CER 2

Game Theory

Part 1

Date: 23/09/2024

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Fundamental Engineering Sciences

Prosit Heading: Energy Challenge for the Paris Olympics

Date: 23/09/2024

Roles:

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1) Understanding the Topic and Clarifying

1.1) Keywords:

- ~~Blackout~~
  - **Definition**: A blackout refers to a complete power failure over a large area or an electrical grid, caused by overload, equipment failure, or natural disasters.
  - **Example**: In 2003, the Northeast blackout affected parts of the U.S. and Canada, leaving 55 million people without power.
  - **Theorem/Principle**: Power systems adhere to the principle of balance between supply and demand. When supply cannot meet demand, voltage drops, potentially leading to a system-wide failure.
  - **Application**: Energy companies invest in infrastructure to prevent blackouts through load balancing and smart grids.

~~Energy Grid~~

- **Definition**: An energy grid (or power grid) is a network of transmission lines, transformers, and distribution systems that deliver electricity from power plants to homes and businesses.

- **Example**: The United States has three major grids: the Eastern Interconnection, the Western Interconnection, and the Texas Interconnection.
- **Theorem**: Kirchhoff's Laws govern electrical circuits, which are essential in designing and managing energy grids.
- **Application**: Grid optimization models use algorithms to minimize transmission losses and ensure efficient distribution of electricity across regions.

Equilibrium

- **Definition**: In general, equilibrium refers to a state of balance. In economics, it refers to the state where supply equals demand. In physics, it's the state in which all forces are balanced, leading to no net change.
- **Example**: In a market equilibrium, the quantity of goods supplied equals the quantity demanded at a certain price level.
- **Theorem**: Nash Equilibrium in game theory refers to a situation where no player can benefit by changing their strategy while the other players keep their strategies unchanged.
- **Application**: In economics, equilibrium prices and quantities help determine the optimal distribution of resources.

Profit

- **Definition**: Profit is the financial gain obtained when revenue from selling goods or services exceeds the costs associated with production.
- **Example**: A company sells a product for $10, and its production cost is $7. The profit is $3 per unit.
- **Formula**: Profit = Total Revenue – Total Costs
- **Application**: Businesses use profit margins to evaluate the efficiency of their operations and pricing strategies.

Losses

- **Definition**: Losses occur when the total costs exceed the total revenue, resulting in a negative financial outcome.
- **Example**: If a company spends $100,000 in production but earns $80,000 in sales, the loss is $20,000.
- **Theorem**: In finance, the break-even point theorem defines the point where profits equal losses, implying no net financial change.

- **Application**: Firms aim to minimize losses through cost-cutting, operational efficiency, or changes in strategy.

Optimal Strategy

- **Definition**: An optimal strategy is the best possible plan of action designed to achieve a goal, maximizing benefits or minimizing costs, given the constraints.
- **Example**: In chess, an optimal strategy maximizes the chance of winning based on the current board position.
- **Theorem**: In game theory, the concept of a **dominant strategy** refers to a strategy that always provides better outcomes regardless of the opponent's actions.
- **Application**: Businesses use optimal strategies in pricing, marketing, and production to maximize profits.

Simultaneously

- **Definition**: Simultaneously means occurring at the same time.
- **Example**: In a simultaneous game in game theory, all players make their decisions without knowing the others' choices.
- **Theorem**: The **simultaneous move game** principle states that in certain games, players choose strategies at the same time, leading to a strategic interdependence.
- **Application**: In markets, companies often set prices simultaneously based on competitors' expected reactions, influencing overall market dynamics.

Game Theoretic Problem

- **Definition**: A game theoretic problem involves decision-making scenarios where the outcome for each participant depends not only on their actions but also on the actions of others. Players use strategies to maximize their outcomes.
- **Example**: The **Prisoner's Dilemma** is a classic game-theoretic problem where two individuals must decide whether to cooperate or betray without knowing the other's choice.
- **Theorem**: Nash Equilibrium is a central theorem in game theory, where each player's strategy is optimal given the strategies of the others, leading to a stable outcome.
- **Application**: Game theory is used in economics, military strategy, political science, and biology to study competition, cooperation, and strategic behaviour among entities.

1.2) Context:

For the Paris 2024 Olympic. There is a high demand in electricity, so we need to make sure that there is no blackout during the event.

2) Needs Analysis:

2.1) Issues:

How to determine an optimal strategic equilibrium for two parties?

2.2) Constraints:

- Time and cost
- Both parties have personal interest
- Limited by the data.
- table of data (disorganized)

2.3) Deliverable:

A Visual response e.g. graph

A report showing how those 2 parties can have a gain.

