02562 Rendering - Introduction

BRDF, Fresnel Reflectance, and Absorption

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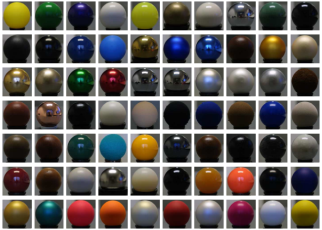
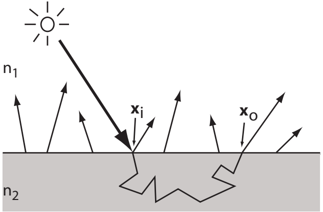
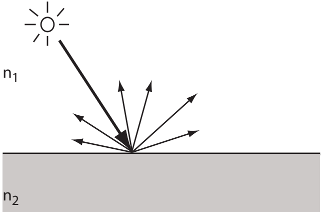
Materials (scattering and absorption of light)

- Optical properties (index of refraction, $n(\lambda) = n'(\lambda) + i\, n''(\lambda)$).
- Reflectance distribution functions, $f(\boldsymbol{x}_i, \vec{\omega}_i; \boldsymbol{x}_o, \vec{\omega}_o)$.







The rendering equation

- The equation we are solving is [Nicodemus 1965, Kajiya 1986]

$$L_o(\boldsymbol{x}, \vec{\omega}_o) = L_e(\boldsymbol{x}, \vec{\omega}_o) + \int_\Omega f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\boldsymbol{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\omega_i \,,$$

- where

L_o is observed radiance,
L_e is emitted radiance,
L_i is incident radiance,
$\boldsymbol{x}$ is a surface position,
$\vec{\omega}_o$ is the direction toward the observer (direction of observation),
$\vec{\omega}_i$ is the direction toward the light source (direction of incidence),
f_r is the bidirectional reflectance distribution function (BRDF),
$\mathrm{d}\omega_i$ is a differential element of solid angle,
Ω is the 2π solid angle around the surface normal $\vec{n}$ at $\boldsymbol{x}$,
θ_i is the angle between $\vec{\omega}_i$ and the surface normal $\vec{n}$ at $\boldsymbol{x}$, such that $\cos\theta_i = \vec{\omega}_i \cdot \vec{n}$.

References

- Nicodemus, F. E. Directional reflectance and emissivity of an opaque surface. *Applied Optics 4*(7), pp. 767–775. July 1965.
- Kajiya, J. The rendering equation. *Computer Graphics (SIGGRAPH 86) 20*(4), pp. 143–150. August 1986.

Reflected radiance

- The Rendering Equation:

$$\begin{array}{ccccc} L_o & = & L_e & + & L_r \\ \text{observed} & \text{is} & \text{emitted} & \text{plus} & \text{reflected} \end{array}$$

- Reflected radiance: $L_r(\boldsymbol{x}, \vec{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\boldsymbol{x}, \vec{\omega}_i) \cos\theta_i \,\mathrm{d}\omega_i$.
- The definitions of radiance and irradiance:

$$L = \frac{\mathrm{d}^2\Phi}{\cos\theta \,\mathrm{d}A \,\mathrm{d}\omega}\,, \quad E = \frac{\mathrm{d}\Phi_i}{\mathrm{d}A}\,,$$

  where Φ is radiant flux and A is surface area.
- An element of directional irradiance is then $\mathrm{d}E = L_i \cos\theta_i \,\mathrm{d}\omega_i$ and

$$L_r(\boldsymbol{x}, \vec{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) \,\mathrm{d}E(\boldsymbol{x}, \vec{\omega}_i)\,.$$

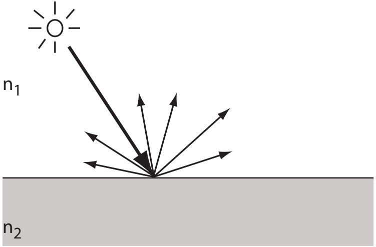
Bidirectional Reflectance Distribution Function

- Reflected radiance: $L_r(\boldsymbol{x}, \vec{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o)\,\mathrm{d}E(\boldsymbol{x}, \vec{\omega}_i)\,.$

- The definition of the BRDF: $f_r = \dfrac{\mathrm{d}L_r}{\mathrm{d}E}\,.$

- The definition of reflectance: $\rho = \dfrac{\mathrm{d}\Phi_r}{\mathrm{d}\Phi_i}$

  (the ratio of reflected to incident radiant flux).



