Supplementary materials to
Codimension-two bifurcation analysis in a three-dimensional discrete prey-predator model with Holling type II functional response

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1 The proof of Theorem 1

Proof The Jacobian matrix at $A_0(0, 0, 0)$ is

$$J(A_0) = \begin{pmatrix} e^{r_1} & -a_{12}e^{r_1} & 0 \\ 0 & e^{-d_2} & 0 \\ 0 & 0 & e^{-d_3} \end{pmatrix}.$$

This matrix has three eigenvalues $\lambda_1 = e^{r_1} > 1$, $\lambda_2 = e^{-d_2} < 1$ and $\lambda_3 = e^{-d_3} < 1$. Thus, $A_0(0, 0)$ is unstable. □

2 The proof of Theorem 2

Proof The Jacobian matrix at $A_1(u_1, 0, 0)$ is

$$J(A_1) = \begin{pmatrix} 1 - r_1 & -a_{12} & -\frac{r_1 a_{13}}{a_{11} + \alpha r_1} \\ 0 & e^{\left(\frac{r_1 a_{21}}{a_{11}} - d_2\right)} & 0 \\ 0 & 0 & e^{\left(\frac{r_1 a_{31}}{a_{11}} - d_3\right)} \end{pmatrix}.$$

This matrix has three eigenvalues $\lambda_1 = 1 - r_1$, $\lambda_2 = e^{\left(\frac{r_1 a_{21}}{a_{11}} - d_2\right)}$ and $\lambda_3 = e^{\left(\frac{r_1 a_{31}}{a_{11}} - d_3\right)}$. If $r_1 < 1$, $\frac{r_1 a_{21}}{a_{11}} < d_2$ and $\frac{r_1 a_{31}}{a_{11}} < d_3$, then we have $\lambda_i < 1 (i = 1, 2, 3)$. Thus, A_1 is locally asymptotically stable. If one of the following inequalities holds: (i)$r_1 > 1$, (ii)$\frac{r_1 a_{21}}{a_{11}} > d_2$, (iii)$\frac{r_1 a_{31}}{a_{11}} > d_3$, then we have that at least one of $\lambda_i (i = 1, 2, 3)$ is greater than 1. Thus, A_1 is unstable. □

3 The proof of Theorem 3

Proof The Jacobian matrix at $A_2(u_2, v_2, 0)$ is

$$J(A_2) = \begin{pmatrix} 1 - a_{11}u_2 & -a_{12} & -\frac{a_{13}u_2}{1+\alpha u_2} \\ a_{21}v_2 & 1 - a_{22}v_2 & -a_{23}v_2 \\ 0 & 0 & e^{\left(\frac{a_{31}u_2}{1+\alpha u_2}+a_{32}v_2-d_3\right)} \end{pmatrix}.$$

The characteristic equation of $J(A_2)$ is

$$F(\lambda) = \left[\lambda - e^{\left(\frac{a_{31}u_2}{1+\alpha u_2}+a_{32}v_2-d_3\right)}\right](\lambda^2 + b_1\lambda + b_2) = 0,$$

where

$$b_1 = a_{11}u_2 + a_{22}v_2 - 2, \quad b_2 = (1 - a_{11}u_2)(1 - a_{22}v_2) + a_{12}a_{21}v_2.$$

According to the Jury-criterion [1], the local stability conditions of the fixed point A_2 are

$$(i)\ \frac{a_{31}u_2}{1 + \alpha u_2} + a_{32}v_2 < d_3, \quad (ii)\ |b_1| < b_2 + 1, \quad (iii)\ b_2 < 1.$$

$\square$

4 The proof of Theorem 4

Proof The Jacobian matrix at $A_3(u_3, 0, w_3)$ is

$$J(A_3) = \begin{pmatrix} 1 - a_{11}u_3 + \frac{a_{13}\alpha u_3 w_3}{(1+\alpha u_3)^2} & -a_{12} & -\frac{a_{13}u_3}{1+\alpha u_3} \\ 0 & e^{(a_{21}u_3-a_{23}w_3-d_2)} & 0 \\ \frac{a_{31}w_3}{(1+\alpha u_3)^2} & a_{32}w_3 & 1 - a_{33}w_3 \end{pmatrix}.$$

The characteristic equation of $J(A_3)$ is

$$F(\lambda) = \left[\lambda - e^{(a_{21}u_3-a_{23}w_3-d_2)}\right](\lambda^2 + b_3\lambda + b_4) = 0,$$

where

$$b_3 = a_{11}u_3 + a_{33}w_3 - \frac{a_{13}\alpha u_3 w_3}{(1 + \alpha u_3)^2},$$

$$b_4 = (1 - a_{33}w_3)\left(1 - a_{11}u_3 + \frac{a_{13}\alpha u_3 w_3}{(1 + \alpha u_3)^2}\right) + \frac{a_{13}a_{31}u_3 w_3}{(1 + \alpha u_3)^3}.$$

The local stability conditions of the fixed point A_3 are

$$(i)\ a_{21}u_3 < a_{23}w_3 + d_2, \quad (ii)\ |b_3| < b_4 + 1, \quad (iii)\ b_4 < 1.$$

$\square$

5 The proof of Theorem 5

Proof The Jacobian matrix at $A_*(u_*, v_*, w_*)$ is

$$J(A_*) = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}, \tag{1}$$

where

$$m_{11} = 1 - a_{11}u_* + \frac{a_{13}\alpha u_* w_*}{(1 + \alpha u_*)^2}, \quad m_{12} = -a_{12}u_*, \quad m_{13} = -\frac{a_{13}u_*}{1 + \alpha u_*},$$

$$m_{21} = a_{21}v_*, \quad m_{22} = 1 - a_{22}v_*, \quad m_{23} = -a_{23}v_*,$$

$$m_{31} = \frac{a_{31}v_*}{(1 + \alpha u_*)^2}, \quad m_{32} = a_{32}w_*, \quad m_{33} = 1 - a_{33}w_*.$$

The characteristic equation of $J(A_*)$ is

$$F(\lambda) = \lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0,$$

where

$$B_1 = -(m_{11} + m_{22} + m_{33}),$$

$$B_2 = m_{11}m_{22} + m_{11}m_{33} + m_{22}m_{33} - m_{12}m_{21} - m_{13}m_{31} - m_{23}m_{32},$$

$$B_3 = m_{11}m_{23}m_{32} + m_{12}m_{21}m_{33} + m_{13}m_{22}m_{31}$$
$$- m_{11}m_{22}m_{33} - m_{12}m_{23}m_{31} - m_{13}m_{21}m_{32}.$$

Based on the Jury-criterion [1], the local stability conditions of the fixed point A_* are

$$(i)\ |B_1 + B_3| < B_2 + 1, \quad (ii)\ |B_3| < 1, \quad (iii)\ |B_2 - B_1B_3| < |1 - B_3^2|.$$

$\square$

6 $f_{jkl}(\epsilon)$, $g_{jkl}(\epsilon)$ and $h_{jkl}(\epsilon)(0 \le j + k + l \le 5)$ in (3.2) and (3.13)

$$\Psi_1 = \exp(-\hat{a}_{12}v_*), \quad \Psi_2 = \exp(\hat{a}_{21}u_*), \quad G_1 = -a_{11} + \frac{a_{13}\alpha w_*}{(1 + \alpha u_*)^2},$$

$$f_{000}(\epsilon) = u_*(\Psi_1 - 1), \quad f_{100}(\epsilon) = (1 - u_*G_1)\Psi_1,$$

$$f_{010}(\epsilon) = -(\tilde{a}_{12} + \hat{a}_{12})u_*\Psi_1, \quad f_{001}(\epsilon) = -\frac{a_{13}u_*}{1 + \alpha u_*}\Psi_1,$$

$$f_{200}(\epsilon) = \left[G_1 + \frac{1}{2}u_*G_1 - \frac{a_{13}\alpha^2 u_* w_*}{(1 + \alpha u_*)^3}\right]\Psi_1, \quad f_{020}(\epsilon) = \frac{1}{2}(\tilde{a}_{12} + \hat{a}_{12})^2 u_*\Psi_1,$$

$$f_{002}(\epsilon) = \frac{a_{13}^2 u_*}{2(1 + \alpha u_*)^2}\Psi_1, \quad f_{110}(\epsilon) = -(\tilde{a}_{12} + \hat{a}_{12})(1 + u_*G_1)\Psi_1,$$

$$f_{101}(\epsilon) = \left[\frac{a_{13}\alpha u_*}{(1 + \alpha u_*)^2} - \frac{a_{13}(1 + u_*G_1)}{1 + \alpha u_*}\right]\Psi_1, \quad f_{011}(\epsilon) = \frac{a_{13}u_*(\tilde{a}_{12} + \hat{a}_{12})}{1 + \alpha u_*}\Psi_1,$$

$$f_{300}(\epsilon) = \left[\frac{a_{13}\alpha^3 u_* w_*}{(1 + \alpha u_*)^4} - \frac{a_{13}\alpha^2 w_*(1 + u_*G_1)}{(1 + \alpha u_*)^3} - \frac{1}{6}G_1^2(2 + u_*G_1)\right]\Psi_1,$$

$$f_{030}(\epsilon) = -\frac{1}{6}(\tilde{a}_{12} + \hat{a}_{12})^3 u_*\Psi_1, \quad f_{003}(\epsilon) = -\frac{a_{13}^3 u_*}{6(1 + \alpha u_*)^3}\Psi_1,$$

$$f_{210}(\epsilon) = \frac{1}{2}(\tilde{a}_{12} + \hat{a}_{12})(2 + u_*G_1)G_1\Psi_1, \quad f_{021}(\epsilon) = \frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})u_*}{2(1 + \alpha u_*)}\Psi_1,$$

$$f_{201}(\epsilon) = \left[\frac{2a_{13}^2\alpha^2 u_* w_*}{(1 + \alpha u_*)^4} - \frac{a_{13}\alpha^2 u_*}{(1 + \alpha u_*)^3} + \frac{a_{13}\alpha(1 + u_*G_1)}{(1 + \alpha u_*)^2} - \frac{a_{13}(1 + u_*G_1)G_1}{2(1 + \alpha u_*)}\right]\Psi_1,$$

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$$f_{120}(\epsilon) = \frac{1}{2}(\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)\Psi_1, \quad f_{012}(\epsilon) = -\frac{a_{13}^2(\tilde{a}_{12} + \hat{a}_{12})u_*}{2(1 + \alpha u_*)^2}\Psi_1,$$

$$f_{102}(\epsilon) = \left[\frac{a_{13}^2(1 + u_* G_1)}{2(1 + \alpha u_*)^2} - \frac{a_{13}^2 \alpha u_*}{(1 + \alpha u_*)^3}\right]\Psi_1,$$

$$f_{111}(\epsilon) = \left[\frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)}{1 + \alpha u_*} - \frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})\alpha u_*}{(1 + \alpha u_*)^2}\right]\Psi_1,$$

$$f_{400}(\epsilon) = \left[\frac{a_{13}^2 \alpha^4 u_* w_*^2}{2(1 + \alpha u_*)^6} - \frac{a_{13} \alpha^4 u_* w_*}{(1 + \alpha u_*)^5} + \frac{a_{13} \alpha^3 (1 + u_* G_1) w_*}{(1 + \alpha u_*)^4}\right.$$
$$\left. - \frac{a_{13} \alpha^2 (1 + u_* G_1) G_1 w_*}{2(1 + \alpha u_*)^3} + \frac{(4 + u_* G_1) G_1^3}{24}\right]\Psi_1,$$

$$f_{040}(\epsilon) = \frac{1}{24}(\tilde{a}_{12} + \hat{a}_{12})^4 u_* \Psi_1, \quad f_{004}(\epsilon) = \frac{a_{13}^4 u_*}{24(1 + \alpha u_*)^4}\Psi_1,$$

$$f_{310}(\epsilon) = \left[-\frac{a_{13} \alpha^3 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*}{(1 + \alpha u_*)^4} + \frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1) w_*}{(1 + \alpha u_*)^3}\right.$$
$$\left. - \frac{G_1^2 (\tilde{a}_{12} + \hat{a}_{12})(3 + u_* G_1)}{6}\right]\Psi_1,$$

