Algorithms: COMP3121/3821/9101/9801

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LECTURE 5: THE GREEDY METHOD

Activity selection problem

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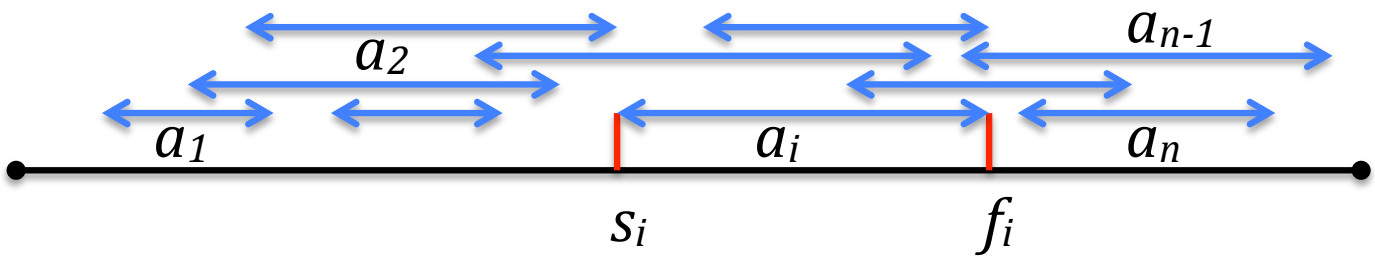

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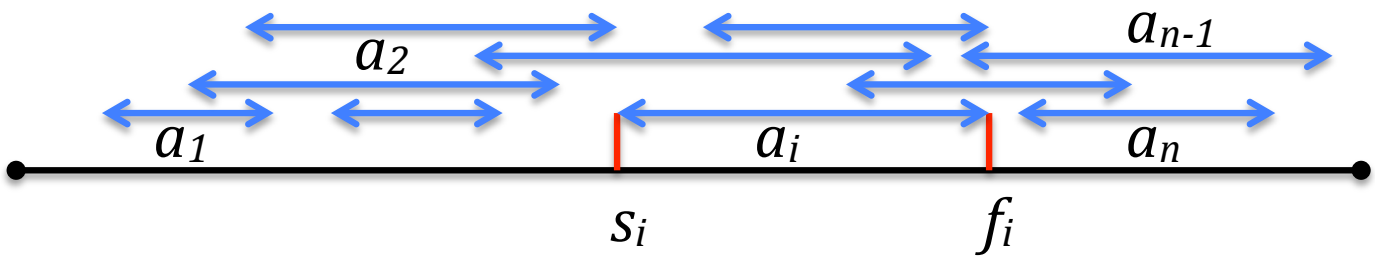

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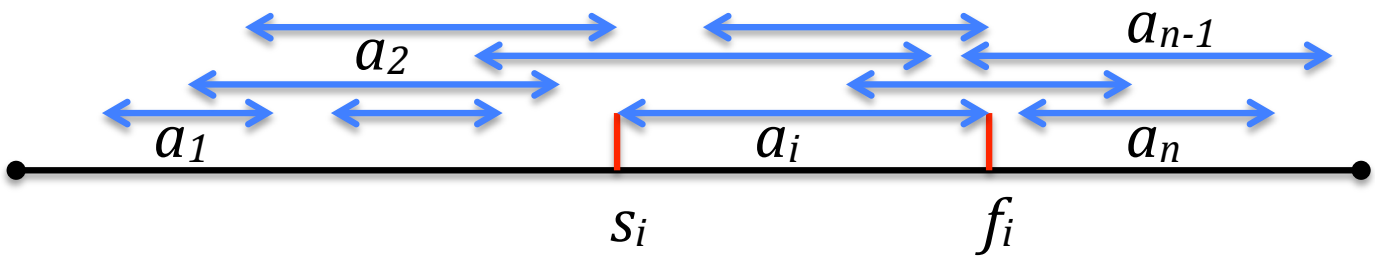

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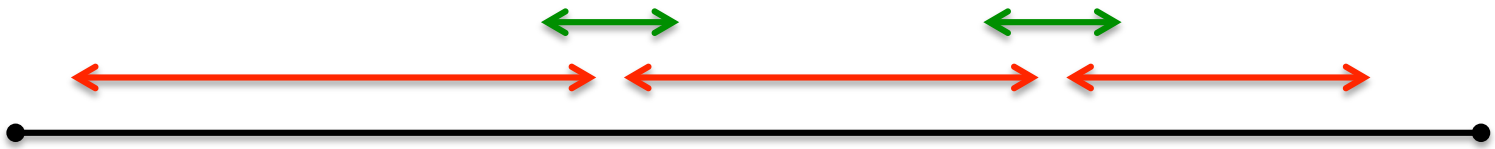

