Fundamental Laws of Continuum Mechanics

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Conservation of mass

$$\frac{D\rho}{Dt} + \rho\frac{\partial u_i}{\partial x_i} = 0,$$

Balance of momentum

$$\rho\frac{Du_i}{Dt} = \rho g_i + \frac{\partial \tau_{ji}}{\partial x_j},$$

Balance of angular momentum

$$\tau_{jk} = \tau_{kj},$$

Balance of energy

$$\rho\frac{D}{Dt}\left[\frac{u_i u_i}{2} + e\right] = \rho g_i u_i + \frac{\partial(\tau_{ji}u_i)}{\partial x_j} - \frac{\partial q_i}{\partial x_i},$$

$$\rho\frac{De}{Dt} = \tau_{ji}\frac{\partial u_i}{\partial x_j} - \frac{\partial q_i}{\partial x_i},$$

Balance of entropy

$$\rho\frac{Ds}{Dt} = \frac{\Phi}{T} - \frac{q_i}{T^2}\frac{\partial T}{\partial x_i} - \frac{\partial}{\partial x_i}\left[\frac{q_i}{T}\right]$$