$$f_{301}(\epsilon) = \left[-\frac{2a_{13}^2 \alpha^3 u_* w_*}{(1 + \alpha u_*)^5} + \frac{(a_{13}^2 \alpha^2 u_* w_* G_1 + a_{13}^2 \alpha^2 w_* + a_{13} \alpha^3 u_*)}{(1 + \alpha u_*)^4}\right.$$
$$\left. - \frac{(a_{13} \alpha^2 u_* G_1 + a_{13} \alpha^2)}{(1 + \alpha u_*)^3} + \frac{(a_{13} \alpha u_* + 2a_{13} \alpha G_1)}{2(1 + \alpha u_*)^2} - \frac{(a_{13} u_* G_1^3 + 3a_{13} G_1^2)}{6(1 + \alpha u_*)}\right]\Psi_1,$$

$$f_{031}(\epsilon) = \frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12}) u_*}{6(1 + \alpha u_*)}\Psi_1, \quad f_{103}(\epsilon) = \left[\frac{a_{13}^2 \alpha u_*}{2(1 + \alpha u_*)^4} - \frac{a_{13}^3 (1 + u_* G_1)}{(1 + \alpha u_*)^3}\right]\Psi_1,$$

$$f_{130}(\epsilon) = -\frac{1}{6}(\tilde{a}_{12} + \hat{a}_{12})^3 (1 + u_* G_1)\Psi_1, \quad f_{013}(\epsilon) = \frac{a_{13}^3 (\tilde{a}_{12} + \hat{a}_{12}) u_*}{6(1 + \alpha u_*)^3}\Psi_1,$$

$$f_{211}(\epsilon) = \left[-\frac{a_{13}^2 \alpha^2 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*}{(1 + \alpha u_*)^4} + \frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12}) u_*}{(1 + \alpha u_*)^3}\right.$$
$$\left. - \frac{a_{13} \alpha (\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)}{(1 + \alpha u_*)^2} + \frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})(2G_1 + u_* G_1)}{2(1 + \alpha u_*)}\right]\Psi_1,$$

$$f_{121}(\epsilon) = \left[\frac{a_{13} \alpha (\tilde{a}_{12} + \hat{a}_{12})^2 u_*}{2(1 + \alpha u_*)^2} - \frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})^2 (1 + u_* G_1)}{2(1 + \alpha u_*)}\right]\Psi_1,$$

$$f_{112}(\epsilon) = \left[\frac{a_{13}^2 \alpha (\tilde{a}_{12} + \hat{a}_{12}) u_*}{(1 + \alpha u_*)^3} - \frac{a_{13}^2 (\tilde{a}_{12} + \hat{a}_{12})^2 (1 + u_* G_1)}{(1 + \alpha u_*)^2}\right]\Psi_1,$$

$$f_{220}(\epsilon) = \left[-\frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})^2 u_* w_*}{2(1 + \alpha u_*)^3} + \frac{1}{4}(\tilde{a}_{12} + \hat{a}_{12})^2 (2G_1 + u_* G_1^2)\right]\Psi_1,$$

$$f_{202}(\epsilon) = \left[-\frac{a_{13}^3 \alpha^2 u_* w_*}{2(1 + \alpha u_*)^5} + \frac{3a_{13}^2 \alpha^2 u_*}{2(1 + \alpha u_*)^4} - \frac{a_{13}^2 \alpha (1 + u_* G_1)}{(1 + \alpha u_*)^3} + \frac{a_{13}^2 (2G_1 + u_* G_1^2)}{4(1 + \alpha u_*)^2}\right]\Psi_1,$$

$$f_{022}(\epsilon) = \frac{a_{13}^2 (\tilde{a}_{12} + \hat{a}_{12}) u_*}{4(1 + \alpha u_*)^2}\Psi_1,$$

$$f_{500}(\epsilon) = \left[-\frac{a_{13}^2 \alpha^5 u_* w_*^2}{(1+\alpha u_*)^7} + \frac{a_{13} \alpha^4 w_* (a_{13} w_* + \alpha u_* + a_{13} u_* w_* G_1)}{2(1+\alpha u_*)^6} \right.$$

$$-\frac{a_{13} \alpha^4 (1 + u_* G_1) w_*}{(1+\alpha u_*)^5} + \frac{a_{13} \alpha^3 (2 + u_* G_1^2) w_*}{2(1+\alpha u_*)^4} - \frac{a_{13} \alpha^2 (3 + u_* G_1^3) w_*}{6(1+\alpha u_*)^3}$$

$$\left. + \frac{G_1^4 (5 + u_* G_1)}{120} \right] \Psi_1,$$

$$f_{050}(\epsilon) = -\frac{1}{120} (\tilde{a}_{12} + \hat{a}_{12})^5 u_* \Psi_1, \quad f_{005}(\epsilon) = -\frac{a_{13}^5 u_*}{120(1+\alpha u_*)^5} \Psi_1,$$

$$f_{311}(\epsilon) = \left[\frac{2 a_{13}^2 \alpha^3 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*}{(1+\alpha u_*)^5} - \frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})(a_{13} w_* + \alpha u_* + a_{13} u_* w_* G_1)}{(1+\alpha u_*)^4} \right.$$

$$+ \frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)}{(1+\alpha u_*)^3} - \frac{a_{13} \alpha (\tilde{a}_{12} + \hat{a}_{12})(2\alpha + u_* G_1)}{2(1+\alpha u_*)^2}$$

$$\left. + \frac{a_{13} (\tilde{a}_{12} + \hat{a}_{12})(6 G_1 + u_* G_1^3)}{6(1+\alpha u_*)} \right] \Psi_1,$$

$$f_{131}(\epsilon) = \left[-\frac{a_{13} \alpha (\tilde{a}_{12} + \hat{a}_{12})^3 u_*}{6(1+\alpha u_*)^2} + \frac{a_{13} (\tilde{a}_{12} + \hat{a}_{12})^3 (1 + u_* G_1)}{6(1+\alpha u_*)} \right] \Psi_1,$$

$$f_{113}(\epsilon) = \left[-\frac{a_{13}^2 \alpha (\tilde{a}_{12} + \hat{a}_{12}) u_*}{2(1+\alpha u_*)^4} + \frac{a_{13}^3 (\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)}{6(1+\alpha u_*)^3} \right] \Psi_1,$$

$$f_{122}(\epsilon) = \left[-\frac{a_{13}^2 \alpha (\tilde{a}_{12} + \hat{a}_{12})^2 u_*}{2(1+\alpha u_*)^3} + \frac{a_{13}^2 (\tilde{a}_{12} + \hat{a}_{12})^2 (1 + u_* G_1)}{4(1+\alpha u_*)^2} \right] \Psi_1,$$

$$f_{212}(\epsilon) = \left[\frac{a_{13}^2 \alpha^2 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*}{4(1+\alpha u_*)^5} - \frac{3 a_{13}^2 \alpha^2 (\tilde{a}_{12} + \hat{a}_{12}) u_*}{2(1+\alpha u_*)^4} \right.$$

$$\left. + \frac{a_{13}^2 \alpha (\tilde{a}_{12} + \hat{a}_{12})(1 + u_* G_1)}{(1+\alpha u_*)^3} + \frac{a_{13}^2 (\tilde{a}_{12} + \hat{a}_{12})(2 G_1 + u_* G_1^2)}{4(1+\alpha u_*)^2} \right] \Psi_1,$$

$$f_{221}(\epsilon) = \left[\frac{a_{13}^2 \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})^2 u_* w_*}{2(1+\alpha u_*)^4} - \frac{a_{13} \alpha^2 (\tilde{a}_{12} + \hat{a}_{12})^2 u_*}{2(1+\alpha u_*)^3} \right.$$

$$\left. + \frac{a_{13} \alpha (\tilde{a}_{12} + \hat{a}_{12})^2 (1 + u_* G_1)}{2(1+\alpha u_*)^2} - \frac{a_{13} (\tilde{a}_{12} + \hat{a}_{12})^2 (2 G_1 + u_* G_1^2)}{4(1+\alpha u_*)} \right] \Psi_1,$$

$$f_{410}(\epsilon) = \left[-\frac{a_{13}^2 \alpha^4 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*^2}{2(1+\alpha u_*)^6} + \frac{a_{13} \alpha^4 (\tilde{a}_{12} + \hat{a}_{12}) u_* w_*}{(1+\alpha u_*)^5} \right.$$

$$-\frac{a_{13} \alpha^3 (1 + u_* G_1) w_*}{(1+\alpha u_*)^4} + \frac{a_{13} \alpha^2 (2 G_1 + u_* G_1^2) w_*}{2(1+\alpha u_*)^3}$$

$$\left. - \frac{G_1^3 (4 + u_* G_1)(\tilde{a}_{12} + \hat{a}_{12})}{24} \right] \Psi_1,$$

$$f_{401}(\epsilon) = \left[-\frac{a_{13}^3 \alpha^4 u_* w_*^2}{2(1+\alpha u_*)^7} + \frac{3 a_{13}^2 \alpha^4 u_* w_*}{(1+\alpha u_*)^6} + \frac{a_{13} \alpha^3 (a_{13} w_* + 2\alpha u_* + 4 a_{13} u_* w_* G_1)}{2(1+\alpha u_*)^5} \right.$$

$$\left. - \frac{a_{13} \alpha^2 (2\alpha + 2 a_{13} w_* G_1 + 2\alpha u_* G_1 + a_{13} u_* w_* G_1^2)}{2(1+\alpha u_*)^4} \right.$$

$$- \frac{a_{13}\alpha^2(2G_1 + u_*G_1^2)}{2(1+\alpha u_*)^3} + \frac{a_{13}\alpha(3G_1^2 + u_*G_1^3)}{6(1+\alpha u_*)^2} - \frac{a_{13}G_1^3(4 + u_*G_1)}{24(1+\alpha u_*)}\Big]\Psi_1,$$

$$f_{041}(\epsilon) = -\frac{a_{13}(\tilde{a}_{12} + \hat{a}_{12})^4 u_*}{24(1+\alpha u_*)}\Psi_1, \quad f_{140}(\epsilon) = \frac{1}{24}(\tilde{a}_{12} + \hat{a}_{12})^4(1 + u_*G_1)\Psi_1,$$

$$f_{104}(\epsilon) = \Big[-\frac{a_{13}^4\alpha u_*}{6(1+\alpha u_*)^5} + \frac{a_{13}^4(1 + u_*G_1)}{24(1+\alpha u_*)^4}\Big]\Psi_1, \quad f_{014}(\epsilon) = -\frac{a_{13}^4(\tilde{a}_{12} + \hat{a}_{12})u_*}{24(1+\alpha u_*)^4}\Psi_1,$$

$$f_{320}(\epsilon) = \Big[\frac{a_{13}\alpha^3(\tilde{a}_{12} + \hat{a}_{12})^2 u_* w_*}{2(1+\alpha u_*)^4} - \frac{a_{13}\alpha^2(\tilde{a}_{12} + \hat{a}_{12})^2(1 + u_*G_1)w_*}{2(1+\alpha u_*)^3}$$

$$+ \frac{G_1^2(\tilde{a}_{12} + \hat{a}_{12})^2(3 + u_*G_1)}{12}\Big]\Psi_1,$$

$$f_{302}(\epsilon) = \Big[\frac{3a_{13}^3\alpha^3 u_* w_*}{2(1+\alpha u_*)^6} - \frac{a_{13}^2\alpha^2(a_{13}w_* + 4\alpha u_* + a_{13}u_* w_*G_1)}{2(1+\alpha u_*)^5}$$

$$+ \frac{3a_{13}^2\alpha^2(1 + u_*G_1)}{2(1+\alpha u_*)^4} - \frac{a_{13}^2\alpha(2G_1 + u_*G_1^2)}{2(1+\alpha u_*)^3} + \frac{a_{13}^2(3G_1^2 + u_*G_1^3)}{12(1+\alpha u_*)^2}\Big]\Psi_1,$$

$$f_{032}(\epsilon) = -\frac{a_{13}^2(\tilde{a}_{12} + \hat{a}_{12})^3 u_*}{12(1+\alpha u_*)^2}\Psi_1,$$

$$f_{230}(\epsilon) = \Big[\frac{a_{13}\alpha^2(\tilde{a}_{12} + \hat{a}_{12})^3 u_* w_*}{6(1+\alpha u_*)^3} - \frac{1}{12}(\tilde{a}_{12} + \hat{a}_{12})^3(2G_1 + u_*G_1^2)\Big]\Psi_1,$$

