HW#5 solutions

Chapter 3

Exercise 14

Use the formula to obtain 6.84 years.

Exercise 16

(a)

$$\begin{aligned} P_A &= 885.84, \\ P_B &= 771.68, \\ P_C &= 657.52, \\ P_D &= 869.57 \end{aligned}$$

(b)

$$\begin{aligned} D_A &= 2.72, \\ D_B &= 2.84, \\ D_C &= 3.00, \\ D_D &= 1.00 \end{aligned}$$

(c) C is most sensitive to a change in yield.

(d)

$$\begin{aligned} V_A + V_B + V_C + V_D &= NPV, \\ D_A V_A + D_B V_B + D_C V_C + D_D V_D &= 2NPV, \end{aligned}$$

where NPV is the present value of the obligation.

(e) Use bond D.

$$\begin{aligned} V_C + V_D &= NPV, \\ D_C V_C + D_D V_D &= 2NPV, \end{aligned}$$

where

$$NPV = \frac{2,000}{1.15^2} = \$1,512.29.$$

Solving $V_C = \$756.15$ and $V_D = \$756.15$.

(f) None

Chapter 4

Exercise 1

(One forward rate)

$$f_{1,2} = \frac{(1+s_2)^2}{(1+s_1)} - 1 = \frac{1.069^2}{1.063} - 1 = 7.5\%$$

Exercise 2

(Spot Update)
Use

$$f_{1,k} = \left[\frac{(1+s_k)^k}{1+s_1}\right]^{1/(k-1)} - 1$$

.
Hence, for example,

$$f_{1,k} = \left[\frac{(1.061)^6}{1.05}\right]^{1/5} - 1 = 6.32\%$$

.
All values are

$f_{1,2}$	$f_{1,3}$	$f_{1,4}$	$f_{1,5}$	$f_{1,6}$
5.60	5.90	6.07	6.25	6.32

Exercise 3

(Construction of a zero)
Use a combination of the two bonds: let x be the number of 9% bonds, and y teh number of 7% bonds. Select x and y to satisfy

$$\begin{aligned} 9x + 7y &= 0, \\ x + y &= 1. \end{aligned}$$

The first equation makes the net coupon zero. The second makes the face value equal to 100. These equations give $x = -3.5$, and $y = 4.5$, respectively. The price is $P = -3.5 \times 101.00 + 4.5 \times 93.20 = 65.90$.

Exercise 13

(a) $D_m = -\frac{1}{P}\frac{dP}{d\lambda} \approx -\frac{1}{P}\frac{\Delta P}{\Delta\lambda} = \frac{1}{990}\frac{(1000-990)}{(0.10-0.105)} = 2.0202$
(b) $D = D_m(1+\frac{\lambda}{m}) = 2.0202(1+\frac{1.05}{365}) = 2.02075$

Extra Problem Solution

Let x_i = amount of bond i purchased, i = 1,2,3

y_j = amount of excess cash generated at the end of jth year, j =1,2.

$$
\begin{aligned}
\text{minimize } & 102x_1 + 99x_2 + 98x_3 \\
\text{subject to } & 105x_1 + 3.5x_2 + 3.5x_3 - y_1 = 12000 \\
& 103.5x_2 + 3.5x_3 + 1.02y_1 - y_2 = 18000 \\
& 103.5x_3 + 1.02y_2 = 20000 \\
& x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad y_1 \geq 0, \quad y_2 \geq 0, \quad y_3 \geq 0
\end{aligned}
$$

Solve the model by following Matlab code:

```
%% Exercise 1.14
% excess cash generation
r = .02;%interest rate of reinvestment
c = [102; 99; 98; 0; 0;];
Aeq = [105   3.5   3.5   -1    0;
         0 103.5   3.5 (1+r)  -1;
         0     0 103.5    0 (1+r);];
beq = [12000; 18000; 20000;];
lb = [0;0;0;0;0];
[x_bond, f_bond] = linprog(c, [],[], Aeq,beq, lb,[]);
```

The constructed portfolio is

	x_1	x_2	x_3	y_1	y_2
Investment	102.265208	167.378469	193.236715	0.000000	0.000000
Minimum Total cost	$45938.717690				

The advantage of reinvestment is that the interest can offset partial cost if the interest rate is good enough, for example, if interest is r = 5%, the total cost will reduce to $45876.972625 or bonds that mature earlier may be cheaper and there fore carryover extra purchases could result in lower costs in meeting liabilities.