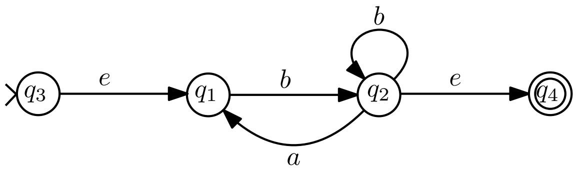
Theory of Computation, Fall 2023
Assignment 3 Solutions

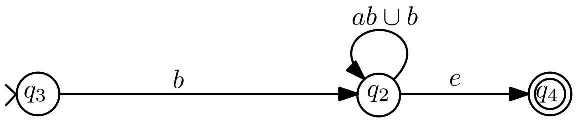
Q1. (a) Ture.
(b) False. $R\emptyset = \emptyset$.
(c) False. When $R = \emptyset$, $R \cup \emptyset^* = \{e\} \neq \emptyset$.
(d) Ture.

Q2. $(a^*ba^*ba^*ba^*)^* \cup a^*$

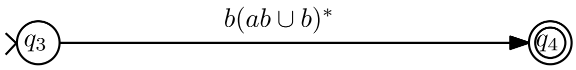
Q3. The regular expression is $b(ab \cup b)^*$



(a) Step 1



(b) Step 2



(c) Step 3

Q4. Let $L = \{ww : w \in a, b^*\}$. Suppose, for the sake of contradiction, that this language is regular. Let p be the pumping length given by the pumping theorem. Consider the string $w = a^p b a^p b \in L$. By pumping theorem, w can be written as $w = xyz$ such that

(i) $xy^i z \in L$ for any $i \geq 0$

(ii) $|y| \geq 0$, and

(iii) $|xy| \leq p$

(ii) and (iii) imply that $y = a^k$ for some $k \geq 0$. Consider the string $xy^0z = a^{p-k}ba^pb$. Clearly, this string does not belong to L. This contradicts with (i). Therefore, L is non-regular.

Q5. Assume that $L_1 = \{0^m 1^n : m \neq n\}$ is regular. Since $L = L(0^*1^*)$ is regular, by the closure property of regular languages, $L - L_1 = \{0^n 1^n : n \geq 0\}$ is regular (because $L - L_1 = L \cap \overline{L}_1$). It is, however, known that $\{0^n 1^n : n \geq 0\}$ is not regular. Contradiction.

Q6. As A is regular, there is a DFA $M = (K, \Sigma, \delta, s, F)$ that accepts A. Consider the following DFA $M' = (K', \Sigma', \delta', s', F')$ where

- $K' = K, \Sigma' = \Sigma, \delta' = \delta, s' = s$ and
- $F' = \{p \in K : (p, w) \vdash_M^* (q, e)$ for some $w \in B$ and some $q \in F\}$

It is straightforward that M' accepts A/B, so A/B is regular.

* Remark: One may notice that, we **cannot** acctually construct M' (in particular, the set F') because B may contain an infinite number of strings. This does not invalidate our proof. To prove that A/B is regular, it suffices to show that existence of M'. There is no need to actually construct it.