- Since flux is both incident and reflected at $\mathrm{d}A$, we have the following relation between the BRDF and the reflectance:

$$f_r = \frac{\mathrm{d}L_r}{\mathrm{d}E} = \frac{\mathrm{d}(\mathrm{d}^2\Phi_r/(\cos\theta_r\,\mathrm{d}A\,\mathrm{d}\omega_r))}{\mathrm{d}(\mathrm{d}\Phi_i/\mathrm{d}A)} = \frac{\mathrm{d}\rho}{\cos\theta_r\,\mathrm{d}\omega_r}$$

The BRDF of perfectly diffuse materials

- Perfectly diffuse materials (Lambertian materials) have $L_r = L_{r,d}$ constant, then $f_r = f_{r,d}$ is constant.
- Using the relation $f_r = \frac{\mathrm{d}\rho}{\cos\theta_r \,\mathrm{d}\omega}$, we can express the BRDF of Lambertian materials in terms of the diffuse reflectance:

$$\rho_d = \int_{2\pi} f_{r,d} \cos\theta_r \,\mathrm{d}\omega_r = f_{r,d} \int_{2\pi} \cos\theta_r \,\mathrm{d}\omega_r \quad .$$

- The integral of a cosine weighted hemisphere is

$$\begin{aligned}
\int_{2\pi} \cos\theta_r \,\mathrm{d}\omega_r &= \int_0^{2\pi} \int_0^{\pi/2} \cos\theta_r \sin\theta_r \,\mathrm{d}\theta_r \,\mathrm{d}\phi_r = 2\pi \int_0^{\pi/2} \cos\theta_r \sin\theta_r \,\mathrm{d}\theta_r \\
&= 2\pi \int_0^1 \cos\theta_r \,\mathrm{d}(\cos\theta_r) = 2\pi \frac{1}{2} = \pi \quad .
\end{aligned}$$

- The BRDF of Lambertian materials is then

$$f_{r,d} = \frac{\rho_d}{\pi} \quad .$$

Examples of diffuse material environments

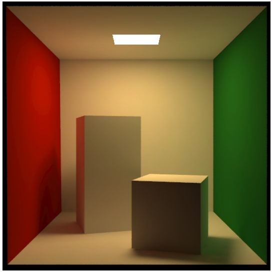


Figure 1: *Global illumination images from 'Monsters U.' These images render more than 30× faster with radiosity caching.* © *Disney/Pixar.*

References

- Cohen, M. F., and Wallace, J. R. *Radiosity and Realistic Image Synthesis*. Academic Press/Morgan Kaufmann, 1993.
- Christensen, P. H., Harker, G., Shade, J., Schubert, B., and Batali, D. Multiresolution radiosity caching for efficient preview and final quality global illumination in movies. Pixar Technical Memo #12-06, July 2012.

Reflectance of perfectly specular materials

- The law of reflection is:

  The angle of reflection equals the angle of incidence ($\theta_r = \theta$) and the reflected ray lies in the plane of incidence ($\phi_r = \phi$)

- This means that $\cos\theta_r \, \mathrm{d}A \, \mathrm{d}\omega = \cos\theta \, \mathrm{d}A \, \mathrm{d}\omega'$, or

$$\rho_s = \frac{\mathrm{d}\Phi_r}{\mathrm{d}\Phi_i} = \frac{L_{r,s}}{L_i} \quad \Leftrightarrow \quad L_{r,s} = \rho_s L_i \ .$$

- Thus the integral over all directions across the hemisphere cancels out for perfectly specular materials.
- The wave theory of light provides a relation between ρ_s and the indices of refraction (n_i in the medium of incidence and n_t in the medium of transmission).
- This relation is referred to as the *Fresnel equations*:

$$\tilde{r}_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \quad , \quad \tilde{r}_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} \quad , \quad R = \frac{1}{2}\left(|\tilde{r}_\perp|^2 + |\tilde{r}_\parallel|^2\right)$$

  and we say that $R = \rho_s$ is the *Fresnel reflectance*.

