

3D Vision: Coordinate Spaces



A lot of slides from Noah Snavely +
Shree Nayar's YT series: First principals of Computer Vision

CS180: Intro to Computer Vision and Comp. Photo
Angjoo Kanazawa & Alexei Efros, UC Berkeley, Fall 2023

House keeping

Project 4 Part 1 due tomorrow

Final Project

Easy path: Pre-canned

- Group of 1 : 2 projects
- Group of 2 : 3 projects

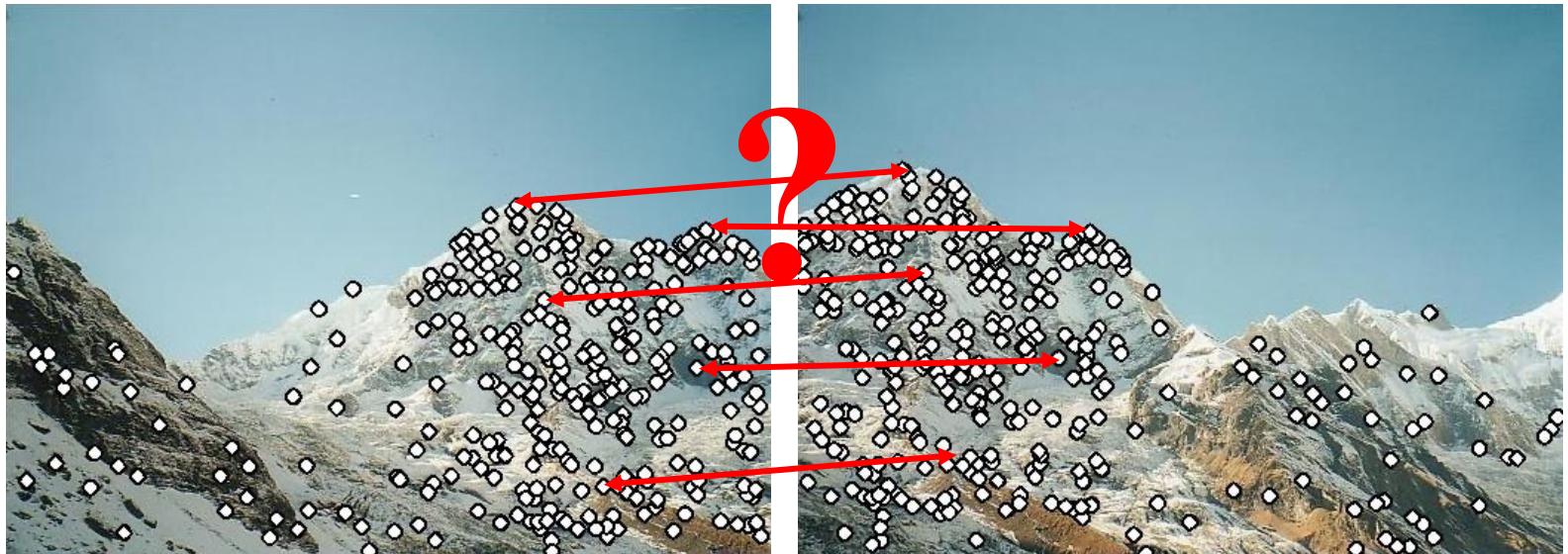
Grad students: Your own project

- 1 page Proposal with pictures **due 11/9**

Recap: Feature descriptors

We know how to detect points

Next question: **How to match them?**



Point descriptor should be:

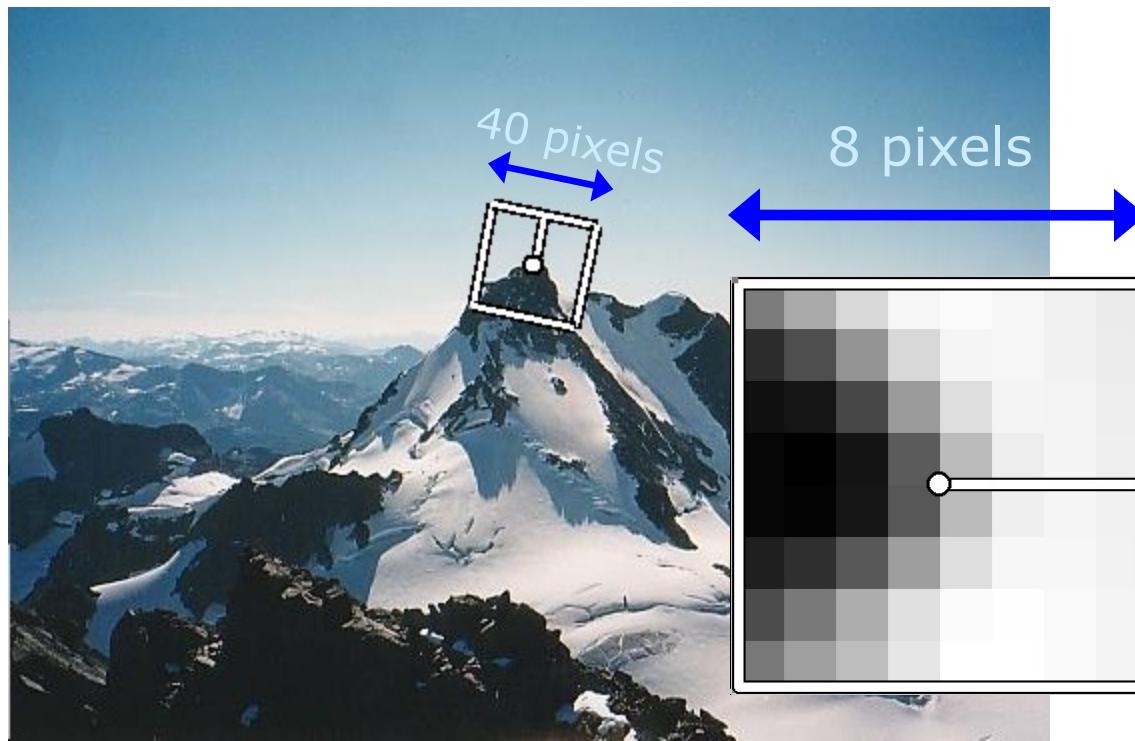
1. Invariant
2. Distinctive

MOPS descriptor vector

8x8 oriented patch

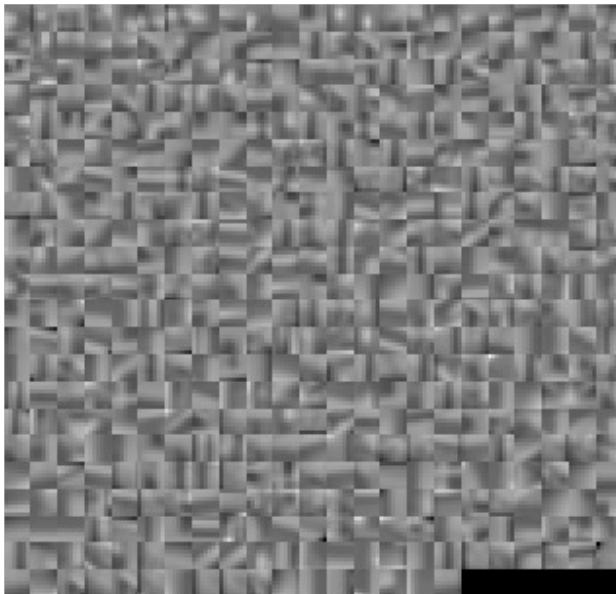
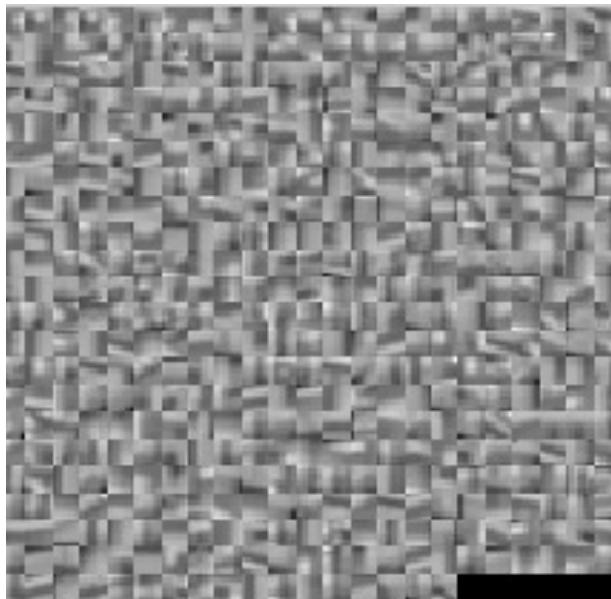
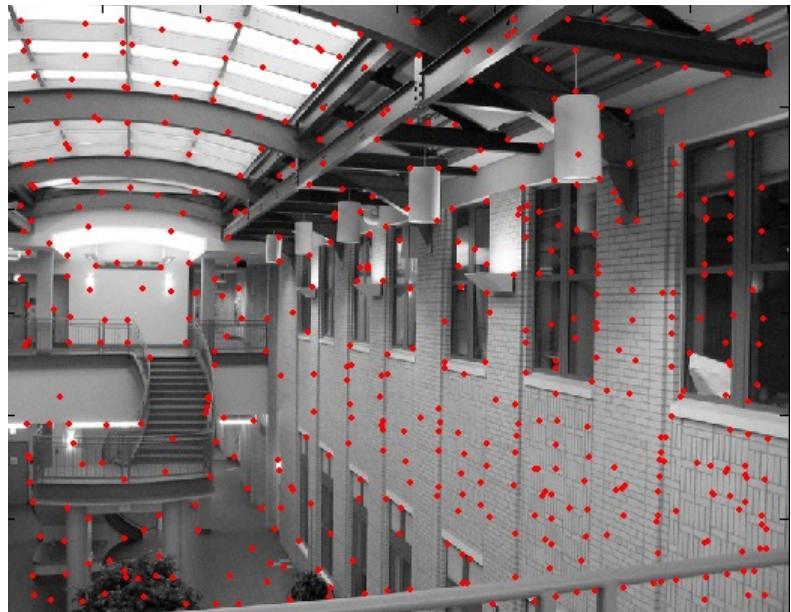
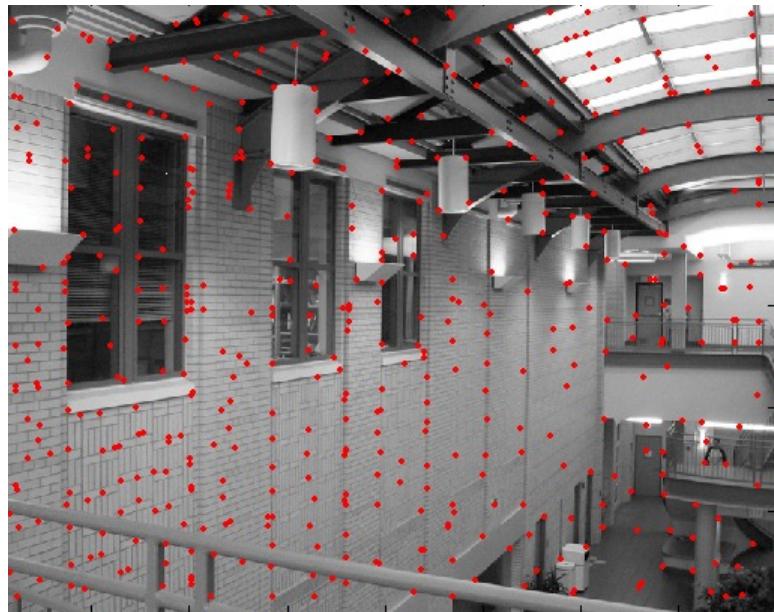
- Sampled at 5 x scale

Bias/gain normalisation: $I' = (I - \mu)/\sigma$



Automatic Feature Matching

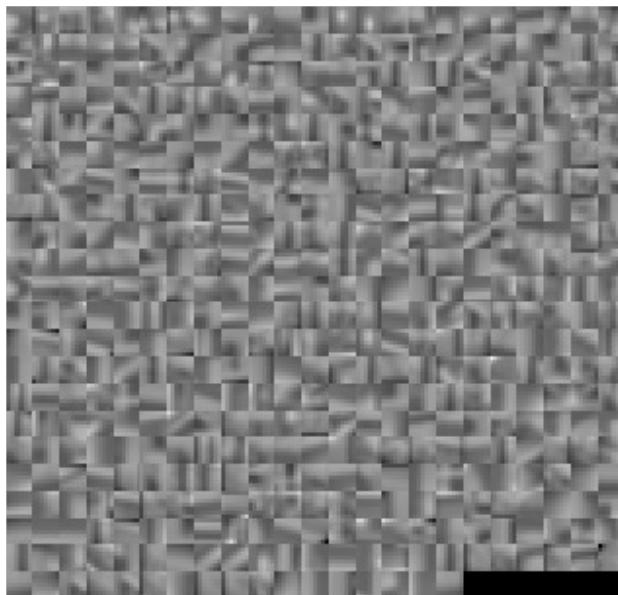
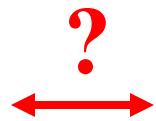
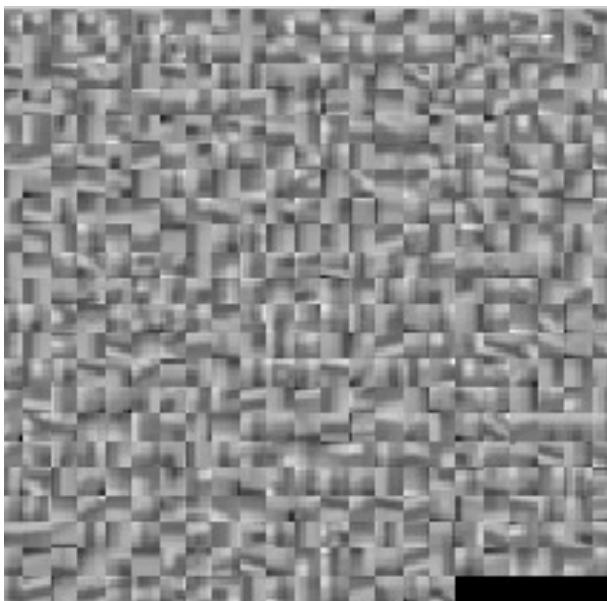
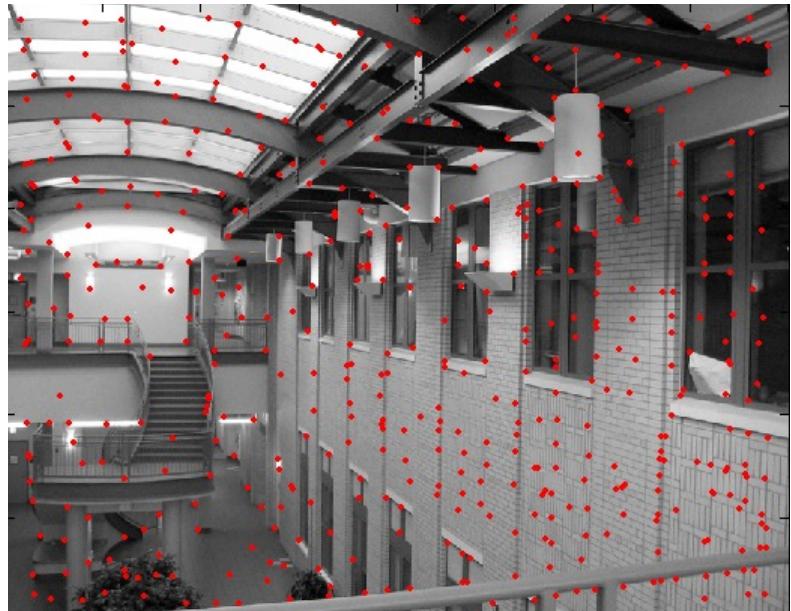
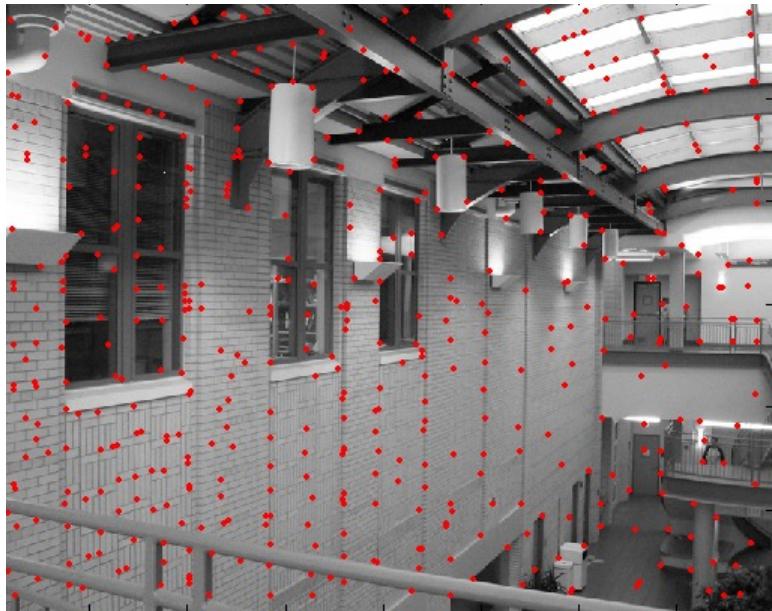
Feature matching



Feature matching

- Pick best match!
 - For every patch in image 1, find the most similar patch (e.g. by SSD).
 - Called “nearest neighbor” in machine learning
- Can do various speed ups:
 - Hashing
 - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
 - Fast Nearest neighbor techniques
 - kd -trees and their variants
 - Clustering / Vector quantization
 - So called “visual words”

What about outliers?

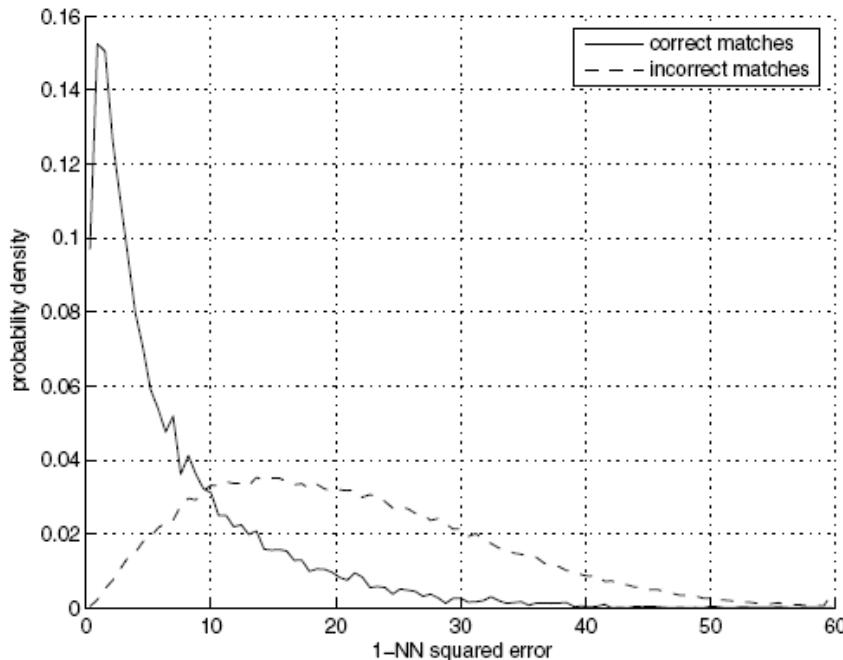


Feature-space outlier rejection

Let's not match all features, but only these that have “similar enough” matches?

How can we do it?

- $\text{SSD}(\text{patch1}, \text{patch2}) < \text{threshold}$
- How to set threshold?



Feature-space outlier rejection: symmetry

Let's not match all features, but only these that have “similar enough” matches?

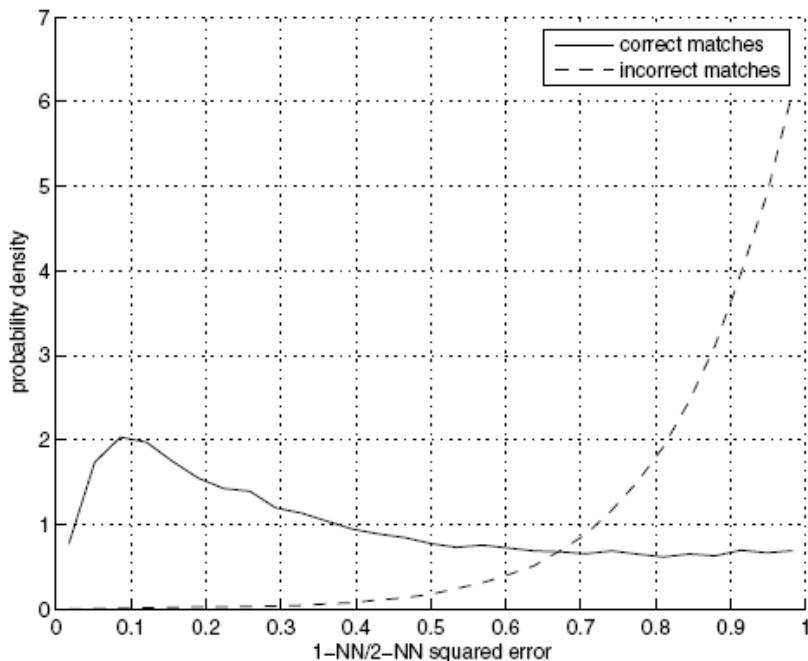
How can we do it?

- Symmetry: x 's NN is y , and y 's NN is x

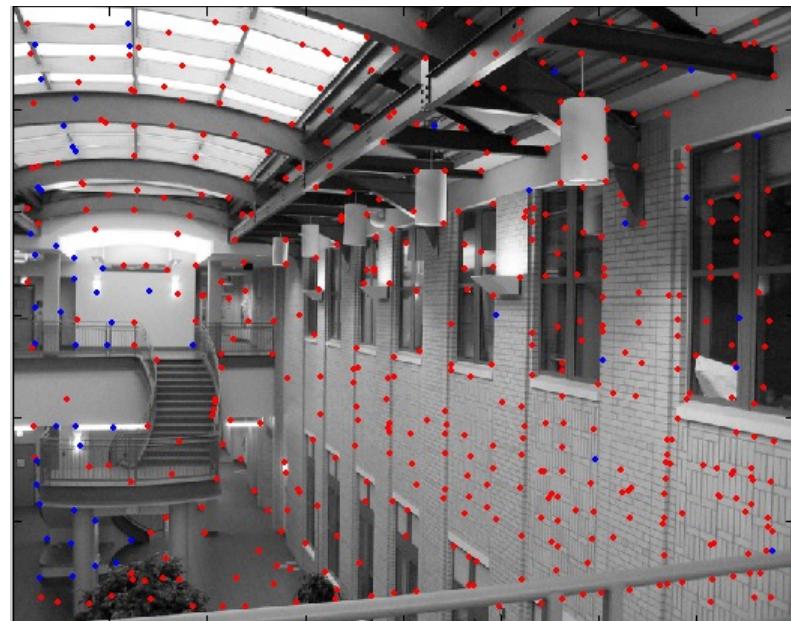
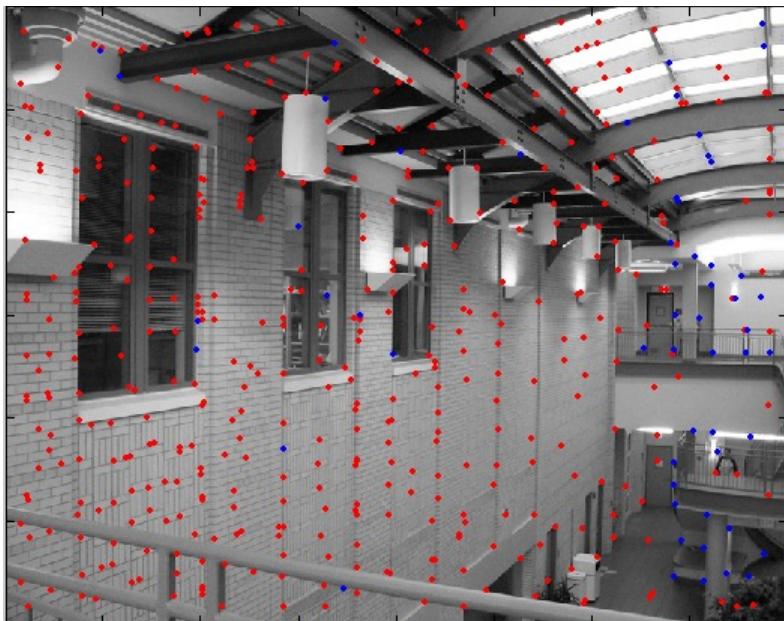
Feature-space outlier rejection: Lowe's trick

A better way [Lowe, 1999]:

- 1-NN: SSD of the closest match
- 2-NN: SSD of the second-closest match
- Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
- That is, is our best match so much better than the rest?