$$f_{203}(\epsilon) = \Big[\frac{a_{13}^4\alpha^2 u_* w_*}{12(1+\alpha u_*)^6} - \frac{a_{13}^3\alpha^2 u_*}{(1+\alpha u_*)^5} + \frac{a_{13}^3\alpha(1 + u_*G_1)}{2(1+\alpha u_*)^4} - \frac{a_{13}^3(2G_1 + u_*G_1^2)}{12(1+\alpha u_*)^3}\Big]\Psi_1,$$

$$f_{023}(\epsilon) = -\frac{a_{13}^3(\tilde{a}_{12} + \hat{a}_{12})^2 u_*}{12(1+\alpha u_*)^3}\Psi_1,$$

$$g_{000}(\epsilon) = v_*(\Psi_2 - 1), \quad g_{100}(\epsilon) = (\tilde{a}_{21} + \hat{a}_{21})v_*\Psi_2, \quad g_{010}(\epsilon) = (1 - a_{22}v_*)\Psi_2,$$

$$g_{001}(\epsilon) = -a_{23}v_*\Psi_2, \quad g_{200}(\epsilon) = \frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})v_*\Psi_2,$$

$$g_{020}(\epsilon) = -\frac{1}{2}a_{22}(2 - a_{22}v_*)\Psi_2, \quad g_{002}(\epsilon) = \frac{1}{2}a_{23}^2 v_*\Psi_2,$$

$$g_{110}(\epsilon) = (\tilde{a}_{21} + \hat{a}_{21})(1 - a_{22}v_*)\Psi_2, \quad g_{101}(\epsilon) = -(\tilde{a}_{21} + \hat{a}_{21})a_{23}v_*\Psi_2,$$

$$g_{011}(\epsilon) = -(1 - a_{22}v_*)a_{23}\Psi_2, \quad g_{300}(\epsilon) = \frac{1}{6}(\tilde{a}_{21} + \hat{a}_{21})^3 v_*\Psi_2,$$

$$g_{030}(\epsilon) = \frac{1}{6}(3 - a_{22}v_*)a_{22}^2\Psi_2, \quad g_{003}(\epsilon) = -\frac{1}{6}a_{23}^3 v_*\Psi_2,$$

$$g_{210}(\epsilon) = \frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})^2(1 - a_{22}v_*)\Psi_2, \quad g_{201}(\epsilon) = -\frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})a_{23}v_*\Psi_2,$$

$$g_{021}(\epsilon) = \frac{1}{2}a_{22}a_{23}(2 - a_{22}v_*)\Psi_2, \quad g_{120}(\epsilon) = -\frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})(2 - a_{22}v_*)a_{22}\Psi_2,$$

$$g_{102}(\epsilon) = \frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})a_{23}^2 v_*\Psi_2, \quad g_{012}(\epsilon) = \frac{1}{2}(1 - a_{22}v_*)a_{23}^2\Psi_2,$$

$$g_{111}(\epsilon) = -(\tilde{a}_{21} + \hat{a}_{21})(1 - a_{22}v_*)a_{23}\Psi_2, \quad g_{400}(\epsilon) = \frac{1}{24}(\tilde{a}_{21} + \hat{a}_{21})^4 v_*\Psi_2,$$

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$$g_{040}(\epsilon) = -\frac{a_{22}^3}{24}(4 - a_{22}v_*)\Psi_2, \quad g_{004}(\epsilon) = \frac{1}{24}a_{23}^4 v_* \Psi_2,$$

$$g_{310}(\epsilon) = \frac{1}{6}(\tilde{a}_{21} + \hat{a}_{21})^3(1 - a_{22}v_*)\Psi_2, \quad g_{301}(\epsilon) = -\frac{1}{6}(\tilde{a}_{21} + \hat{a}_{21})^3 a_{23}v_* \Psi_2,$$

$$g_{031}(\epsilon) = -\frac{1}{6}a_{22}^2 a_{23}(3 - a_{22}v_*)\Psi_2, \quad g_{130}(\epsilon) = \frac{1}{6}(\tilde{a}_{21} + \hat{a}_{21})(3 - a_{22}v_*)a_{22}^2 \Psi_2,$$

$$g_{103}(\epsilon) = -\frac{1}{6}(\tilde{a}_{21} + \hat{a}_{21})a_{23}^3 v_* \Psi_2, \quad g_{013}(\epsilon) = -\frac{1}{6}a_{23}^3(1 - a_{22}v_*)\Psi_2,$$

$$g_{211}(\epsilon) = -\frac{1}{2}(\tilde{a}_{21} + \hat{a}_{21})^2(1 - a_{22}v_*)a_{23}\Psi_2,$$

$$g_{121}(\epsilon) = \frac{1}{2}a_{22}a_{23}(\tilde{a}_{21} + \hat{a}_{21})(2 - a_{22}v_*)\Psi_2,$$

$$g_{112}(\epsilon) = \frac{1}{2}a_{23}^2(\tilde{a}_{21} + \hat{a}_{21})(1 - a_{22}v_*)\Psi_2,$$

$$g_{220}(\epsilon) = -\frac{1}{4}a_{22}(\tilde{a}_{21} + \hat{a}_{21})^2(2 - a_{22}v_*)\Psi_2,$$

$$g_{202}(\epsilon) = \frac{1}{4}(\tilde{a}_{21} + \hat{a}_{21})a_{23}^2 v_* \Psi_2, \quad g_{022}(\epsilon) = -\frac{1}{4}a_{22}a_{23}^2(2 - a_{22}v_*)\Psi_2,$$

$$g_{500}(\epsilon) = \frac{1}{120}(\tilde{a}_{21} + \hat{a}_{21})^5 v_* \Psi_2, \quad g_{050}(\epsilon) = \frac{1}{120}a_{22}^4(5 - a_{22}v_*)\Psi_2,$$

$$g_{005}(\epsilon) = -\frac{1}{120}a_{23}^5 v_* \Psi_2, \quad g_{311}(\epsilon) = -\frac{1}{6}a_{23}(\tilde{a}_{21} + \hat{a}_{21})^3(1 - a_{22}v_*)\Psi_2,$$

$$g_{131}(\epsilon) = -\frac{1}{6}a_{22}^2 a_{23}(\tilde{a}_{21} + \hat{a}_{21})(3 - a_{22}v_*)\Psi_2,$$

$$g_{113}(\epsilon) = -\frac{1}{6}a_{23}^3(\tilde{a}_{21} + \hat{a}_{21})(1 - a_{22}v_*)\Psi_2,$$

$$g_{122}(\epsilon) = -\frac{1}{4}a_{22}a_{23}^2(\tilde{a}_{21} + \hat{a}_{21})(2 - a_{22}v_*)\Psi_2,$$

$$g_{212}(\epsilon) = \frac{1}{4}a_{23}^2(\tilde{a}_{21} + \hat{a}_{21})^2(1 - a_{22}v_*)\Psi_2,$$

$$g_{221}(\epsilon) = \frac{1}{4}a_{22}a_{23}(\tilde{a}_{21} + \hat{a}_{21})(2 - a_{22}v_*)\Psi_2,$$

$$g_{410}(\epsilon) = \frac{1}{24}(\tilde{a}_{21} + \hat{a}_{21})^4(1 - a_{22}v_*)\Psi_2, \quad g_{401}(\epsilon) = -\frac{1}{24}(\tilde{a}_{21} + \hat{a}_{21})^4 a_{23}v_* \Psi_2,$$

$$g_{041}(\epsilon) = \frac{1}{24}a_{22}^3 a_{23}(1 - a_{22}v_*)\Psi_2, \quad g_{140}(\epsilon) = -\frac{1}{24}a_{22}^3(\tilde{a}_{21} + \hat{a}_{21})(4 - a_{22}v_*)\Psi_2,$$

$$g_{104}(\epsilon) = \frac{1}{24}(\tilde{a}_{21} + \hat{a}_{21})a_{23}^4 v_* \Psi_2, \quad g_{014}(\epsilon) = \frac{1}{24}a_{23}^4(1 - a_{22}v_*)\Psi_2,$$

$$g_{320}(\epsilon) = -\frac{1}{12}a_{22}(\tilde{a}_{21} + \hat{a}_{21})^3(2 - a_{22}v_*)\Psi_2, \quad g_{302}(\epsilon) = \frac{1}{12}(\tilde{a}_{21} + \hat{a}_{21})^3 a_{23}^3 v_* \Psi_2,$$

$$g_{032}(\epsilon) = \frac{1}{12}a_{22}^2 a_{23}^2(3 - a_{22}v_*)\Psi_2, \quad g_{230}(\epsilon) = \frac{1}{12}a_{22}^2(\tilde{a}_{21} + \hat{a}_{21})(3 - a_{22}v_*)\Psi_2,$$

$$g_{203}(\epsilon) = -\frac{1}{12}(\tilde{a}_{21} + \hat{a}_{21})a_{23}^3 v_* \Psi_2, \quad g_{230}(\epsilon) = \frac{1}{12}a_{22}a_{23}^3(2 - a_{22}v_*)\Psi_2,$$

$$h_{000}(\epsilon) = 0, \quad h_{100}(\epsilon) = \frac{a_{31}w_*}{(1+\alpha u_*)^2}, \quad h_{010}(\epsilon) = a_{32}w_*, \quad h_{001}(\epsilon) = 1 - a_{33}w_*,$$

$$h_{200}(\epsilon) = \frac{a_{31}^2 w_*}{2(1+\alpha u_*)^4} - \frac{a_{31}\alpha w_*}{(1+\alpha u_*)^3}, \quad h_{020}(\epsilon) = \frac{1}{2}a_{32}^2 w_*,$$

$$h_{002}(\epsilon) = \frac{1}{2}(a_{33}w_* - 2)a_{33}, \quad h_{110}(\epsilon) = \frac{a_{31}a_{32}w_*}{(1+\alpha u_*)^2}, \quad h_{101}(\epsilon) = \frac{a_{31}(1 - a_{33}w_*)}{(1+\alpha u_*)^2},$$

$$h_{011}(\epsilon) = a_{32}(1 - a_{33}w_*), \quad h_{300}(\epsilon) = \frac{a_{31}^3 w_*}{6(1+\alpha u_*)^6} - \frac{a_{31}^2 \alpha w_*}{(1+\alpha u_*)^5} + \frac{a_{31}\alpha^2 w_*}{(1+\alpha u_*)^4},$$

$$h_{030}(\epsilon) = \frac{1}{6}a_{32}^3 w_*, \quad h_{003}(\epsilon) = \frac{1}{6}(3 - a_{33}w_*)a_{33}^2,$$

$$h_{210}(\epsilon) = -\frac{a_{31}a_{32}\alpha w_*}{(1+\alpha u_*)^3} + \frac{a_{31}^2 a_{32} w_*}{2(1+\alpha u_*)^4},$$

$$h_{201}(\epsilon) = \frac{1}{2}\Big[\frac{a_{31}^2}{(1+\alpha u_*)^4} + \frac{2a_{31}\alpha w_*}{(1+\alpha u_*)^3}\Big](1 - a_{33}w_*),$$

$$h_{021}(\epsilon) = \frac{1}{2}a_{32}^2(1 - a_{33}w_*), \quad h_{120}(\epsilon) = \frac{a_{31}a_{32}^2 w_*}{2(1+\alpha u_*)^2},$$

$$h_{102}(\epsilon) = -\frac{a_{31}a_{33}}{2(1+\alpha u_*)^2}(2 - a_{33}w_*), \quad h_{012}(\epsilon) = -\frac{1}{2}a_{32}a_{33}(2 - a_{33}w_*),$$

$$h_{111}(\epsilon) = \frac{a_{31}a_{32}}{(1+\alpha u_*)^2}(1 - a_{33}w_*),$$

$$h_{400}(\epsilon) = \frac{a_{31}^4 w_*}{24(1+\alpha u_*)^8} - \frac{a_{31}^3 \alpha w_*}{2(1+\alpha u_*)^7} + \frac{3a_{31}\alpha^2 w_*}{2(1+\alpha u_*)^6} + \frac{a_{31}\alpha^3 w_*}{(1+\alpha u_*)^5},$$

$$h_{040}(\epsilon) = \frac{1}{24}a_{32}^4 w_*, \quad h_{004}(\epsilon) = -\frac{1}{24}a_{33}^3(4 - a_{33}w_*),$$

$$h_{310}(\epsilon) = \frac{a_{31}^3 a_{32} w_*}{6(1+\alpha u_*)^6} + \frac{a_{31}^2 a_{32} \alpha w_*}{(1+\alpha u_*)^5} + \frac{a_{31}a_{32}\alpha^2 w_*}{(1+\alpha u_*)^4},$$

