given by the two equations:

$$g_f = g_i g \qquad \text{or} \qquad g_f = g g_i.$$

The first one is a body-based perspective of the transformation (it asks what transformation is applied at the initial body frame to map to the final frame), whereas the second one is the spatial-based perspective of the transformation (it says, how should the observer frame be pushed forward so that the rigid body now lies at the final configuration). They both have equally valid solutions, it's just a matter of respecting the order of the multiplication. Up until now you have been given the order of the multiplication so this freedom did not exist. Let's work out both solutions. But before doing that, I just want to let you know that the more natural one when talking about moving objects is the body velocity. Moving to spatial is typically done under special circumstances.

The transformation from the initial to the final configuration, in the frame of the initial configuration (body-based) is

$$g_f = g_i g \quad \rightarrow \quad g = g_i^{-1} g_f.$$

This means that,

$$\begin{aligned} g = {} & \left[\begin{array}{c|c} R(\pi/4) & (-3, 2)^T \\ \hline 0 & 1 \end{array}\right]^{-1} \left[\begin{array}{c|c} R(-\pi/3) & (3, -3)^T \\ \hline 0 & 1 \end{array}\right] \\ & = \left[\begin{array}{c|c} R(-7\pi/12) & R(-\pi/4)\left\{\begin{array}{c} 6 \\ -5 \end{array}\right\}^T \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} R(-7\pi/12) & \left\{\begin{array}{c} 0.707 \\ -7.778 \end{array}\right\} \\ \hline 0 & 1 \end{array}\right] \end{aligned}$$

In vector form, the outcome is

$$g = \left\{\begin{array}{c} 0.707 \\ -7.778 \\ \hline -7\pi/12 \end{array}\right\}.$$

We will compute the logarithm such that $\exp(\xi) = g$, meaning that the exponent implicitly flows for $\tau = 1$. The logarithm for our particular g gives

$$\omega = -7\pi/12 \quad \text{and} \quad v = -\frac{7\pi}{12}\mathbb{J}\left(I - R(-7\pi/12)\right)^{-1}(0.707, -7.778)^T = (7.624, -4.821)^T.$$

This is done in body coordinates, since it was done with respect to the initial configuration, hence it's the body velocity. Therefore, $\xi^b = (7.624, -4.821, -7\pi/12)^T$ for $\tau = 1$.

Suppose instead that we had asked for the transformation that moves the spatial frame from its original location to one such that a rigidly attached g_i gets moved to g_f,

$$g_f = g g_i \quad \rightarrow \quad g = g_f g_i^{-1}.$$

The computation for g gives

$$\begin{aligned}
g = &\left[\begin{array}{c|c} R(-\pi/3) & (3,-3)^T \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} R(\pi/4) & (-3,2)^T \\ \hline 0 & 1 \end{array}\right]^{-1} \\
&= \left[\begin{array}{c|c} R(-7\pi/12) & R(-7\pi/12)\left\{\begin{matrix} 3 \\ -2 \end{matrix}\right\} + \left\{\begin{matrix} 3 \\ -3 \end{matrix}\right\} \\ \hline 0 & 1 \end{array}\right] \\
&= \left[\begin{array}{c|c} R(-7\pi/12) & \begin{matrix} 0.292 \\ -5.380 \end{matrix} \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{cc|c} -0.259 & -0.966 & 0.292 \\ 0.966 & -0.259 & -5.380 \\ \hline \multicolumn{2}{c|}{0} & 1 \end{array}\right]
\end{aligned}$$

In vector form, this is

$$g = \left\{\begin{array}{c} 0.292 \\ -5.380 \\ \hline -7\pi/12 \end{array}\right\}$$

The logarithm for this particular g gives

$$\omega = -\pi/2 \quad \text{and} \quad v = -\frac{7\pi}{12}\mathbb{J}\left(I - R(-7\pi/12)\right)^{-1}(0.292, -5.380)^T = (5.1349, -3.5155)^T.$$

This is the spatial velocity $\xi^s = (5.1349, -3.5155, -7\pi/12)^T$.

To expand on this solution, I should be able to get to one from the other using the Adjoint (as in ξ^b from ξ^s, or vice-versa). Hopefully, it's all starting to come together.

