Theory of Computation, Fall 2023
Assignment 6 Solutions

Q1. $M = (K, \Sigma, \delta, s, H)$ where

- $K = K_1 \cup K_2 \cup K_3 \cup \{h\}$,
- $s = s_1$,
- $H = H_2 \cup H_3 \cup \{h\}$, and
- for $q \in K - H$ and for $c \in \Sigma$,

$$\delta(q, c) = \begin{cases} \delta_1(q, c) & \text{if } q \in K_1 - H_1 \\ \delta_2(q, c) & \text{if } q \in K_2 - H_2 \\ \delta_3(q, c) & \text{if } q \in K_3 - H_3 \\ (s_2, a) & \text{if } q \in H_1 \text{ and } c = a \\ (s_3, b) & \text{if } q \in H_1 \text{ and } c = b \\ (h, \leftarrow) & \text{if } q \in H_1 \text{ and } c \in \Sigma - \{a, b\} \end{cases}$$

Q2. The Turing machine is as follows.

$$\begin{tikzcd} > L \arrow[r, "a \neq \sqcup"] \arrow[d, "\sqcup"] & \sqcup R a L_{\sqcup} \arrow[l, bend right] \\ RR_{\sqcup} & \end{tikzcd}$$

Q3. (a) True. Every Turing machine semidecides exactly one language, which is L(M).
(b) False. If a Turing machines does not always halt, then it does not decides any language.

Q4. Since M decides some language, it halts on every input. Therefore, L(M) is the set of all strings over the input alphabet.

Q5. Since L is a recursive language, it is decided by some Turing machine $M = (K, \Sigma, \delta, s, \{y, n\})$. We can obtain a Turing machine that decides $\overline{L}$ by exchanging the role of y and n, so $\overline{L}$ is recursive.

Q6. (a) $A_w = \{w : D \text{ accepts } w\}$

(b) Yes. A_w can be decided by the the following Turing machine.

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M = on input w :
run D on w
if D accepts w
    accept w
else
    reject w
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(c) Note that $A_w = L(D)$. Since D is an arbitrary DFA, it follows that any regular language is recursive.

Q7. Suppose Turing machine M decides EQ_{DFA}. We construct a Turing machine M' that decides A_L as follows.

M' = on input "D" :

1. construct a DFA D_0 with $L(D_0) = L$
2. run M_{EQ} on "D" "D_0"
3. output the result

The reduction is $f(\text{“}D\text{”}) = \text{“}D\text{”}\,\text{“}D_0\text{”}$ where $L(D_0) = L$.