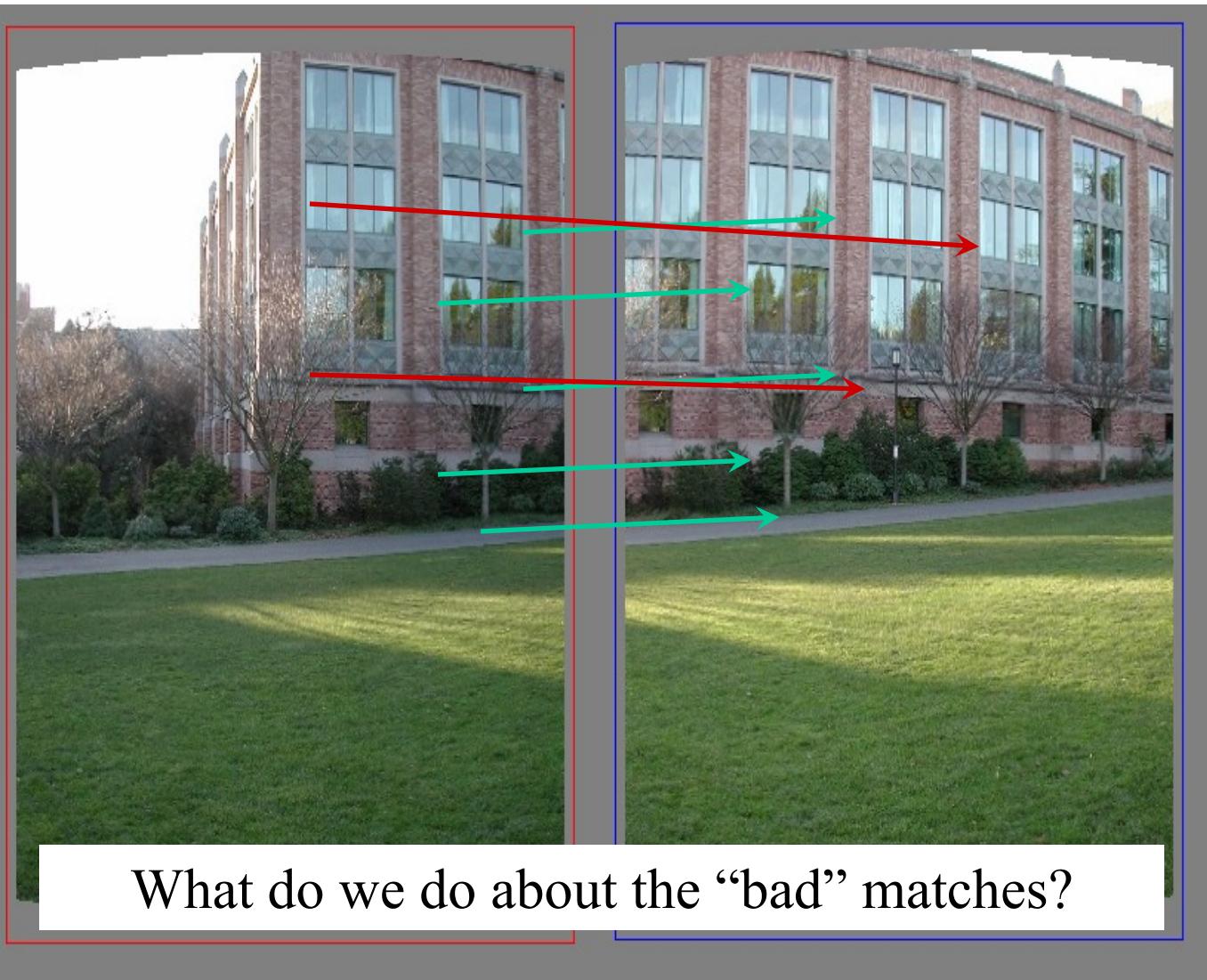
Feature-space outlier rejection



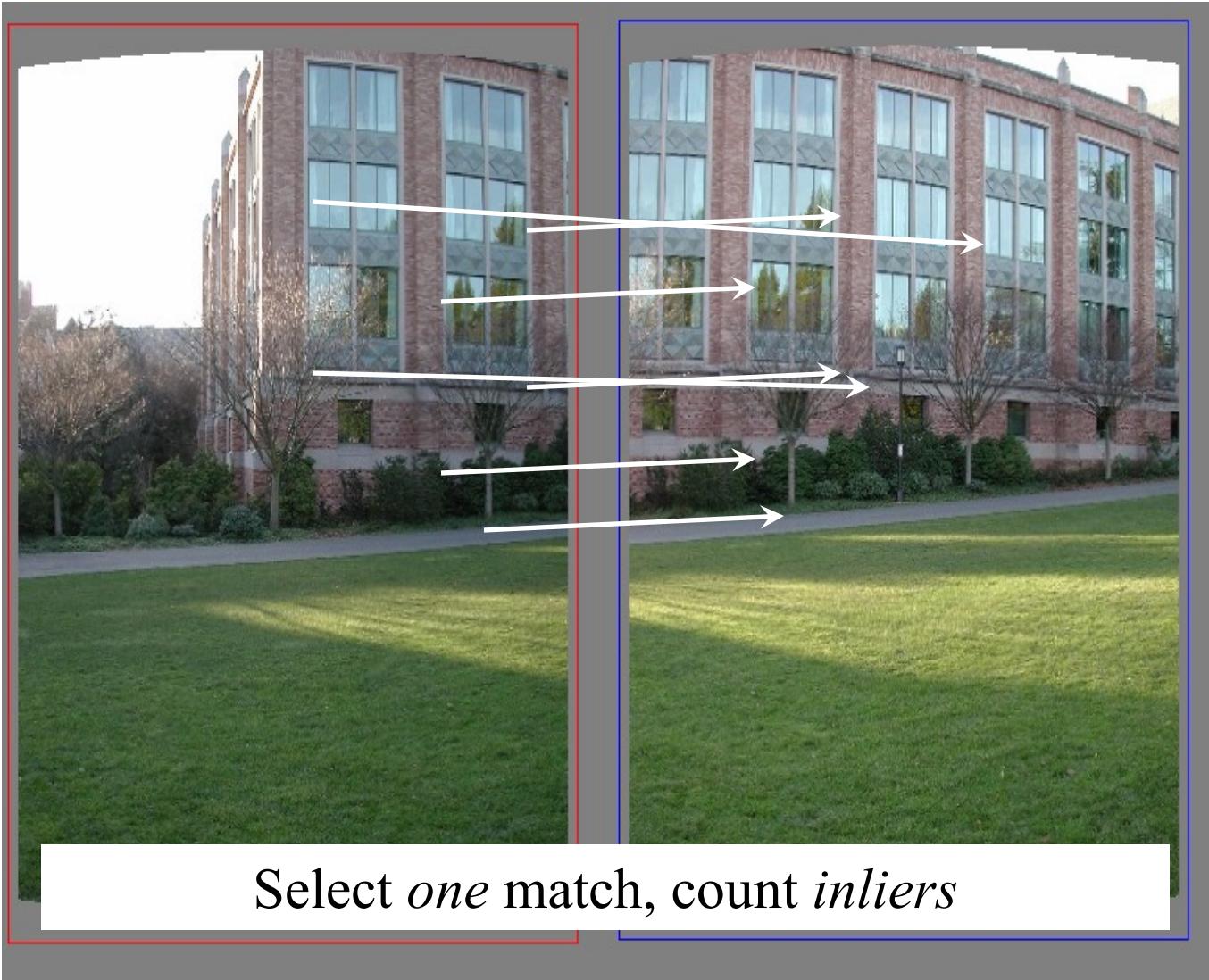
Can we now compute H from the blue points?

- No! Still too many outliers...
- What can we do?

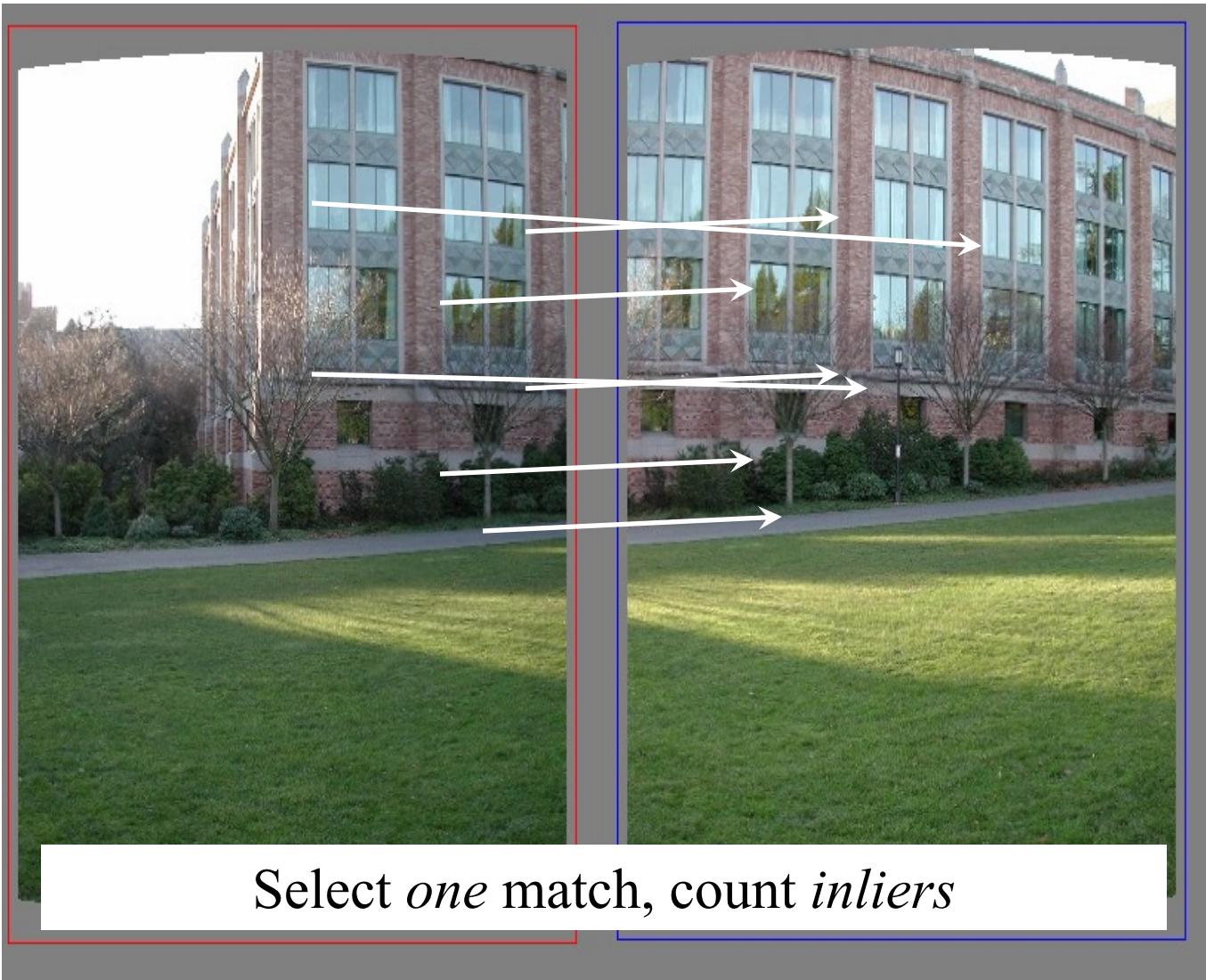
Matching features



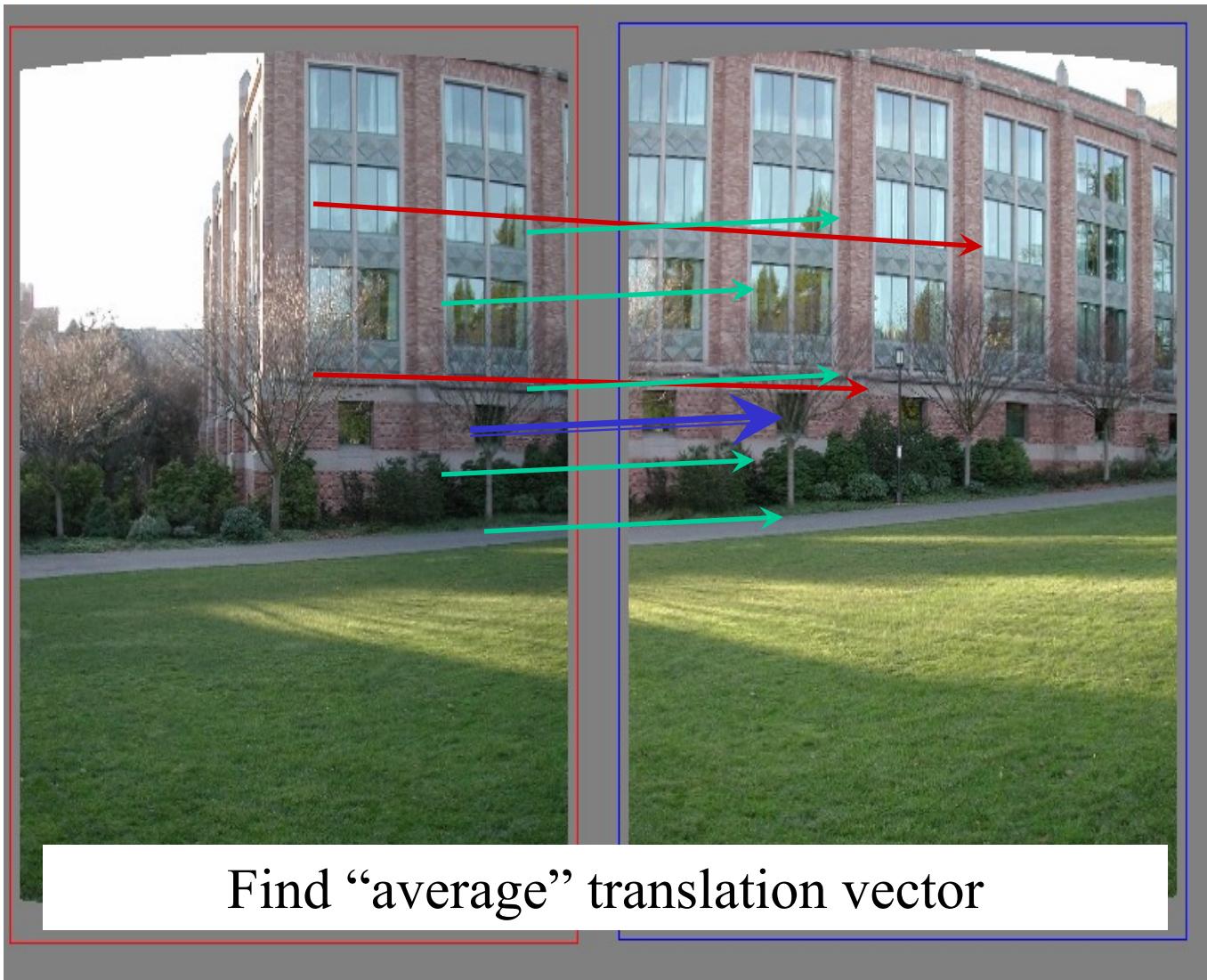
Random Sample Consensus



Random Sample Consensus



Least squares fit

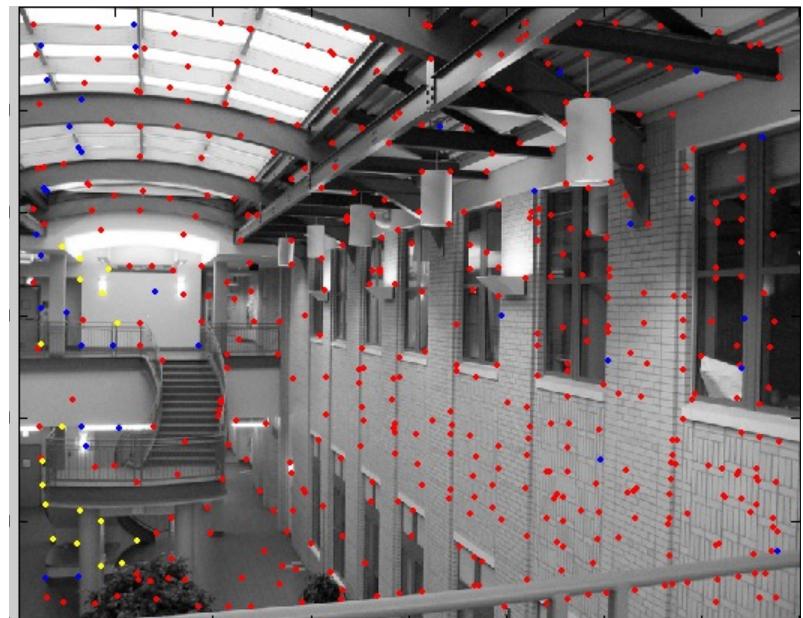
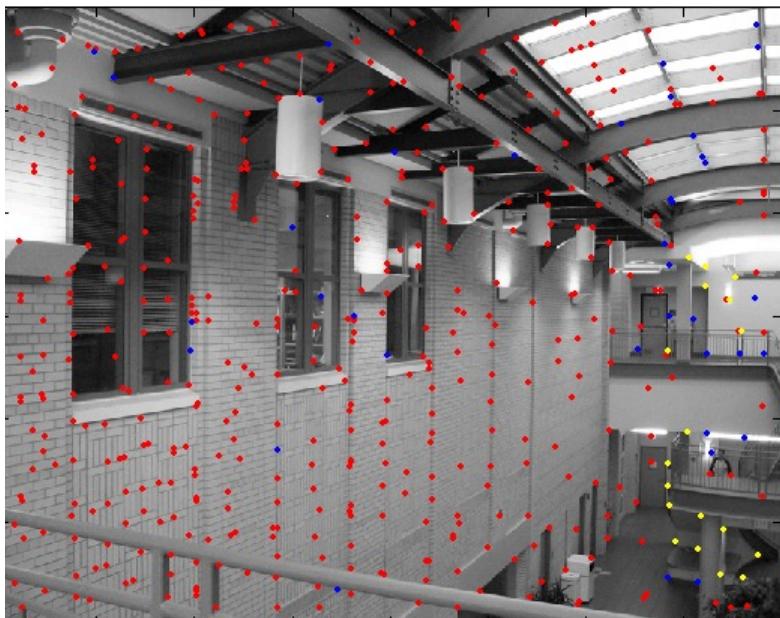


RANSAC for estimating homography

RANSAC loop:

- 
1. Select four feature pairs (at random)
 2. Compute homography H (exact)
 3. Compute *inliers* where $dist(p_i', H p_i) < \varepsilon$
 4. Keep largest set of inliers
 5. Re-compute least-squares H estimate on all of the inliers

RANSAC



Limitations of Alignment

We need to know the global transform
(e.g. affine, homography, etc)

Breaking out of 2D

...now we are ready to break out of 2D



And enter the real world!



on to 3D...

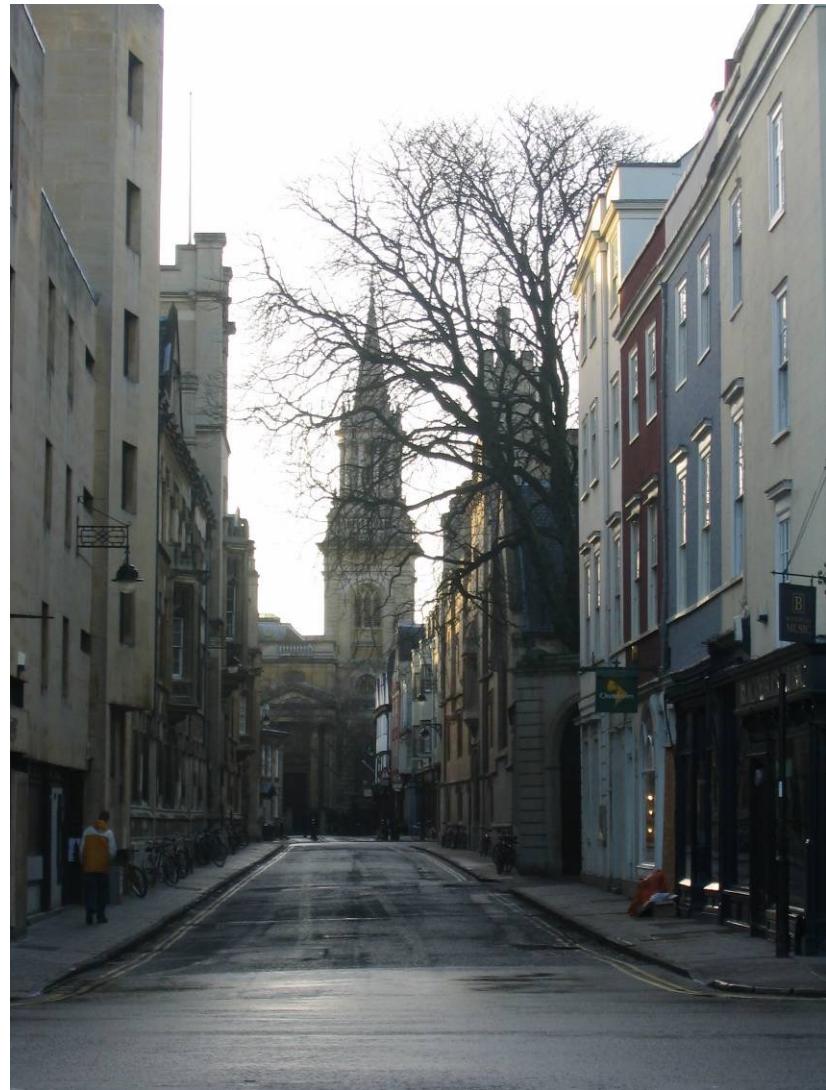
Enough of images!

We want more of the
plenoptic function

We want real 3D scene
walk-throughs:

Camera rotation

Camera translation



3D is super cool!



<https://rd.nytimes.com/projects/reconstructing-journalistic-scenes-in-3d>

3D is super cool!