Problem 3. (15 pts) Compute the exponential of the following $\mathfrak{so}(3)$ rotation matrix vector,

$$\hat{\omega} = \begin{bmatrix} 0 & -1 & -0.25 \\ 1 & 0 & 0.5 \\ 0.25 & -0.5 & 0 \end{bmatrix}$$

for $\tau = 2$.

Solution 3. The solution to this problem is just a matter of working out Rodrigues' formula for the exponential.

$$\exp(\hat{\omega}\tau) = \mathbb{1} + \frac{\hat{\omega}}{||\omega||}\sin(||\omega||\,\tau) + \frac{\hat{\omega}^2}{||\omega||^2}(1 - \cos(||\omega||\,\tau)),$$

for which we compute some of the intermediate quantities first. Given that $\tau = 2$ and $||\omega|| = 1.1456$, we get that

$$\cos(||\omega||\,\tau) = -0.6598 \qquad \text{and} \qquad \sin(||\omega||\,\tau) = 0.7515\,. \tag{2}$$

With these computations, the exponent equation becomes

$$\exp(\hat{\omega}\tau) = \mathbb{1} + \frac{0.7515}{1.1456}\begin{bmatrix} 0.0000 & -1.0000 & -0.2500 \\ 1.0000 & 0.0000 & 0.5000 \\ 0.2500 & -0.5000 & 0.0000 \end{bmatrix} + \frac{-0.6598}{1.3125}\begin{bmatrix} -1.0625 & 0.1250 & -0.5000 \\ 0.1250 & -1.2500 & -0.2500 \\ -0.5000 & -0.2500 & -0.3125 \end{bmatrix}.$$

Working out the matrix addition results in

$$\exp(\hat{\omega}\tau) = \begin{bmatrix} -0.3436 & -0.4979 & -0.7963 \\ 0.8140 & -0.5807 & 0.0118 \\ -0.4683 & -0.6441 & 0.6048 \end{bmatrix}.$$

Problem 4. (25 pts) If the initial and final configurations of an object in $SE(3)$ are

$$g_i = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 1 & 0 & 0 & 2 \\ \hline & 0 & & 1 \end{array}\right] \qquad \text{and} \qquad g_f = \left[\begin{array}{ccc|c} \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 6 \\ \frac{1}{2} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & -3 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 9 \\ \hline & 0 & & 1 \end{array}\right].$$

What is the Lie algebra element, $\xi \in \mathfrak{se}(3)$ associated with this transformation? Verify your solution by computing the exponential of it and showing that $g_f = g_i e^{\hat{\xi}\tau}$.

Solution 4. To compute the logarithm, we need to start with the transformation g, which is obtained from $g = g_i^{-1} g_f$ and works out to be

$$g = \left[\begin{array}{ccc|c} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 7 \\ \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 3 \\ \frac{1}{2} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & -8 \\ \hline & 0 & & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 0.000 & 0.866 & -0.500 & 7.000 \\ 0.866 & 0.250 & 0.433 & 3.000 \\ 0.500 & -0.433 & -0.750 & -8.000 \\ \hline & 0 & & 1 \end{array}\right],$$

given that

$$g_i^{-1} = \left[\begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ \hline & 0 & & 1 \end{array}\right].$$

Now, it is simply a matter of computing the logarithm of g. The logarithm is found in two steps, **(i)** find $\hat{\omega}$ and $||\omega||$, then **(ii)** find v. Here, since it is not given, the time τ is implictly 1.

To get $||\omega||$, we use

$$||\omega|| = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right) = 2.4189.$$

To get $\hat{\omega}$, we use

$$\hat{\omega} = \frac{||\omega||}{2\sin(||\omega||\,\tau)}(R - R^T) = \begin{bmatrix} 0.0000 & 0.0000 & -1.8285 \\ 0.0000 & 0.0000 & 1.5835 \\ 1.8285 & -1.5835 & 0.0000 \end{bmatrix}$$

leading to $\omega = (-1.584, -1.828, 0.000)^T$.