$$h_{301}(\epsilon) = \frac{a_{31}^3(1 - a_{33}w_*)}{6(1+\alpha u_*)^6} + \frac{a_{31}^2\alpha(1 - a_{33}w_*)}{(1+\alpha u_*)^5} + \frac{a_{31}\alpha^2(1 - a_{33}w_*)}{(1+\alpha u_*)^4},$$

$$h_{031}(\epsilon) = \frac{1}{6}a_{32}^3(1 - a_{33}w_*), \quad h_{130}(\epsilon) = \frac{a_{31}a_{32}^3 w_*}{6(1+\alpha u_*)^2},$$

$$h_{103}(\epsilon) = \frac{a_{31}a_{33}^2(3 - a_{33}w_*)}{6(1+\alpha u_*)^2}, \quad h_{013}(\epsilon) = \frac{1}{6}a_{32}a_{33}^2(3 - a_{33}w_*),$$

$$h_{211}(\epsilon) = \frac{a_{31}^2 a_{32}}{2(1+\alpha u_*)^4} - \frac{a_{31}a_{32}\alpha(1 - a_{33}w_*)}{(1+\alpha u_*)^3} + \frac{a_{31}^2 a_{32} a_{33} w_*}{2(1+\alpha u_*)^2},$$

$$h_{121}(\epsilon) = \frac{a_{31}a_{32}^2(1 - a_{33}w_*)}{2(1+\alpha u_*)^2}, \quad h_{112}(\epsilon) = -\frac{a_{31}a_{32}a_{33}(2 - a_{33}w_*)}{2(1+\alpha u_*)^2},$$

$$h_{220}(\epsilon) = \frac{a_{31}^2 a_{32}^2 w_*}{4(1+\alpha u_*)^4} + \frac{a_{31}a_{32}^2 \alpha w_*}{2(1+\alpha u_*)^3},$$

$$h_{202}(\epsilon) = -\frac{a_{31}^2 a_{33}(2 - a_{33}w_*)}{4(1+\alpha u_*)^4} - \frac{a_{31}a_{33}\alpha(2 - a_{33}w_*)}{2(1+\alpha u_*)^3},$$

$$h_{022}(\epsilon) = -\frac{1}{4}a_{32}^2 a_{33}(2 - a_{33}w_*),$$

$$h_{500}(\epsilon) = \frac{a_{31}^5 w_*}{120(1 + \alpha u_*)^{10}} + \frac{a_{31}^4 \alpha w_*}{6(1 + \alpha u_*)^9} + \frac{a_{31}^3 \alpha^2 w_*}{(1 + \alpha u_*)^8} + \frac{a_{31}^2 \alpha^3 w_*}{(1 + \alpha u_*)^7},$$

$$h_{050}(\epsilon) = \frac{1}{120}a_{32}^5 w_*, \quad h_{005}(\epsilon) = \frac{1}{120}a_{33}^5(5 - a_{33}w_*),$$

$$h_{311}(\epsilon) = \frac{a_{31}^3 a_{32}(1 - a_{33}w_*)}{6(1 + \alpha u_*)^6} + \frac{a_{31}^2 a_{32}\alpha(1 - a_{33}w_*)}{(1 + \alpha u_*)^5} + \frac{a_{31}a_{32}\alpha^2(1 - a_{33}w_*)}{(1 + \alpha u_*)^4},$$

$$h_{131}(\epsilon) = \frac{a_{31}a_{32}^3(1 - a_{33}w_*)}{6(1 + \alpha u_*)^2}, \quad h_{113}(\epsilon) = \frac{a_{31}a_{32}a_{33}^2(3 - a_{33}w_*)}{6(1 + \alpha u_*)^2},$$

$$h_{122}(\epsilon) = -\frac{a_{31}a_{32}^2 a_{33}(2 - a_{33}w_*)}{4(1 + \alpha u_*)^2},$$

$$h_{212}(\epsilon) = \frac{a_{31}^2 a_{32}a_{33}(2 - a_{33}w_*)}{4(1 + \alpha u_*)^4} + \frac{a_{31}a_{32}a_{33}(1 - 2a_{33}w_*)}{4(1 + \alpha u_*)^3},$$

$$h_{221}(\epsilon) = \frac{a_{31}^2 a_{32}^2(1 - a_{33}w_*)}{4(1 + \alpha u_*)^4} + \frac{a_{31}a_{32}^2(1 - a_{33}w_*)}{2(1 + \alpha u_*)^3},$$

$$h_{410}(\epsilon) = \frac{a_{31}^4 a_{32}w_*}{24(1 + \alpha u_*)^8} + \frac{a_{31}^3 a_{32}\alpha w_*}{2(1 + \alpha u_*)^7} + \frac{a_{31}^2 a_{32}\alpha^2 w_*}{(1 + \alpha u_*)^6} + \frac{a_{31}a_{32}\alpha^3 w_*}{(1 + \alpha u_*)^5},$$

$$h_{401}(\epsilon) = \frac{a_{31}^4(1 - a_{33}w_*)}{24(1 + \alpha u_*)^8} + \frac{a_{31}^3 \alpha(1 - a_{33}w_*)}{2(1 + \alpha u_*)^7} + \frac{3a_{31}^2 \alpha^2(1 - a_{33}w_*)}{2(1 + \alpha u_*)^6}$$
$$+ \frac{a_{31}\alpha^3(1 - a_{33}w_*)}{(1 + \alpha u_*)^5},$$

$$h_{041}(\epsilon) = \frac{1}{24}a_{32}^4(1 - a_{33}w_*), \quad h_{140}(\epsilon) = \frac{a_{31}a_{32}^4 w_*}{24(1 + \alpha u_*)^2},$$

$$h_{104}(\epsilon) = \frac{a_{31}a_{33}^3(4 - a_{33}w_*)}{24(1 + \alpha u_*)^2}, \quad h_{014}(\epsilon) = -\frac{1}{24}a_{32}a_{33}^3(4 - a_{33}w_*),$$

$$h_{320}(\epsilon) = \frac{a_{31}^3 a_{32}^2 w_*}{12(1 + \alpha u_*)^6} + \frac{a_{31}^2 a_{32}^2 \alpha w_*}{2(1 + \alpha u_*)^5} + \frac{a_{31}a_{32}^2 \alpha^2 w_*}{2(1 + \alpha u_*)^4},$$

$$h_{302}(\epsilon) = -\frac{a_{31}^3 a_{33}(1 - a_{33}w_*)}{6(1 + \alpha u_*)^6} - \frac{a_{31}^2 a_{33}\alpha(2 - a_{33}w_*)}{2(1 + \alpha u_*)^5} - \frac{a_{31}a_{33}\alpha^2(1 - a_{33}w_*)}{2(1 + \alpha u_*)^4},$$

$$h_{032}(\epsilon) = -\frac{1}{12}a_{32}^3 a_{33}(1 - a_{33}w_*), \quad h_{230}(\epsilon) = \frac{a_{31}^2 a_{32}^3 w_*}{12(1 + \alpha u_*)^4} + \frac{a_{31}a_{32}^3 \alpha w_*}{6(1 + \alpha u_*)^3},$$

$$h_{203}(\epsilon) = \frac{a_{31}^2 a_{33}^3(3 - a_{33}w_*)}{12(1 + \alpha u_*)^4} + \frac{a_{31}a_{33}^3 \alpha(3 - a_{33}w_*)}{6(1 + \alpha u_*)^3},$$

$$h_{023}(\epsilon) = \frac{1}{12}a_{32}^2 a_{33}^2(3 - a_{33}w_*).$$

7 $m_{200}(0)$, $m_{011}(0)$, $m_{300}(0)$, $m_{111}(0)$, $s_{110}(0)$, $s_{210}(0)$ and $s_{021}(0)$ in (3.5)

$$m_{200}(0) = \xi_{11}^2 H_{200} + \xi_{12}^2 H_{020} + \xi_{13}^2 H_{002} + \xi_{11}\xi_{12}H_{110} + \xi_{11}\xi_{13}H_{101} + \xi_{12}\xi_{13}H_{011},$$

$$m_{011}(0) = \frac{1}{2}\xi_{21}^2 H_{200} + \frac{1}{2}\xi_{22}^2 H_{020} + \frac{1}{2}\xi_{23}^2 H_{002} + (\xi_{21}\bar{\xi}_{22} + \xi_{22}\bar{\xi}_{21})H_{110}$$
$$+ (\xi_{21}\bar{\xi}_{23} + \xi_{23}\bar{\xi}_{21})H_{101} + (\xi_{22}\bar{\xi}_{23} + \xi_{23}\bar{\xi}_{22})H_{011},$$

$$m_{300}(0) = \xi_{11}^3 H_{300} + \xi_{12}^3 H_{030} + \xi_{13}^3 H_{003} + \xi_{11}\xi_{12}\xi_{13}H_{111} + \xi_{11}^2\xi_{12}H_{210} + \xi_{11}^2\xi_{13}H_{201}$$
$$+ \xi_{12}^2\xi_{13}H_{021} + \xi_{11}\xi_{12}^2 H_{120} + \xi_{11}\xi_{13}^2 H_{102} + \xi_{12}\xi_{13}^2 H_{012},$$

$$m_{111}(0) = 2(\xi_{11}\xi_{21}\bar{\xi}_{22} + \xi_{11}\xi_{22}\bar{\xi}_{21} + \xi_{12}\xi_{21}^2)H_{210} + 2(\xi_{11}\xi_{21}\bar{\xi}_{23} + \xi_{11}\xi_{23}\bar{\xi}_{21}$$
$$+ \xi_{13}\xi_{21}^2)H_{201} + 2(\xi_{12}\xi_{22}\bar{\xi}_{23} + \xi_{12}\xi_{23}\bar{\xi}_{22} + \xi_{13}\xi_{22}^2)H_{021} + 2(\xi_{12}\xi_{21}\bar{\xi}_{22}$$
$$+ \xi_{12}\xi_{22}\bar{\xi}_{21} + \xi_{11}\xi_{22}^2)H_{120} + 2(\xi_{13}\xi_{21}\bar{\xi}_{23} + \xi_{13}\xi_{33}\bar{\xi}_{21} + \xi_{11}\xi_{23}^2)H_{102}$$
$$+ 2(\xi_{13}\xi_{22}\bar{\xi}_{23} + \xi_{13}\xi_{23}\bar{\xi}_{22} + \xi_{12}\xi_{23}^2)H_{012} + 6\xi_{11}\xi_{21}^2 H_{300} + 6\xi_{12}\xi_{22}^2 H_{030}$$
$$+ 6\xi_{13}\xi_{23}^2 H_{003} + (\xi_{11}\xi_{22}\bar{\xi}_{23} + \xi_{11}\xi_{23}\bar{\xi}_{22} + \xi_{12}\xi_{21}\bar{\xi}_{23} + \xi_{12}\xi_{23}\bar{\xi}_{21}$$
$$+ \xi_{13}\xi_{21}\bar{\xi}_{22} + \xi_{13}\xi_{22}\bar{\xi}_{21})H_{111},$$

$$s_{210}(0) = (\xi_{11}^2\xi_{22} + 2\xi_{11}\xi_{12}\xi_{21})K_{210} + (\xi_{11}^2\xi_{23} + 2\xi_{11}\xi_{13}\xi_{21})K_{201}$$
$$+ (\xi_{12}^2\xi_{23} + 2\xi_{12}\xi_{13}\xi_{23})K_{021} + (\xi_{12}^2\xi_{21} + 2\xi_{11}\xi_{12}\xi_{23})K_{120}$$
$$+ (\xi_{13}^2\xi_{21} + 2\xi_{11}\xi_{13}\xi_{23})K_{102} + (\xi_{13}^2\xi_{22} + 2\xi_{12}\xi_{13}\xi_{23})K_{012}$$
$$+ 3\xi_{11}^2\xi_{21}K_{300} + 3\xi_{12}^2\xi_{22}K_{030} + 3\xi_{13}^2\xi_{23}K_{003}$$
$$+ (\xi_{11}\xi_{12}\xi_{23} + \xi_{11}\xi_{13}\xi_{22} + \xi_{12}\xi_{13}\xi_{21})K_{111},$$

