```
% ECE 4560 - Homework 13.1
% Caitlyn Caggia

clear all; close all;

syms a1 a2 a3 l1 l2 l3 t
T = [l1*cos(a1) + l2*cos(a1+a2) + l3*cos(a1+a2+a3);
    l1*sin(a1) + l2*sin(a1+a2) + l3*sin(a1+a2+a3)];
Jac = jacobian(T, [a1 a2 a3]);
x = 1 - 0.5*cos(t);
xdot = 0.5*sin(t);
y = -0.75 + 0.49*sin(t);
ydot = 0.49*cos(t);

%part a
 ------------------------------------------------------------------
ti = 0; tf = 2*pi; time = linspace(ti, tf, 50000);
l1 = 1; l2 = 0.5; l3 = 0.25;

ai = [-0.2987; -2.4189; 1.1468];
[alphaA, posA] = resolvedRate(ai, time, 2, 0);

%plot alphas vs time
figure
hold on
plot(time, alphaA)
plot(time, zeros(1, length(time))); %to find singularity graphically
title('Part A Joint Angles')
legend('alpha1', 'alpha2', 'alpha3', 'zero')


%plot position vs time
figure
plot(time, posA)
title('Part A End Effector Position')
legend('x','y')

%parametric plot
tend = length(time);
figure
hold on
planarR3_display([alphaA(1,1); alphaA(2,1); alphaA(3,1)], [l1; l2;
 l3])
planarR3_display([alphaA(1,0.2*tend); alphaA(2,0.2*tend);
 alphaA(3,0.2*tend)], [l1; l2; l3])
planarR3_display([alphaA(1,0.4*tend); alphaA(2,0.4*tend);
 alphaA(3,0.4*tend)], [l1; l2; l3])
planarR3_display([alphaA(1,0.6*tend); alphaA(2,0.6*tend);
 alphaA(3,0.6*tend)], [l1; l2; l3])
planarR3_display([alphaA(1,0.8*tend); alphaA(2,0.8*tend);
 alphaA(3,0.8*tend)], [l1; l2; l3])
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```
