APS 502 Financial Engineering Final Exam Winter 2018

Instructions: Closed book and closed notes except for both sides of a 3 by 5 inch notecard. Only a simple scientific non-financial calculator with no programming capability is allowed. Please write neatly as this will aid in providing maximum partial credit. **IMPORTANT: Interpretation of the exam questions is part of the exam and so no questions will be taken that ask for clarification of the question. You must turn in this question sheet with your answer booklet or else your exam will NOT be marked. SHOW ALL WORK.**

Problem 1 (15 points, 3 points each)

You are the manager of a pension plan that pays out 10 million dollars a year forever. The first payment is exactly one year from now. The term structure is flat at 5% i.e. $s_t = 0.05$ for all years t.

(a) Compute the present value of the pension liabilities.

(b) Suppose that the interest rate goes down by 0.1%. How does the value of your liability change?

(c) Given your answer to (b), what is the modified duration of the pension liability?

(d) Suppose that the pension plan is fully funded (i.e. the value of your assets equal the value of pension liabilities). You want to invest all your assets in bonds in a way so that you can avoid any interest rate risk. What should the duration of the bond portfolio be?

(e) Suppose that your bond portfolio will consist of a single zero coupon bond. What should the maturity and the face value be?

Problem 2 (12 points, 6 points each)

The yield to maturity of a 10 year zero coupon bond is 4%.

(a) Suppose that you buy the bond today and hold it for 10 years. What is your return? (Note: You must express this return as an annual rate.)

(b) Given only the information provided, can you compute the return on the bond if you hold the bond only for 5 years? If you answered yes, compute the return else explain why you answered no.

Problem 3 (21 points, 3 points each)

Consider a financial market with the three assets A, B, and C where A is the market portfolio and C is the risk-free asset. The expected return of the market portfolio is 12% i.e. $\mu_A = 0.12$ and the standard deviation is 20% i.e. $\sigma_A = 0.20$. The risk-free asset returns 4%. Asset B is a risky asset whose return has a standard deviation of 40% and a beta of 1. Assume that CAPM holds in this financial market.

(a) Compute the expected return of asset B and its covariances with asset A and asset C, respectively.

(b) Consider a portfolio of A and B with weight α in A and $1-\alpha$ in asset B. Compute the expected return and standard deviation of the portfolios that result from setting $\alpha = 0, 0.5$, and 1,respectively, and enter them into the following table:

α	0	0.5	1
expected return			
standard deviation			

(c) Can you rank the three portfolios from (b)? Explain.

(d) Consider a portfolio with equal weights in asset B and C. Denote this portfolio as asset D. Compute the expected return and standard deviation of asset D.

(e) Consider a portfolio of asset A and C. Find the portfolio weights of A and C such that its standard deviation is the same as in asset D from part (d) above. What is the expected return of this portfolio?

(f) Are any of the assets A, B, and C efficient? Explain.

(g) Construct an efficient portfolio from the assets A, B, and C with an expected return of 10%.

Problem 4 (17 points)

You wish to build a portfolio of securities A, B, and C.The expected returns for each security are $\mu_A = 20\%, \mu_B = 15\%, \mu_C = 10\%$. The co-variance of the securities are given in the following matrix

$$\begin{bmatrix} \sigma_{ij} & A & B & C \\ A & 0.3600 & 0.0840 & 0.1050 \\ B & 0.0840 & 0.1225 & 0.0700 \\ C & 0.1050 & 0.0700 & 0.0625 \end{bmatrix}$$

You wish to find an optimal portfolio that minimizes portfolio variance subject to the goal of getting 16% expected return and to the budget constraint. Also, you will also allow short selling. Prove or disprove that the optimal portfolio is $x_A = 0.2265, x_B = 0.7471$, and $x_C = 0.0264$.

Problem 5 (17 points)

Ms. York's entire stock portfolio currently worth \$500,000 is invested in an index fund (portfolio) that tracks the S&P 500 index. The expected rate of return of the index fund is 9.5% and standard deviation is 18% per year. The one year risk-free rate is 2%.

Now Ms. York receives a strongly favorable security analyst's report on Blueberry Inc. The analyst projects a return of 25%. Blueberry Inc. has a high volatility (40% standard deviation) but its correlation coefficient with the S&P 500 is only 0.3.

Assume the analyst's forecast is unbiased and assume that the index fund tracks the S&P 500 very well. Should Ms. York sell part of her index fund holdings and invest in Blueberry Inc.? If so, how much should she invest in Blueberry? Note that Ms. York can borrow or lend at the 2% risk-free rate.

Problem 6 (10 points, 5 points each)

(a) Portfolios with many assets (i.e. when n = number of assets is large) can be tedious to manage. One way to control the number of assets is to impose a restriction on the number of assets in portfolio optimization models. For example, one could add a cardinality constraint to MVO (mean-variance optimization) models that enforces that exactly k assets out of n assets are in a portfolio where $k < n$. Note that the optimization is over n assets as usual but that feasible portfolios will have exactly k assets. Formulate the MVO model with cardinality constraints. Define any additional variables and other constraints that you need to implement the cardinality constraint.

(b) Consider the cardinality constrained MVO model in (a) for the case $n = 4$ and $k = 2$. Describe (in words) the efficient frontier that one would obtain using such a model and draw a typical efficient frontier.

Problem 7 (8 points, 4 points each)

(a) Consider the following strategy which is executed simultaneously: you buy one share of a stock (underlying asset) and buy one European put option on the same stock with strike price K, and you sell one European call option on the stock with strike price K. Note: The options have the same maturity and suppose the stock does not pay out any cash i.e. dividends.

Calculate the payoff and explain the risk (i.e. the possiblity of losing money) of the strategy.

(b) Consider a European call option and a European put option on the same underlying asset X with both options having a maturity of 3 months, and same strike prices. Suppose each of these 3 month options cost \$10 and that the strike price is \$60.

Which is worth more a 6 month call option on X with strike price \$60 or a 6 month put option on X with strike price \$60. Also, assume X does not pay out any cash during any of the maturities. Prove your answer mathematically.