@capturingreality

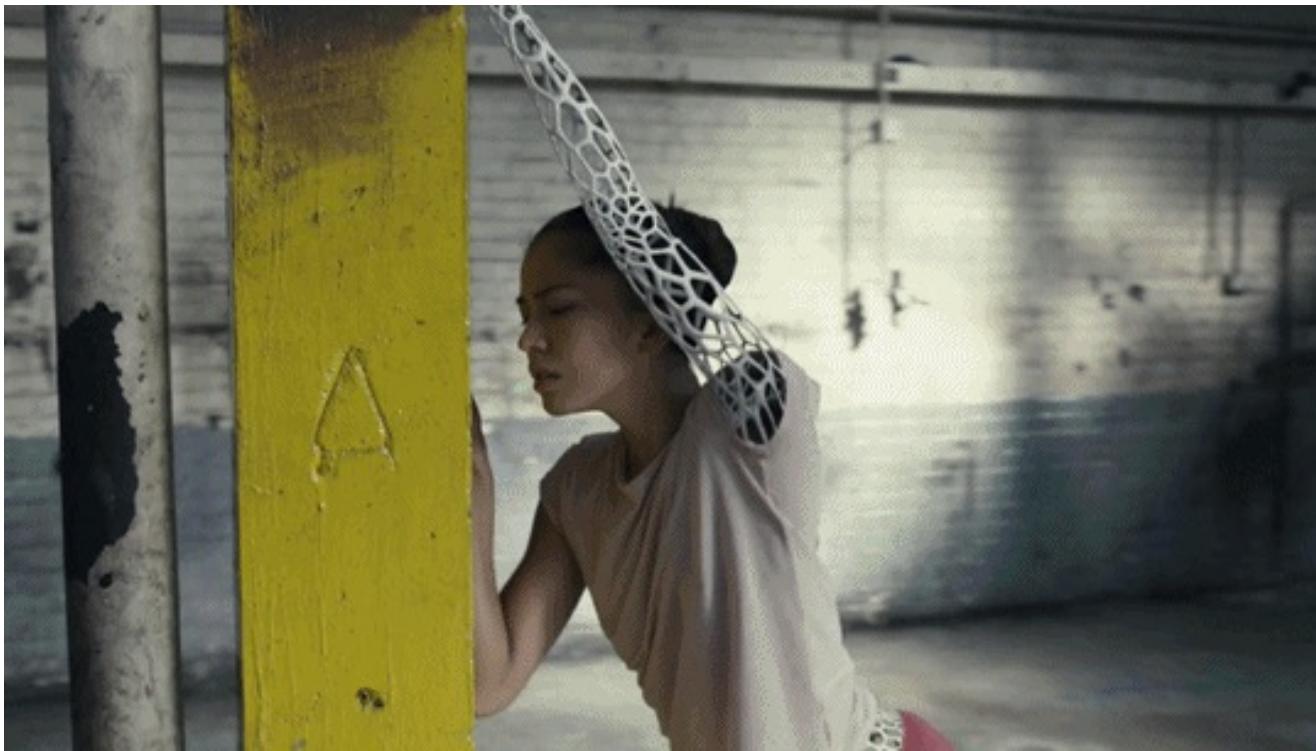


[@organiccomputer](#)

NeRF in the wild (will get to in few more lectures)



Not just about 3D reconstruction



[The Chemical Brothers - Wide Open ft. Beck, MV]

3D for video editing



@blottermedia

My Research

Single-View 3D Human Mesh Recovery



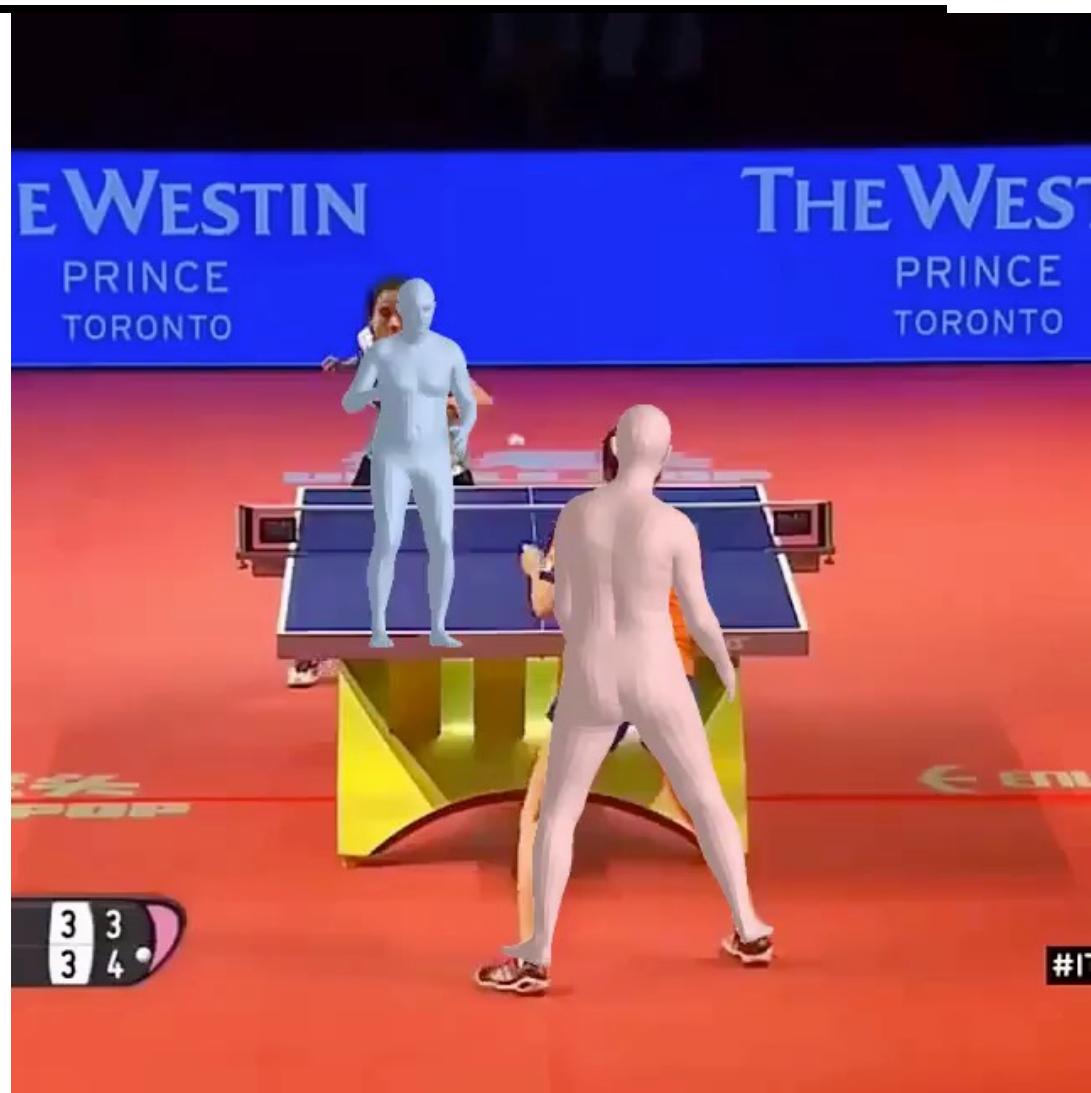
[Bogo*, Kanazawa*, Lassner, Gehler, Romero, Black ECCV '16]

In everyday photos



Kanazawa, Black, Jacobs, Malik. CVPR 2018

Or from Video



Kanazawa, Zhang, and Felsen et al. CVPR 2019

In more detail

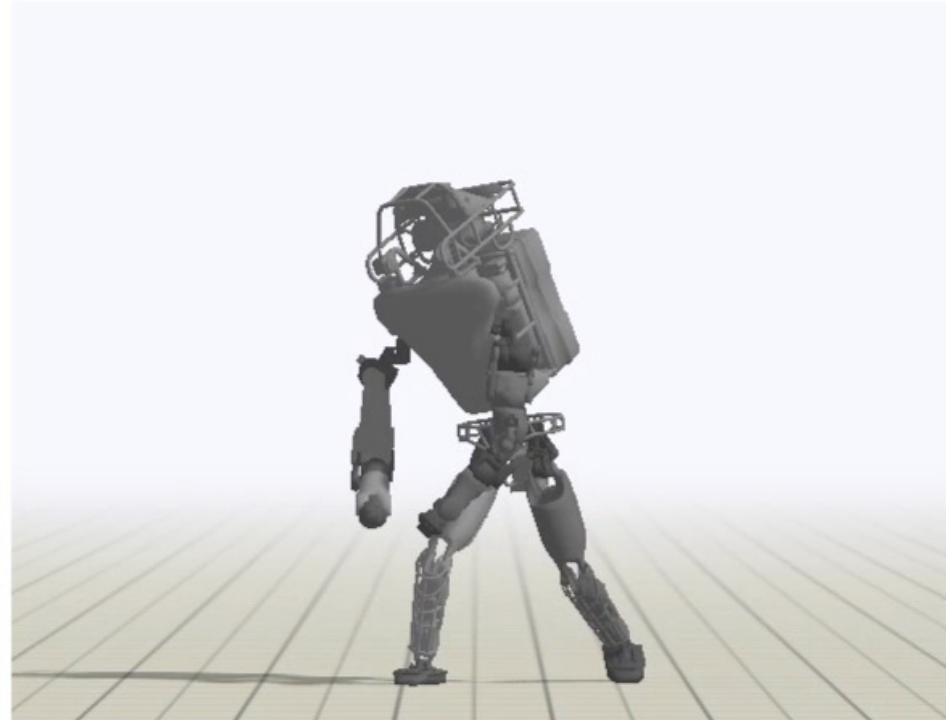


Pixel-Aligned Implicit Function for High-Resolution Clothed Human Digitization,
Saito, Huang, Natsume, Morishima, **Kanazawa**, Li, ICCV 2019

Teaching robots how to dance from watching YouTube



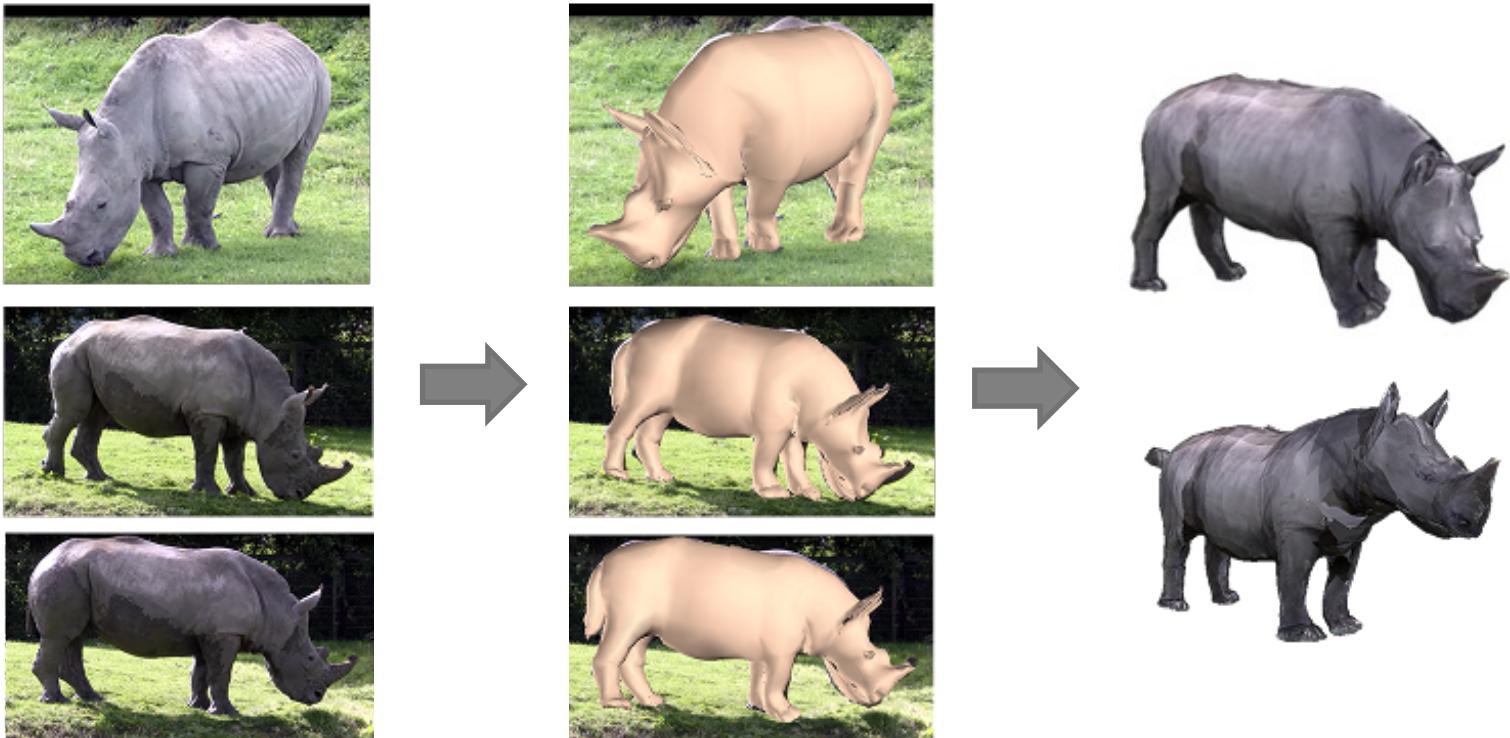
Video



Policy

Peng, Kanazawa, Malik, Abbeel, Levine
"SFV: Reinforcement Learning of Physical Skills from Videos", SIGGRAPH Asia 2018

Reconstructing Animals with Human Input



Zuffi, Kanazawa, Black, "Lions and Tigers and Bears: Capturing Non-Rigid, 3D, Articulated Shape from Images", CVPR 2018

Print it!!



[Kanazawa*, Tulsiani*, Efros, Malik, ECCV 2018]

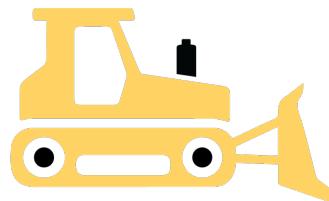


Zuffi, Kanazawa, Black, "Lions and Tigers and Bears: Capturing Non-Rigid, 3D, Articulated Shape from Images", CVPR 2018

Flying into an image



Infinite Nature: Perpetual View Generation of Natural Scenes from a Single Image, ICCV 2021

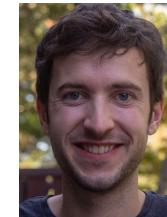


nerfstudio

Matthew Tancik*, Ethan Weber*, Evonne Ng*, Ruilong Li, Brent Yi,
Terrance Wang, Alexander Kristoffersen, Jake Austin, Kamyar Salahi,
Abhik Ahuja, David McAllister, Angjoo Kanazawa



100+ additional Github
contributors



Matt



Ethan



Evonne

3D Capture

GETTING STARTED GITHUB DOCUMENTATION

VIEWPORT RENDER VIEW

► RESUME TRAINING

Show Scene Show Images

Refresh Page

Resolution: 640x1024px
Time Allocation: 100% spent on viewer

Server Connected | Render Connected

CONTROLS RENDER SCENE

LOAD PATH EXPORT PATH

Height: 1080 Width: 1920 FOV: 50

Seconds: 4 FPS: 24

Smoothness: ▲ 0.00 0 1 2 3

ADD CAMERA 🔍 ⚙

CAMERA 0 🔍 🗑

This image shows the nerfstudio interface for 3D capture. The central area displays a rendered view of a living room interior. The room features a fireplace, a sofa, a coffee table, and several paintings on the walls. The interface includes a top navigation bar with links to 'GETTING STARTED', 'GITHUB', and 'DOCUMENTATION'. Below the navigation is a tab bar with 'VIEWPORT' and 'RENDER VIEW'. To the right of the render view are buttons for 'RESUME TRAINING', 'Show Scene', 'Show Images', and 'Refresh Page'. A status bar indicates 'Resolution: 640x1024px' and 'Time Allocation: 100% spent on viewer'. The main workspace is divided into three sections: 'CONTROLS', 'RENDER', and 'SCENE'. The 'RENDER' section contains buttons for 'LOAD PATH' and 'EXPORT PATH', and sliders for 'Height' (1080), 'Width' (1920), 'FOV' (50), 'Seconds' (4), and 'FPS' (24). The 'SCENE' section includes a camera icon for 'ADD CAMERA', a search icon, and a settings icon. At the bottom, there's a slider for 'Smoothness' (set to 0.00) and a row of navigation icons for camera control.



so on to 3D...

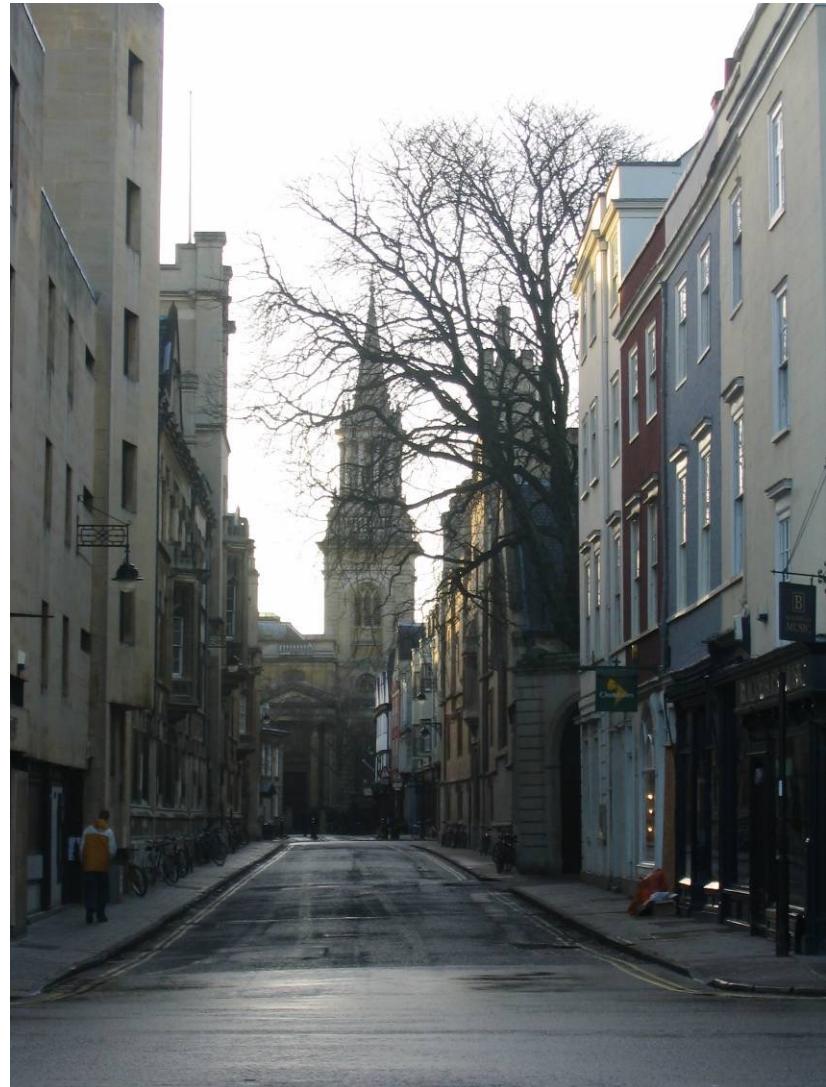
Enough of images!

We want more of the
plenoptic function

We want real 3D scene
walk-throughs:

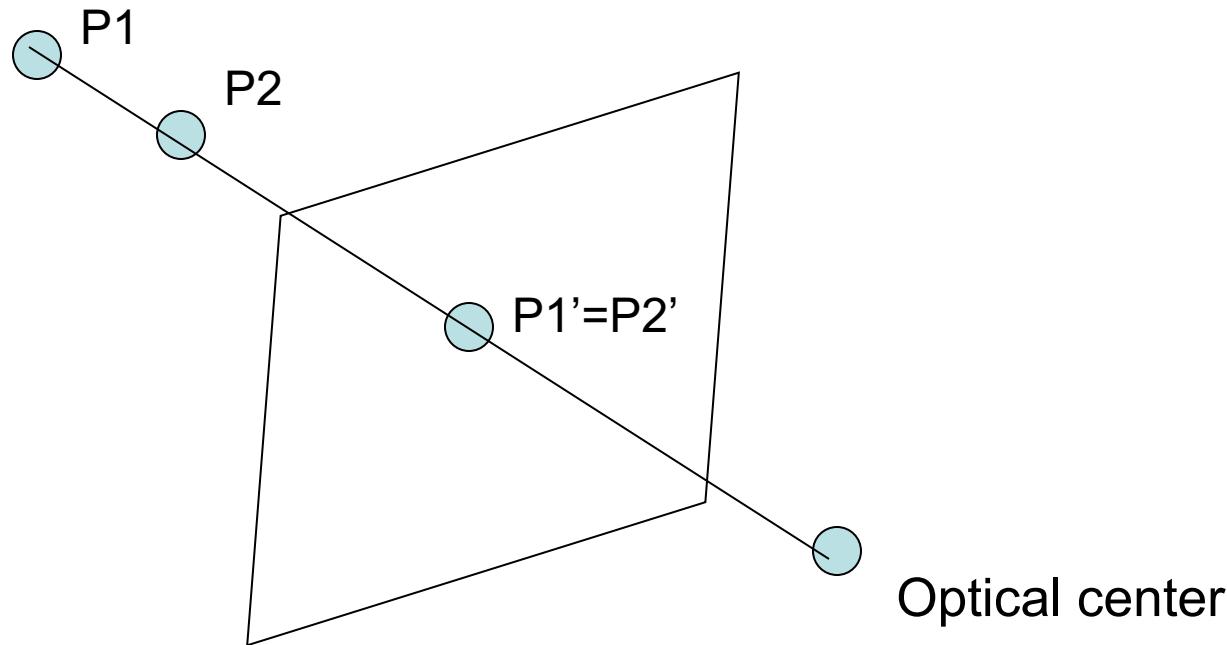
- Camera rotation
- Camera translation

Can we do it from a single
photograph?



Why multiple views?

- Structure and depth are inherently ambiguous from single views.



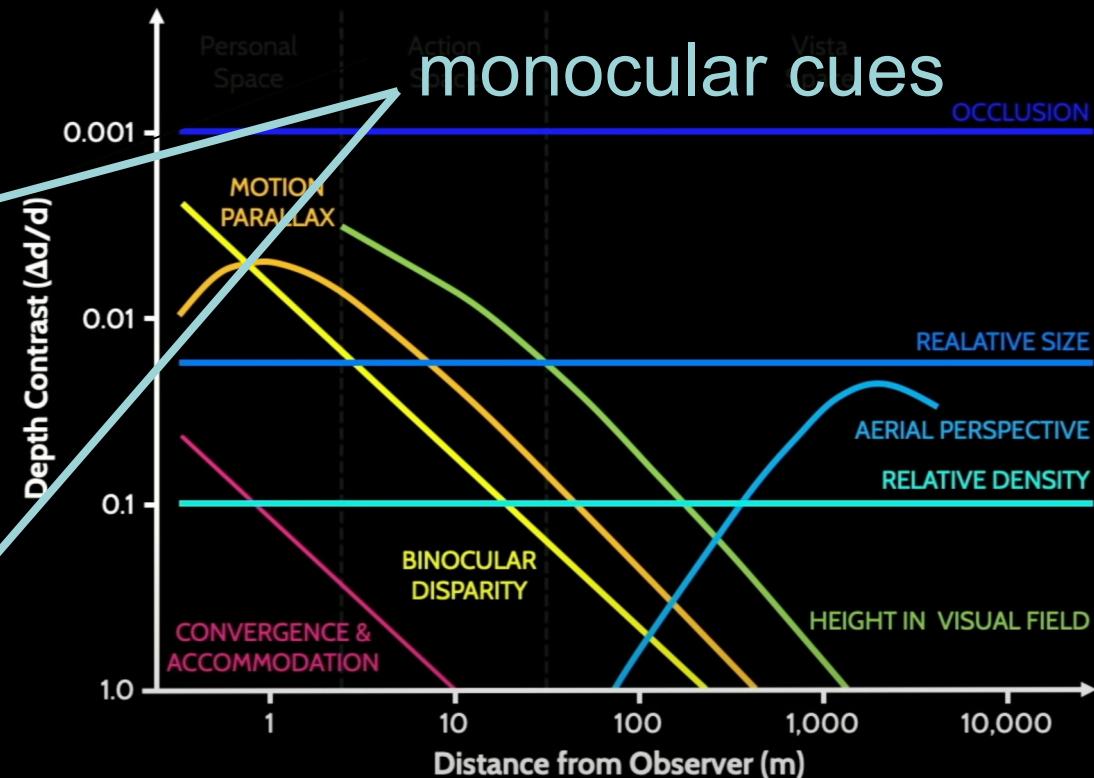
Why multiple views?

- Structure and depth are inherently ambiguous from single views.



Human Depth Cues

OCCLUSION
RELATIVE SIZE
AERIAL PERSPECTIVE
RELATIVE DENSITY
HEIGHT IN VISUAL FIELD
BINOCULAR DISPARITY
MOTION PARALLAX
CONVERGENCE & ACCOMMODATION



Cutting and Vishton. Perceiving layout and knowing distances. 1995.

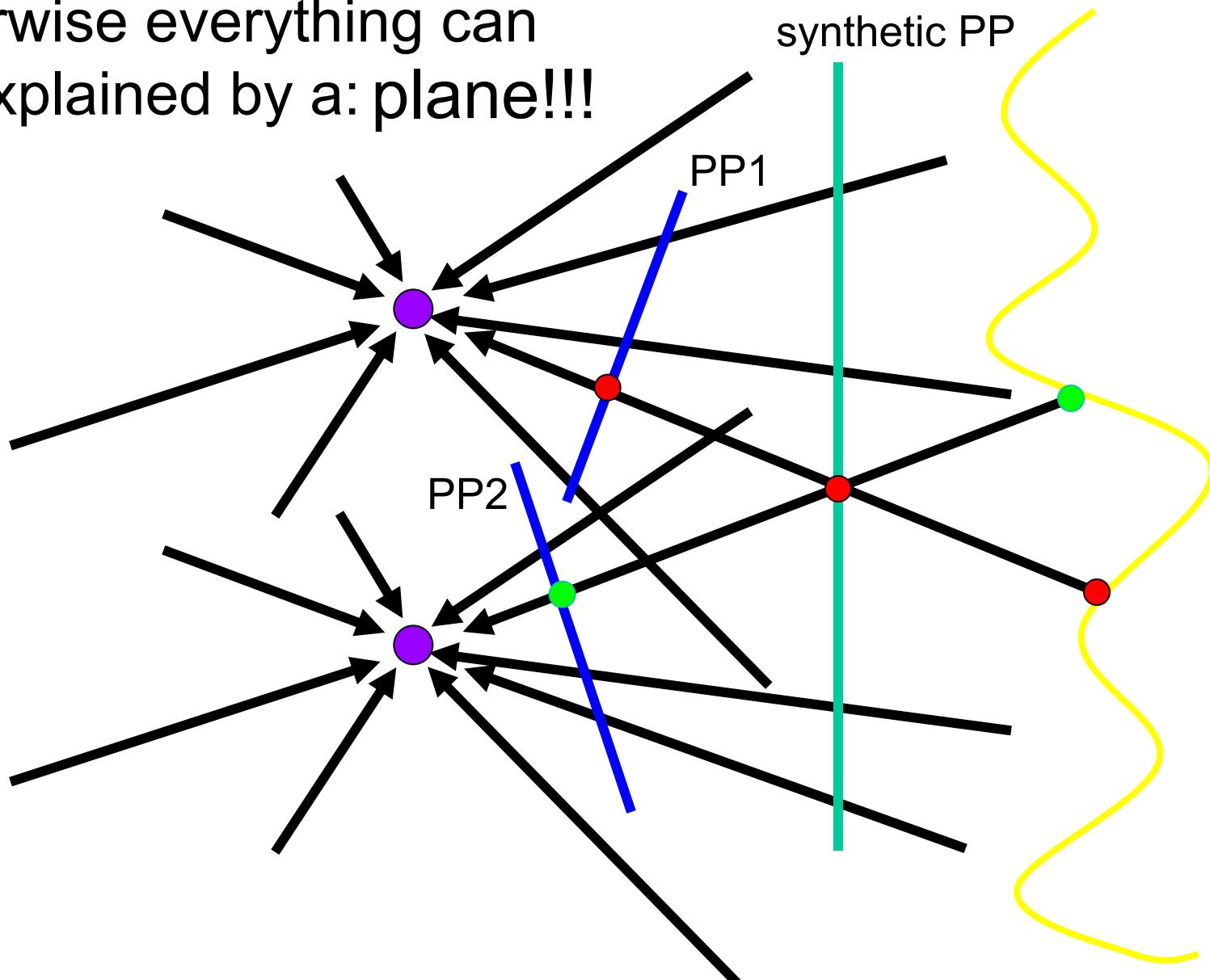
<https://youtu.be/LQwMAI9bGNY?si=6rf4F9NmB7o8vfqY>

Geometric Depth Understanding

- Ambiguous from a single image
- Why?