planarR3_display([alphaA(1,tend); alphaA(2,tend); alphaA(3,tend)],
 [l1; l2; l3])
plot(posA(1,:), posA(2,:)) %compare to desired path
title('Part A Parametric Plot')

%part b
 ---------------------------------------------------------------------
[alphaB1, posB1] = resolvedRate(ai, time, 'd', 0.05);
%plot alphas vs time
figure
plot(time, alphaB1)
title('Part B Joint Angles, p^2 = 0.05')
legend('alpha1', 'alpha2', 'alpha3')
%plot position vs time
figure
plot(time, posB1)
title('Part B End Effector Position, p^2 = 0.05')
legend('x','y')
%parametric plot
figure
hold on
planarR3_display([alphaB1(1,1); alphaB1(2,1); alphaB1(3,1)], [l1; l2;
 l3])
planarR3_display([alphaB1(1,0.2*tend); alphaB1(2,0.2*tend);
 alphaB1(3,0.2*tend)], [l1; l2; l3])
planarR3_display([alphaB1(1,0.4*tend); alphaB1(2,0.4*tend);
 alphaB1(3,0.4*tend)], [l1; l2; l3])
planarR3_display([alphaB1(1,0.6*tend); alphaB1(2,0.6*tend);
 alphaB1(3,0.6*tend)], [l1; l2; l3])
planarR3_display([alphaB1(1,0.8*tend); alphaB1(2,0.8*tend);
 alphaB1(3,0.8*tend)], [l1; l2; l3])
planarR3_display([alphaB1(1,tend); alphaB1(2,tend); alphaB1(3,tend)],
 [l1; l2; l3])
plot(posB1(1,:), posB1(2,:)) %actual path
plot(posA(1,:), posA(2,:)) %compare to desired path
title('Part B Parametric Plot, p^2 = 0.05')

[alphaB2, posB2] = resolvedRate(ai, time, 'd', 0.005);
%plot alphas vs time
figure
plot(time, alphaB2)
title('Part B Joint Angles, p^2 = 0.005')