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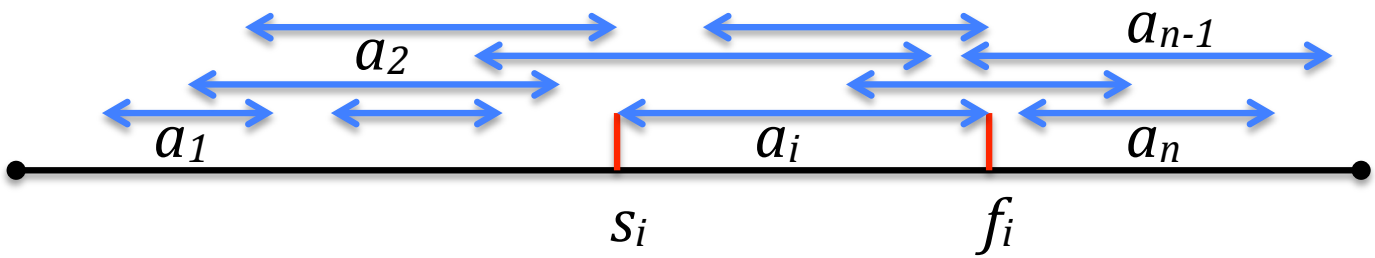

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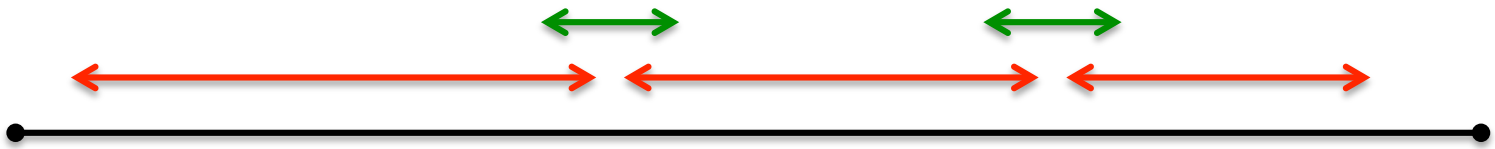

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(chosen activities in green, conflicting in red)

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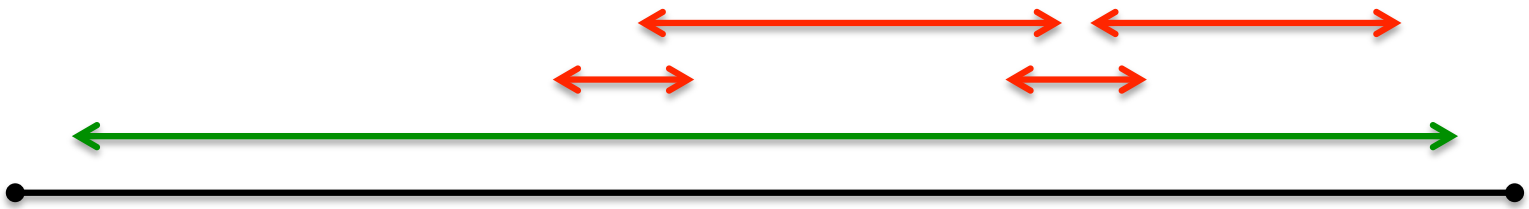

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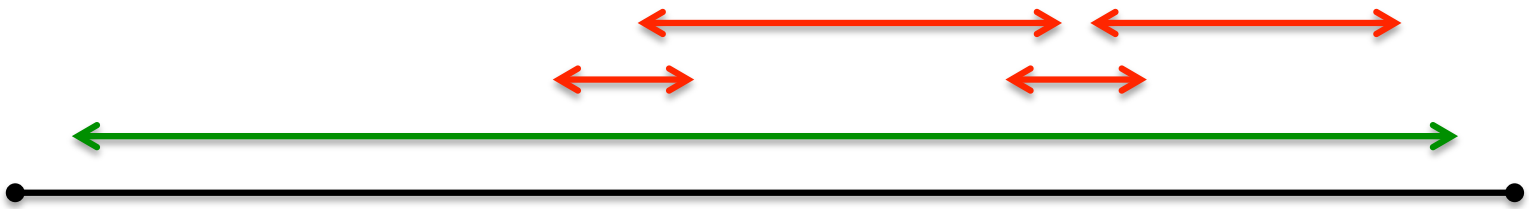