Secondly, v is found through the equation,

$$v = ||\omega||^2 \left((\mathbb{1} - R)\hat{\omega} + \omega\omega^T\tau\right)^{-1} d = (-1.679, 10.517, -7.681)^T.$$

The final solution for the body velocity is

$$\xi = (-1.679, 10.517, -7.681, -1.584, -1.828, 0.000)^T. \quad \text{and} \quad \tau = 1.$$

Now, we need to test things out and verify that what we computed was correct. Yup, it's time to exponentiate the Lie algebra element ξ. (Let the fun begin!) The first step is to apply Rodrigues' formula to get the rotation matrix. For that we will need $\hat{\omega}$. Using the vector ξ we get $\hat{\omega}$,

$$\hat{\omega} = \begin{bmatrix} 0 & -\xi^3 & \xi^2 \\ \xi^3 & 0 & -\xi^1 \\ -\xi^2 & \xi^1 & 0 \end{bmatrix} = \begin{bmatrix} 0.0000 & 0.0000 & -1.8285 \\ 0.0000 & 0.0000 & 1.5835 \\ 1.8285 & -1.5835 & 0.0000 \end{bmatrix}$$

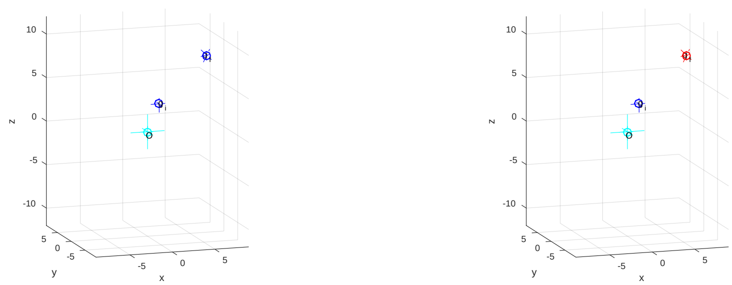

Figure 1: Setup and verification of Problem 4 (see text for details).

So, applying Rorigues' formula with $\tau = 1$,

$$\exp(\hat{\omega}\tau) = \mathbb{1} + \frac{\hat{\omega}}{||\omega||}\sin(||\omega||\,\tau) + \frac{\hat{\omega}^2}{||\omega||^2}(1-\cos(||\omega||\,\tau)) = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{1}{2} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} \end{bmatrix} = \begin{bmatrix} 0.000 & 0.866 & -0.500 \\ 0.866 & 0.250 & 0.433 \\ 0.500 & -0.433 & -0.750 \end{bmatrix},$$

which agrees with the g we computed, so that means we are on the right track. Next up is the computation of the displacement,

$$T = (\mathbb{1} - \exp(\hat{\omega}\tau))\frac{\hat{\omega}\,v}{||\omega||^2} + \frac{\omega\omega^T}{||\omega||^2}v\tau = \left\{ \begin{matrix} 7 \\ 3 \\ -8 \end{matrix} \right\}.$$

Awesome, it agrees with g. The next step is to get that $g_f = g_i\,g$, which I leave to you since we already know that we are on the right track given that $g = \exp(\xi)$. Just so you know that I really did verify, check out Figure 1, which depicts the geometry of the problem (left plot) and then verifies by plotting my solution (right plot) and seeing that the verification configuration (in red) overwrites the final configuration as it should (because they should be the same).

Suppose that you accidentally did it in spatial form,

$$g = g_f g_i^{-1} = \left[\begin{array}{ccc|c} 0.250 & 0.433 & 0.866 & 1.353 \\ -0.433 & -0.750 & 0.500 & 1.049 \\ 0.866 & -0.500 & 0.000 & 8.902 \\ \hline & 0 & & 1 \end{array}\right]$$

then the spatial velocity/twist is

$$\xi^s = (2.599, -6.588, 7.463, -1.828, 0.000, -1.584)^T.$$

The exponential should give back the g computed just above.

Problem 5. (10 pts)

(a) Let's get the actions (what I call products) of group elements with vectors (or velocities) coded up so we don't have to worry about it. In particular, we will have to extend the left action to apply to either a point or a vector representation for the velocity. The good thing is that a point is a 2x1 vector and a vector representation of the velocity is a 3x1 vector (translational + rotational velocity).

The code had an `if` statement with two versions. Maybe you coded up both, or just one of them. Add in the missing code. Verify that your code gives the same answers as in the solutions, both for the group times homogeneous translation vector and the group times vector-form of velocity for the planar case.

(b) Also amend the adjoint to operate on vectors as well. Modify the adjoint code reposted and reproduce both the vector solution and the homogeneous solution. Hopefully you see that this code can be verified by reproducing the solution to the Adjoint times vector-form of velocity problem from the last homework. Demonstrate that you have verified the code.