legend('alpha1', 'alpha2', 'alpha3')
%plot position vs time
figure
plot(time, posB2)
title('Part B End Effector Position, p^2 = 0.005')
legend('x','y')
%parametric plot
figure
hold on
planarR3_display([alphaB2(1,1); alphaB2(2,1); alphaB2(3,1)], [l1; l2;
 l3])
```

```
planarR3_display([alphaB2(1,0.2*tend); alphaB2(2,0.2*tend);
 alphaB2(3,0.2*tend)], [l1; l2; l3])
planarR3_display([alphaB2(1,0.4*tend); alphaB2(2,0.4*tend);
 alphaB2(3,0.4*tend)], [l1; l2; l3])
planarR3_display([alphaB2(1,0.6*tend); alphaB2(2,0.6*tend);
 alphaB2(3,0.6*tend)], [l1; l2; l3])
planarR3_display([alphaB2(1,0.8*tend); alphaB2(2,0.8*tend);
 alphaB2(3,0.8*tend)], [l1; l2; l3])
planarR3_display([alphaB2(1,tend); alphaB2(2,tend); alphaB2(3,tend)],
 [l1; l2; l3])
plot(posB2(1,:), posB2(2,:)) %actual path
plot(posA(1,:), posA(2,:)) %compare to desired path
title('Part B Parametric Plot, p^2 = 0.005')

%part c
 ------------------------------------------------------------------
finalposA = posA(:, end)
finalposB1 = posB1(:, end)
finalposB2 = posB2(:, end)

%extra credit
 ------------------------------------------------------------
disp('Singularity occurs when a2 and a3 are zero, as rank is lost.')
disp('Graphically, we can examine joint angles in Figure 1.')
disp('This shows we are closest to singularity just before 4
 seconds.')
disp('More specifically, we get closest to singularity when a2 is at a
 maximum.')