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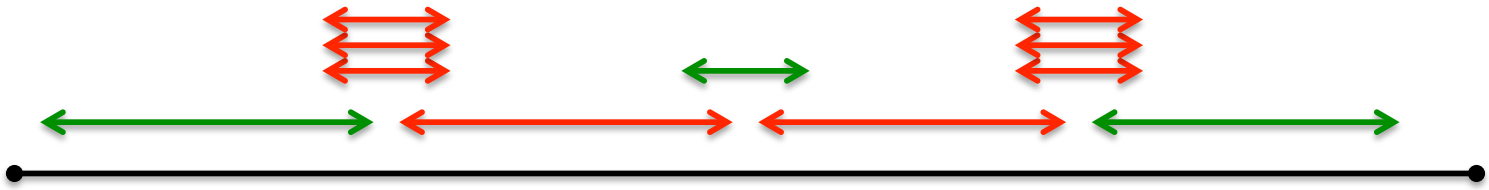

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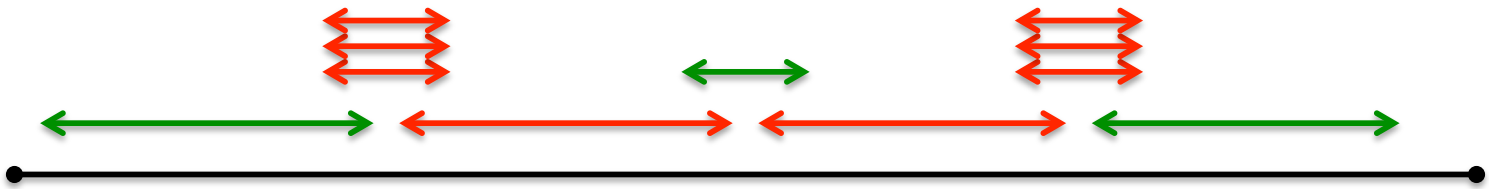

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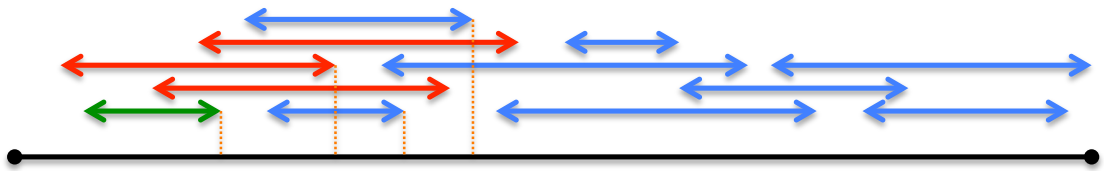

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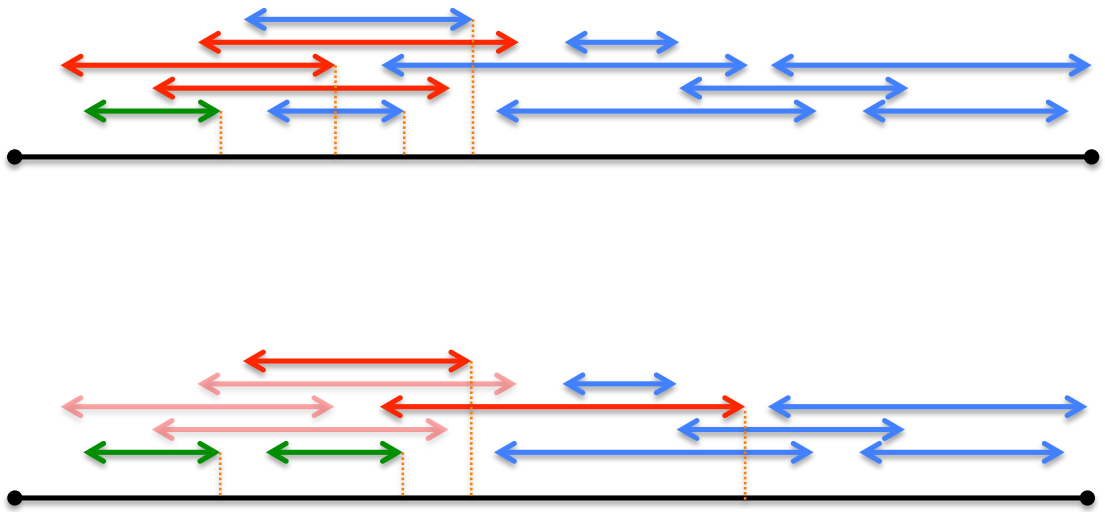