$$s_{110}(0) = \frac{1}{2}\xi_{11}\xi_{21}K_{200} + \frac{1}{2}\xi_{12}\xi_{22}K_{020} + \frac{1}{2}\xi_{13}\xi_{23}K_{002} + (\xi_{11}\xi_{22} + \xi_{12}\xi_{21})K_{110}$$
$$+ (\xi_{11}\xi_{23} + \xi_{13}\xi_{21})K_{101} + (\xi_{12}\xi_{23} + \xi_{13}\xi_{22})K_{011},$$

$$s_{021}(0) = (\xi_{21}^2\bar{\xi}_{22} + 2\xi_{21}\xi_{22})K_{210} + (\xi_{21}^2\bar{\xi}_{23} + 2\xi_{21}\xi_{23})K_{201}$$
$$+ (\xi_{22}^2\bar{\xi}_{23} + 2\xi_{22}\xi_{13})K_{021} + (\xi_{22}^2\bar{\xi}_{21} + 2\xi_{21}\xi_{22})K_{120}$$
$$+ (\xi_{23}^2\bar{\xi}_{21} + 2\xi_{21}\xi_{23})K_{102} + (\xi_{23}^2\bar{\xi}_{22} + 2\xi_{21}\xi_{23})K_{012}$$
$$+ 3\xi_{21}^2\xi_{21}K_{300} + 3\xi_{22}^2\xi_{22}K_{030} + 3\xi_{23}^2\xi_{23}K_{003}$$
$$+ (\xi_{21}\xi_{22}\bar{\xi}_{23} + \xi_{21}\xi_{23}\bar{\xi}_{22} + \xi_{22}\xi_{23}\bar{\xi}_{21})K_{111},$$

$$H_{jkl} = \xi_{31}(0)f_{jkl}(0) + \xi_{32}(0)g_{jkl}(0) + \xi_{33}(0)h_{jkl}(0)(2 \le j+k+l \le 3),$$
$$K_{jkl} = \xi_{41}(0)f_{jkl}(0) + \xi_{42}(0)g_{jkl}(0) + \xi_{43}(0)h_{jkl}(0)(2 \le j+k+l \le 3).$$

8 $\widetilde{m}_{300}(0)$, $\widetilde{m}_{111}(0)$, $\widetilde{m}_{500}(0)$, $\widetilde{m}_{311}(0)$, $\widetilde{m}_{122}(0)$, $\widetilde{s}_{210}(0)$, $\widetilde{s}_{021}(0)$, $\widetilde{s}_{410}(0)$, $\widetilde{s}_{221}(0)$ and $\widetilde{s}_{032}(0)$ in (3.16)

$$\widetilde{m}_{300}(0) = \varsigma_{11}^3 \widetilde{H}_{300} + \varsigma_{12}^3 \widetilde{H}_{030} + \varsigma_{13}^3 \widetilde{H}_{003} + \varsigma_{11}\varsigma_{12}\varsigma_{13}\widetilde{H}_{111} + \varsigma_{11}^2\varsigma_{12}\widetilde{H}_{210} + \varsigma_{11}^2\varsigma_{13}\widetilde{H}_{201}$$

$$+ \varsigma_{12}^2\varsigma_{13}\widetilde{H}_{021} + \varsigma_{11}\varsigma_{12}^2\widetilde{H}_{120} + \varsigma_{11}\varsigma_{13}^2\widetilde{H}_{102} + \varsigma_{12}\varsigma_{13}^2\widetilde{H}_{012},$$

$$
\begin{aligned}
\widetilde{m}_{111}(0) =& 2(\varsigma_{11}\varsigma_{21}\bar{\varsigma}_{22} + \varsigma_{11}\varsigma_{22}\bar{\varsigma}_{21} + \varsigma_{12}\varsigma_{21}^2)H_{210} + 2(\varsigma_{11}\varsigma_{21}\bar{\varsigma}_{23} + \varsigma_{11}\varsigma_{23}\bar{\varsigma}_{21} \\
& + \varsigma_{13}\varsigma_{21}^2)H_{201} + 2(\varsigma_{12}\varsigma_{22}\bar{\varsigma}_{23} + \varsigma_{12}\varsigma_{23}\bar{\varsigma}_{22} + \varsigma_{13}\varsigma_{22}^2)H_{021} + 2(\varsigma_{12}\varsigma_{21}\bar{\varsigma}_{22} \\
& + \varsigma_{12}\varsigma_{22}\bar{\varsigma}_{21} + \varsigma_{11}\varsigma_{22}^2)H_{120} + 2(\varsigma_{13}\varsigma_{21}\bar{\varsigma}_{23} + \varsigma_{13}\varsigma_{33}\bar{\varsigma}_{21} + \varsigma_{11}\varsigma_{23}^2)H_{102} \\
& + 2(\varsigma_{13}\varsigma_{22}\bar{\varsigma}_{23} + \varsigma_{13}\varsigma_{23}\bar{\varsigma}_{22} + \varsigma_{12}\varsigma_{23}^2)H_{012} + 6\varsigma_{11}\varsigma_{21}^2 H_{300} + 6\varsigma_{12}\varsigma_{22}^2 H_{030} \\
& + 6\varsigma_{13}\varsigma_{23}^2 H_{003} + (\varsigma_{11}\varsigma_{22}\bar{\varsigma}_{23} + \varsigma_{11}\varsigma_{23}\bar{\varsigma}_{22} + \varsigma_{12}\varsigma_{21}\bar{\varsigma}_{23} + \varsigma_{12}\varsigma_{23}\bar{\varsigma}_{21} \\
& + \varsigma_{13}\varsigma_{21}\bar{\varsigma}_{22} + \varsigma_{13}\varsigma_{22}\bar{\varsigma}_{21})H_{111},
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{m}_{500}(0) =& \varsigma_{11}^5\widetilde{H}_{500} + \varsigma_{12}^5\widetilde{H}_{050} + \varsigma_{13}^5\widetilde{H}_{005} + \varsigma_{11}^3\varsigma_{12}\varsigma_{13}\widetilde{H}_{311} + \varsigma_{11}\varsigma_{12}^3\varsigma_{13}\widetilde{H}_{131} \\
& + \varsigma_{11}\varsigma_{12}\varsigma_{13}^3\widetilde{H}_{113} + \varsigma_{11}\varsigma_{12}^2\varsigma_{13}^2\widetilde{H}_{122} + \varsigma_{11}^2\varsigma_{12}\varsigma_{13}^2\widetilde{H}_{212} + \varsigma_{11}^2\varsigma_{12}^2\varsigma_{13}\widetilde{H}_{221} \\
& + \varsigma_{11}^4\varsigma_{12}\widetilde{H}_{410} + \varsigma_{11}^4\varsigma_{13}\widetilde{H}_{401} + \varsigma_{12}^4\varsigma_{13}\widetilde{H}_{041} + \varsigma_{11}\varsigma_{12}^4\widetilde{H}_{140} + \varsigma_{11}\varsigma_{13}^4\widetilde{H}_{104} \\
& + \varsigma_{12}\varsigma_{13}^4\widetilde{H}_{014} + \varsigma_{11}^3\varsigma_{12}^2\widetilde{H}_{320} + \varsigma_{11}^3\varsigma_{13}^2\widetilde{H}_{302} + \varsigma_{12}^3\varsigma_{13}^2\widetilde{H}_{032} + \varsigma_{11}^2\varsigma_{12}^3\widetilde{H}_{230} \\
& + \varsigma_{11}^2\varsigma_{13}^3\widetilde{H}_{203} + \varsigma_{12}^2\varsigma_{13}^3\widetilde{H}_{023},
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{s}_{210}(0) =& 3\varsigma_{21}^3\widetilde{K}_{030} + 3\varsigma_{22}^3\widetilde{K}_{003} + 3\varsigma_{23}^3\widetilde{K}_{003} \\
& (\varsigma_{21}^2\bar{\varsigma}_{22} + 2\varsigma_{21}^2\varsigma_{22})\widetilde{K}_{210} + (\varsigma_{21}^2\bar{\varsigma}_{23} + 2\varsigma_{21}^2\varsigma_{23})\widetilde{K}_{201} \\
& (\varsigma_{22}^2\bar{\varsigma}_{23} + 2\varsigma_{22}^2\varsigma_{23})\widetilde{K}_{021} + (\varsigma_{22}^2\bar{\varsigma}_{21} + 2\varsigma_{21}^2\varsigma_{22})\widetilde{K}_{120} \\
& (\varsigma_{23}^2\bar{\varsigma}_{21} + 2\varsigma_{21}^2\varsigma_{23})\widetilde{K}_{102} + (\varsigma_{23}^2\bar{\varsigma}_{22} + 2\varsigma_{22}^2\varsigma_{23})\widetilde{K}_{012} \\
& + (\varsigma_{21}\varsigma_{22}\bar{\varsigma}_{23} + \varsigma_{21}\varsigma_{23}\bar{\varsigma}_{22} + \varsigma_{22}\varsigma_{23}\bar{\varsigma}_{21})\widetilde{K}_{111},
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{m}_{311}(0) =& 20\varsigma_{11}^3\varsigma_{21}^2\widetilde{H}_{500} + 20\varsigma_{12}^3\varsigma_{22}^2\widetilde{H}_{050} + 20\varsigma_{13}^3\varsigma_{23}^2\widetilde{H}_{005} + (\varsigma_{11}^3\varsigma_{22}\bar{\varsigma}_{23} + \varsigma_{11}^3\varsigma_{23}\bar{\varsigma}_{22} \\
& + 3\varsigma_{11}^2\varsigma_{12}\varsigma_{21}\bar{\varsigma}_{23} + 3\varsigma_{11}^2\varsigma_{12}\varsigma_{23}\bar{\varsigma}_{21} + 3\varsigma_{11}^2\varsigma_{13}\varsigma_{21}\bar{\varsigma}_{22} + 3\varsigma_{11}^2\varsigma_{13}\varsigma_{22}\bar{\varsigma}_{21} \\
& + 6\varsigma_{11}\varsigma_{12}\varsigma_{13}\varsigma_{21}^2)\widetilde{H}_{311} + (\varsigma_{12}^3\varsigma_{21}\varsigma_{23} + \varsigma_{12}^3\varsigma_{21}\bar{\varsigma}_{23} + 3\varsigma_{11}\varsigma_{12}^2\varsigma_{22}\bar{\varsigma}_{23} \\
& + 3\varsigma_{11}\varsigma_{12}^2\varsigma_{23}\bar{\varsigma}_{22} + 3\varsigma_{12}^2\varsigma_{13}\varsigma_{21}\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{13}\varsigma_{21}\bar{\varsigma}_{22} + 6\varsigma_{11}\varsigma_{12}\varsigma_{13}\varsigma_{22}^2)\widetilde{H}_{131} \\
& + (\varsigma_{13}^3\varsigma_{21}\varsigma_{22} + \varsigma_{13}^3\varsigma_{21}\varsigma_{23} + 3\varsigma_{11}\varsigma_{13}^2\varsigma_{22}\bar{\varsigma}_{23} + 3\varsigma_{11}\varsigma_{13}^2\varsigma_{23}\bar{\varsigma}_{22} \\
& + 3\varsigma_{12}\varsigma_{13}^2\varsigma_{21}\varsigma_{23} + 3\varsigma_{12}\varsigma_{13}^2\varsigma_{21}\bar{\varsigma}_{22} + 6\varsigma_{11}\varsigma_{12}\varsigma_{13}\varsigma_{23}^2)\widetilde{H}_{113} \\
& + (4\varsigma_{11}^3\varsigma_{21}\varsigma_{22} + 4\varsigma_{11}^3\varsigma_{21}\bar{\varsigma}_{22} + 12\varsigma_{11}^2\varsigma_{12}\varsigma_{21}^2)\widetilde{H}_{410} \\
& + (4\varsigma_{11}^3\varsigma_{21}\varsigma_{23} + 4\varsigma_{11}^3\varsigma_{21}\bar{\varsigma}_{23} + 12\varsigma_{11}^2\varsigma_{13}\varsigma_{21}^2)\widetilde{H}_{401} \\
& + (4\varsigma_{13}^3\varsigma_{22}\varsigma_{23} + 4\varsigma_{13}^3\varsigma_{22}\bar{\varsigma}_{23} + 12\varsigma_{12}^2\varsigma_{13}\varsigma_{23}^2)\widetilde{H}_{041} \\
& + (4\varsigma_{12}^3\varsigma_{21}\varsigma_{22} + 4\varsigma_{12}^3\varsigma_{21}\bar{\varsigma}_{22} + 12\varsigma_{11}\varsigma_{12}^2\varsigma_{22}^2)\widetilde{H}_{140} \\
& + (4\varsigma_{13}^3\varsigma_{21}\varsigma_{23} + 4\varsigma_{13}^3\varsigma_{21}\bar{\varsigma}_{23} + 12\varsigma_{11}\varsigma_{13}^2\varsigma_{23}^2)\widetilde{H}_{104} \\
& + (4\varsigma_{13}^3\bar{\varsigma}_{22}\varsigma_{23} + 4\varsigma_{13}^3\varsigma_{22}\bar{\varsigma}_{23} + 12\varsigma_{12}\varsigma_{13}^2\varsigma_{23}^2)\widetilde{H}_{014} \\
& + (2\varsigma_{11}^3\varsigma_{23}^2 + 6\varsigma_{11}^2\varsigma_{13}\varsigma_{21}\varsigma_{23} + 6\varsigma_{11}^2\varsigma_{13}\varsigma_{21}\bar{\varsigma}_{23} + 6\varsigma_{11}\varsigma_{13}^2\varsigma_{21}^2)\widetilde{H}_{302} \\
& + (2\varsigma_{11}^3\varsigma_{22}^2 + 6\varsigma_{11}^2\varsigma_{12}\varsigma_{21}\varsigma_{22} + 6\varsigma_{11}^2\varsigma_{12}\varsigma_{21}\bar{\varsigma}_{22} + 6\varsigma_{11}\varsigma_{12}^2\varsigma_{21}^2)\widetilde{H}_{320} \\
& + (2\varsigma_{12}^3\varsigma_{23}^2 + 6\varsigma_{12}^2\varsigma_{13}\varsigma_{21}\varsigma_{23} + 6\varsigma_{12}^2\varsigma_{13}\varsigma_{21}\bar{\varsigma}_{23} + 6\varsigma_{12}\varsigma_{13}^2\varsigma_{22}^2)\widetilde{H}_{032}
\end{aligned}
$$