disp('Thus, singularity occurs at...')
[value, index] = max(alphaA(2,:));
singularity_time = index*(time(2) - time(1))


% function to integrate over resolved rate
 -------------------------------
function [a, pos] = resolvedRate(ai, time, invtype, p)

a1 = ai(1); a2 = ai(2); a3 = ai(3);
a = zeros(3,length(time));
a(1,1) = a1; a(2,1) = a2; a(3,1) = a3;

pos = zeros(2,length(time));
pos(:,1) = forwardkin(a(:,1));

deltat = time(2) - time(1);

for i = 2:length(time)

    %     x = 1 - 0.5*cos(t); xdot = 0.5*sin(t);
    %     y = -0.75 + 0.49*sin(t); ydot = 0.49*cos(t);

    t = time(i);

    %store old values
```

```
    aold = [a1; a2; a3];
    a(:,i) = aold;
    pos(:,i) = forwardkin(aold);

    %calculate velocity
    xi = [0.5*sin(time(i)); 0.49*cos(time(i))];

    %calculate new alphas
    Jp = pseudoInv(aold, invtype, p);
    alphanew = aold + deltat*Jp*xi;
    a1 = alphanew(1); a2 = alphanew(2); a3 = alphanew(3);

end

end

% function to compute pseudoinverse
 -------------------------------------
function Jp = pseudoInv(alphas, type, p2)

l1 = 1; l2 = 0.5; l3 = 0.25;
a1 = alphas(1); a2 = alphas(2); a3 = alphas(3);

%calculate Jacobian
J11 = -l2*sin(a1+a2) - l1*sin(a1) - l3*sin(a1+a2+a3);
J12 = -l2*sin(a1+a2) - l3*sin(a1+a2+a3);
J13 = -l3*sin(a1+a2+a3);
J21 = l2*cos(a1+a2) + l1*cos(a1) + l3*cos(a1+a2+a3);
J22 = l2*cos(a1+a2) + l3*cos(a1+a2+a3);
J23 = l3*cos(a1+a2+a3);
Jac = [J11 J12 J13; J21 J22 J23];