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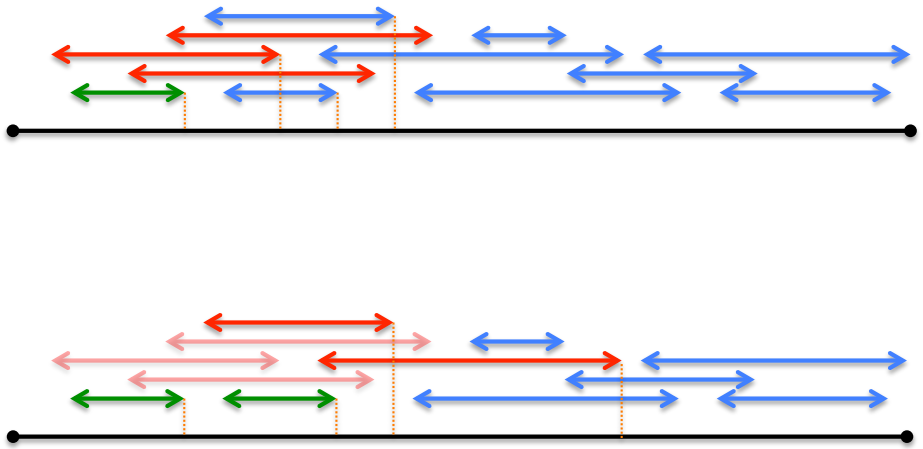

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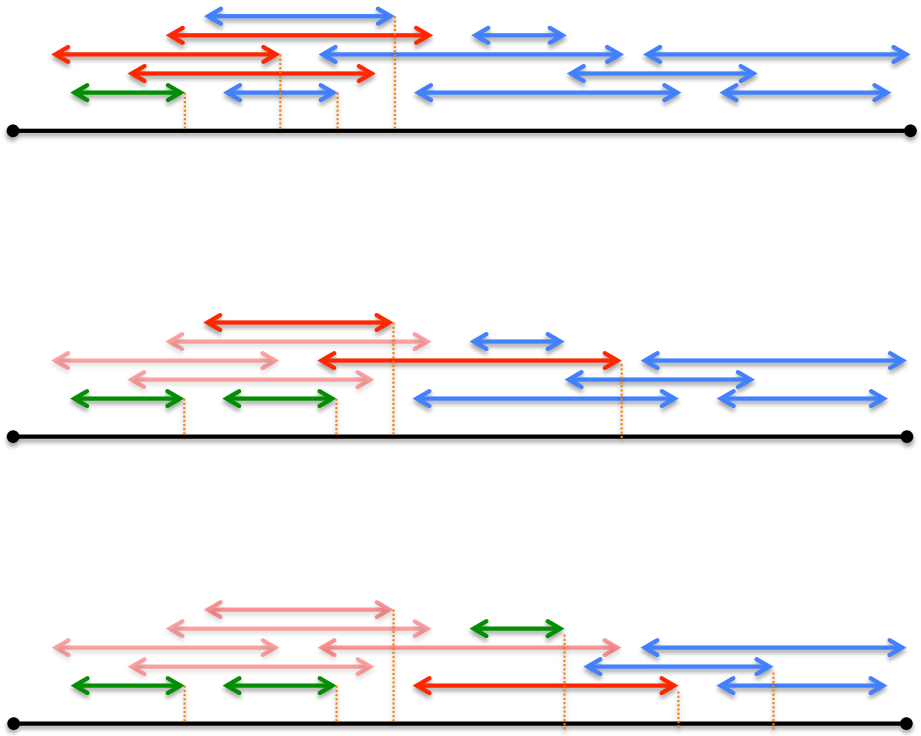