(c) To make life easier, create a new section at the bottom of the SE2 class that performs the hat and unhat operations. At the end of the class definition add the following:

```
methods(Static)

  %------------------------- hat -------------------------
  %
  %  Perform the hat operation with a vector form of se(2).
  %  Output is the homogeneous matrix form of se(2).
  %
  %
  function xiHat = hat(xiVec)

  CODE HERE;

  end

  %------------------------ unhat ------------------------
  %
  %  Perform the unhat operation with a matrix form of se(2).
  %  Output is the vector form of se(2).
  %
  %
  function xiHat = hat(xiVec)

  CODE HERE;

  end
end
```

Use your new code to then show that $(\mathrm{Ad}_g \xi)^\wedge = \mathrm{Ad}_g \hat{\xi}$ and vice-versa $\left(\mathrm{Ad}_g \hat{\xi}\right)^\vee = \mathrm{Ad}_g \hat{\xi}$. You can make up the g and ξ elements. The important thing is to have the right code and to also show that the outputs agree, in vector form for the first version and homogeneous form for the second version.

The `hat` and `unhat` functions are accessed as follows:

```
xiHat = SE2.hat(xiVec);
xiVec = SE2.unhat(xiHat);
```

Static functions don't need an actual group element, but they do need to be referred to by the class they belong to. It's kind of neat. Pretty soon we'll redo this coding practice for $SE(3)$.

Solution 5.

(a) The modified left action function is:

```
function x2 = leftact(g, x, type)

R = g.M(1:2,1:2);
d = g.M(1:2,3);

if ( (size(x,1) == 2) && (size(x,2) == 1) )
  x2 = d + R*x;
```

```
elseif ( (size(x,1) == 3) && (size(x,2) == 1) )
  x2 = [R*x(1:2); x(3)];
end
```

and the associated code is:

```
gOA = SE2([5;2], pi/3);
gOB = SE2([2;7], pi/2);
gBC = SE2([0;3],-pi/6);

gBA = inv(gOB)*gOA;
gOC = gOB * gBC;

xi1A = [2;5;-pi/12];
xi1B = gBA .* xi1A
xi1O = gOA .* xi1A

xi2C = [1;-4; pi/15];
xi2B = gBC .* xi2C
xi2O = gOC .* xi2C
```

The output of the code is:

```
xi1B =

    4.2321
    3.3301
   -0.2618

xi1O =

   -3.3301
    4.2321
   -0.2618

xi2B =

   -1.1340
   -3.9641
    0.2094

xi2O =

    3.9641
   -1.1340
    0.2094
```

(b) The modified function is:

```
function z = adjoint(g, x)


if (isa(x,'SE2'))
  z = g*x*inv(g);
```

```
elseif ( (size(x,1) == 3) && (size(x,2) == 1) )
  R = g.M(1:2,1:2);
  d = g.M(1:2,3);
  JJ = [0 1;-1 0];
   z = [R , JJ*d;0 0 1]*x;
elseif ( (size(x,1) == 3) && (size(x,2) == 3) )
   z = g.M*x*inv(g.M);
end

end
```

and the associated code is:

```
gBC = SE2([0;3],-pi/6);
xiBB = [2;-3; pi/9];
xiCC = adjoint(inv(gBC),xiBB)
```

The output of the code is:

```
xiCC =

    2.3252
   -2.1217
    0.3491
```

which agree with result from before.

Problem 6. [10 pts] It's time to start moving into the 3D world. Get the Lie group product, the left action, and the inverse functions written. The Lie group element should be stored as a homogeneous matrix. Also fill out the functions that get the translation and rotation components individually, plus the helper functions for whatever has been covered already. Code stubs are on t-square.