$$+ (2\varsigma_{12}^3\varsigma_{21}^2 + 6\varsigma_{11}^2\varsigma_{12}\varsigma_{21}\varsigma_{22} + 6\varsigma_{11}^2\varsigma_{12}\varsigma_{21}\bar\varsigma_{22} + 6\varsigma_{11}\varsigma_{12}^2\varsigma_{22}^2)\widetilde{H}_{230}$$
$$+ (2\varsigma_{13}^3\varsigma_{21}^2 + 6\varsigma_{11}^2\varsigma_{13}\varsigma_{21}\varsigma_{23} + 6\varsigma_{11}^2\varsigma_{13}\varsigma_{21}\bar\varsigma_{23} + 6\varsigma_{11}\varsigma_{13}^2\varsigma_{23}^2)\widetilde{H}_{203}$$
$$+ (2\varsigma_{13}^3\varsigma_{22}^2 + 6\varsigma_{12}^2\varsigma_{13}\varsigma_{23}\varsigma_{22} + 6\varsigma_{12}^2\varsigma_{13}\varsigma_{23}\bar\varsigma_{22} + 6\varsigma_{12}\varsigma_{13}^2\varsigma_{23}^2)\widetilde{H}_{023},$$

$$\widetilde{m}_{122}(0) = 30\varsigma_{11}\varsigma_{21}^4\widetilde{H}_{500} + 30\varsigma_{12}\varsigma_{22}^4\widetilde{H}_{050} + 30\varsigma_{13}\varsigma_{23}^4\widetilde{H}_{005}$$
$$+ (3\varsigma_{11}\varsigma_{21}^2\bar\varsigma_{22}\bar\varsigma_{23} + 6\varsigma_{11}\varsigma_{21}^2\varsigma_{22}\bar\varsigma_{23} + 6\varsigma_{11}\varsigma_{21}^2\varsigma_{23}\bar\varsigma_{22} + 3\varsigma_{11}\varsigma_{22}^2\varsigma_{23}\bar\varsigma_{22}$$
$$+ 3\varsigma_{12}\varsigma_{21}^3\bar\varsigma_{23} + 3\varsigma_{12}\varsigma_{21}^3\varsigma_{23} + 3\varsigma_{13}\varsigma_{21}^3\bar\varsigma_{22} + 3\varsigma_{13}\varsigma_{21}^3\varsigma_{22})\widetilde{H}_{311}$$
$$+ (3\varsigma_{12}\varsigma_{21}\varsigma_{22}^2\bar\varsigma_{23} + 6\varsigma_{13}\varsigma_{21}\varsigma_{22}^2\varsigma_{23} + 6\varsigma_{13}\varsigma_{21}\varsigma_{22}^2\varsigma_{23} + 3\varsigma_{12}\varsigma_{21}\varsigma_{23}\bar\varsigma_{22}^2$$
$$+ 3\varsigma_{11}\varsigma_{22}^3\bar\varsigma_{23} + 3\varsigma_{11}\varsigma_{22}^3\varsigma_{23} + 3\varsigma_{13}\varsigma_{21}\varsigma_{22}^3 + 3\varsigma_{13}\varsigma_{21}\varsigma_{22}^3)\widetilde{H}_{131}$$
$$+ (3\varsigma_{13}\varsigma_{21}\varsigma_{22}\bar\varsigma_{23}^2 + 6\varsigma_{13}\varsigma_{21}\varsigma_{22}\varsigma_{23} + 6\varsigma_{13}\varsigma_{21}\varsigma_{23}\bar\varsigma_{22}^2 + 3\varsigma_{11}\varsigma_{22}\varsigma_{23}^3$$
$$+ 3\varsigma_{11}\varsigma_{23}^3\bar\varsigma_{22} + 3\varsigma_{12}\varsigma_{21}\varsigma_{23}^3 + 3\varsigma_{12}\varsigma_{21}\varsigma_{23}^3 + 3\varsigma_{13}\varsigma_{21}\varsigma_{23}^3)\widetilde{H}_{113}$$
$$+ (12\varsigma_{11}\varsigma_{21}^3\varsigma_{22} + 12\varsigma_{11}\varsigma_{21}^3\bar\varsigma_{22} + 6\varsigma_{12}\varsigma_{21}^4)\widetilde{H}_{410}$$
$$+ (12\varsigma_{11}\varsigma_{21}^3\varsigma_{23} + 12\varsigma_{11}\varsigma_{21}^3\bar\varsigma_{23} + 6\varsigma_{13}\varsigma_{21}^4)\widetilde{H}_{401}$$
$$+ (12\varsigma_{12}\varsigma_{22}^3\varsigma_{23} + 12\varsigma_{12}\varsigma_{22}^3\bar\varsigma_{23} + 6\varsigma_{13}\varsigma_{22}^4)\widetilde{H}_{041}$$
$$+ (24\varsigma_{11}\varsigma_{21}\varsigma_{22}^3 + 6\varsigma_{11}\varsigma_{22}^4)\widetilde{H}_{140} + (24\varsigma_{13}\varsigma_{21}\varsigma_{23}^3 + 6\varsigma_{11}\varsigma_{23}^4)\widetilde{H}_{104}$$
$$+ (12\varsigma_{13}\varsigma_{22}\varsigma_{23}^3 + 12\varsigma_{13}\bar\varsigma_{22}\varsigma_{23}^3 + 6\varsigma_{12}\varsigma_{23}^4)\widetilde{H}_{014}$$
$$+ (15\varsigma_{11}\varsigma_{21}^2\varsigma_{22}^2 + 3\varsigma_{11}\varsigma_{21}^2\bar\varsigma_{22}^2 + 6\varsigma_{12}\varsigma_{21}^3\varsigma_{22} + 6\varsigma_{12}\varsigma_{12}^3\bar\varsigma_{21})\widetilde{H}_{320}$$
$$+ (15\varsigma_{11}\varsigma_{21}^2\varsigma_{23}^2 + 3\varsigma_{11}\varsigma_{21}^2\bar\varsigma_{23}^2 + 6\varsigma_{13}\varsigma_{21}^3\varsigma_{23} + 6\varsigma_{13}\varsigma_{21}^3\bar\varsigma_{23})\widetilde{H}_{302}$$
$$+ (12\varsigma_{12}\varsigma_{22}^2\varsigma_{23}^2 + 3\varsigma_{12}\varsigma_{22}^2\bar\varsigma_{23}^2 + 3\varsigma_{12}\varsigma_{23}^2\bar\varsigma_{22}^2 + 6\varsigma_{13}\varsigma_{22}^3\varsigma_{23} + 6\varsigma_{13}\varsigma_{22}^3\varsigma_{23})\widetilde{H}_{032}$$
$$+ (12\varsigma_{11}\varsigma_{21}\varsigma_{22}^3 + 15\varsigma_{12}\varsigma_{21}^2\varsigma_{22}^2 + 3\varsigma_{12}\varsigma_{21}^3\bar\varsigma_{22})\widetilde{H}_{230}$$
$$+ (12\varsigma_{11}\varsigma_{21}\varsigma_{23}^3 + 15\varsigma_{13}\varsigma_{21}^2\varsigma_{23}^2 + 3\varsigma_{13}\varsigma_{21}^3\bar\varsigma_{23})\widetilde{H}_{203}$$
$$+ (12\varsigma_{12}\varsigma_{22}\varsigma_{23}^3 + 3\varsigma_{13}\varsigma_{22}^2\bar\varsigma_{23}^2 + 12\varsigma_{13}\varsigma_{22}^2\varsigma_{23}^2 + 3\varsigma_{13}\varsigma_{23}^3\bar\varsigma_{22}^3)\widetilde{H}_{023},$$

$$\widetilde{s}_{410}(0) = 5\varsigma_{11}^4\varsigma_{21}\widetilde{K}_{500} + 5\varsigma_{12}^4\varsigma_{22}\widetilde{K}_{050} + 5\varsigma_{13}^4\varsigma_{23}\widetilde{K}_{005}$$
$$+ (\varsigma_{11}\varsigma_{12}^3\varsigma_{23} + \varsigma_{11}^3\varsigma_{13}\varsigma_{22} + 3\varsigma_{11}^2\varsigma_{12}\varsigma_{13}\varsigma_{21})\widetilde{K}_{311}$$
$$+ (\varsigma_{11}\varsigma_{12}^3\varsigma_{23} + \varsigma_{12}\varsigma_{13}^3\varsigma_{21} + 3\varsigma_{11}\varsigma_{12}^2\varsigma_{13}\varsigma_{22})\widetilde{K}_{131}$$
$$+ (\varsigma_{11}\varsigma_{13}^3\varsigma_{22} + \varsigma_{12}\varsigma_{13}^3\varsigma_{21} + 3\varsigma_{11}\varsigma_{12}\varsigma_{13}^2\varsigma_{23})\widetilde{K}_{113}$$
$$+ (\varsigma_{11}^4\varsigma_{22} + 4\varsigma_{11}^3\varsigma_{12}\varsigma_{21})\widetilde{K}_{410} + (\varsigma_{11}^4\varsigma_{23} + 4\varsigma_{11}^3\varsigma_{13}\varsigma_{21})\widetilde{K}_{401}$$
$$+ (\varsigma_{12}^4\varsigma_{23} + 4\varsigma_{12}^3\varsigma_{13}\varsigma_{22})\widetilde{K}_{041} + (\varsigma_{12}^4\varsigma_{21} + 4\varsigma_{11}^3\varsigma_{12}\varsigma_{22})\widetilde{K}_{140}$$
$$+ (\varsigma_{13}^4\varsigma_{21} + 4\varsigma_{11}^3\varsigma_{13}\varsigma_{23})\widetilde{K}_{104} + (\varsigma_{13}^4\varsigma_{22} + 4\varsigma_{12}^3\varsigma_{13}\varsigma_{23})\widetilde{K}_{014}$$
$$+ (2\varsigma_{11}^3\varsigma_{12}\varsigma_{22} + 3\varsigma_{11}^2\varsigma_{12}^2\varsigma_{21})\widetilde{K}_{320} + (2\varsigma_{11}^3\varsigma_{13}\varsigma_{23} + 3\varsigma_{11}^2\varsigma_{13}^2\varsigma_{21})\widetilde{K}_{302}$$
$$+ (2\varsigma_{11}^3\varsigma_{12}\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{13}^2\varsigma_{22})\widetilde{K}_{032} + (2\varsigma_{11}\varsigma_{12}^3\varsigma_{21} + 3\varsigma_{11}^2\varsigma_{12}^2\varsigma_{22})\widetilde{K}_{230}$$
$$+ (2\varsigma_{11}\varsigma_{13}^3\varsigma_{21} + 3\varsigma_{11}^2\varsigma_{13}^2\varsigma_{23})\widetilde{K}_{203} + (2\varsigma_{12}\varsigma_{13}^3\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{13}^2\varsigma_{23})\widetilde{K}_{023},$$