Angular dependency of specular reflectance

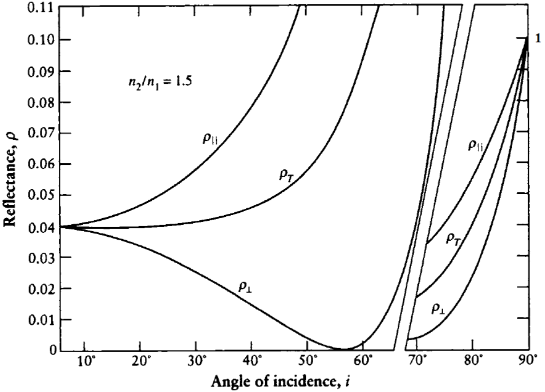


FIGURE 15.8

The Fresnel reflectance for an air-glass boundary with index of refraction 1.5. We show the two polarized components and the term for unpolarized light. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.2, p. 400.

References

- Glassner, A. S. *Principles of Digital Image Synthesis*. Morgan Kaufmann, 1995. Two volumes set.

Total internal reflection

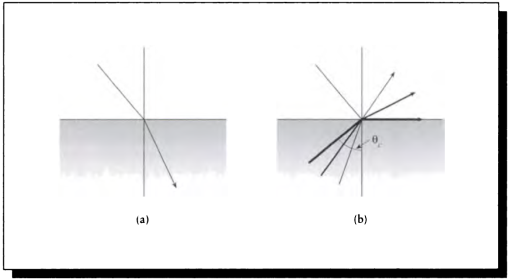


FIGURE 11.18

Refraction. (a) Transmission into a denser medium. (b) Transmission into a rarer medium.

- The Fresnel equations involve the cosine of the angle of refraction. We get this from the law of refraction:

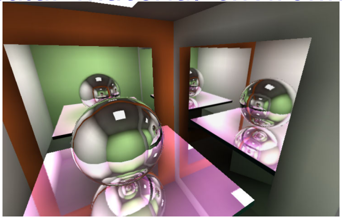
$$\cos^2\theta_t = 1 - \sin^2\theta_t = 1 - \frac{n_i^2}{n_t^2}\sin^2\theta_i = 1 - \frac{n_i^2}{n_t^2}(1 - \cos^2\theta_i)\,.$$

- If $\cos^2\theta_t$ is negative, we have passed the critical angle (θ_c), and we have *total internal reflection*: $\rho_s = R = 1$.

References

- Glassner, A. S. *Principles of Digital Image Synthesis*. Morgan Kaufmann, 1995. Two volumes set.

Examples of specular material environments



References

- Nielsen, K. H., and Christensen, N. J. Real-time recursive specular reflections on planar and curved surfaces using graphics hardware. *Journal of WSCG 10*(3), pp. 91–98, 2002.
- Glassner, A. S. Soap bubbles: Part 2. *IEEE Computer Graphics and Applications 20*(6), pp. 99–109, November/December 2000.
- Christensen, P. H., Fong, J., Laur, D. M., Batali, D. Ray tracing for the movie 'Cars'. In *Proceedings of the IEEE Symposium on Interactive Ray Tracing 2006*, pp. 1–6, September 2006.

Transmitted radiance

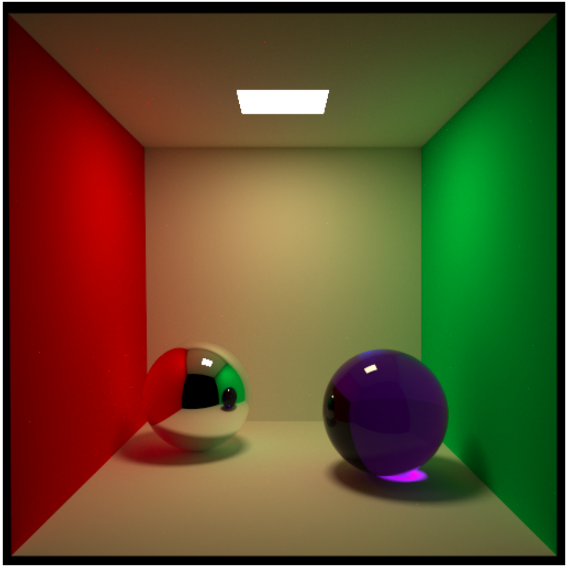
- Flux not reflected off the surface of a medium is transmitted into it: $\rho_t = 1 - \rho_s$.
- For perfectly specular materials, $T = 1 - R$ is called the *Fresnel transmittance*.
- If the medium is not a strong absorber, some transmitted flux reemerges.
- If the medium scatters light, the point of emergence may be any surface point.
- If the medium is only a weak absorber, it is called transparent.
- Light transmitted into a transparent material travels along the refracted ray and is attenuated according to *Bouguer's law* of exponential attenuation:

$$L = L_0 T_r = L_0 \exp(-\sigma_t s)\,,$$

where

L_0 is the radiance transmitted into the medium,
σ_t is the extinction coefficient,
s is the distance traveled along the ray, and
T_r is called the *beam transmittance*.

Example reference image



Microfacet surfaces

- A microfacet surface is modelled by a BRDF that scatters light in more than one direction.
- One way to describe this is by a distribution of microfacet normals.

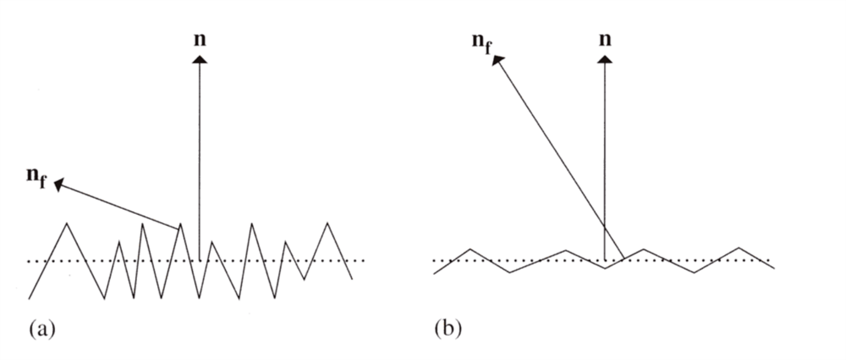


Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004] scalar diffraction by surface elements around a plane

The origins

- The scattering of electromagnetic waves from rough surfaces.
  - Overview by Beckmann and Spizzichino [1963].
    - How to develop facet normal distribution functions.

V-grooves

- Translation to geometrical optics.
  - Theory for off-specular reflection from roughened surfaces by Torrance and Sparrow [1967].
    - Introducing the BRDF model

$$f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) = \frac{FGD}{4(\vec{n} \cdot \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_o)} = \frac{FGD}{4 \cos\theta_i \cos\theta_o} \ ,$$

where
F is the Fresnel reflectance,
G is the geometrical attenuation factor,
D is the facet normal distribution function.

References

- Beckmann, P., and Spizzichino, A. *The Scattering of Electromagnetic Waves from Rough Surfaces*. International Series of Monographs on Electromagnetic waves, Vol. 4, Pergamon Press, 1963.
- Torrance, K. E., and Sparrow, E. M. Theory of off-specular reflection from roughened surfaces. *Journal of the Optical Society of America 57*(9), pp. 1105-1114, September 1967.

Geometrical attenuation

- Important effects to consider:
  (a) Masking.
  (b) Shadowing.
  (c) Interreflections.