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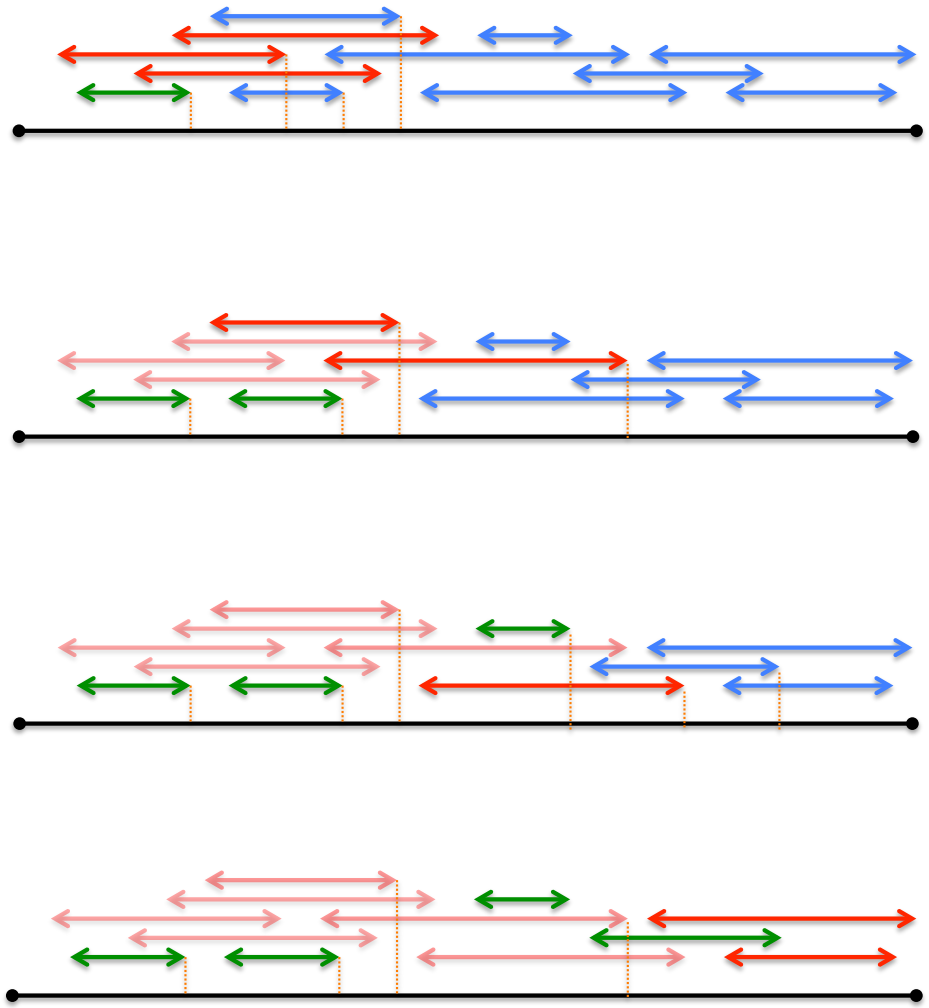
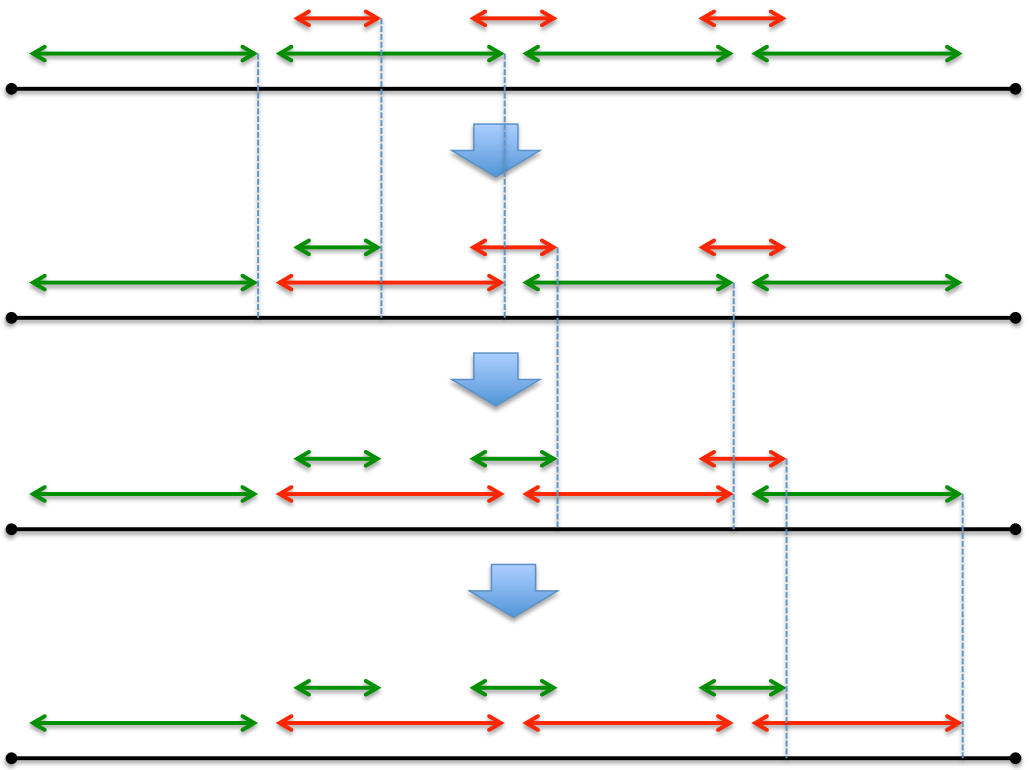