$$\widetilde{s}_{032}(0) = 10\varsigma_{21}^5\widetilde{K}_{500} + 10\varsigma_{22}^5\widetilde{K}_{050} + 10\varsigma_{23}^5\widetilde{K}_{005}$$

$$+ (\varsigma_{21}^3 \bar\varsigma_{22}\bar\varsigma_{23} + 3\varsigma_{21}^3\varsigma_{22}\bar\varsigma_{23} + 3\varsigma_{21}^3\varsigma_{23}\bar\varsigma_{22} + 3\varsigma_{21}^3\varsigma_{22}\varsigma_{23})\widetilde{K}_{311}$$
$$+ (4\varsigma_{21}\varsigma_{22}^3\varsigma_{23} + 6\varsigma_{21}\varsigma_{22}^3\varsigma_{23})\widetilde{K}_{131} + (4\varsigma_{21}\varsigma_{23}^3\varsigma_{22} + 6\varsigma_{21}\varsigma_{22}^3\varsigma_{23})\widetilde{K}_{113}$$
$$+ (4\varsigma_{21}^4\varsigma_{22} + 6\varsigma_{21}^4\bar\varsigma_{22})\widetilde{K}_{410} + (4\varsigma_{21}^4\varsigma_{23} + 6\varsigma_{22}^4\bar\varsigma_{23})\widetilde{K}_{401}$$
$$+ 10\varsigma_{21}\varsigma_{22}^4\widetilde{K}_{041} + 10\varsigma_{21}\varsigma_{23}^4\widetilde{K}_{140} + 10\varsigma_{22}\varsigma_{23}^4\widetilde{K}_{104}$$
$$+ (4\varsigma_{23}^4\bar\varsigma_{22} + 6\varsigma_{22}^4\varsigma_{23})\widetilde{K}_{014} + (9\varsigma_{21}^3\varsigma_{22}^2 + \varsigma_{21}^3\bar\varsigma_{22}^2)\widetilde{K}_{320}$$
$$+ (9\varsigma_{21}^3\varsigma_{23}^2 + \varsigma_{21}^3\bar\varsigma_{23}^2)\widetilde{K}_{302} + (9\varsigma_{22}^3\varsigma_{23}^2 + \varsigma_{22}^3\bar\varsigma_{23}^2)\widetilde{K}_{032}$$
$$+ 10\varsigma_{21}^2\varsigma_{22}^3\widetilde{K}_{230} + 10\varsigma_{21}^2\varsigma_{23}^3\widetilde{K}_{203} + 10\varsigma_{22}^2\varsigma_{23}^3\widetilde{K}_{023},$$

$$\widetilde{s}_{221}(0) = 30\varsigma_{11}^2\varsigma_{21}^3\widetilde{K}_{500} + 30\varsigma_{12}^2\varsigma_{22}^3\widetilde{K}_{050} + 30\varsigma_{13}^2\varsigma_{23}^3\widetilde{K}_{005} + (3\varsigma_{11}^2\varsigma_{21}\varsigma_{22}\bar\varsigma_{23}$$
$$+ 3\varsigma_{11}^2\varsigma_{21}\varsigma_{23}\bar\varsigma_{22} + 3\varsigma_{11}^2\varsigma_{22}\varsigma_{23}\bar\varsigma_{21} + 3\varsigma_{11}\varsigma_{12}\varsigma_{21}^2\bar\varsigma_{23} + 6\varsigma_{11}\varsigma_{12}^2\varsigma_{21}\varsigma_{23}$$
$$+ 3\varsigma_{11}\varsigma_{13}\varsigma_{21}^2\bar\varsigma_{22} + 6\varsigma_{11}\varsigma_{13}\varsigma_{21}^2\varsigma_{22} + 3\varsigma_{12}\varsigma_{13}\varsigma_{21}^3)\widetilde{K}_{311} + (3\varsigma_{11}\varsigma_{12}\varsigma_{22}^2\bar\varsigma_{23}$$
$$+ 6\varsigma_{11}\varsigma_{12}\varsigma_{22}^2\varsigma_{23} + 3\varsigma_{12}^2\varsigma_{21}\varsigma_{22}\varsigma_{23} + 3\varsigma_{12}^2\varsigma_{21}\varsigma_{22}\bar\varsigma_{23} + 6\varsigma_{12}^2\varsigma_{21}\varsigma_{23}\bar\varsigma_{22}$$
$$+ 3\varsigma_{12}\varsigma_{13}\varsigma_{21}^2\varsigma_{22} + 6\varsigma_{12}\varsigma_{13}\varsigma_{21}^2\varsigma_{22} + 3\varsigma_{11}\varsigma_{13}\varsigma_{22}^3)\widetilde{K}_{131} + (6\varsigma_{11}\varsigma_{13}\varsigma_{22}^2\varsigma_{23}$$
$$+ 3\varsigma_{11}\varsigma_{13}\varsigma_{23}^2\bar\varsigma_{22} + 3\varsigma_{12}\varsigma_{13}\varsigma_{21}\varsigma_{23}^2 + 6\varsigma_{12}\varsigma_{13}\varsigma_{21}\varsigma_{23}^2 + 3\varsigma_{12}^2\varsigma_{13}\varsigma_{21}\varsigma_{22}$$
$$+ 3\varsigma_{13}^2\varsigma_{21}\varsigma_{22}\bar\varsigma_{23} + 6\varsigma_{13}^2\varsigma_{21}\varsigma_{23}\bar\varsigma_{22} + 3\varsigma_{11}\varsigma_{12}\varsigma_{23}^3)\widetilde{K}_{113}$$
$$+ (12\varsigma_{11}^2\varsigma_{21}^2\varsigma_{22} + 6\varsigma_{11}^2\varsigma_{21}^2\bar\varsigma_{22} + 12\varsigma_{11}\varsigma_{12}\varsigma_{21}^3)\widetilde{K}_{410}$$
$$+ (12\varsigma_{11}^2\varsigma_{21}^2\varsigma_{23} + 6\varsigma_{11}^2\varsigma_{21}^2\bar\varsigma_{23} + 12\varsigma_{11}\varsigma_{13}\varsigma_{21}^3)\widetilde{K}_{401}$$
$$+ (12\varsigma_{12}^2\varsigma_{22}^2\varsigma_{23} + 6\varsigma_{12}^2\varsigma_{22}^2\bar\varsigma_{23} + 12\varsigma_{12}\varsigma_{13}\varsigma_{22}^3)\widetilde{K}_{041}$$
$$+ (12\varsigma_{11}\varsigma_{12}\varsigma_{22}^3 + 6\varsigma_{12}^2\varsigma_{21}\varsigma_{22}^2 + 12\varsigma_{12}^2\varsigma_{21}\varsigma_{22}^2)\widetilde{K}_{140}$$
$$+ (12\varsigma_{11}\varsigma_{13}\varsigma_{22}^3 + 6\varsigma_{13}^2\varsigma_{21}\varsigma_{23}^2 + 12\varsigma_{13}^2\varsigma_{21}\varsigma_{23}^2)\widetilde{K}_{104}$$
$$+ (12\varsigma_{12}\varsigma_{13}\varsigma_{23}^3 + 6\varsigma_{13}^2\varsigma_{23}\varsigma_{22}^2 + 12\varsigma_{13}^2\varsigma_{22}\varsigma_{23}^2)\widetilde{K}_{014}$$
$$+ (9\varsigma_{11}^2\varsigma_{21}^2\varsigma_{23}^2 + 12\varsigma_{11}\varsigma_{12}\varsigma_{21}^2\varsigma_{22} + 6\varsigma_{11}\varsigma_{12}\varsigma_{21}^2\bar\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{21}^3)\widetilde{K}_{320}$$
$$+ (9\varsigma_{11}^2\varsigma_{21}^2\varsigma_{23}^2 + 12\varsigma_{11}\varsigma_{13}\varsigma_{21}^2\varsigma_{23} + 6\varsigma_{11}\varsigma_{13}\varsigma_{21}^2\bar\varsigma_{23} + 3\varsigma_{13}^2\varsigma_{21}^3)\widetilde{K}_{302}$$
$$+ (6\varsigma_{12}^2\varsigma_{23}^2\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{23}^2\bar\varsigma_{22} + 6\varsigma_{12}\varsigma_{13}\varsigma_{22}^2\bar\varsigma_{23} + 12\varsigma_{12}\varsigma_{13}\varsigma_{22}^2\varsigma_{23} + 3\varsigma_{13}^2\varsigma_{22}^3)\widetilde{K}_{032}$$
$$+ (3\varsigma_{11}^2\varsigma_{22}^3 + 18\varsigma_{11}\varsigma_{12}\varsigma_{21}\varsigma_{22}^2 + 6\varsigma_{12}^2\varsigma_{21}^2\varsigma_{22} + 3\varsigma_{12}^2\varsigma_{21}^2\bar\varsigma_{22})\widetilde{K}_{230}$$
$$+ (3\varsigma_{11}^2\varsigma_{23}^3 + 18\varsigma_{11}\varsigma_{13}\varsigma_{21}\varsigma_{23}^2 + 6\varsigma_{13}^2\varsigma_{21}^2\varsigma_{23} + 3\varsigma_{13}^2\varsigma_{21}^2\bar\varsigma_{23})\widetilde{K}_{203}$$
$$+ (3\varsigma_{12}^2\varsigma_{23}^3 + 12\varsigma_{12}\varsigma_{13}\varsigma_{22}\varsigma_{23}^2 + 6\varsigma_{12}\varsigma_{13}\varsigma_{23}^2\bar\varsigma_{22} + 3\varsigma_{13}^2\varsigma_{22}^2\varsigma_{23} + 6\varsigma_{13}^2\varsigma_{22}^2\varsigma_{23})\widetilde{K}_{023},$$

$$\widetilde{s}_{021}(0) = 3\varsigma_{11}^2\varsigma_{21}\widetilde{K}_{030} + 3\varsigma_{12}^2\varsigma_{22}\widetilde{K}_{003} + 3\varsigma_{13}^2\varsigma_{23}\widetilde{K}_{003}$$
$$(\varsigma_{11}^2\varsigma_{22} + 2\varsigma_{11}\varsigma_{12}\varsigma_{21})\widetilde{K}_{210} + (\varsigma_{11}^2\varsigma_{23} + 2\varsigma_{11}\varsigma_{13}\varsigma_{21})\widetilde{K}_{201}$$
$$(\varsigma_{12}^2\varsigma_{23} + 2\varsigma_{12}\varsigma_{13}\varsigma_{23})\widetilde{K}_{021} + (\varsigma_{12}^2\varsigma_{21} + 2\varsigma_{11}\varsigma_{12}\varsigma_{23})\widetilde{K}_{120}$$
$$(\varsigma_{13}^2\varsigma_{21} + 2\varsigma_{11}\varsigma_{13}\varsigma_{23})\widetilde{K}_{102} + (\varsigma_{13}^2\varsigma_{22} + 2\varsigma_{12}\varsigma_{13}\varsigma_{23})\widetilde{K}_{012}$$
$$+ (\varsigma_{11}\varsigma_{12}\varsigma_{23} + \varsigma_{11}\varsigma_{13}\varsigma_{22} + \varsigma_{12}\varsigma_{13}\varsigma_{21})\widetilde{K}_{111},$$

$$\widetilde{H}_{jkl} = \varsigma_{31}(0)f_{jkl}(0) + \varsigma_{32}(0)g_{jkl}(0) + \varsigma_{33}(0)h_{jkl}(0)(3 \le j + k + l \le 5),$$

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$$\widetilde{K}_{jkl} = \varsigma_{41}(0)f_{jkl}(0) + \varsigma_{42}(0)g_{jkl}(0) + \varsigma_{43}(0)h_{jkl}(0)(3 \leq j + k + l \leq 5).$$