Verify that it works by reproducing the following:

```
>> g1 = SE3([1;2;3],[1 0 0;0 0 -1; 0 1 0]);
>> g2 = SE3([2;-1;-1], [sqrt(2)/2 0 -sqrt(2)/2;0 1 0;sqrt(2)/2 0 sqrt(2)/2]);
>> p1 = [4;5;6];
>> p2 = [4;5;6;1];
>> g3 = g1*g2

g3 =

    0.7071         0   -0.7071    3.0000
   -0.7071         0   -0.7071    3.0000
         0    1.0000         0    2.0000
         0         0         0    1.0000

>> g3.*p1

ans =

    1.5858
   -4.0711
    7.0000

>> g3.*p1
```

```
ans =

    1.5858
   -4.0711
    7.0000

>> g3.*p2

ans =

    1.5858
   -4.0711
    7.0000
    1.0000

>> inv(g3)

ans =

    0.7071   -0.7071         0         0
         0         0    1.0000   -2.0000
   -0.7071   -0.7071         0    4.2426
         0         0         0    1.0000

>>
```

Solution 6. OK, here goes the code.

```
function g = SE3(d, R)

if (nargin == 0)
  d = zeros([3 1]);
  R = eye(3);
elseif ( (size(d,1) == 1) && (size(d,2) == 3) ) % if row vector.
  d = d'; %   make column.
elseif ( (size(d,1) ~= 3) || (size(d,2) ~= 1) )
  error('The homogeneous group element incorrect dimensions');
end

g.M = [R, d;0 0 0 1];

end

function g = mtimes(g1, g2)

g = SE3();
g.M = g1.M*g2.M;

end

function x2 = leftact(g, x, type)

if ( (size(x,1) == 3) && (size(x,2) == 1) )     % If three vector.
```

```
  x2 = g.M(1:3,:)*[x;ones(1,size(x,2))]; %  treat like a point.
elseif ( (size(x,1) == 4) && (size(x,2) == 1) ) % else it is homogeneous.
  x2 = g.M*x;                 %  do the right thing.
end

end

function invg = inverse(g)

invg = SE3();
invg.M = inv(g.M);

end

function d = getTranslation(g)
d = g.M(1:3,4);
end

function R = getRotation(g)
R = g.M(1:3,1:3);
end
```

Problem 7. (30 pts) This week, the objective is to work out the forward kinematics for the Piktul manipulators. Work out Module Set 0, part 2 from the class wiki.

Solution 7. This problem is similar to an earlier problem on forward kinematics, so the full solution won't be covered. What matters is that the link lengths are 4.5 and 4 inches for the first link and the second link, respectively. The shoulder joint is designed to be above the origin that is etched into the cardboard. Also, the first joint raises and lowers the stage, so it is the only one responsible for the height (other than the totip variable).

In millimeters, the link lengths are 114.3 and 101.6 mm, for the first and second links, respectively. I write it just in case you kept the millimeter to inches conversion. The difference in height between the tip of the grippers and the center of the gripper is about 0.5 inches (or 12.7 mm). Thus, when the totip option is set to false, then the returned height should be 0.5 inches higher.

Hopefully during the demo the calibration was rigorously checked by whoever was doing the check-off. Otherwise, there may be problems downstream. I think that's it.

Problem 7. [30 pts] For the turtlebot track, work out the first two parts of Module Set 1, "Basic ROS Commands" and also "Moving the turtlebot."

Solution 7. There is no solution here. For the most part, those who I interacted with got feedback from me directly. Hopefully those who interacted with the TA also got feedback. I will operate under the assumption that the demo time is being used to correct things and arrive at a reasonable submission, as well as to answer general questions regarding what it being demoed. You do have to also answer the "Adventure and Explore" questions in the wiki, so hopefully you did that.

Problem 7. (30 pts) For the biped track, work out Module Set 1, Part 1. It explores the kinematics for different frames, and plotting of the kinematic chain relative to them.

Solution 7. There is no solution here. You've got your group contact who will provide the feedback necessary to correct and advance. For sure, I would recommend getting in the habit of demoing a little early to course correct should there be minor details that were missed. The better state things are in at the official/final demo, the easier it will be to proceed.

Problem 7. (30 pts)**.** If you are working on a group project, then your group contact will review the submissions associated to the prior tasks and provide a new set of tasks. You should work out whatever baby steps this group contact assigns the group as opposed to the lab problem. It is your responsibility to turn in the results to the group contact. There should be sufficient output for them to assess progress.

Solution 7. For those in the group project, this step involved accomplishing what was requested, then turning in some kind of document or video as demonstration that the learning tasks were accomplished. The expectation is that the group response should be similar in details as the solutions to the homeworks are, which means that not only should they communicate accomplishments, but they should also convey the overall procedure involved.