Need two different camera center

otherwise everything can
be explained by a plane!!!



Fundamental Depth Ambiguity in 2D → 3D



Original Image



Same 2D Projection

Huge object,
far from camera

Tiny object,
close to camera

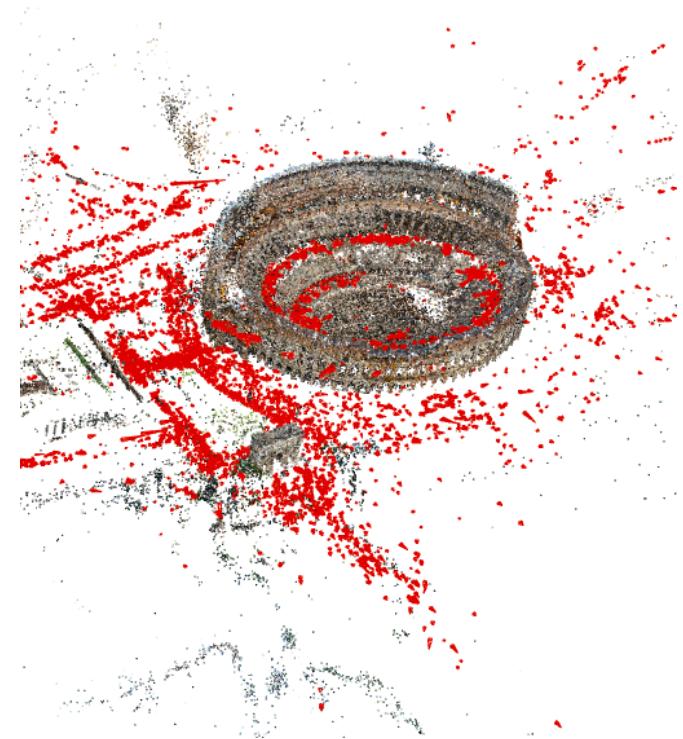


Camera

Infinite Possible 3D Interpretations

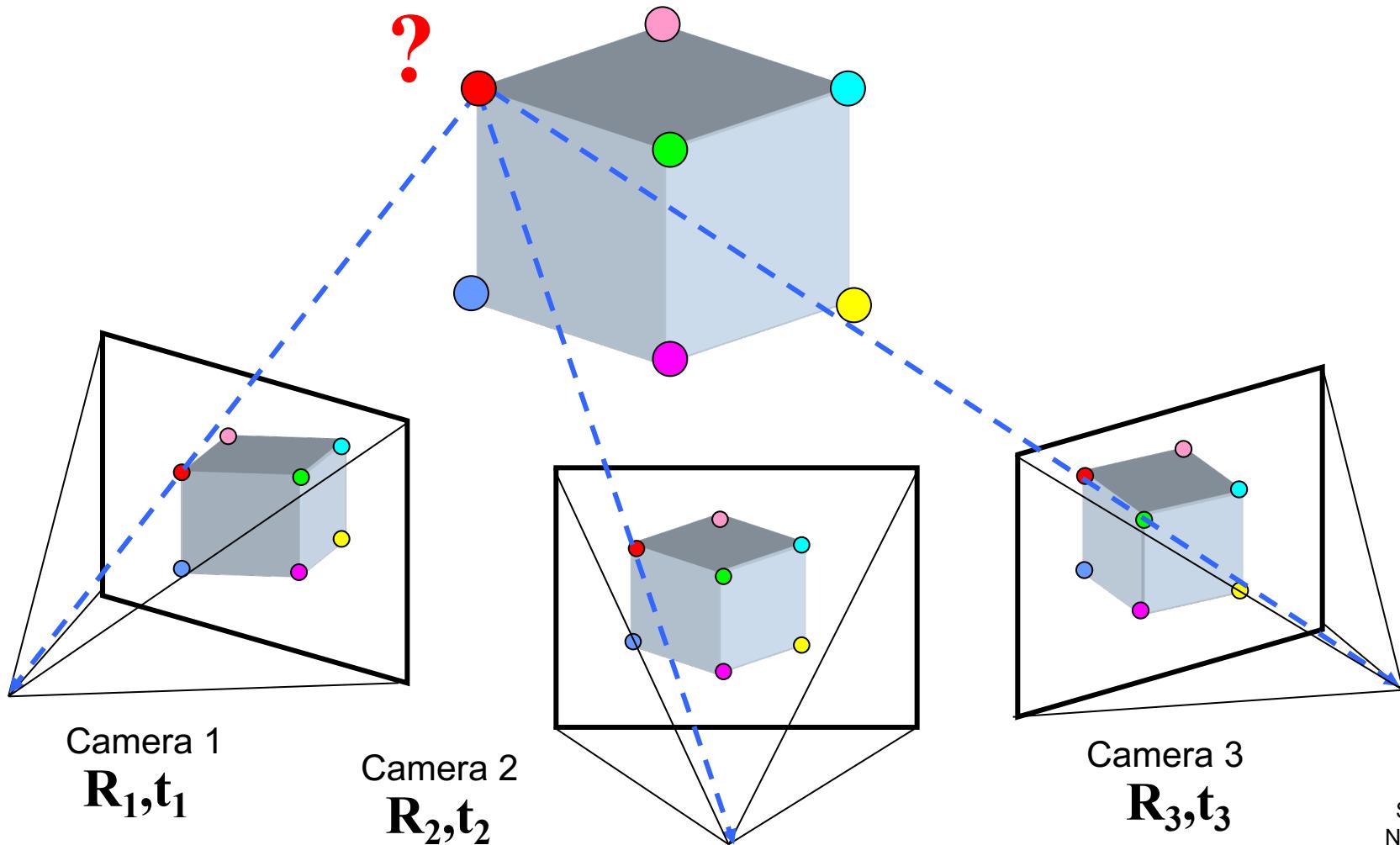
2.5D vs 3D

- is 3D = depth from a single image?



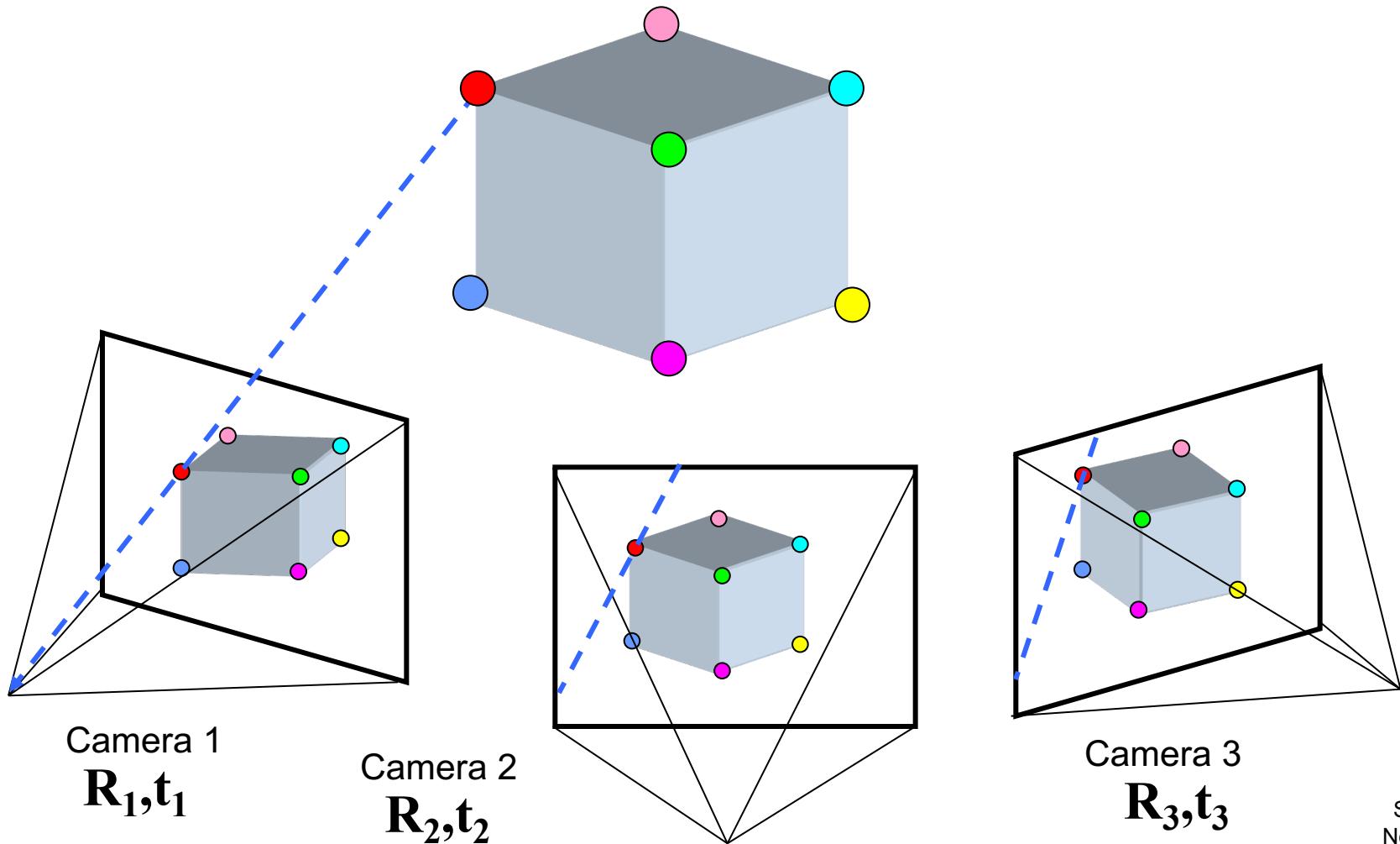
Multi-view geometry problems

- **Structure:** What is the 3D coordinate of a point that can be seen in multiple images?



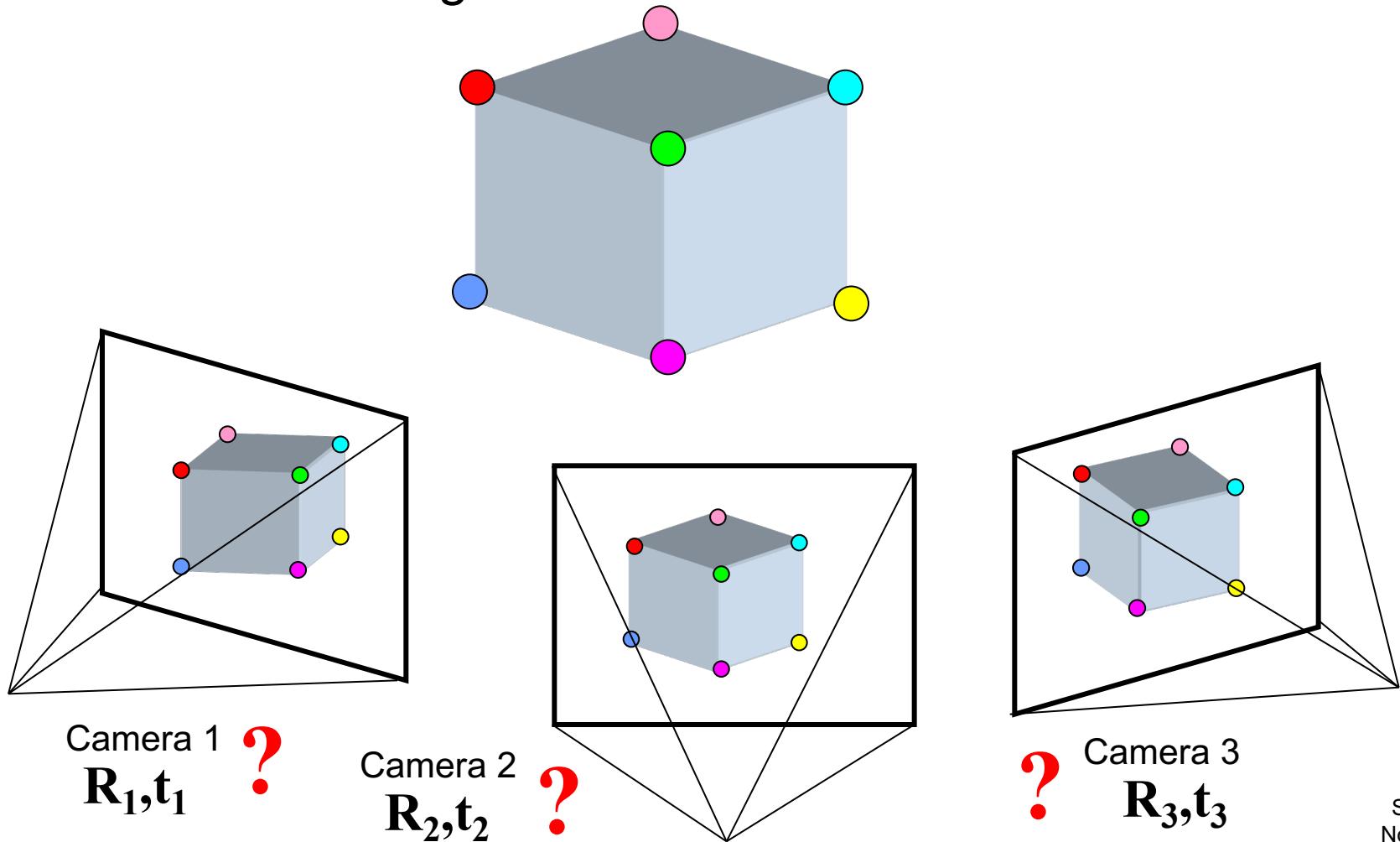
Multi-view geometry problems

- **Correspondence:** Given a point in one of the images, where are the corresponding points in the other images?



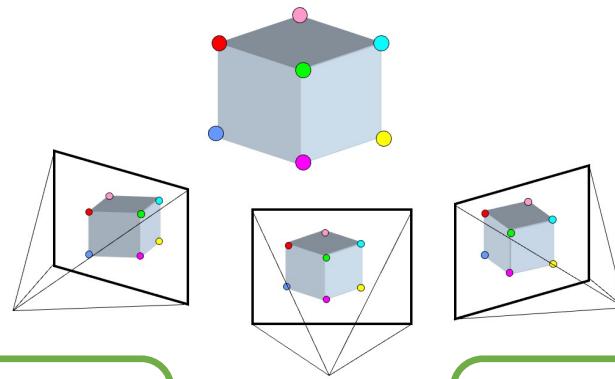
Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, what is the relative camera parameters between the images?



Big picture: 3 key components in 3D

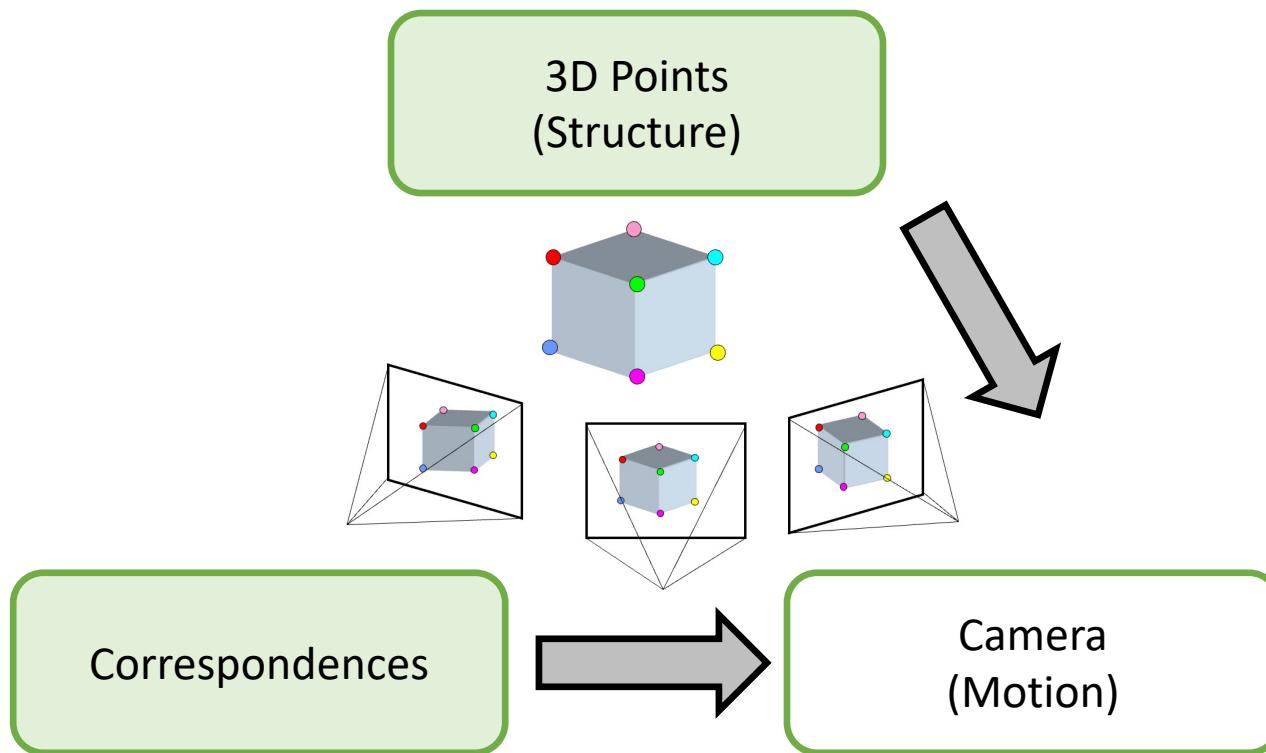
3D Points
(Structure)



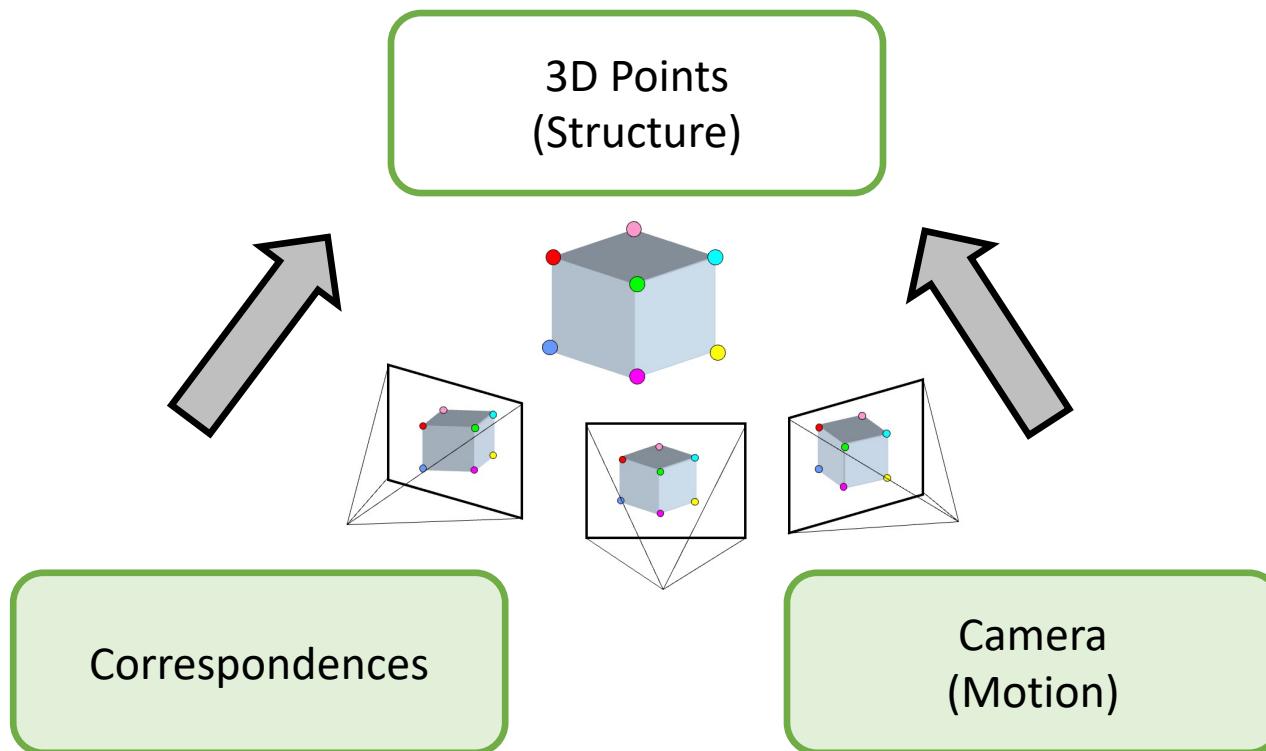
Correspondences

Camera
(Motion)

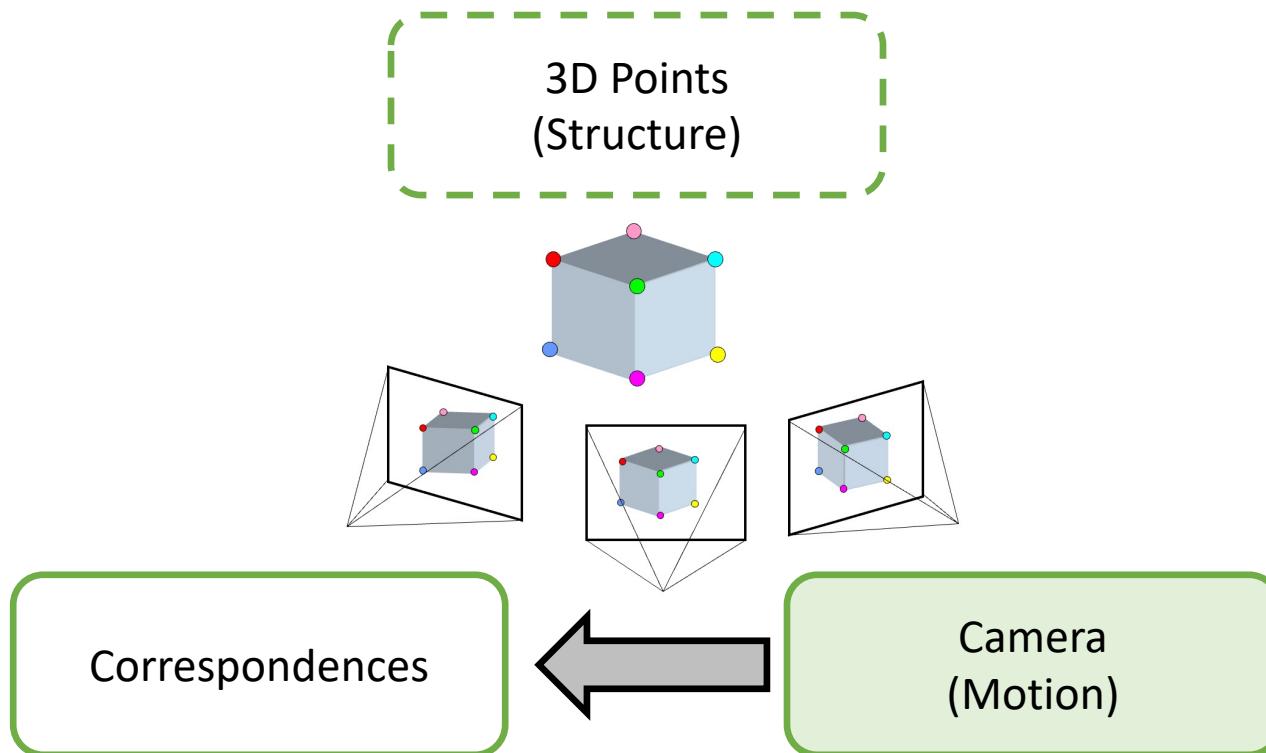
Big picture: 3 key components in 3D



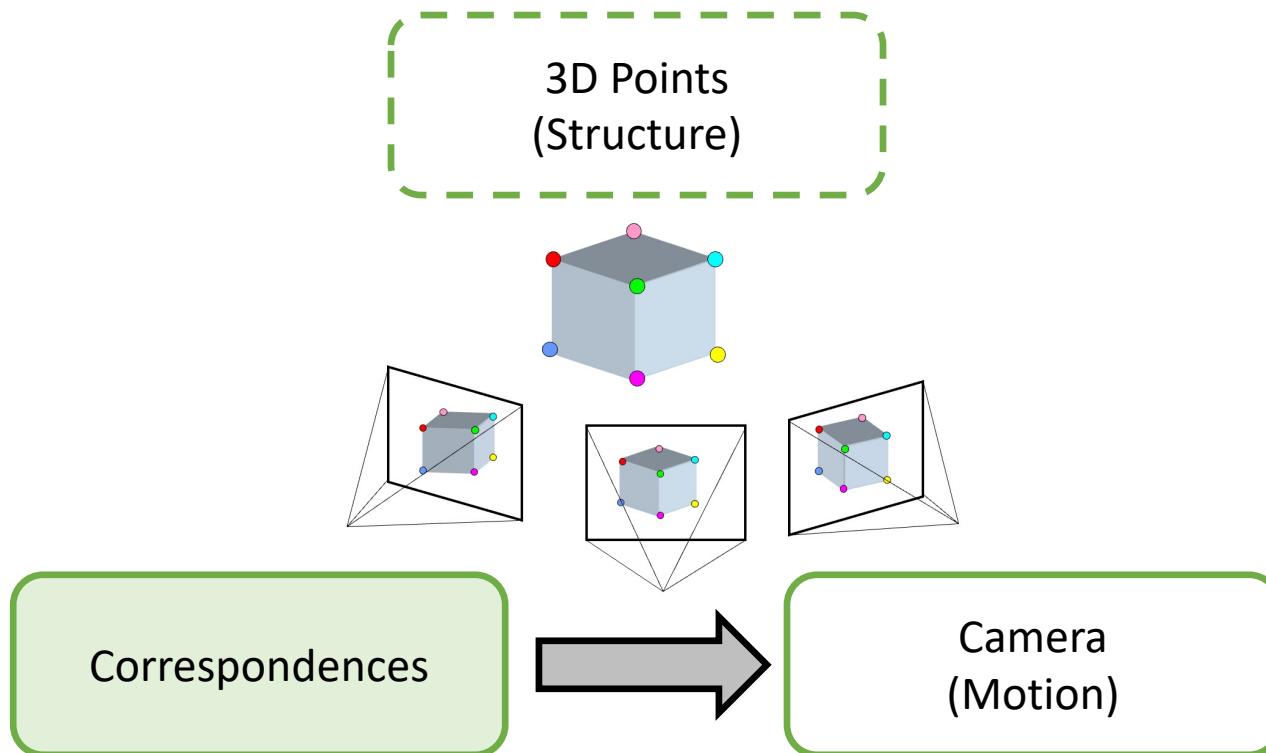
Big picture: 3 key components in 3D



Big picture: 3 key components in 3D



Big picture: 3 key components in 3D



From pixels to the 3D world

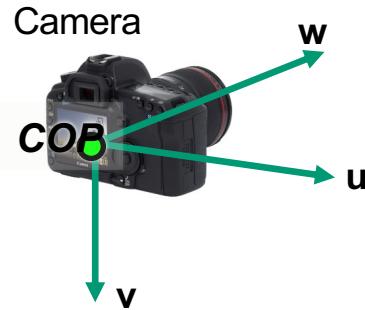
To go from pixels to 3D location in the **world coordinates**, we need to know two things about the camera:

1. Position of the camera with respect to the world (extrinsics)
2. How the camera maps a point in the world to image (intrinsics)

Problem setup

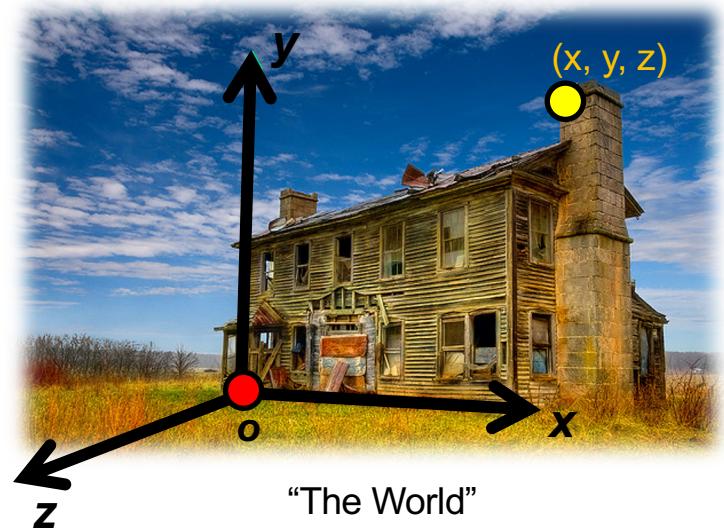
There is a world coordinate frame and camera looking at the world

How can we model the geometry of a camera?



Three important coordinate systems:

1. *World coordinates*
2. *Camera coordinates*
3. *Image coordinates*

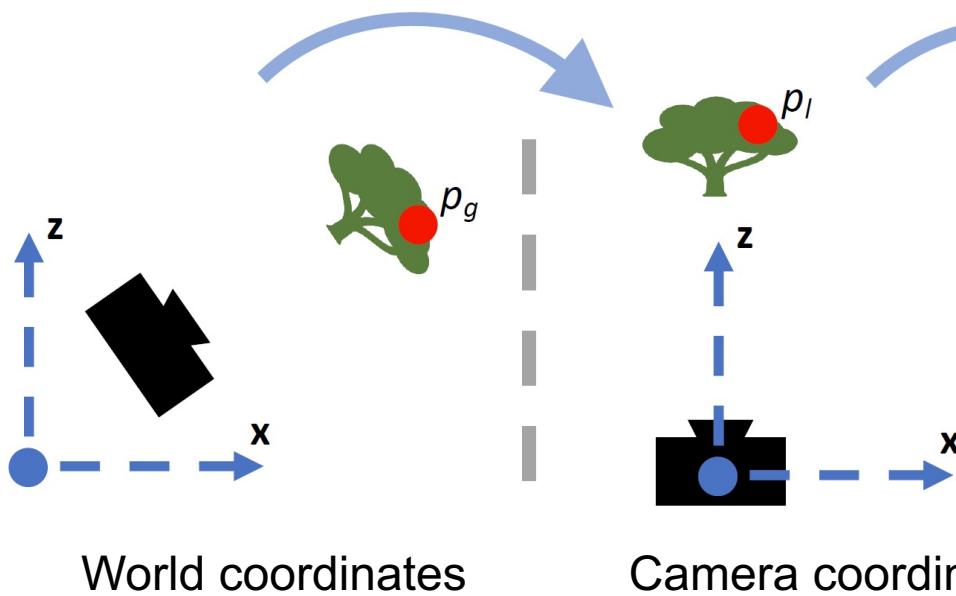


Coordinate frames + Transforms

Orientation + Location of
the camera in the World

How the camera maps a
point in 3D to image

Extrinsics (R, T)



Intrinsics (K)

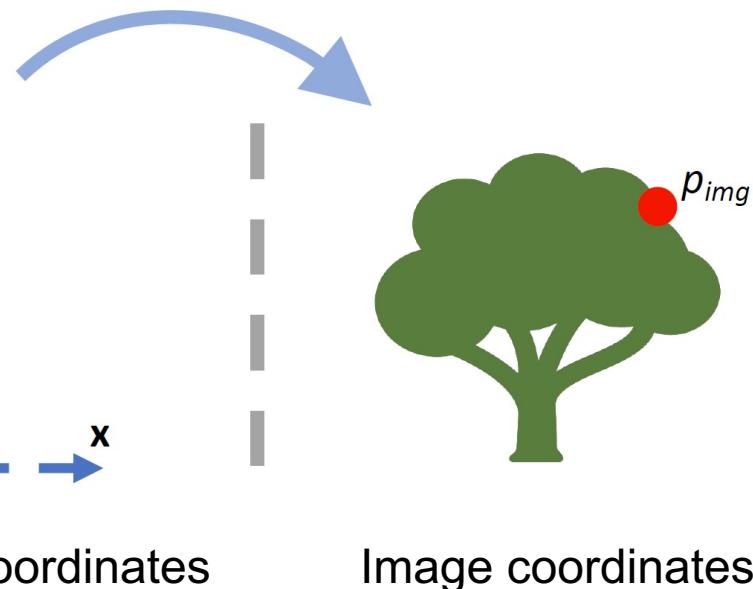


Figure credit: Peter Hedman

Camera: Specifics

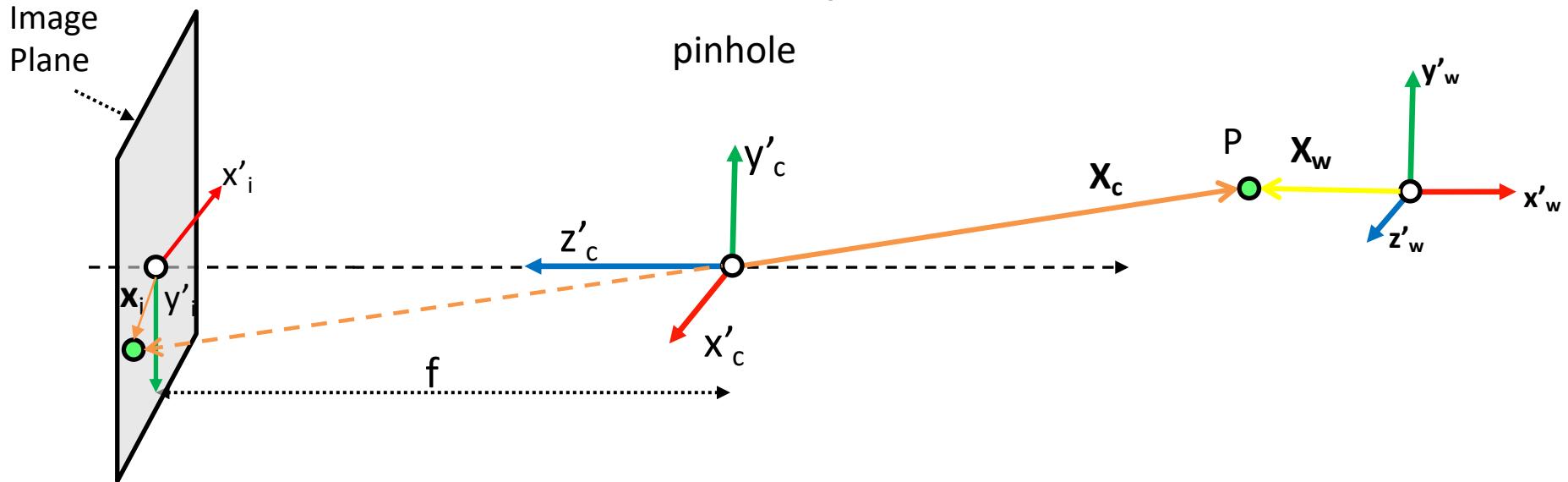


Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Perspective
Projection
(3D to 2D)

Camera Coordinates

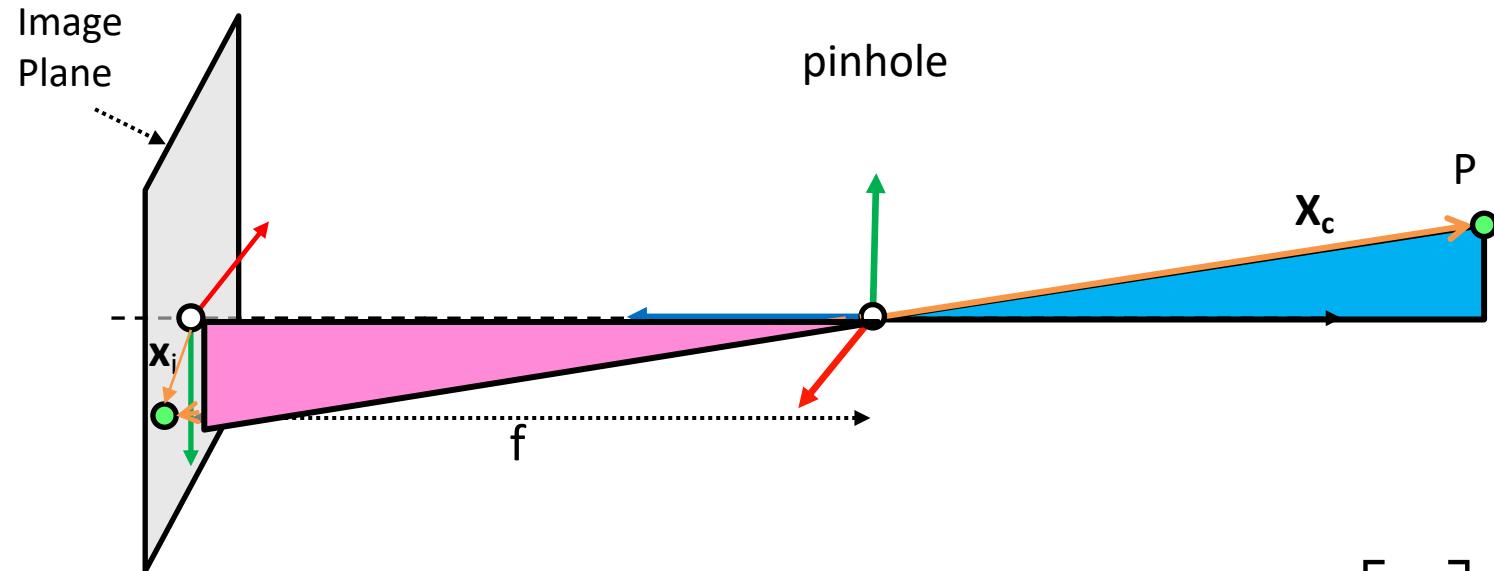
$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Coordinate
Transformation
(3D to 3D)

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Perspective Projection



$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Image Coordinates

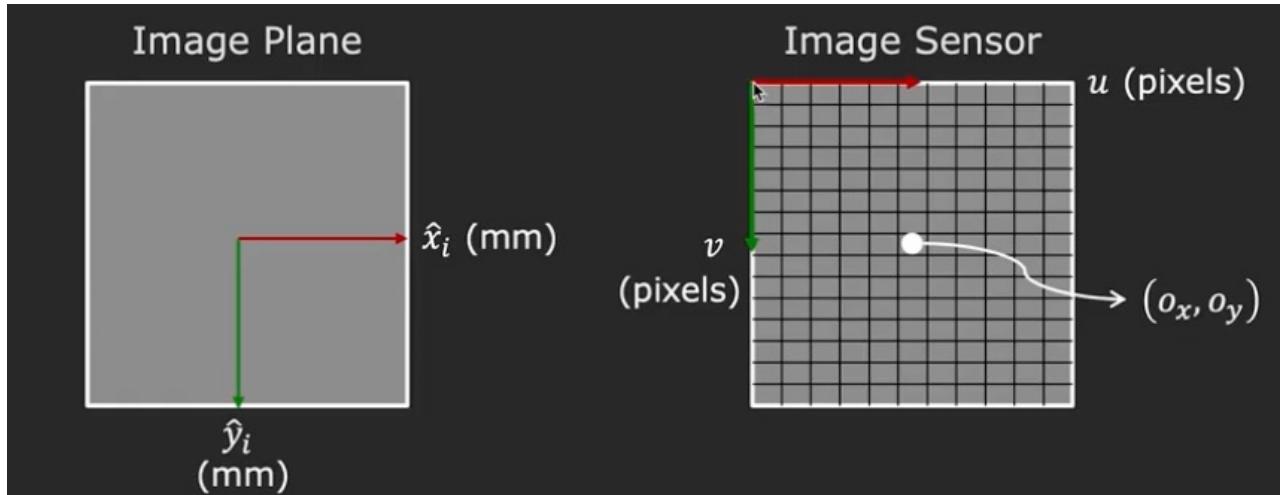
$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

$$\mathbf{x}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Camera Coordinates

$$x_i = f \frac{x_c}{z_c}$$

Image Plane to Image Sensor Mapping



1. Account for pixel density (pixel/mm) & aspect ratio by scalars: $[m_x, m_y]$

$$m_x x_i, m_y y_i$$

2. Usually the top left corner is the origin. But in the image plane, the origin is where the optical axis pierces the plane! Need to shift by:

$$(o_x, o_y)$$

$$u_i = \alpha_x x_i + o_x = \alpha_x f \frac{x_c}{z_c} + o_x$$

where $[f_x, f_y] = [m_x f, m_y f]$

Pixel Coordinates:

$$u_i = \boxed{f_x} \frac{x_c}{z_c} + \boxed{o_x} \quad v_i = \boxed{f_y} \frac{y_c}{z_c} + \boxed{o_y}$$

With homogeneous coordinates

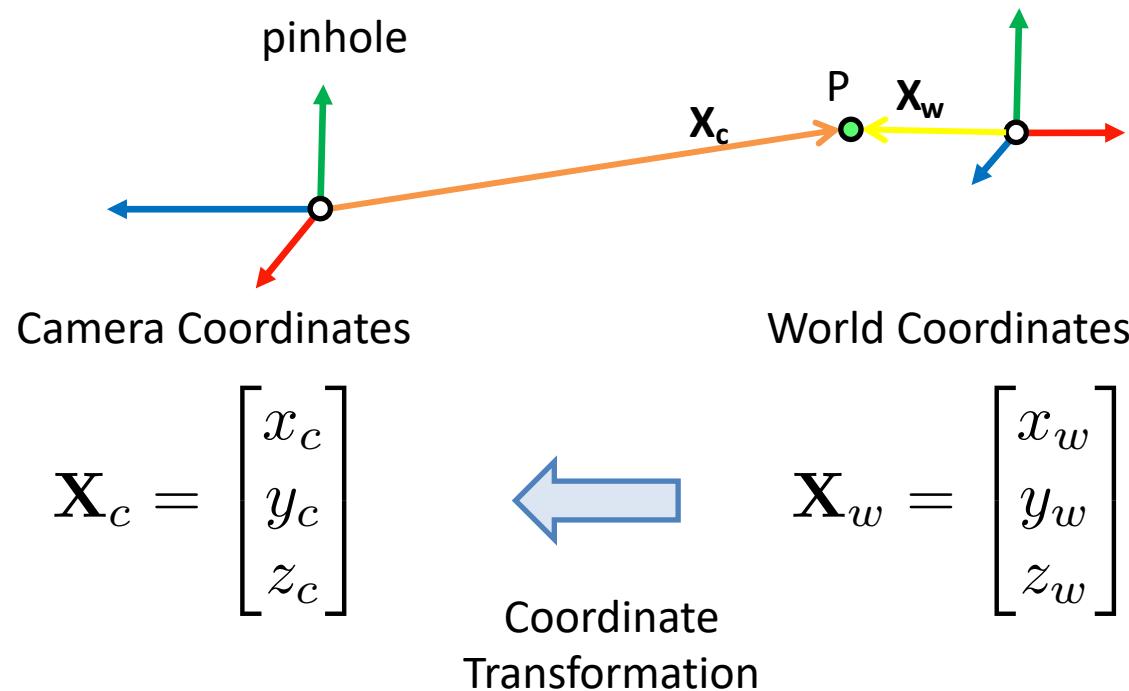
Perspective projection + Transformation to Pixel Coordinates:

$$u_i = f_x \frac{x_c}{z_c} + o_x \quad v_i = f_y \frac{y_c}{z_c} + o_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Intrinsic Matrix

Camera Transformation (3D-to-3D)



$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix

Putting it all together

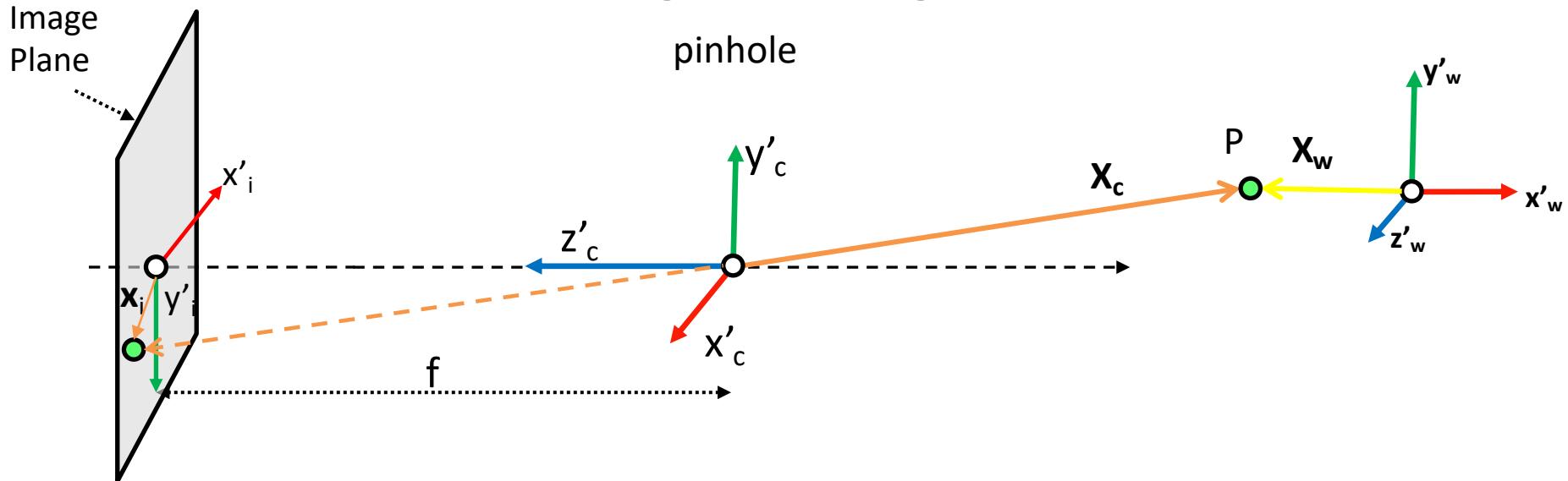


Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

←

Perspective
Projection

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

←

Coordinate
Transformation

$$\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

Projection Matrix

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

3 x 4 Projection matrix

For completeness, we need to add **skew** (this is 0 unless pixels are shaped like rhombi/parallelograms)

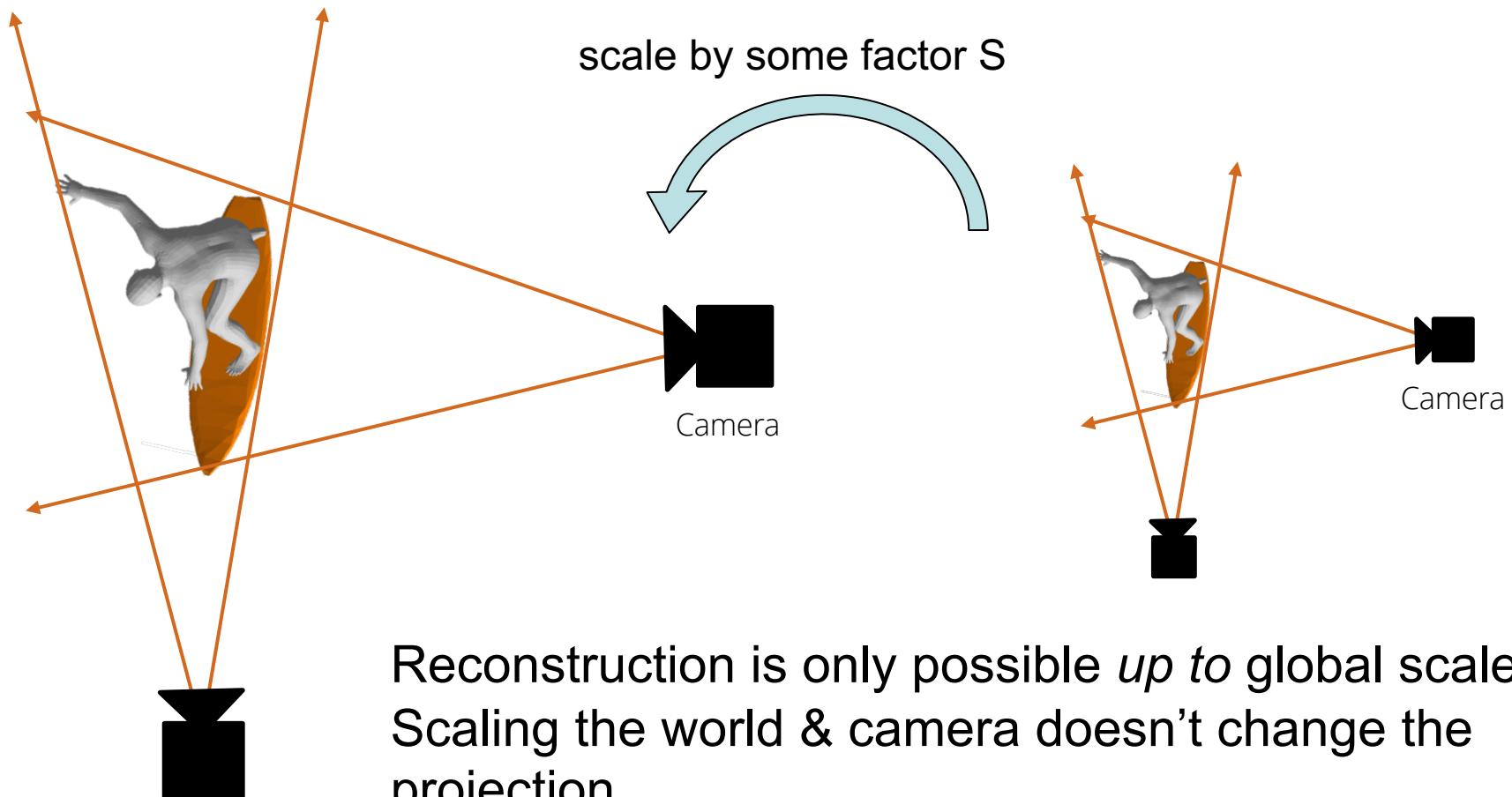
$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

What's the Degrees of Freedom?