3) Generalization:

Game theory

4) Solution Tracks:

The safest for Marc is to rent the generator for 70K.

The safest for Nicole is to rent the generator for 70K.

There is a 0 percent chance of a blackout.

Consider the probability of cloudy skies.

Calculate the cost of the potential breakdown.

5) Developing the Action Plan:

1. Define keywords.
2. Exercise Basket / WS.
3. Resource Analysis.
4. Calculate the gain and losses for every parties
5. Calculate the risk for both side
6. Get optimal strategic equilibrium
7. Validation of assumptions.
8. Conclusion.

6) Resource Analysis:

2.1) Dominance

In **game theory**, **dominance** refers to a comparison between strategies for players, determining whether one strategy is better than another, regardless of what the other players do. A strategy is said to be **dominant** if it always results in a better (or at least no worse) payoff for a player, no matter what the other players' strategies are. There are two types of dominance:

1. Strict Dominance

- **Definition**: A strategy strictly dominates another if it always results in a strictly better payoff for the player, regardless of what the other players choose.
- **Example**: Suppose two players can choose between two strategies: A and B. If choosing strategy, A always results in a better outcome for Player 1, no matter what Player 2 does, then strategy A strictly dominates strategy B.
- **Application**: Players can eliminate strictly dominated strategies from consideration, simplifying the decision-making process.

2. Weak Dominance

- **Definition**: A strategy weakly dominates another if it always results in a payoff at least as good as the other strategy and sometimes results in a better payoff, depending on what the other players do.
- **Example**: If Strategy A sometimes leads to a higher payoff than Strategy B and never leads to a lower payoff, then A weakly dominates B.
- **Application**: Although weakly dominated strategies might not be eliminated outright, rational players tend to prefer strategies that dominate others.

Dominant Strategy

- **Definition**: A strategy is a **dominant strategy** if it is the best strategy for a player, regardless of the strategies chosen by other players.
- **Example**: In the **Prisoner's Dilemma**, both prisoners have a dominant strategy to betray the other, as betraying yields a better outcome for each prisoner, no matter what the other does.

- **Application**: In many economic or competitive scenarios, the concept of a dominant strategy helps predict player behaviour, as rational players tend to follow dominant strategies when they exist.

Dominance and Nash Equilibrium

- **Relationship**: Nash equilibria often arise when each player's strategy is optimal given the others' strategies. In games where players have dominant strategies, a Nash equilibrium can be easily identified because each player will choose their dominant strategy.
- **Example**: In a two-player game where both players have dominant strategies, the intersection of these strategies forms a **Nash equilibrium**, as neither player has an incentive to deviate from their strategy.

		Player 2 (Steven)	
		Split	Steal
Player 1 (Sarah)	Split	3 3	2 4
	Steal	4 2	2 2

		Player 2 (Steven)	
		Split	Steal
Player 1 (Sarah)	Split	4 3	2 4
	Steal	3 2	1 2

2.2) Nash equilibrium

A **Nash Equilibrium** is a concept in game theory where no player can improve their payoff by unilaterally changing their strategy, assuming all other players keep their strategies unchanged. In other words, a Nash equilibrium is a stable state where each player's strategy is the best response to the strategies of the other players.

At Nash equilibrium, every player is playing their optimal strategy given the strategies of the others, and no player has an incentive to deviate from their chosen strategy.

Types of Nash Equilibria

1. **Pure Strategy Nash Equilibrium**:
   - Players choose a single strategy with certainty.
   - **Example**: In a pricing competition between two firms, if both firms set the same price and neither can increase profit by changing prices, this is a pure strategy Nash equilibrium.

2. **Mixed Strategy Nash Equilibrium**:
   - Players randomize over their possible strategies, with certain probabilities.
   - **Example**: In **Rock-Paper-Scissors**, no pure strategy can be an equilibrium because each strategy can be defeated. However, if each player chooses Rock, Paper, or Scissors with a probability of 1/3, it forms a mixed strategy Nash equilibrium, as no player can do better by unilaterally changing their probability distribution.