% m = dimension of task space
% n = number of independent variables (joints)

%find pseudoinverse
if(type == 1)
    %true inverse m = n, kinematially sufficient
    Jp = inv(Jac);
elseif (type == 2)
    %pseudoinverse m < n, kinematically redundant
    Jp = Jac' * inv(Jac*Jac');
elseif (type == 3)
    %pseudoinverse m > n, kinematically insufficient
    Jp = inv(Jac' * Jac) * Jac';
elseif(type == 'd')
    %damped pseudo inverse
    Jp = Jac' * inv(Jac*Jac' + p2*eye(2));
end

end

%function to compute forward kinematics
 ---------------------------------
```

```
function T = forwardkin(a)

l1 = 1; l2 = 0.5; l3 = 0.25;
g1 = SE2([0,0],a(1));
g2 = SE2([l1,0],a(2));
g3 = SE2([l2,0],a(3));
g4 = SE2([l3,0],0);
ge = g1 * g2 * g3 * g4;
T = getTranslation(ge);

end
```

```
finalposA =

    0.4999
   -0.7499


finalposB1 =

    0.3700
   -0.8084

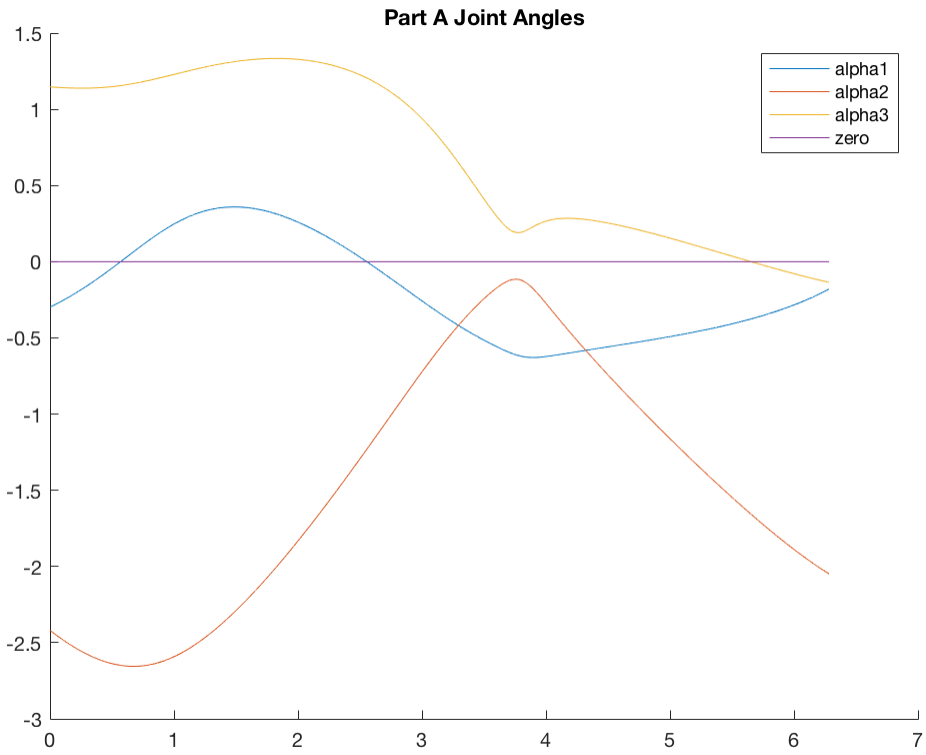

finalposB2 =

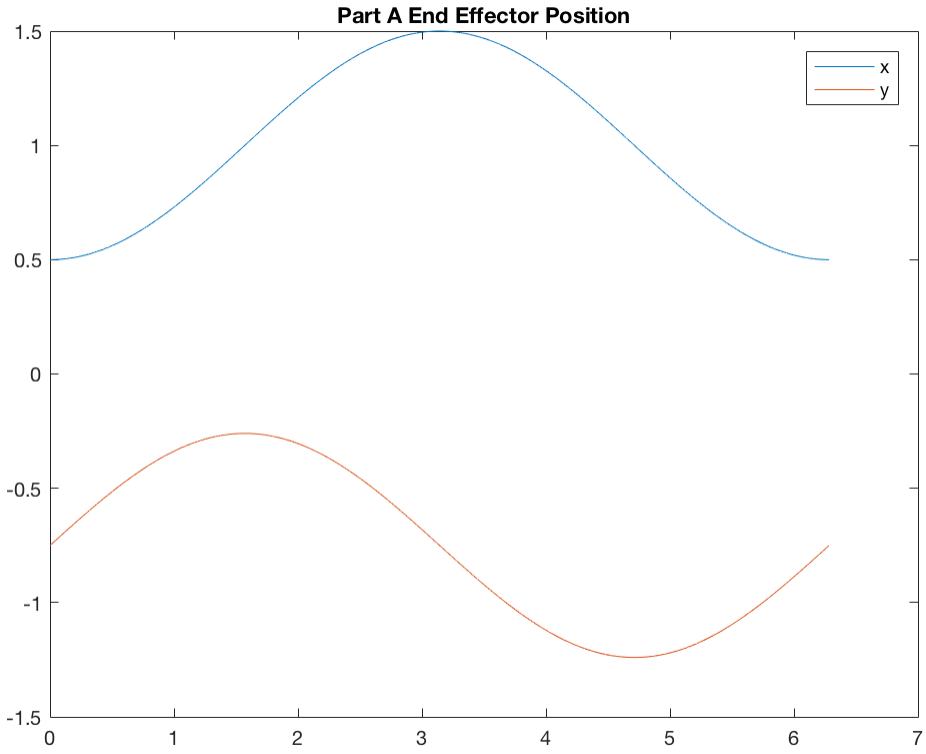
    0.4759
   -0.7573

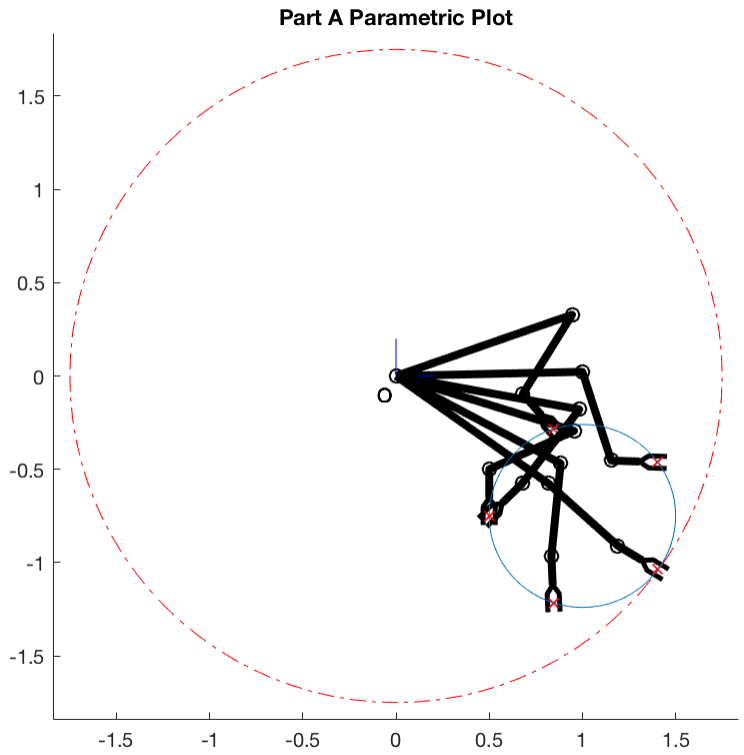
Singularity occurs when a2 and a3 are zero, as rank is lost.
Graphically, we can examine joint angles in Figure 1.
This shows we are closest to singularity just before 4 seconds.
More specifically, we get closest to singularity when a2 is at a
 maximum.
Thus, singularity occurs at...

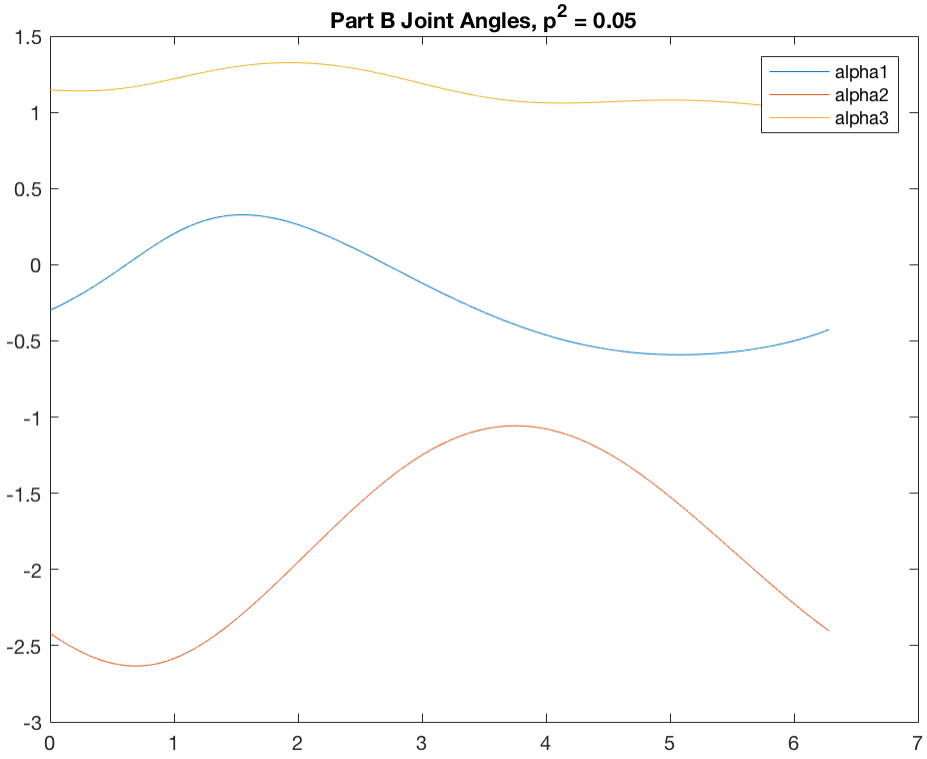
singularity_time =

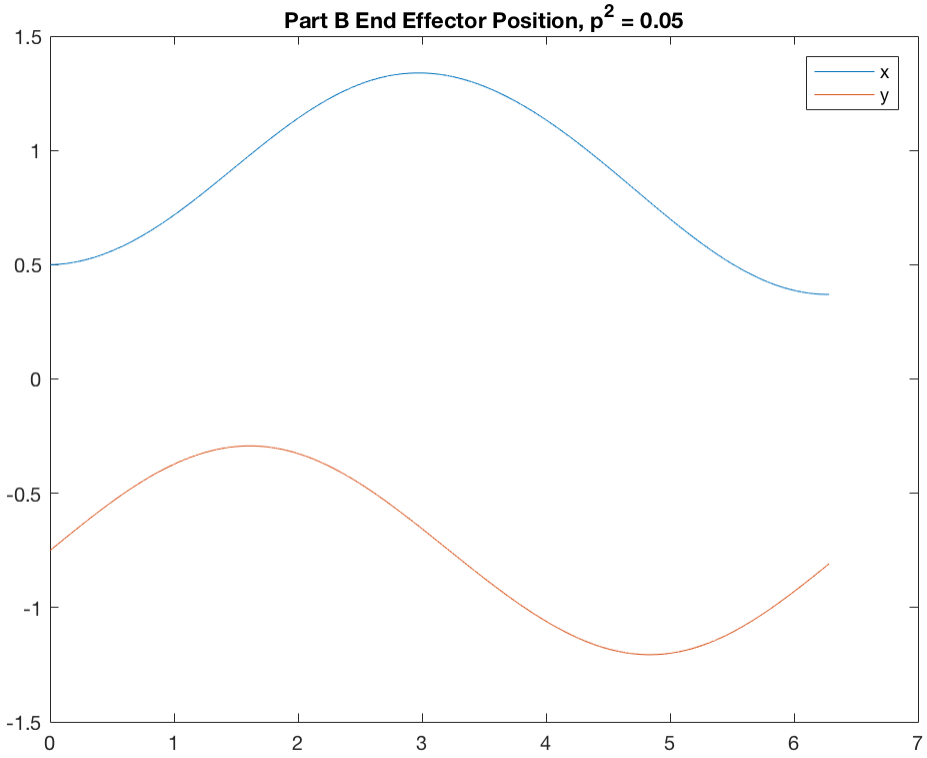
    3.7569
```

