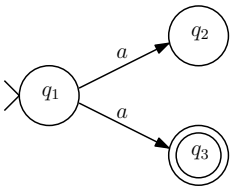
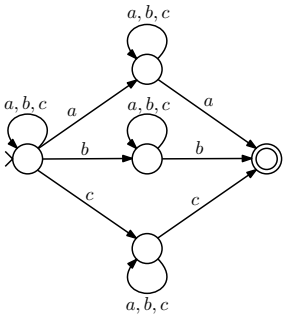
Theory of Computation, Fall 2023
Assignment 2 Solutions

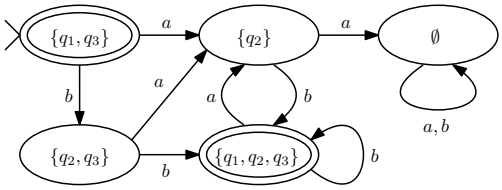
Q1. No. Let M denotes the following NFA over $\Sigma = \{a\}$, then $L(M) \cap L(M') = \{a\} \neq \emptyset$.



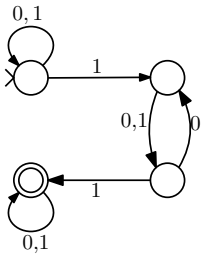
Q2. The NFA guesses at the last symbol of w.



Q3. The DFA is as follows.



Q4. Whenever a 1 is read, it may or may not be the first 1 in the pair. The NFA makes a guess.



Q5. (a) Let $A = \{(p, q) : \delta(p, a) = q \text{ for some } a \in \Sigma\}$. We construct a NFA as $M' = (K, \Sigma, \Delta, s, F)$, where $\Delta = \{(p, a, q) : (p, q) \in A, a \in \Sigma\}$.

Basically, if the DFA M can go from state p to state q when it reads some symbol in Σ, then the NFA M' can go from state p to state q when it reads any symbol in Σ. One can see that

if some string u is accepted by M, then any string with the same length as u will be accepted by M', and vice versa.

(b) One can observe that $D = B \cap C$. Since both B and C are regular, by the closure properties, D is regular.