The Greedy Method

- Transforming any optimal solution to the greedy solution with equal number of activities: find the first place where the chosen activity violates the greedy choice and show that replacing that activity with the greedy choice produces a non conflicting selection with the same number of activities. Continue in this manner till you "morph" your optimal solution into the greedy solution, thus proving the greedy solution is also optimal.



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- **Question:** What happens if the activities are not all of the same duration?
- Greedy strategy no longer works - we will need a more sophisticated technique.

Minimising job lateness

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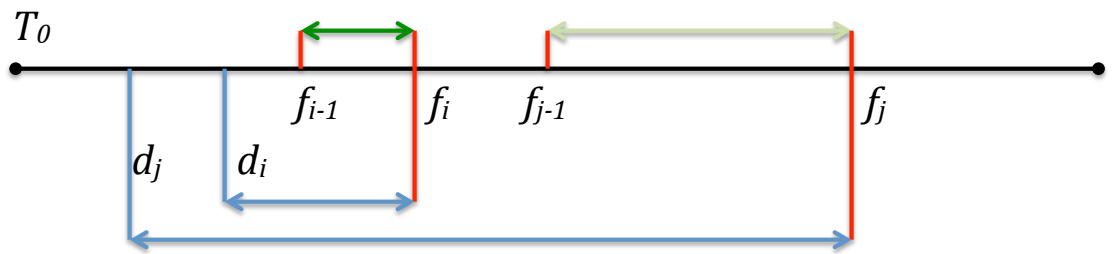
- **Solution:** Ignore job durations and schedule jobs in the increasing order of deadlines.

- **Optimality:** Consider any optimal solution. We say that jobs a_i and jobs a_j form an inversion if job a_i is scheduled before job a_j but $d_j < d_i$.



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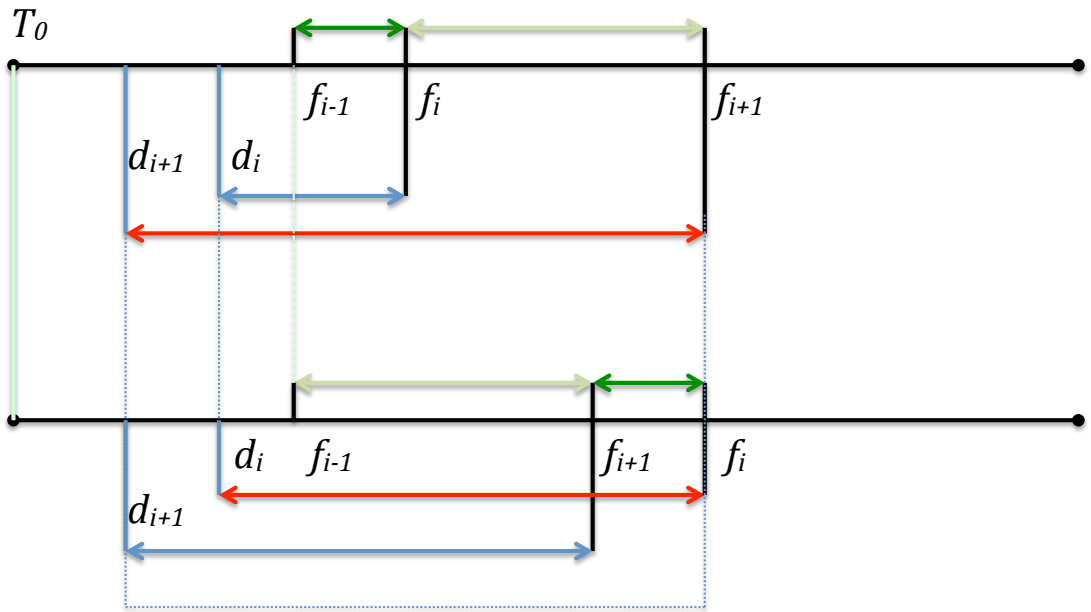