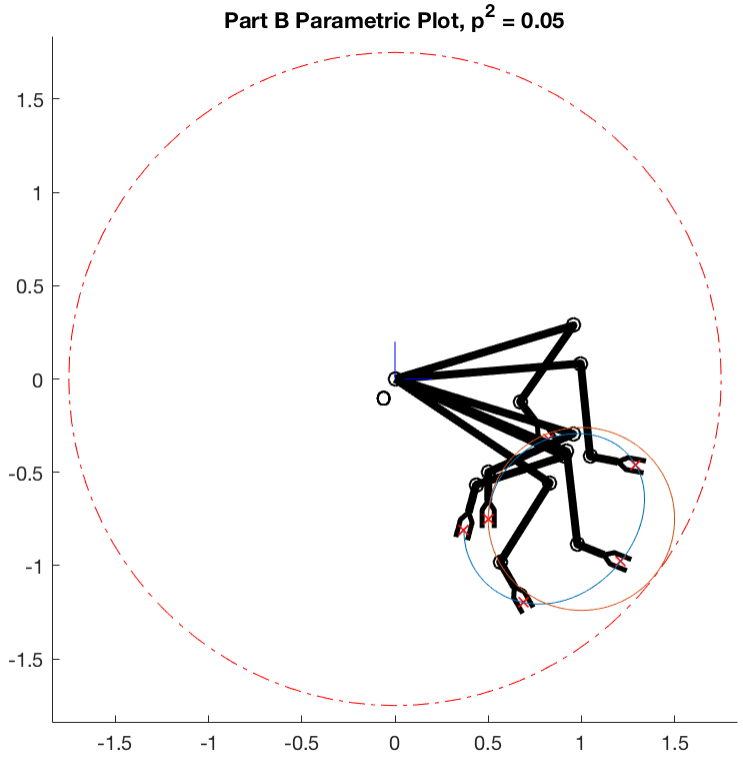

Part A Joint Angles
alpha1
alpha2
alpha3
zero
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
1
2
3
4
5
6
7

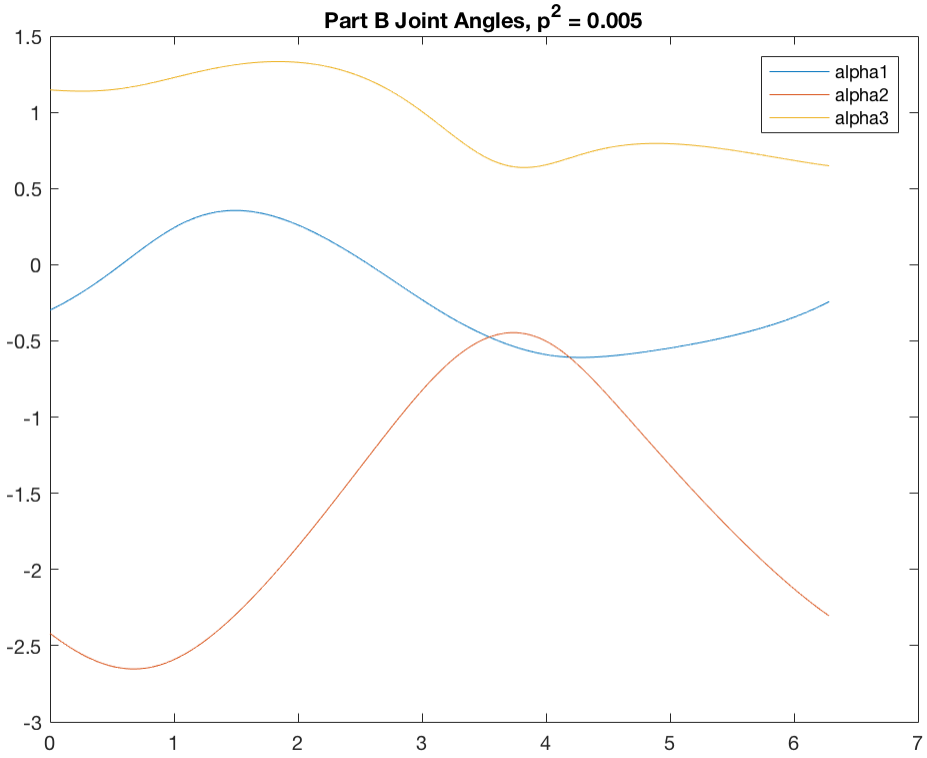

Part A End Effector Position
x
y
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7