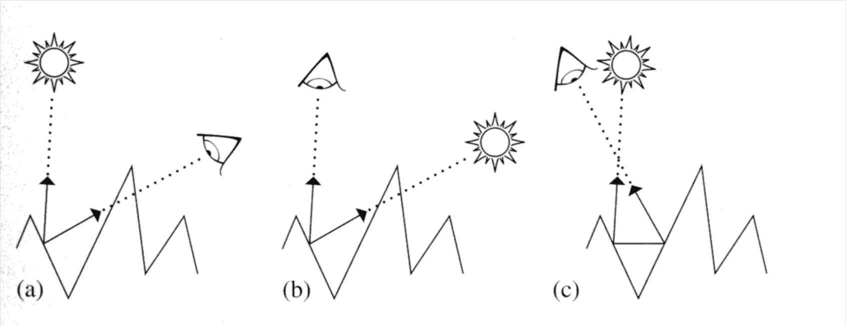


Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

Microfacet models in graphics

- Introduced by Blinn [1977].
- The Torrance-Sparrow model with different microfacet distributions (D):
  - The modified Phong [1975] model for D (cosine lobe distribution using half-vector).
  - The Torrance-Sparrow [1967] model for D (Gaussian distribution).
  - A model by Trowbridge and Reitz [1975] for D (microfacets as ellipsoids of revolution).
- There are other options as well.
  - See Cook and Torrance [1981].

References

- Blinn, J. F. Models of light reflection for computer synthesized pictures. *Computer Graphics (SIGGRAPH '77) 11*(2), pp. 192-198, 1977.
- Phong, B. T. Illumination for computer generated images. *Communications of the ACM 18*(6), pp. 311–317, June 1975.
- Torrance, K. E., and Sparrow, E. M. Theory of off-specular reflection from roughened surfaces. *Journal of the Optical Society of America 57*(9), pp. 1105-1114, September 1967.
- Trowbridge, T. S., and Reitz, K. P. Average irregularity representation of a roughened surface for ray reflection. *Journal of the Optical Society of America 65*(5), pp. 531-536, 1975.
- Cook, R. L., and Torrance, K. E. A reflectance model for computer graphics. *Computer Graphcis (SIGGRAPH '81) 15*(3), pp. 307-316, August 1981.

The Torrance-Sparrow model

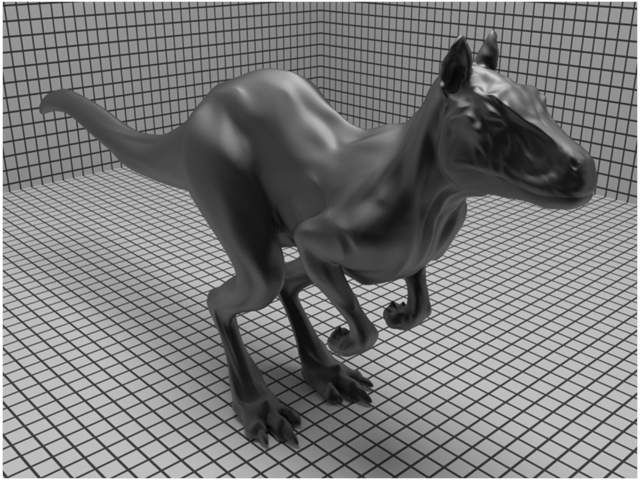


Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

- Using the Blinn-Phong microfacet distribution.

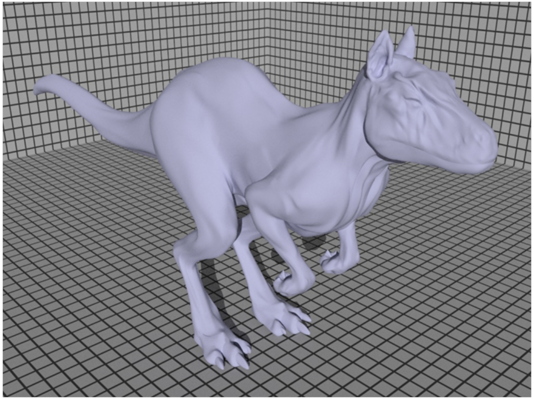
Newer microfacet models

- Oren-Nayar [1994].
  - Using Lambertian microfacets.
- Lafortune [1997].
  - Using multiple Phong lobes.
- Ashikmin-Shirley [2000].
  - Two Phong lobes and Fresnel reflectance.
- Weidlich and Wilkie [2007, 2009]
  - Layered microfacet models.