9 $M_i(\Gamma, \Sigma)$ and $W_i(\Gamma, \Sigma, \Upsilon)$ in (3.23)

$$M(\Gamma, \Sigma) = (M_1, M_2, M_3)^T, \quad W(\Gamma, \Sigma, \Upsilon) = (W_1, W_2, W_3)^T,$$

where

$$M_1 = \hat{f}_{200}\hat{x}_1\hat{y}_1 + \hat{f}_{020}\hat{x}_2\hat{y}_2 + \hat{f}_{002}\hat{x}_3\hat{y}_3 + \hat{f}_{110}\hat{x}_1\hat{y}_2 + \hat{f}_{101}\hat{x}_1\hat{y}_3 + \hat{f}_{011}\hat{x}_2\hat{y}_3,$$

$$M_2 = \hat{g}_{200}\hat{x}_1\hat{y}_1 + \hat{g}_{020}\hat{x}_2\hat{y}_2 + \hat{g}_{002}\hat{x}_3\hat{y}_3 + \hat{g}_{110}\hat{x}_1\hat{y}_2 + \hat{g}_{101}\hat{x}_1\hat{y}_3 + \hat{g}_{011}\hat{x}_2\hat{y}_3,$$

$$M_3 = \hat{h}_{200}\hat{x}_1\hat{y}_1 + \hat{h}_{020}\hat{x}_2\hat{y}_2 + \hat{h}_{002}\hat{x}_3\hat{y}_3 + \hat{h}_{110}\hat{x}_1\hat{y}_2 + \hat{h}_{101}\hat{x}_1\hat{y}_3 + \hat{h}_{011}\hat{x}_2\hat{y}_3,$$

$$W_1 = \hat{f}_{300}\hat{x}_1\hat{y}_1\hat{z}_1 + \hat{f}_{030}\hat{x}_2\hat{y}_2\hat{z}_2 + \hat{f}_{003}\hat{x}_3\hat{y}_3\hat{z}_3 + \hat{f}_{210}\hat{x}_1\hat{y}_1\hat{z}_2 + \hat{f}_{201}\hat{x}_1\hat{y}_1\hat{z}_3 + \hat{f}_{021}\hat{x}_2\hat{y}_2\hat{z}_3$$
$$+ \hat{f}_{120}\hat{x}_1\hat{y}_2\hat{z}_2 + \hat{f}_{102}\hat{x}_1\hat{y}_3\hat{z}_3 + \hat{f}_{012}\hat{x}_2\hat{y}_3\hat{z}_3 + \hat{f}_{111}\hat{x}_1\hat{y}_2\hat{z}_3,$$

$$W_2 = \hat{g}_{300}\hat{x}_1\hat{y}_1\hat{z}_1 + \hat{g}_{030}\hat{x}_2\hat{y}_2\hat{z}_2 + \hat{g}_{003}\hat{x}_3\hat{y}_3\hat{z}_3 + \hat{g}_{210}\hat{x}_1\hat{y}_1\hat{z}_2 + \hat{g}_{201}\hat{x}_1\hat{y}_1\hat{z}_3 + \hat{g}_{021}\hat{x}_2\hat{y}_2\hat{z}_3$$
$$+ \hat{g}_{120}\hat{x}_1\hat{y}_2\hat{z}_2 + \hat{g}_{102}\hat{x}_1\hat{y}_3\hat{z}_3 + \hat{g}_{012}\hat{x}_2\hat{y}_3\hat{z}_3 + \hat{g}_{111}\hat{x}_1\hat{y}_2\hat{z}_3,$$

$$W_3 = \hat{h}_{300}\hat{x}_1\hat{y}_1\hat{z}_1 + \hat{h}_{030}\hat{x}_2\hat{y}_2\hat{z}_2 + \hat{h}_{003}\hat{x}_3\hat{y}_3\hat{z}_3 + \hat{h}_{210}\hat{x}_1\hat{y}_1\hat{z}_2 + \hat{h}_{201}\hat{x}_1\hat{y}_1\hat{z}_3 + \hat{h}_{021}\hat{x}_2\hat{y}_2\hat{z}_3$$
$$+ \hat{h}_{120}\hat{x}_1\hat{y}_2\hat{z}_2 + \hat{h}_{102}\hat{x}_1\hat{y}_3\hat{z}_3 + \hat{h}_{012}\hat{x}_2\hat{y}_3\hat{z}_3 + \hat{h}_{111}\hat{x}_1\hat{y}_2\hat{z}_3,$$

and

$$\hat{G}_1 = -a_{11} + \frac{a_{13}\alpha w}{(1+\alpha u)^2}, \quad \hat{f}_{200} = \hat{G}_1 + \frac{1}{2}u\hat{G}_1 - \frac{a_{13}\alpha^2 uw}{(1+\alpha u)^3}, \quad \hat{f}_{020} = \frac{1}{2}a_{12}^2 u,$$

$$\hat{f}_{002} = \frac{a_{13}^2 u}{2(1+\alpha u)^2}, \quad \hat{f}_{110} = -a_{12}(1 + u\hat{G}_1), \quad \hat{f}_{101} = \frac{a_{13}\alpha u}{(1+\alpha u)^2} - \frac{a_{13}(1 + u\hat{G}_1)}{1+\alpha u},$$

$$\hat{f}_{011} = \frac{a_{12}a_{13}u}{1+\alpha u}, \quad \hat{f}_{300} = \frac{a_{13}\alpha^3 uw}{(1+\alpha u)^4} - \frac{a_{13}\alpha^2 w(1 + u\hat{G}_1)}{(1+\alpha u)^3} - \frac{1}{6}\hat{G}_1^2(2 + u\hat{G}_1),$$

$$\hat{f}_{030} = -\frac{1}{6}a_{12}^3 u, \quad \hat{f}_{003} = -\frac{a_{13}^3 u}{6(1+\alpha u)^3}, \quad \hat{f}_{210} = \frac{1}{2}a_{12}(2 + u\hat{G}_1)\hat{G}_1,$$

$$\hat{f}_{021} = \frac{a_{12}a_{13}u}{2(1+\alpha u)},$$

$$\hat{f}_{201} = \frac{2a_{13}^2\alpha^2 uw}{(1+\alpha u)^4} - \frac{a_{13}\alpha^2 u}{(1+\alpha u)^3} + \frac{a_{13}\alpha(1 + u\hat{G}_1)}{(1+\alpha u)^2} - \frac{a_{13}(1 + u\hat{G}_1)G_1}{2(1+\alpha u)},$$

$$\hat{f}_{120} = \frac{1}{2}a_{12}(1 + u\hat{G}_1), \quad \hat{f}_{012} = -\frac{a_{12}a_{13}^2 u}{2(1+\alpha u)^2},$$

$$\hat{f}_{102} = \frac{a_{13}^2(1 + u\hat{G}_1)}{2(1+\alpha u)^2} - \frac{a_{13}^2\alpha u}{(1+\alpha u)^3}, \quad \hat{f}_{111} = \frac{a_{12}a_{13}(1 + u\hat{G}_1)}{1+\alpha u} - \frac{a_{12}a_{13}\alpha u}{(1+\alpha u)^2},$$

$$\hat{g}_{200} = \frac{1}{2}a_{21}v, \quad \hat{g}_{020} = -\frac{1}{2}a_{22}(2 - a_{22}v), \quad \hat{g}_{002} = \frac{1}{2}a_{23}^2 v,$$

$$\hat{g}_{110} = a_{21}(1 - a_{22}v), \quad \hat{g}_{101} = -a_{21}a_{23}v, \quad \hat{g}_{011} = -(1 - a_{22}v)a_{23},$$

$$\hat{g}_{300} = \frac{1}{6}a_{21}^3 v, \quad \hat{g}_{030} = \frac{1}{6}(3 - a_{22}v)a_{22}^2, \quad \hat{g}_{003} = -\frac{1}{6}a_{23}^3 v,$$

$$\hat{g}_{210} = \frac{1}{2}a_{21}^2(1 - a_{22}v), \quad \hat{g}_{201} = -\frac{1}{2}a_{21}a_{23}v, \quad \hat{g}_{021} = \frac{1}{2}a_{22}a_{23}(2 - a_{22}v),$$

$$\hat{g}_{120} = -\frac{1}{2}a_{21}a_{22}(2 - a_{22}v), \quad \hat{g}_{102} = \frac{1}{2}a_{21}a_{23}^2 v, \quad \hat{g}_{012} = \frac{1}{2}(1 - a_{22}v)a_{23}^2,$$

$$\hat{g}_{111} = -a_{21}(1 - a_{22}v)a_{23}, \quad \hat{h}_{200} = \frac{a_{31}^2 w}{2(1 + \alpha u)^4} - \frac{a_{31}\alpha w}{(1 + \alpha u)^3},$$

$$\hat{h}_{020} = \frac{1}{2}a_{32}^2 w, \quad \hat{h}_{002} = \frac{1}{2}(a_{33}w - 2)a_{33}, \quad \hat{h}_{110} = \frac{a_{31}a_{32} w}{(1 + \alpha u)^2},$$

$$\hat{h}_{101} = \frac{a_{31}(1 - a_{33}w)}{(1 + \alpha u)^2}, \quad \hat{h}_{011} = a_{32}(1 - a_{33}w),$$

$$\hat{h}_{300} = \frac{a_{31}^3 w}{6(1 + \alpha u)^6} - \frac{a_{31}^2 \alpha w}{(1 + \alpha u)^5} + \frac{a_{31}\alpha^2 w}{(1 + \alpha u)^4}, \quad \hat{h}_{030} = \frac{1}{6}a_{32}^3 w,$$

$$\hat{h}_{003} = \frac{1}{6}(3 - a_{33}w)a_{33}^2, \quad \hat{h}_{210} = \frac{a_{31}a_{32}\alpha w}{(1 + \alpha u)^3} + \frac{a_{31}^2 a_{32} w}{2(1 + \alpha u)^4},$$

$$\hat{h}_{201} = -\frac{1}{2}\left[\frac{a_{31}^2}{(1 + \alpha u)^4} + \frac{2a_{31}\alpha w}{(1 + \alpha u)^3}\right](1 - a_{33}w), \quad \hat{h}_{021} = \frac{1}{2}a_{32}^2(1 - a_{33}w),$$

$$\hat{h}_{120} = \frac{a_{31}a_{32}^2 w}{2(1 + \alpha u)^2}, \quad \hat{h}_{102} = -\frac{a_{31}a_{33}}{2(1 + \alpha u)^2}(2 - a_{33}w),$$

$$\hat{h}_{012} = -\frac{1}{2}a_{32}a_{33}(2 - a_{33}w), \quad \hat{h}_{111} = \frac{a_{31}a_{32}}{(1 + \alpha u)^2}(1 - a_{33}w).$$

10 e_{20}, e_{11}, e_{02}, e_{30}, e_{12}, e_{03}, e_{21}, d_{20}, d_{11} and d_{02} in (3.27)

$$e_{20} = \frac{1}{2}\langle \widetilde{p}_1, M(\widetilde{q}_1, \widetilde{q}_1)\rangle, \quad e_{02} = \frac{1}{2}\langle \widetilde{p}_1, M(\widetilde{q}_2, \widetilde{q}_2)\rangle, \quad e_{11} = \langle \widetilde{p}_2, M(\widetilde{q}_1, \widetilde{q}_2)\rangle,$$

$$e_{30} = \frac{1}{6}\langle \widetilde{q}_1, W(\widetilde{q}_1, \widetilde{q}_1, \widetilde{q}_1) + 3M(\widetilde{q}_1, d_{20})\rangle,$$

$$e_{12} = \frac{1}{2}\langle \widetilde{q}_1, W(\widetilde{q}_1, \widetilde{q}_2, \widetilde{q}_2) + M(\widetilde{q}_1, d_{02}) + 2M(\widetilde{q}_2, d_{11})\rangle,$$

$$e_{03} = \frac{1}{6}\langle \widetilde{p}_2, W(\widetilde{q}_2, \widetilde{q}_2, \widetilde{q}_2) + 3M(\widetilde{q}_2, d_{02})\rangle,$$

$$e_{21} = \frac{1}{2}\langle \widetilde{p}_2, W(\widetilde{q}_1, \widetilde{q}_1, \widetilde{q}_2) + M(\widetilde{q}_2, d_{20}) + 2M(\widetilde{q}_1, d_{11})\rangle,$$

$$d_{20} = (J - I_3)^{-1}(2e_{20}\widetilde{q}_1 - M(\widetilde{q}_1, \widetilde{q}_1)), \quad d_{11} = (J + I_3)^{-1}(e_{11}\widetilde{q}_2 - M(\widetilde{q}_1, \widetilde{q}_2)),$$

$$d_{02} = (J - I_3)^{-1}(2e_{02}\widetilde{q}_1 - M(\widetilde{q}_2, \widetilde{q}_2)).$$

References

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