- Note that swapping adjacent inverted jobs reduces the larger lateness!

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- **Proof of optimality:** Let d be the distance associated with the first edge of the minimal spanning tree which was not added to our k trees; it is clearly the minimal distance between two vertices belonging to two of our k trees. Clearly, all the edges included in k many trees produced by our algorithm are of length smaller or equal to d.

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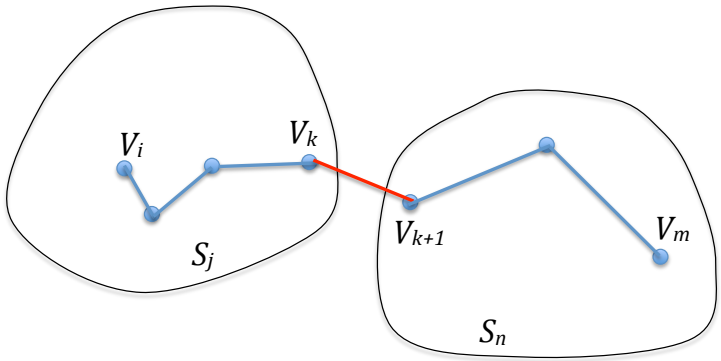
- Consider any partition $\mathcal{S}$ into k subsets different from the one produced by our algorithm. This means that there is a tree produced by our algorithm which contains vertices V_i and V_m from two different subsets from $\mathcal{S}$, say S_j and S_n.



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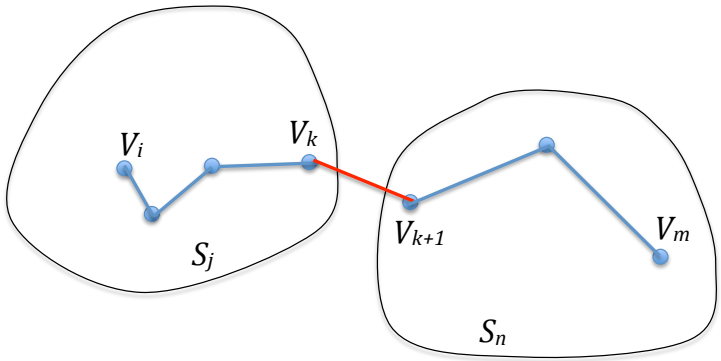

- Since V_i and V_m belong to the same tree, there is a path along the edges of that tree connecting V_i and V_m. Let V_k and V_{k+1} be two consecutive vertices on that path such that V_k belongs to S_j and V_{k+1} belongs to S_n. Note that $d(V_k, V_{k+1}) \leq d$ which implies that the minimal distance between clusters of $\mathcal{S}$ is smaller or equal to the minimal distance d between the k trees produced by our algorithm. Thus, such a partition cannot be a more optimal clustering than the one produced by our algorithm

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- This is minimised if $l_1 \leq l_2 \leq l_3 \leq \ldots \leq l_n$.

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- We now show that this is minimised if the files are ordered in a decreasing order of values of the ratio p_i/l_i.

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- Consequently, $E > E'$ if and only if $p_k/l_k < p_{k+1}/l_{k+1}$, which means that the swap decreases the expected time just in case $p_k/l_k < p_{k+1}/l_{k+1}$, i.e., if there is an inversion: a file f_{i+1} with a larger ratio p_{k+1}/l_{k+1} has been put after a file f_i with a smaller ratio p_k/l_k.

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- For as long as there are inversions there will be inversions of consecutive files and swapping will reduce the expected time. Consequently, the optimal solution is the one with no inversions.

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- Unfortunately there is no easy rule...