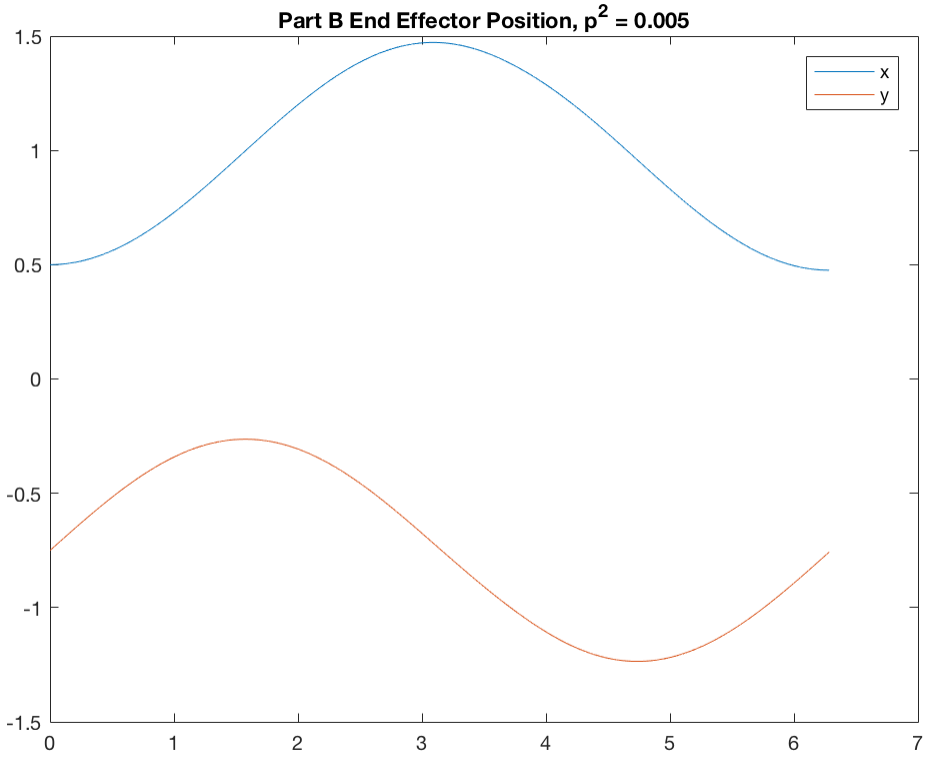

Part A Parametric Plot
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

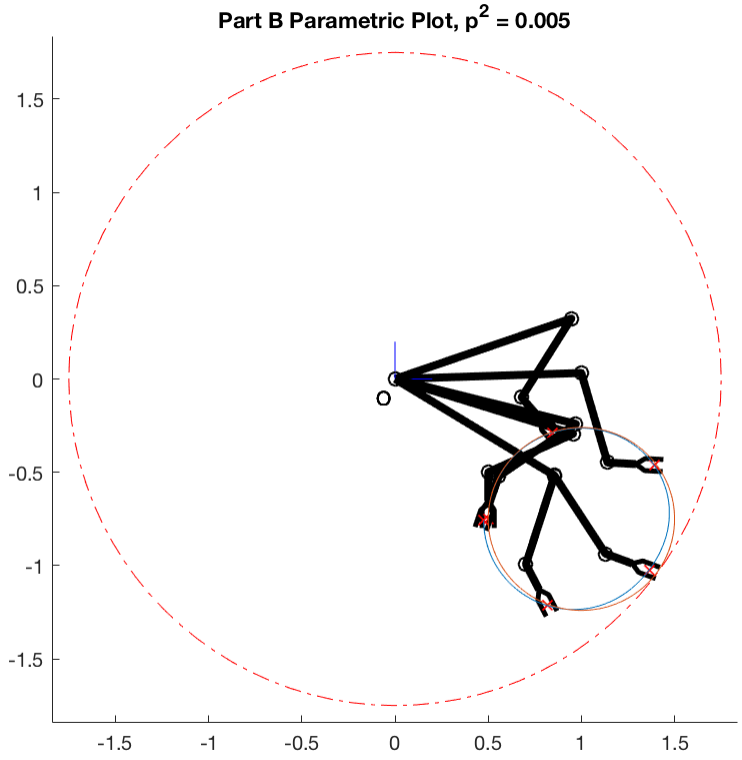

Part B Joint Angles, p2 = 0.05
alpha1
alpha2
alpha3
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
1
2
3
4
5
6
7


Part B End Effector Position, p2 = 0.05
x
y
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7


Part B Parametric Plot, p2 = 0.05
O
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5


Part B Joint Angles, p^2 = 0.005
alpha1
alpha2
alpha3
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
1
2
3
4
5
6
7


Part B End Effector Position, p^2 = 0.005
x
y
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7



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