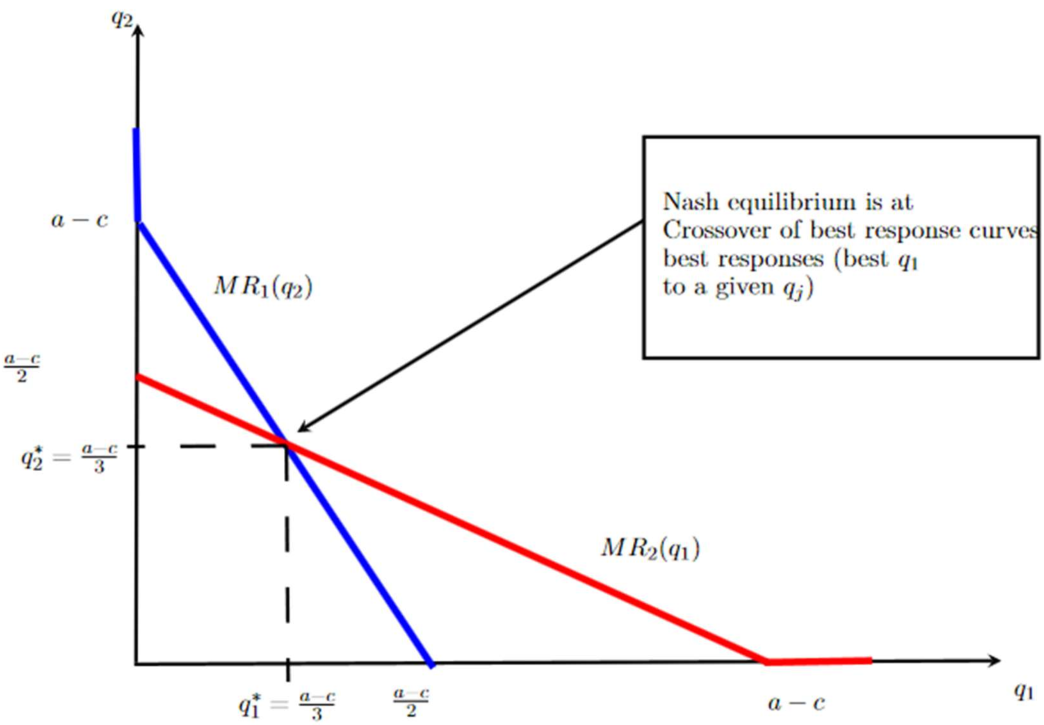

FIGURE 2.1 – Graphical representation of the Cournot-Nash equilibrium

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2.3) Application

Here is an example:

Heads or Tails Game			Player 1	
			Heads	Tails
	Player 2	Heads	(-1,1)	(1,-1)
		Tails	(1,-1)	(-1,1)

* * *	Player 2			
Player 1	* * *	Left	Middle	Right
	Top	(1,0)	(1,2)	(0,1)
	Bottom	(0,3)	(0,1)	(2,0)

For Player 2, $(q, r, 1-q-r)$ is a mixed strategy. The pure strategies are Left, Middle, and Right, which coincide with the probability vectors $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, respectively.

In mixed strategies :

$$U_{J_1}(\text{Top}/q,r) = U_{J_1}(\text{Bottom}/q,r) \Leftrightarrow 1.q + 1.r + 0.(1-q-r) = 2.(1-q-r)$$

$$\Leftrightarrow 1.q + 1.r + 0.(1-q-r) = 2.(1-q-r)$$

$$U_{J_2}(\text{Left}/p) = U_{J_2}(\text{Middle}/p) = U_{J_2}(\text{Right}/p) \Leftrightarrow p = p = 2(1-p) \Rightarrow p = \frac{1}{2}.$$

6) Solution:

		Nicole			
		Accepter 50 k€		Insister sur 70 k€	
Marc	ready to rent fo 50 k€	Marc: avoids breakdown at lower cost if the sky is cloudy OR the sky is clear but pays 50 k€.	Nicole: Loses 5 k€ if the generator is used OR earns 50 k€ if it isn't	$x < y$, so the generator is not rented. Therefore, Marc risks a breakdown if the sky is overcast OR avoids the breakdown at no cost.	Nicole earns 0 in both cases
	ready to rent 70 k€	Marc: is ready to rent the genset for 70 k€ but Nicole accepts the discount: the genset is therefore rented for 50 k€. Marc avoids a breakdown at lower cost if the sky is cloudy OR if the sky is clear, but pays 50 k€.	Nicole: Loses 5 k€ if the generator is used OR earns 50 k€ if it isn't	The generator is rented for 70 k€, so Marc uses the generator in both cases. He therefore avoids a breakdown if the weather is cloudy OR pays 70 k€ for nothing if the weather is clear.	Nicole earns 5 k€ with a **'certain'** probability.

Nash equilibrium:

Used an algorithm => nash.py

Weather forecast to be considered:

Used an algorithm risk.py

7) Conclusion:

In conclusion, there was no Nash equilibrium this means that both strategies stood neither to gain or to lose. The best strategy would be using Marc's strategy because the gain is slighter better than Nicole's strategy.