Analysis 1B — Supplementary Paper 2020

Christian Jones: University of Bath

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Introduction

Here are the solutions to the past paper discussed in the revision session on 9th May 2023. This is designed as a guide to how much to write in the exam, and how you might want to style your solutions. To return to the homepage, click here.

Question 1

Question. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \frac{2x^2 - 2}{x^2 + x + 2}.$$

a) Using the $\epsilon - \delta$ definition, prove that

$$\lim_{x \to 0} f(x) = -1.$$

b) Using the *Algebra of Limits* and the *Sequential Characterisation of Limits*, show that

$$\lim_{x \to \pm 1} f(x) = 0.$$

Solution. a) Fix $\epsilon > 0$ and suppose $0 < |x - 0| < \delta$ for some $\delta > 0$ to be chosen later. Then

$$\begin{aligned} |f(x) - (-1)| &= \left| \frac{2x^2 - 2}{x^2 + x + 2} - (-1) \right|, \\ &= \frac{|x||3x + 1|}{|x^2 + x + 2|}. \end{aligned}$$

Now,

$$\begin{aligned} |3x + 1| &\leq 3|x| + 1 && \text{(by the triangle ineq.)}, \\ &< 4 && \text{(if } \delta < 1\text{)}. \end{aligned}$$