Intrinsics: $4 + 1$ (skew)
Extrinsic: $3 + 3 = 6$

11 unknowns (up to scale)

Fundamental Scale Ambiguity



Reconstruction is only possible *up to* global scale
Scaling the world & camera doesn't change the projection
Unless you know something metric about the scene
e.g. surfboard is 2.1m

Going from World to Camera

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

$$\mathbf{X}_c = T_{w2c} \mathbf{X}_w$$

Going from Camera to World

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

$$T_{w2c}^{-1} \mathbf{X}_c = \mathbf{X}_w$$

Camera to Image

Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Intrinsics Matrix:

$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{x}_i = K\mathbf{X}_c$$

Image to Camera?

Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

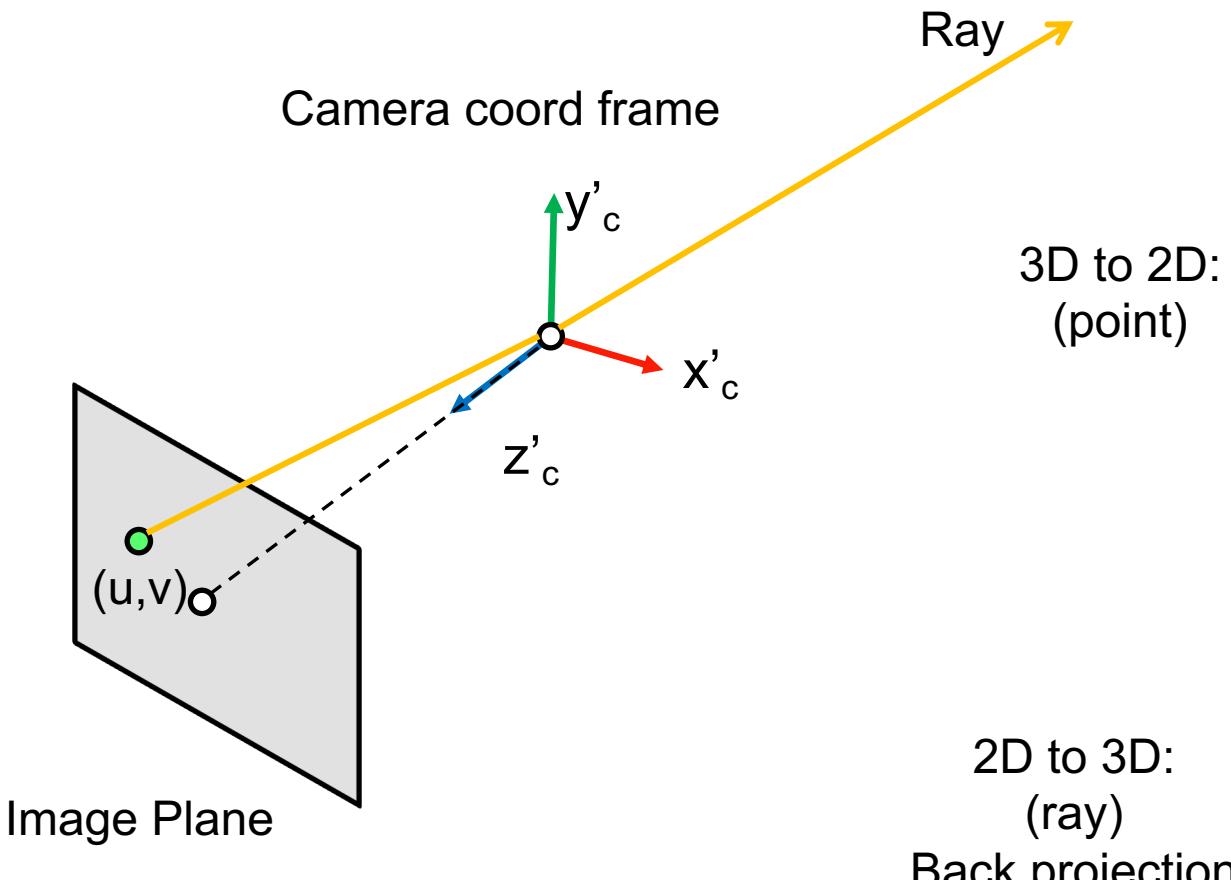
Intrinsics Matrix:

$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$u = f_x \frac{x_c}{z_c} + o_x \quad \longrightarrow \quad x = \frac{z}{f_x} (u - o_x)$$

What's the problem?

We don't know the depth!
but at the least it will be:
on the ray!



$$u = f_x \frac{x_c}{z_c} + o_x$$
$$v = f_y \frac{y_c}{z_c} + o_y$$

$$x = \frac{z}{f_x} (u - o_x)$$
$$y = \frac{z}{f_y} (v - o_y)$$
$$z > 0$$

What is your coordinate space?

- In Project 5 (and in life) always make sure you're in the right coordinate space.
- eg. Which space is the ray defined in?

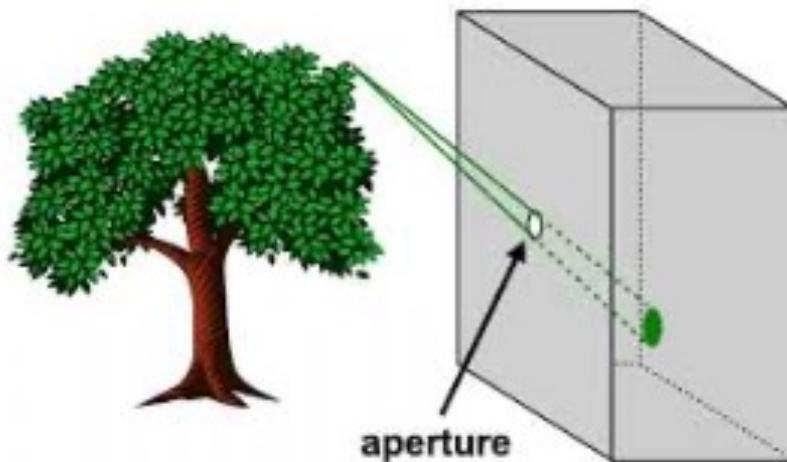
2D to 3D:
(ray)
Back projection

$$x = \frac{z}{f_x} (u - o_x)$$

$$y = \frac{z}{f_y} (v - o_y)$$

$$z > 0$$

Watch these 5 min videos



<https://www.youtube.com/watch?v=F5WA26W4JaM>
<https://www.youtube.com/watch?v=g7Pb8mrwcJ0>

Where are my cameras?

How to calibrate the camera?

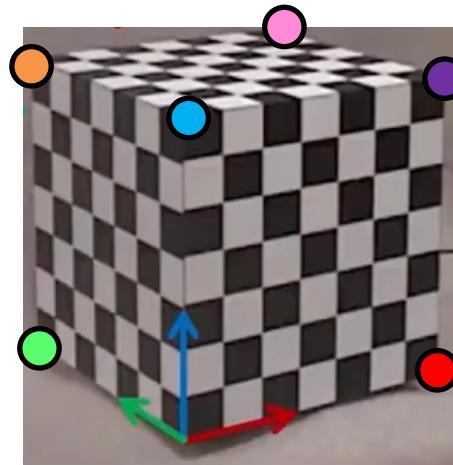
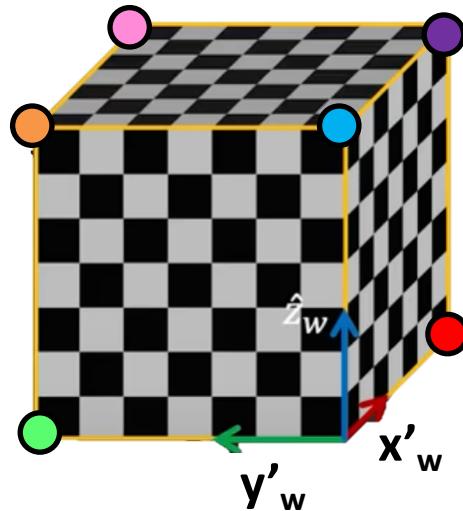
$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

If we know the points in 3D we can estimate the camera!!

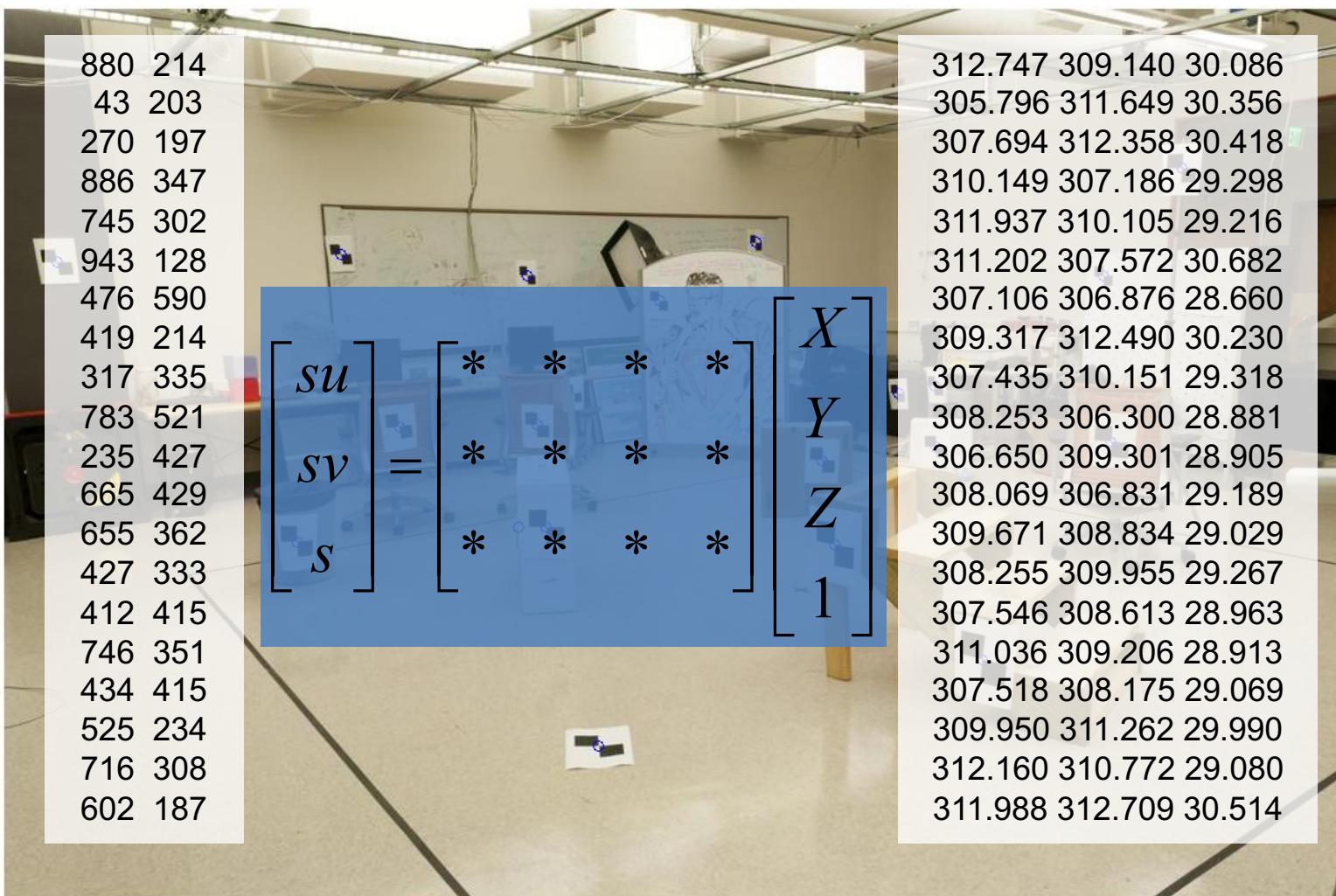
Step 1: With a known 3D object

1. Take a picture of an object with known 3D geometry



2. Identify correspondences

How do we calibrate a camera?



Method: Set up a linear system

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Solve for m's entries using linear least squares

Ax=0 form

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & \vdots & & & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix}$$

Similar to how you solved for homography!

Can we factorize M back to K [R | T]?

- Yes.
- Why? because K and R have a very special form:

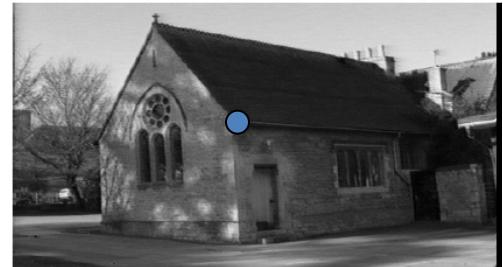
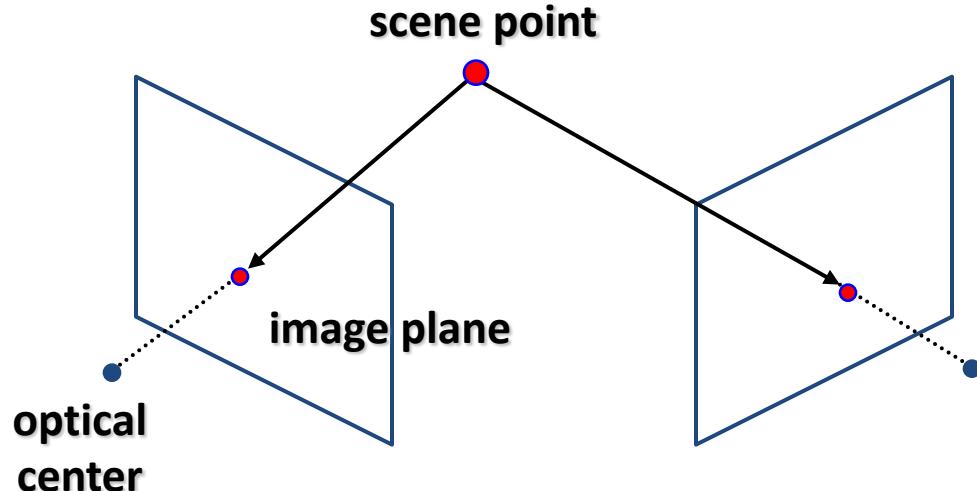
$$\begin{bmatrix} f_x & s & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

- QR decomposition
- Practically, use camera calibration packages (there is a good one in OpenCV)

Now that our cameras are calibrated, can we find the 3D scene point of a pixel?

Estimating depth with stereo

- **Stereo:** shape from “motion” between **two views**
- We’ll need to consider:
 - 1. Camera pose (“calibration”)
 - 2. Image point correspondences



Stereo vision



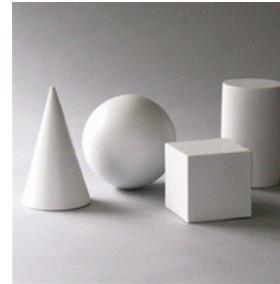
Two cameras, simultaneous views



Single moving camera and static scene

Simple Stereo Setup

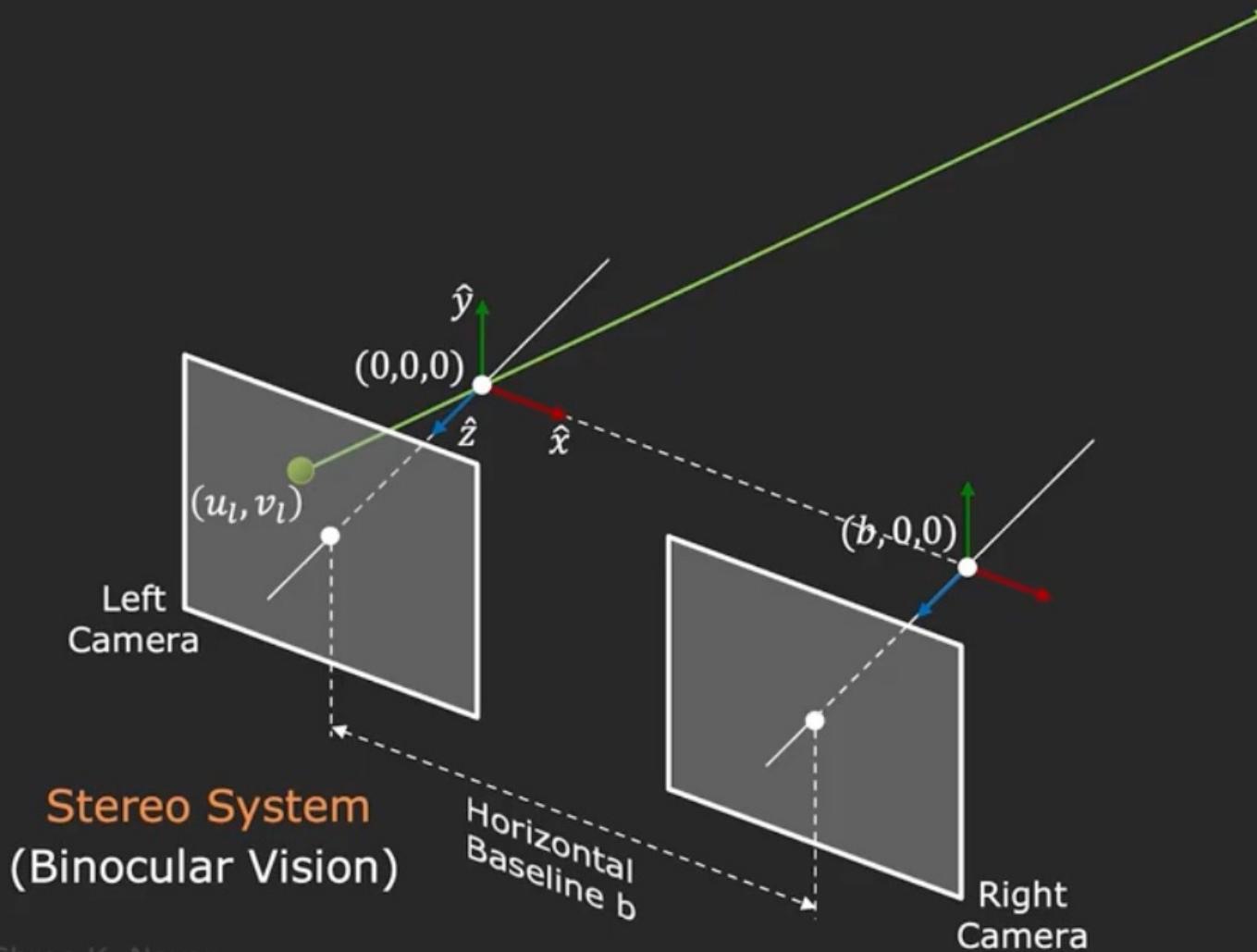
- Assume **parallel** optical axes
- Two cameras are calibrated
- Find relative depth



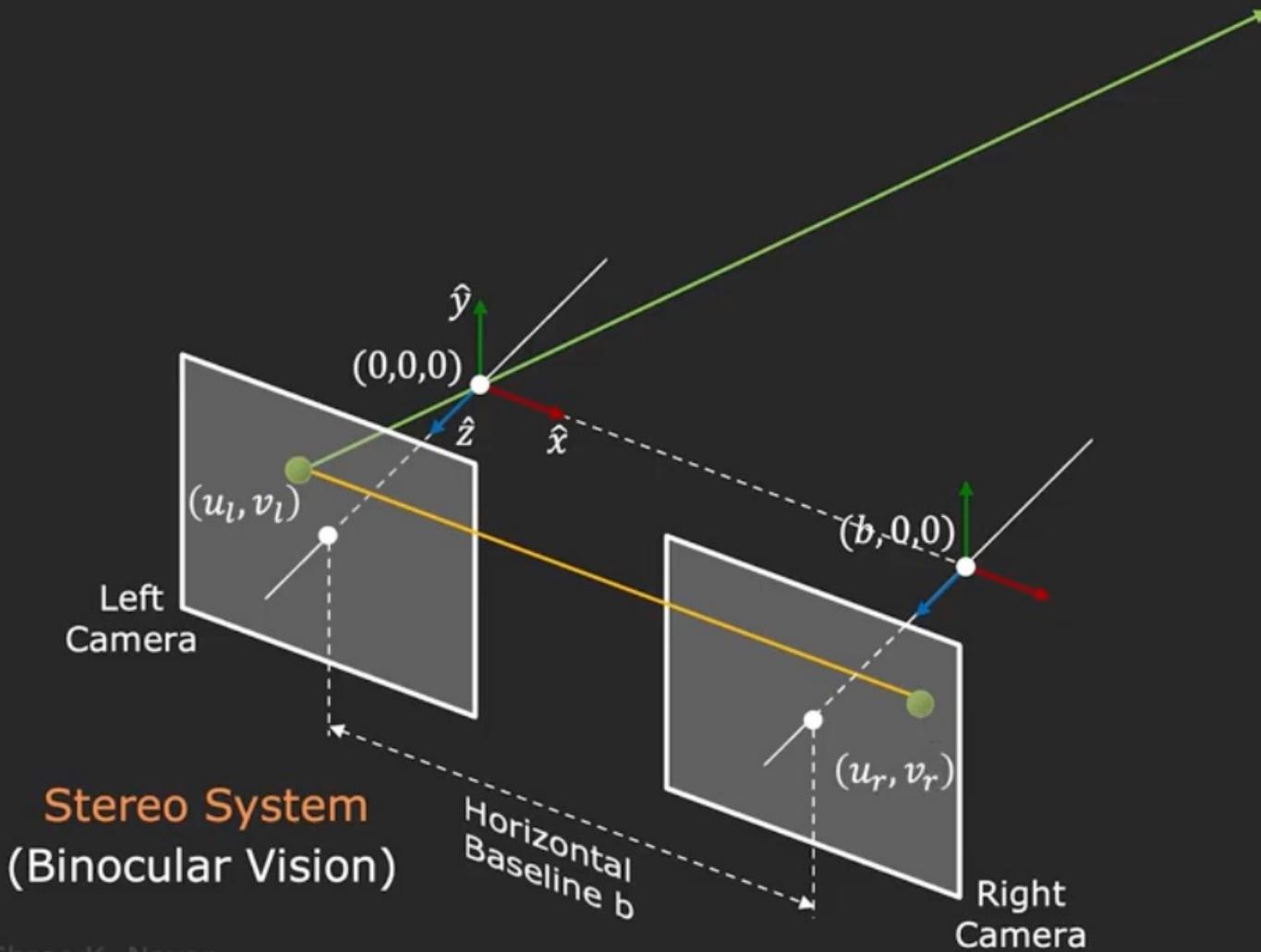
Key Idea: difference in corresponding points to understand shape

Slide credit: Noah Snavely

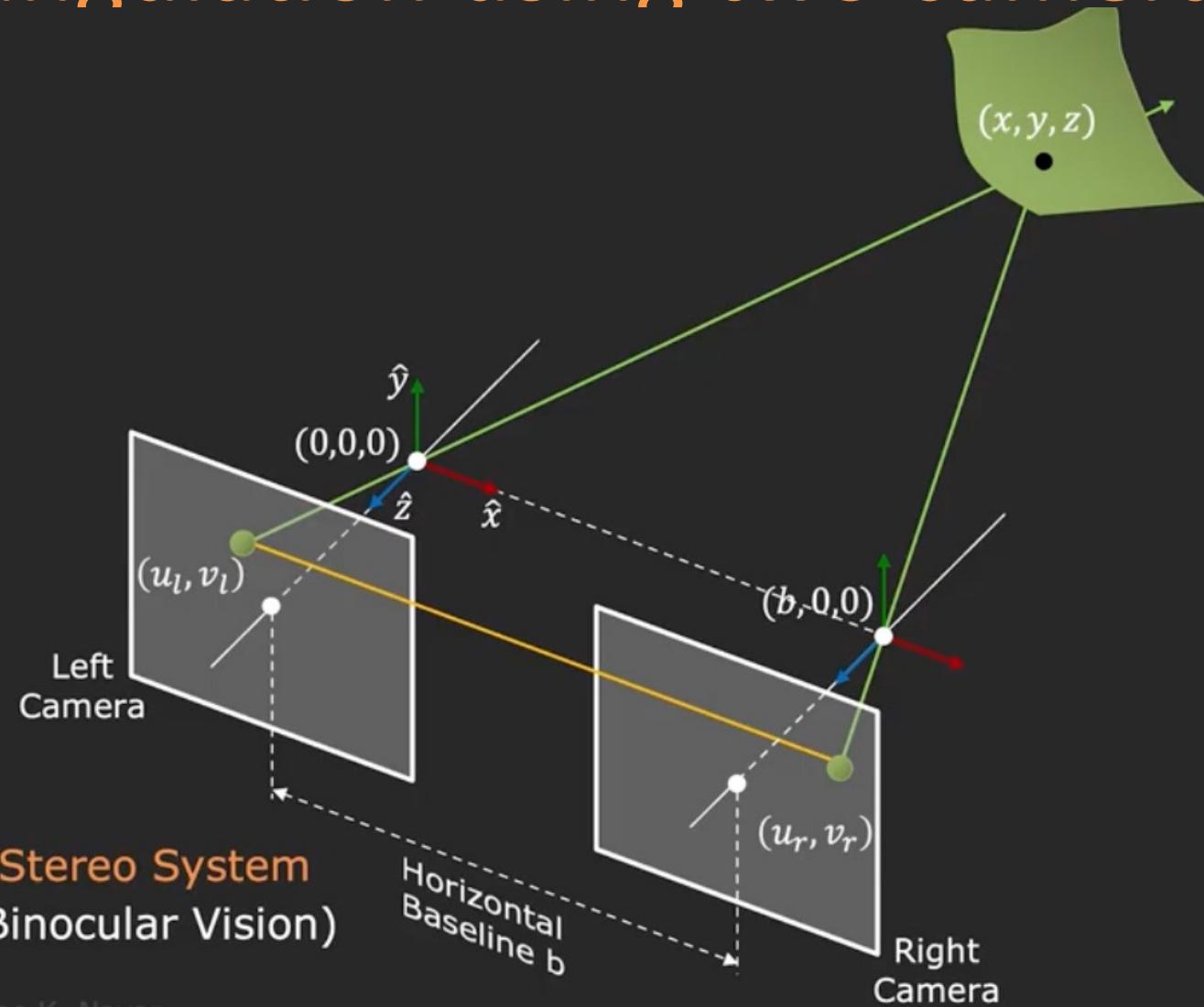
Triangulation using two cameras



Triangulation using two cameras



Triangulation using two cameras



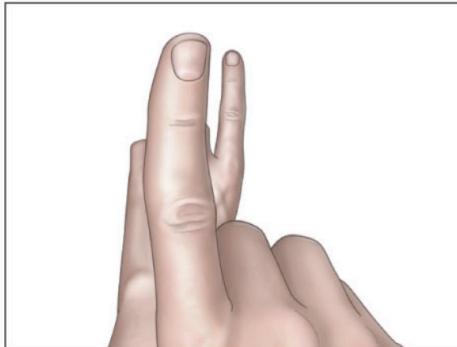
We are equipped with binocular vision.