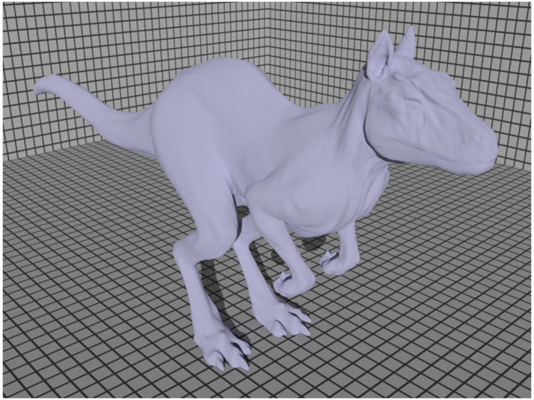
References

- Oren, M., and Nayar, S. K. Generalization of Lambert's reflectance model. In *Proceedings of SIGGRAPH '94*, pp. 239-246, 1994.
- Lafortune, E. P. F., Foo, S.-C., Torrance, K. E., and Greenberg, D. P. Non-linear approximation of reflectance functions. In *Proceedings of SIGGRAPH '97*, pp. 117-126, 1997.
- Ashikmin, M., and Shirley, P. An anisotropic Phong BRDF model. *Journal of Graphics Tools 5*(2), pp. 25-32, 2000.
- Weidlich, A., and Wilkie, A. Arbitrarily layered micro-facet surfaces. In *Proceedings of GRAPHITE 2007*, pp. 171–178, ACM, 2007.
- Weidlich, A., and Wilkie, A. Exploring the potential of layered BRDF models. *SIGGRAPH Asia 2009 Course Notes*, ACM Press, 2009.

The Oren-Nayar model



Lambertian



Oren-Nayar

Images by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The Lafortune model

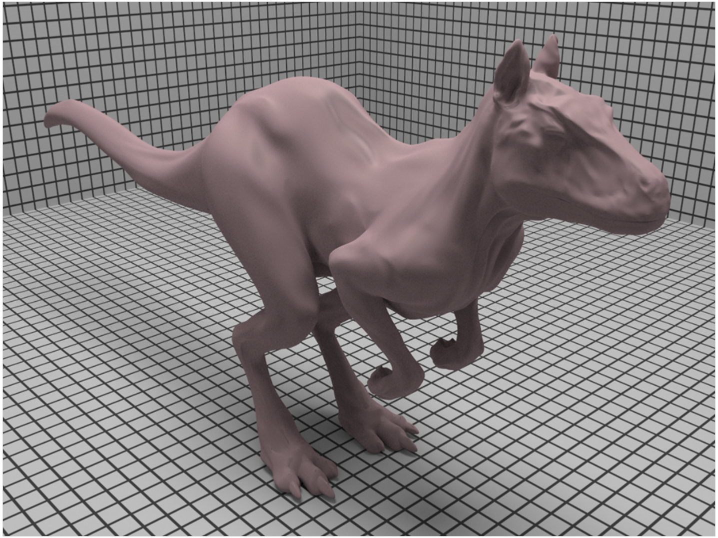


Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The Ashikmin-Shirley model

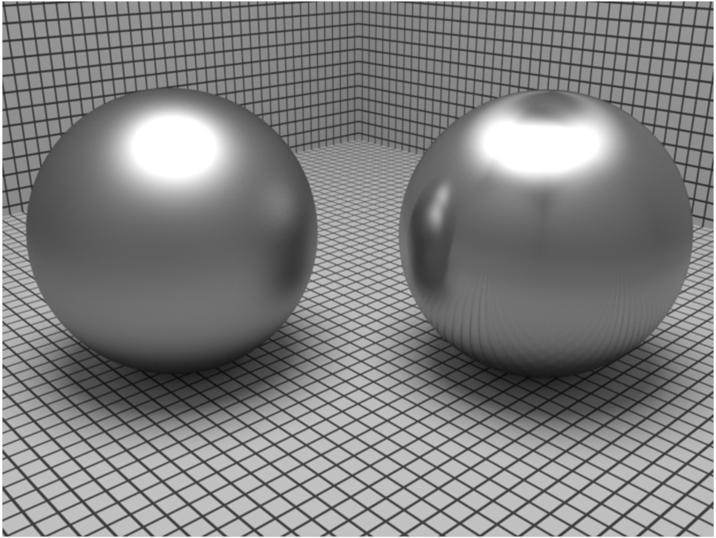


Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The layered model

Image by Weidlich and Wilkie [2007]