Discrete Optimization - Assignment 2

The Vehicle Routing Problem

Introduction

In this assignment you are asked to design an algorithm to solve The Vehicle Routing Problem (VRP). A delivery company needs to deliver goods to many different customers. The deliveries are made by dispatching a fleet of vehicles from a centralized warehouse. The goal of this problem is to design a route for each vehicle (similar to traveling salesman tours) so that all of the customers are served by exactly one vehicle and the travel distance of the vehicles is minimized. The vehicles have a fixed storage capacity and the customers have different demands.

Assignment

Write an algorithm to solve the VRP. The problem is mathematically formulated as follows:
We are given a list of locations `N = 0 ... n-1`, where by convention, location 0 is the location of the warehouse. <u>All vehicles start and end their routes at the warehouse.</u> The remaining locations are customers. Each location is characterized by three values (d_i, x_i, y_i) $i \in N$, i.e. a demand d_i and a point x_i, y_i. The fleet of vehicles `V = 0 ... v-1` is fixed and each vehicle has a limited capacity `c`. All of the demands assigned to a vehicle cannot exceed its capacity `c`. For each vehicle $i \in V$, let T_i be the sequence of customer deliveries made by that vehicle and let $dist(m_1, m_2)$ be the Euclidean distance between two customers. It is assumed that the vehicles can travel in straight lines between each pair of locations.Then the vehicle routing problem is formalized as the following optimization problem:

$$\text{minimize:} \quad \sum_{i \in V} \left(dist(0, T_{i,0}) + \sum_{\langle j,k \rangle \in T_i} dist(j,k) + dist(T_{i,|T_i|-1}, 0) \right)$$

$$\text{subject to:}$$

$$\sum_{j \in T_i} d_j \leq c \quad (i \in V)$$

$$\sum_{i \in V} (j \in T_i) = 1 \quad (j \in N \setminus 0)$$

$$dist(j,k) = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}$$

Data Format

The input consists of |N | + 1 lines. The first line contains 3 numbers: The number of customers |N |, the number of vehicles |V |, and the vehicle capacity c. It is followed by |N | lines, each line represents a location triple (d_i, x_i, y_i), with a demand $d_i \in N$ and a point $x_i, y_j \in R$.

Input Format

```
|N| |V| c
d_0 x_0 y_0
d_1 x_1 y_1
...
d_|N|-1 x_|N|-1 y_|N|-1
```

The output has |V | + 1 lines. The first line contains two values *obj* and *opt*. *obj* is the length of all of the vehicle routes (i.e. the objective value) as a real number. *opt* should be 1 if your algorithm proved optimality and 0 otherwise. The following |V | lines represent the vehicle routes *T* encoding the solution. Each vehicle line starts with warehouse identifier 0 followed by the identifiers of the customers serviced by that vehicle and ends with the warehouse identifier 0. Each vehicle line can contain between 2 and |N |+2 values depending on how many customers that vehicle services. Each customer identifier must appear in one of these vehicle lines.

Output Format:

```
obj opt
0 t_0_1 t_0_2 ... 0
0 t_1_1 t_1_2 ... 0
...
0 t_|V|-1_1 t_|V|-1_2 ... 0
```

Instructions

To be announced soon. For now please start working on this assignment on your computer locally.

Examples

Input Example

```
5 4 10
0 0 0
3 0 10
3 -10 10
3 0 -10
3 10 -10
```

Output Example 1

```
80.6 0
0 1 2 3 0
0 4 0
0 0
0 0
```

This output represents the following routes for each vehicle. Vehicle 0 - $\{0 \to 1, 1 \to 2, 2 \to 3, 3 \to 0\}$; Vehicle 1 - $\{0 \to 4, 4 \to 0\}$; Vehicle 2 - $\{0 \to 0\}$; Vehicle 3 - $\{0 \to 0\}$. Note the following equivalent solution using the same routes with different vehicles.

Output Example 2

```
80.6 0
0 4 0
0 0
0 1 2 3 0
0 0
```