Let's try!

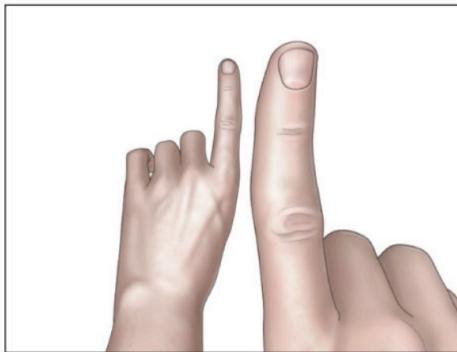
(a)



(b)

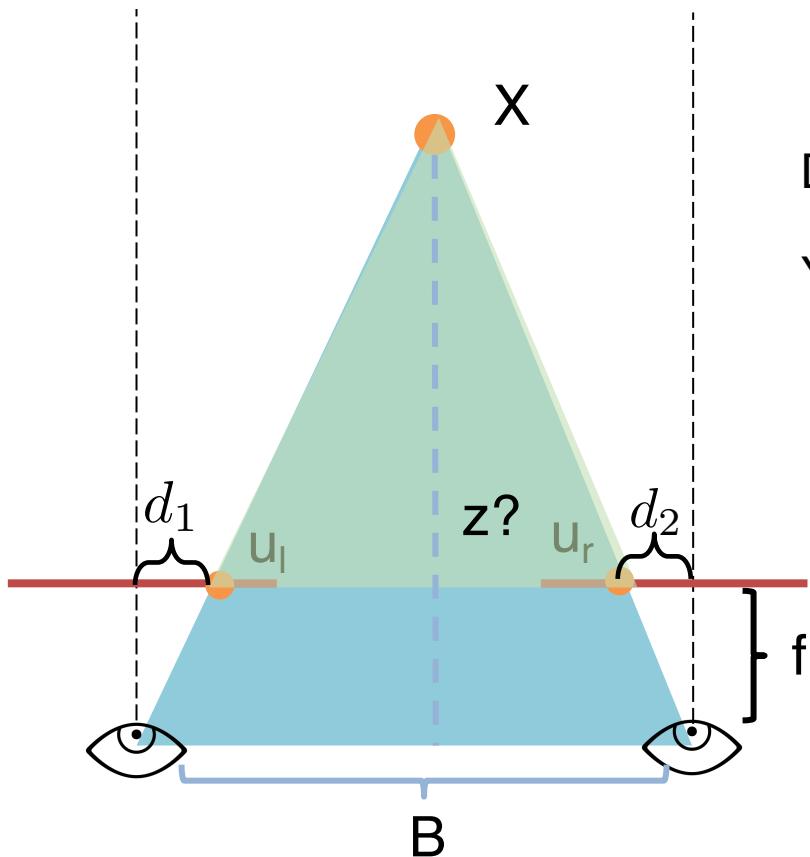


Right retinal image

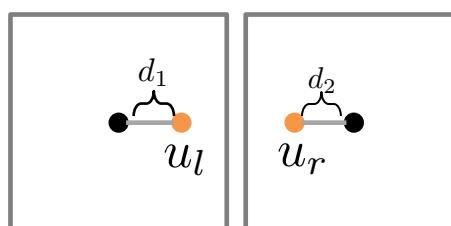


Left retinal image

Solving for Depth in Simple Stereo



Base of : $B - (d_1 + d_2)$
in image coordinates: $= B - (u_l - u_r)$



Do we have enough to know what is Z ?
Yes, similar triangles!

$$\frac{B - (u_l - u_r)}{z - f} = \frac{B}{z}$$

$$z = \frac{fB}{u_l - u_r}$$

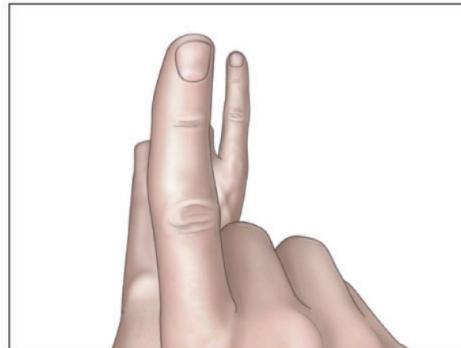
disparity
(how much
corresp. pixels
move)

Try with your hands!

(a)



(b)

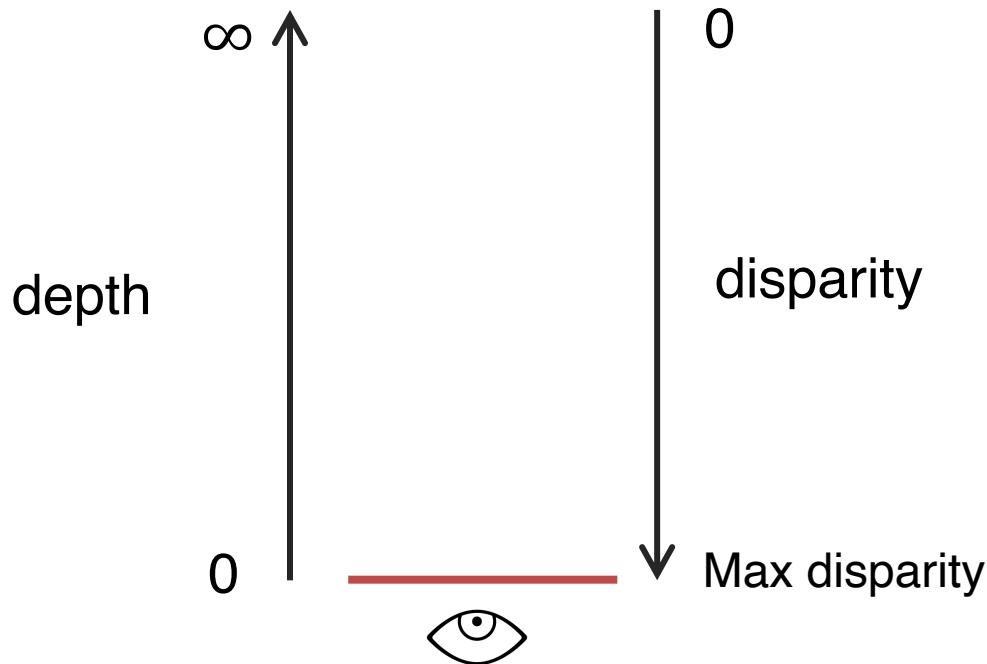


Right retinal image



Left retinal image

Depth is inversely proportional to disparity



$$z = \frac{fB}{u_l - u_r}$$

$$z \propto \frac{1}{u_l - u_r} = \frac{1}{d}$$

what is the disparity of the closer point?

what is the disparity of the far away point?

Disparity gives you the depth information!

Try again

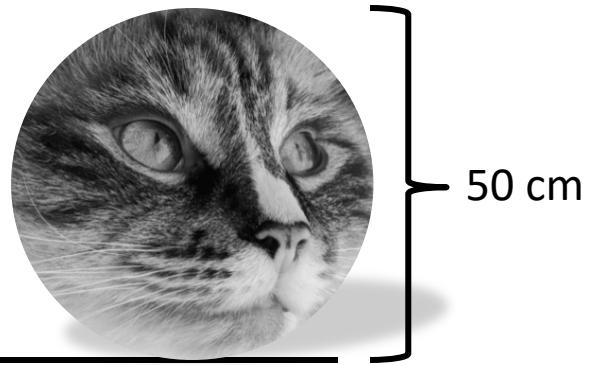
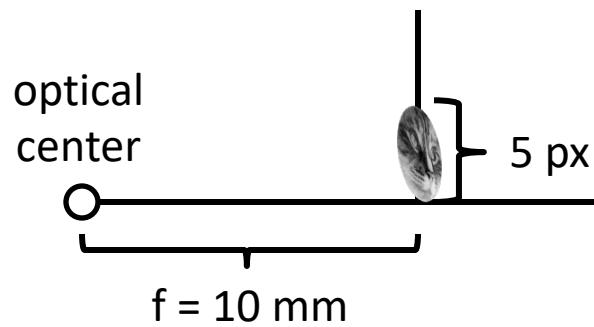
1. Setup so your fingers are on the same line of sight from one eye
2. Now look in the other eye

They move!

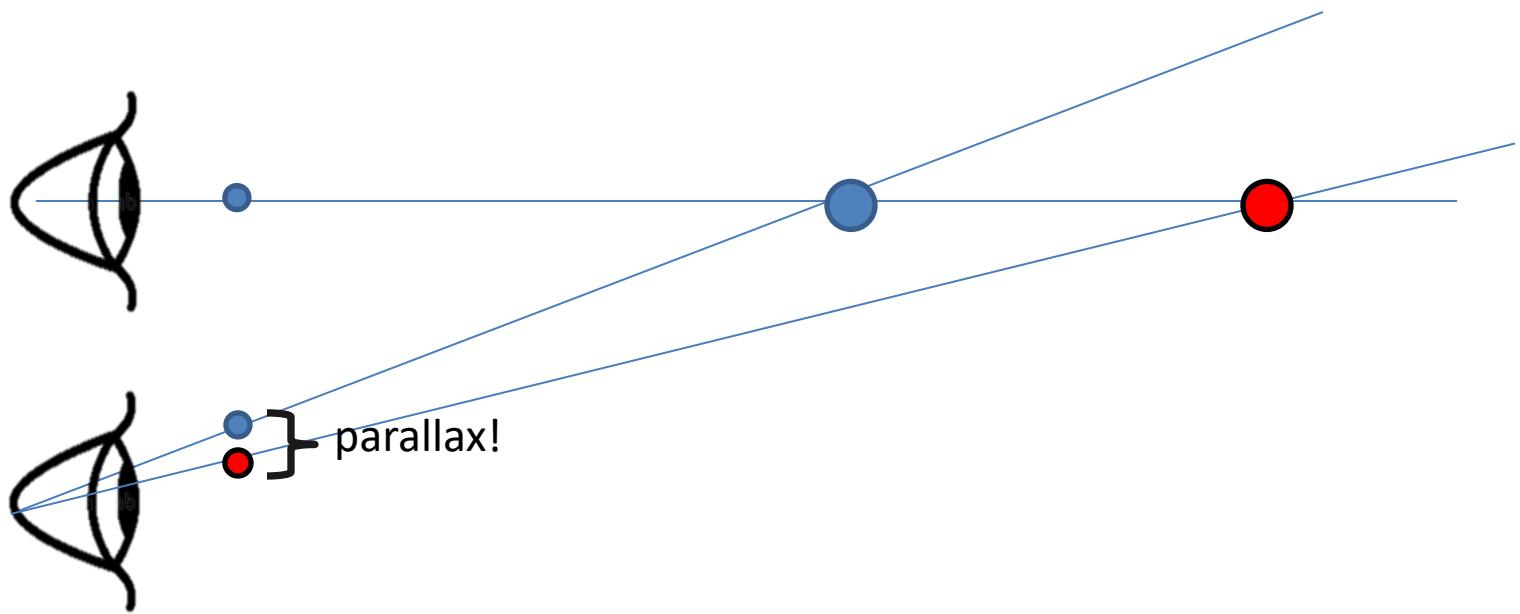
Relative displacement is higher as the relative distance grows

== Parallax





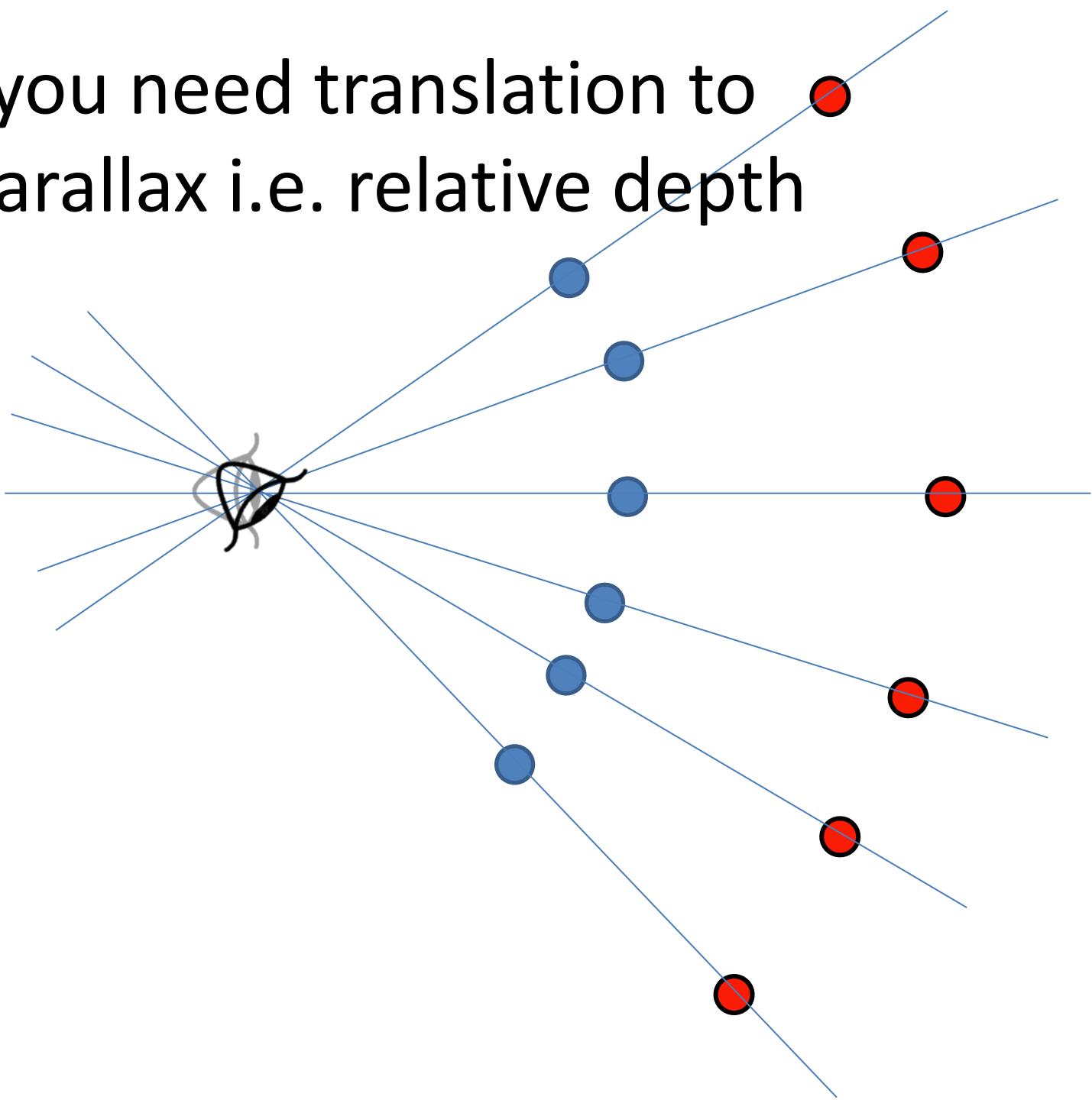
Parallax



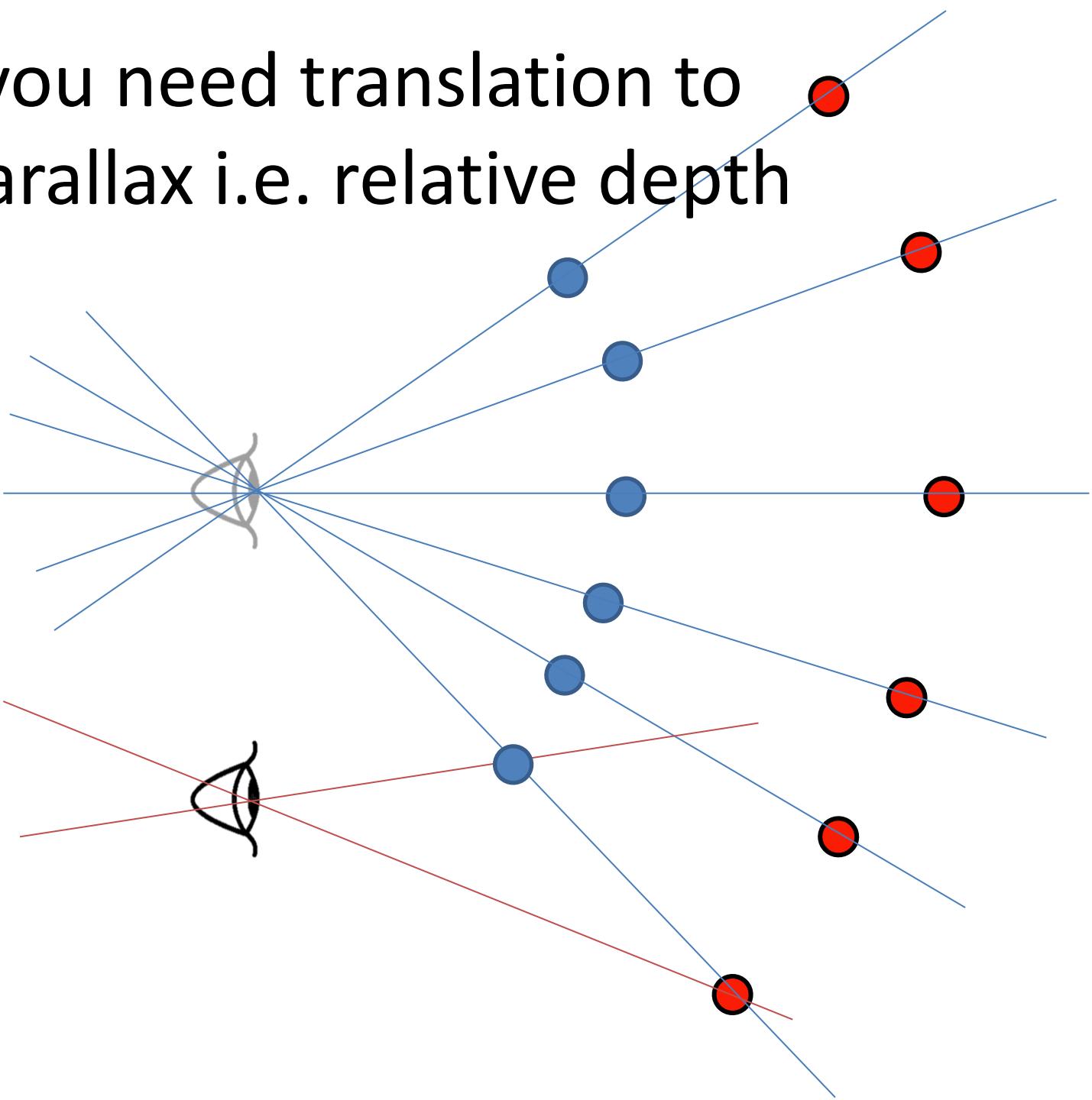
Parallax = *from ancient Greek parállaxis*
 = *Para* (side by side) + *allássō*, (to alter)
 = *Change in position from different view point*

Two eyes give you parallax, you can also move to see more
parallax = “Motion Parallax”

Why you need translation to see parallax i.e. relative depth



Why you need translation to see parallax i.e. relative depth



Stereo Matching: Finding Disparities

Goal: Find the disparity between left and right stereo pairs.



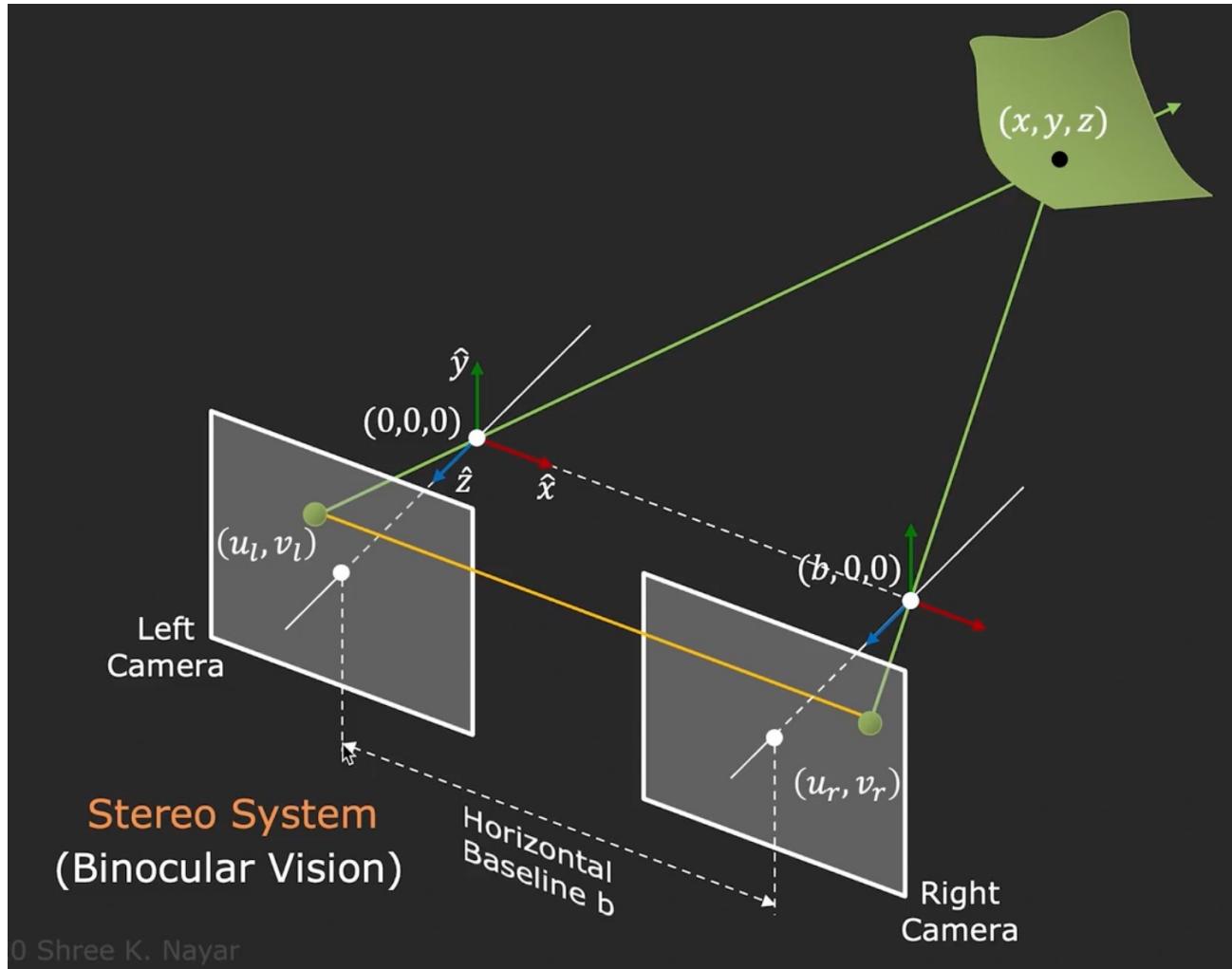
Left/Right Camera Images



Disparity Map (Ground Truth)

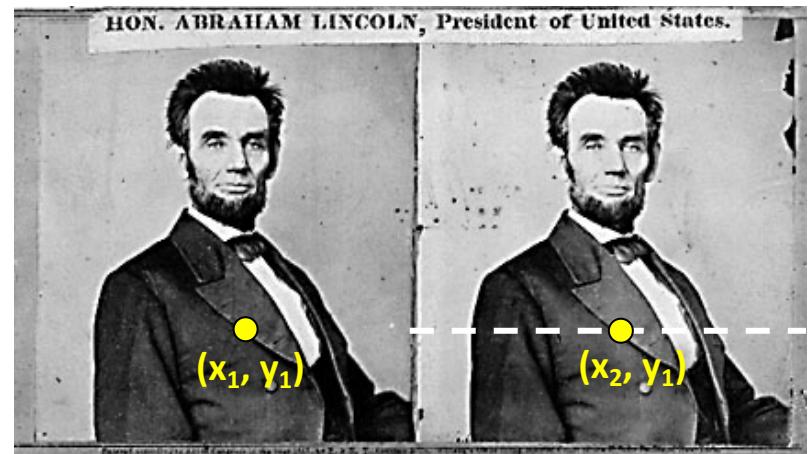
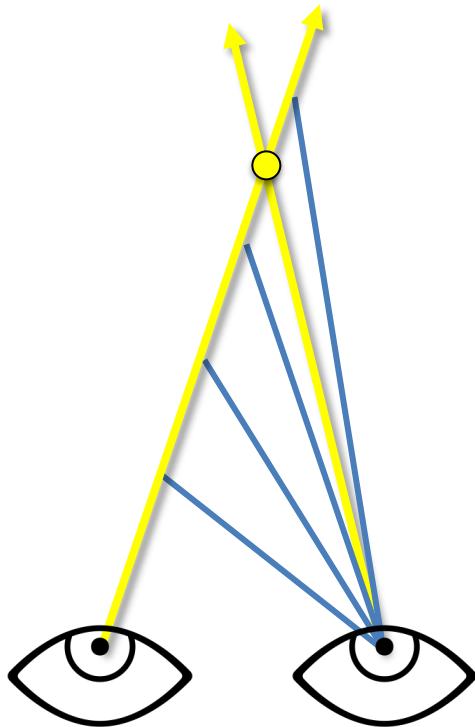
Where is the corresponding point going to be?

