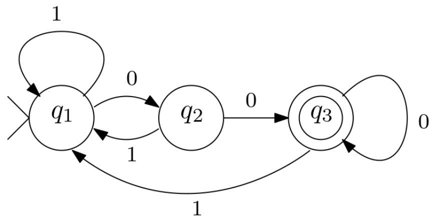
Theory of Computation, Fall 2023
Assignment 1 Solutions

Q1. (a) True. (b) True. (c) False. (d) True. (e) True. (f) True. (g) True.

Q2. L is accepted by the following finite automaton.



Q3. Since A and B are regular, there are two finite automate $M_A = (K_A, \Sigma, \delta_A, s_A, F_A)$ and $M_B = (K_B, \Sigma, \delta_B, s_B, F_B)$ that accept A and B, respectively.

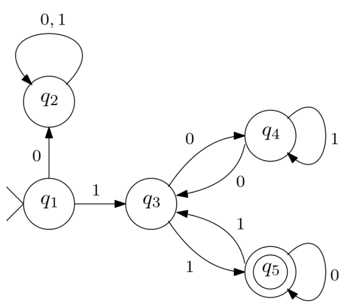
From M_A and M_B, we can construct a finite automaton $M_\cap$ to accept $A \cap B$ as follows. Conceptually, $M_\cap$ runs M_A and M_B in parallel, and accepts the input if both M_A and M_B accept. $M_\cap = (K_\cap, \Sigma, \delta_\cap, s_\cap, F_\cap)$ where

- $K_\cap = K_A \times K_B$,
- $s_\cap = (s_A, s_B)$,
- $F_\cap = F_A \times F_B$, and
- for any $(q_A, q_B) \in K_A \times K_B$ and any $a \in \Sigma$,

$$\delta_\cap((q_A, q_B), a) = (\delta_A(q_A, a), \delta_B(q_B, b)).$$

Q4. Since A is regular, there is a finite automaton $M = (K, \Sigma, \delta, s, F)$ that accepts A. From M, we can construct a finite automaton M' to accept $\overline{A}$ by exchanging the roles of final and non-final states. More precisely, $M' = (K, \Sigma, \delta, s, K \setminus F)$ where $K \setminus F = \{q \in K : q \notin F\}$.

Q5. L is accepted by the following finite automaton.



Q6. Intuitively, we obtain M' from M by adding a new "dead" state q_{dead}. When M' reads a symbol in Σ, it acts exactly the same as M; when M' reads a symbol not in Σ, it enters the dead state. We construct M' as follows.

- $K' = K \cup \{q_{dead}\}$

- $s' = s$
- $F' = F$
- For any $q \in K'$ and any $a \in \Sigma'$,

$$\delta'(q,a) = \begin{cases} q_{dead} & \text{if } q = q_{dead} \text{ or } a \notin \Sigma \\ \delta(q,a) & \text{otherwise} \end{cases}$$

Q7. Let M_A and M_B be two finite automate that accepts A and B, respectively. The finite automaton M_L that accepts L works as follows. Given a input string w, M_L first run M_A. At the moments that the symbol # is read, if M_A is at its final states, M_L switches to M_B. If M_B accepts the remain input, M_L accepts the input. Let $M_A = (K_A, \Sigma, \delta_A, s_A, F_A)$ and $M_B = (K_B, \Sigma, \delta_B, s_B, F_B)$. $M_L = (K_L, \Sigma, \delta_L, s_L, F_L)$ can be constructed as follows.

- $K_L = K_A \cup K_B \cup \{q_{dead}\}$
- $s_L = s_A$
- $F_L = F_B$
- For any $q \in K_L$ and any $a \in \Sigma \cup \{\#\}$,

$$\delta_L(q,a) = \begin{cases} q_{dead} & \text{if } q = q_{dead} \\ \delta_A(q,a) & \text{if } q \in K_A \text{ and } a \neq \# \\ q_{dead} & \text{if } q \notin F_A \text{ and } a = \# \\ s_B & \text{if } q \in F_A \text{ and } a = \# \\ \delta_B(q,a) & \text{if } q \in K_B \text{ and } a \neq \# \end{cases}$$