Hint



Stereo System
(Binocular Vision)

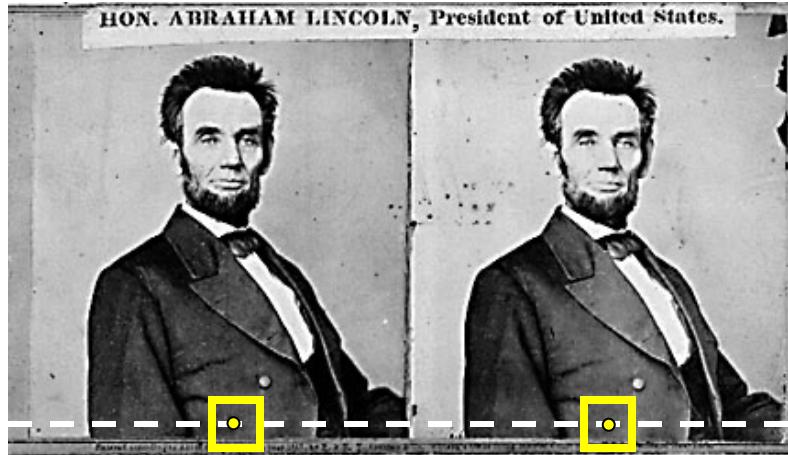
Epipolar Line



Two images captured by a purely horizontal translating camera
(rectified stereo pair)

$$x_1 - x_2 = \text{the disparity of pixel } (x_1, y_1)$$

Your basic stereo algorithm



For every epipolar line:

For each pixel in the left image

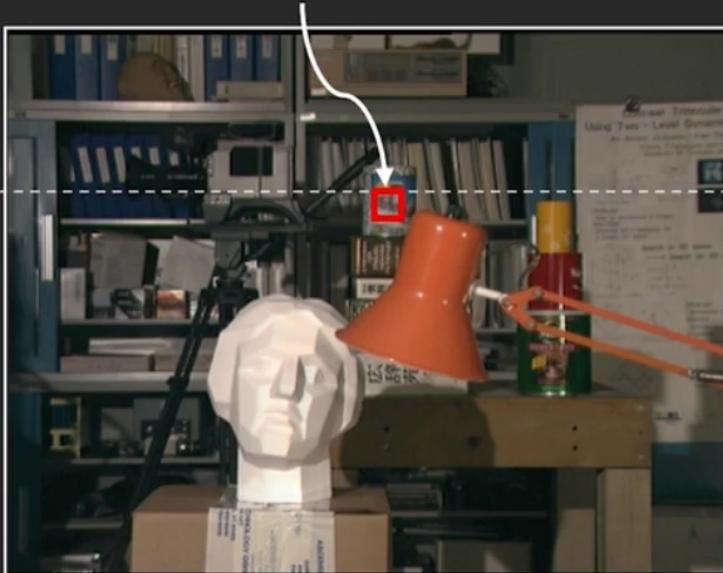
- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match **windows**, + clearly lots of matching strategies

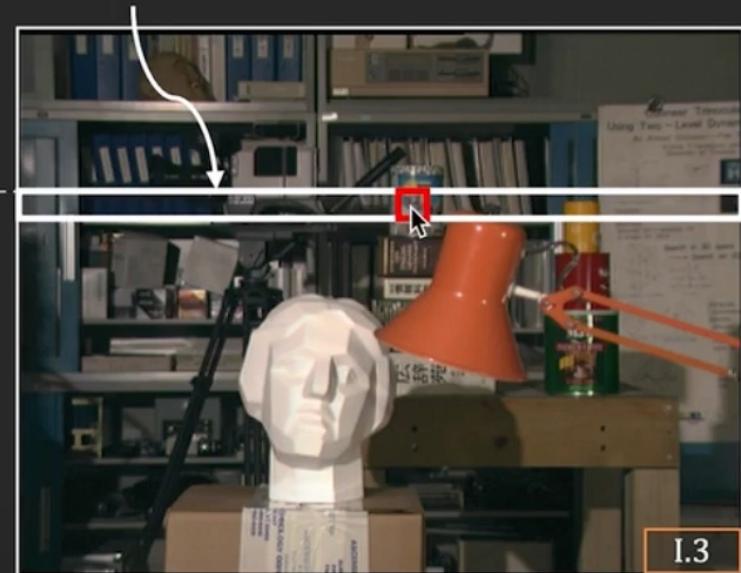
Your basic stereo algorithm

Determine Disparity using **Template Matching**

Template Window T



Search Scan Line L



Left Camera Image E_l

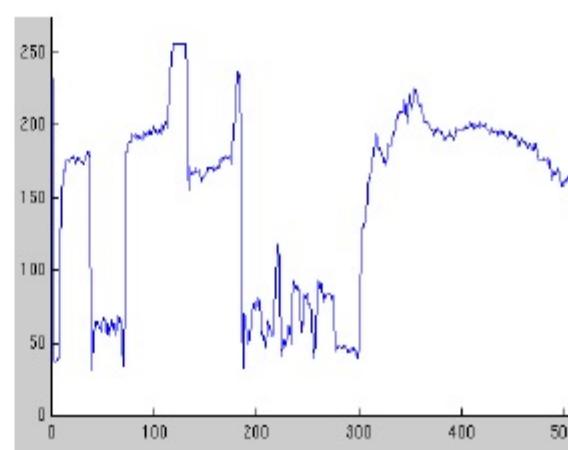
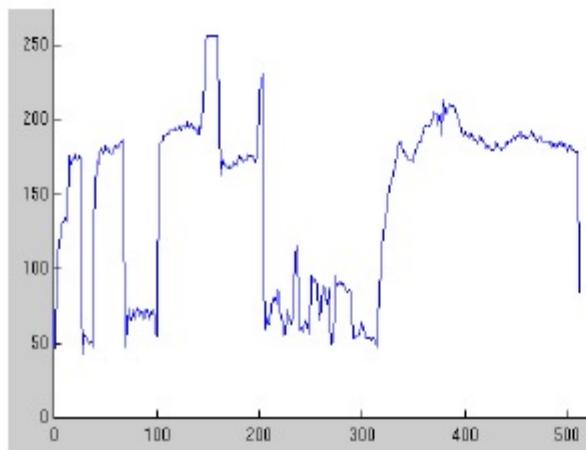
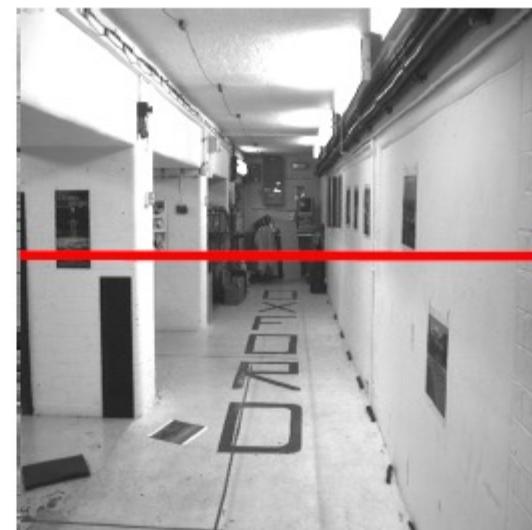
Right Camera Image E_r

Correspondence problem

Parallel camera example – epipolar lines are corresponding rasters

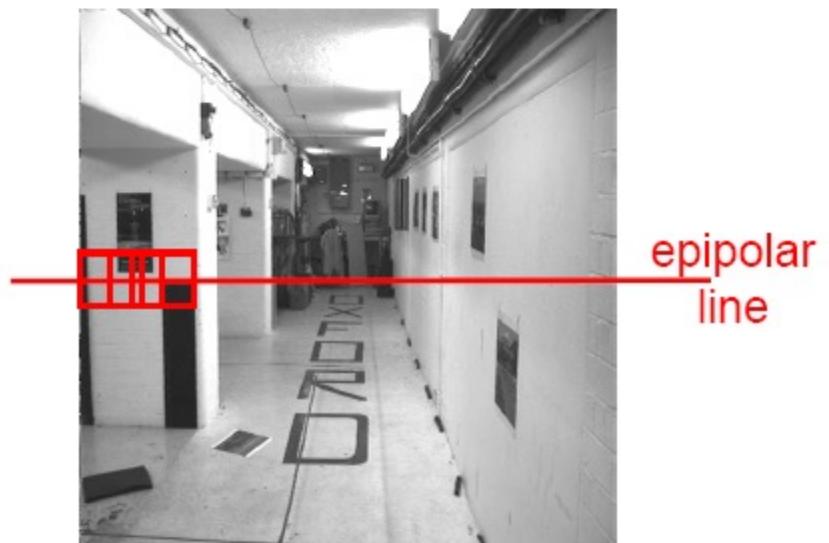
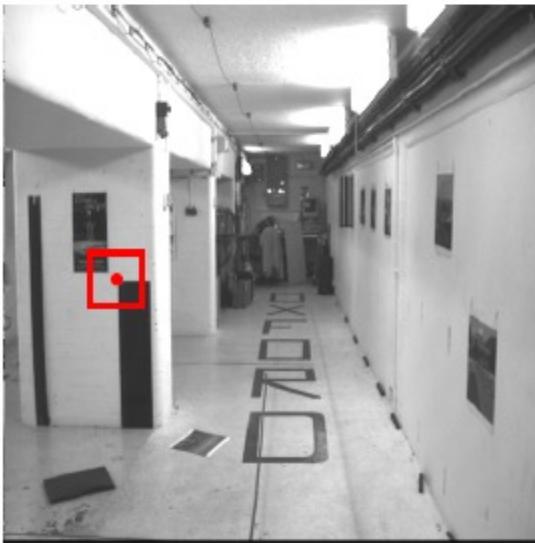


Intensity profiles



- Clear correspondence between intensities, but also noise and ambiguity

Correspondence problem



Neighborhood of corresponding points are similar in intensity patterns.

Normalized cross correlation

subtract mean: $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

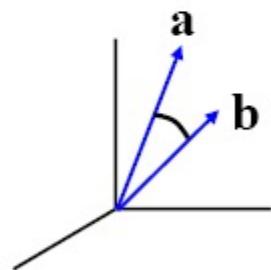
$$\text{NCC} = \frac{\sum_i \sum_j A(i, j)B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors

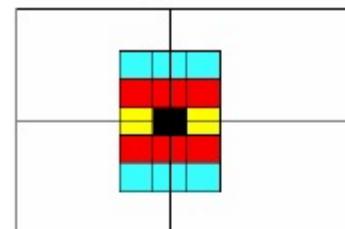
$$A \rightarrow \mathbf{a}, \quad B \rightarrow \mathbf{b}$$

$$\text{NCC} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

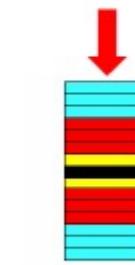
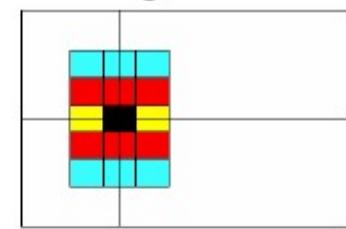
$$-1 \leq \text{NCC} \leq 1$$



region A



region B



vector \mathbf{a}



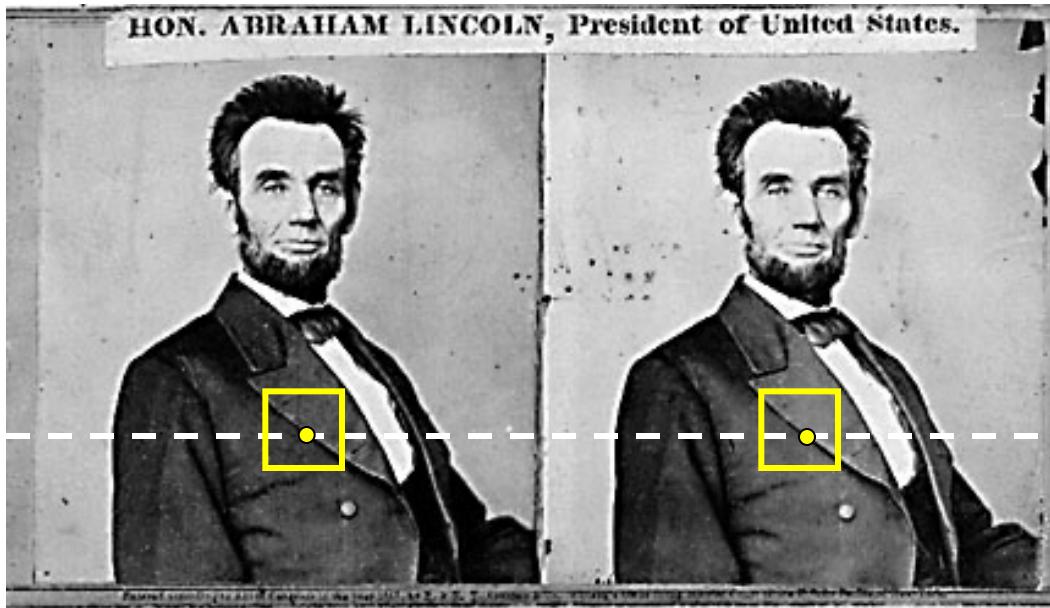
vector \mathbf{b}

Correlation-based window matching



left image band (x)

Dense correspondence search

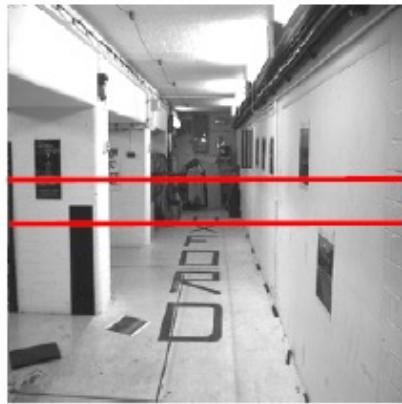


For each epipolar line

For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

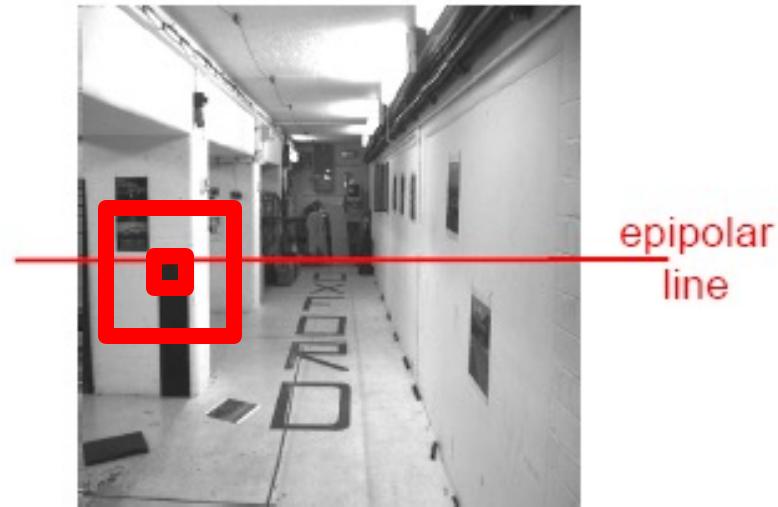
Textureless regions



target region

left image band (x)

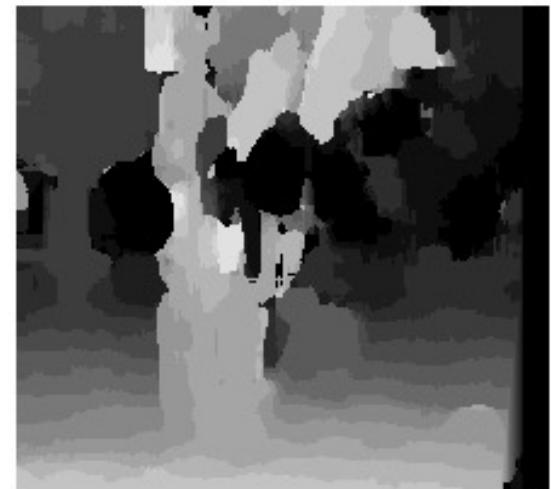
Effect of window size



Effect of window size



$W = 3$



$W = 20$

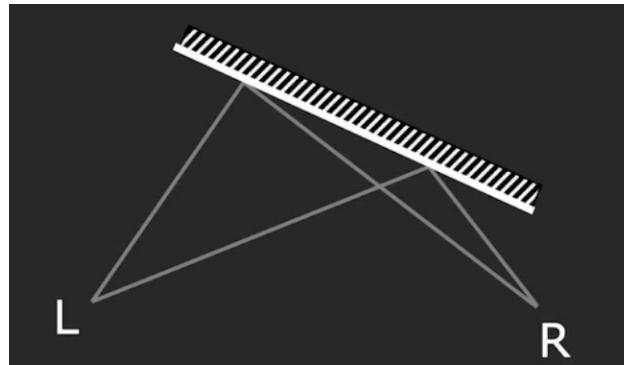
Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Issues with Stereo

- Surface must have non-repetitive texture



- Foreshortening effect makes matching a challenge



Stereo Results

- Data from University of Tsukuba

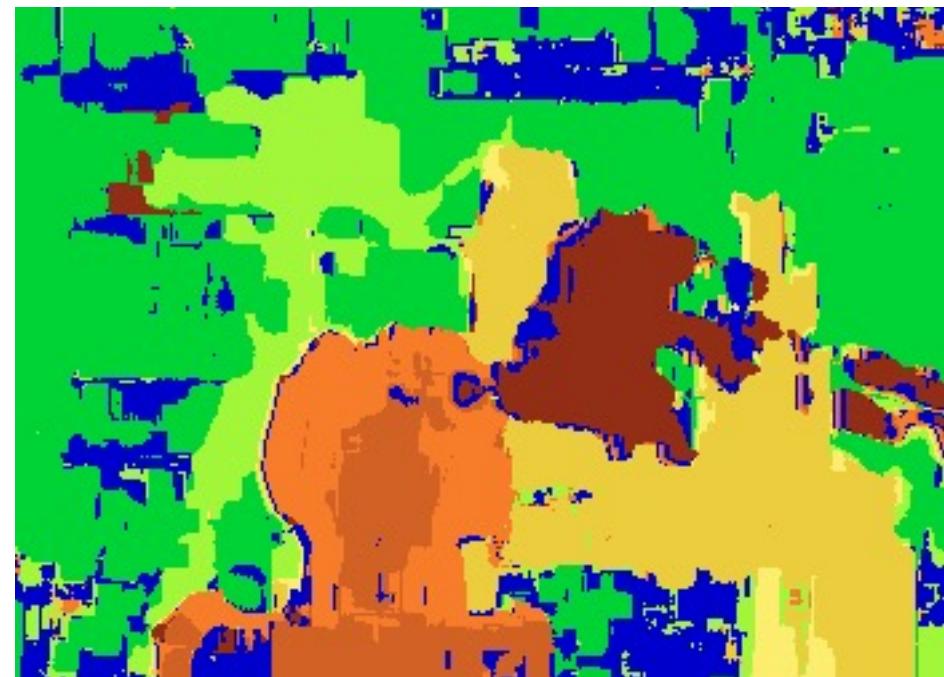


Scene



Ground truth

Results with Window Search



Window-based matching
(best window size)



Ground truth

Better methods exist...



Energy Minimization

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),
International Conference on Computer Vision, September 1999.



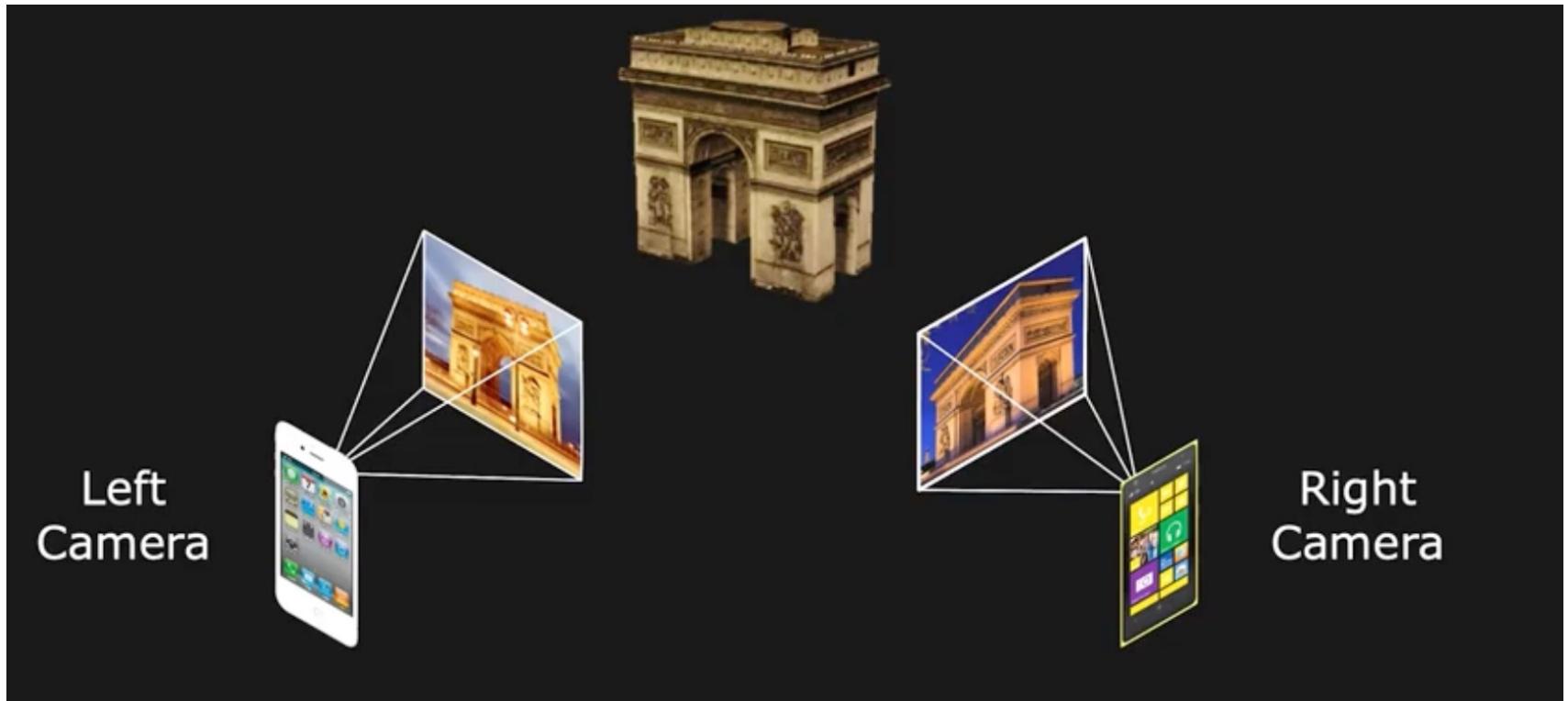
Ground truth

Summary

- With a simple stereo system, **how much pixels move, or “disparity”** give information about the depth
- Correspondences to measure the pixel disparity

Next: Uncalibrated Stereo

- From two arbitrary views



- Assume intrinsics are known (f_x, f_y, o_x, o_y)