

# CS 184: Computer Graphics and Imaging, Spring 2024

## Homework 2: Mesh Edit

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### Overview

In this project, I explored the world of mesh editing through building Bezier curves and surfaces using the de Casteljau algorithm and implementing various mesh operations such as area-weighted vertex normals, edge flip, edge split, and loop subdivision.

### Section I: Bezier Curves and Surfaces

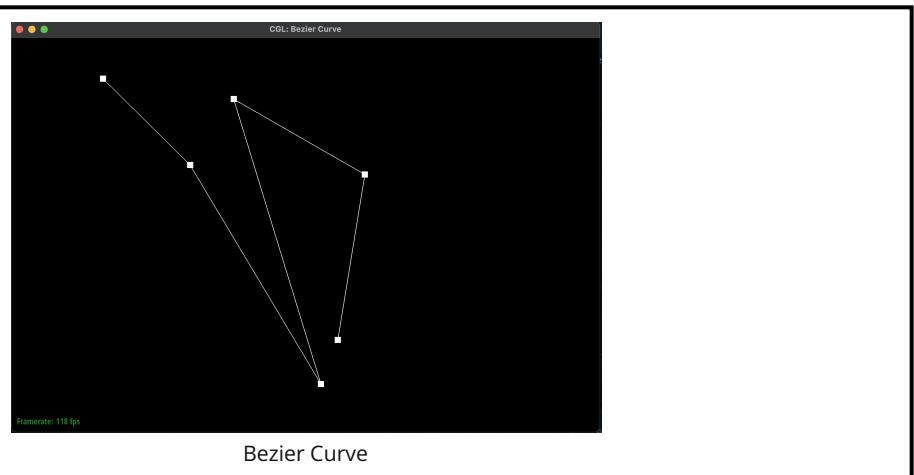
#### Part 1: Bezier Curves with 1D de Casteljau Subdivision

Briefly explain de Casteljau's algorithm and how you implemented it in order to evaluate Bezier curves.

- de Casteljau's Algorithm takes in a set of control points and a parameter  $t$ , a proportion of length along the line and evaluates a Bezier curve by recursively interpolating between each pair of control points. It can repeat this process until the criterion has been met or that the final interpolated point has been calculated. By adjusting this parameter  $t$ , it can find all the points along the curve. I implemented this algorithm by looping through each point and its adjacent point,  $p_i$  and  $p_{i+1}$ , and computing the interpolated point  $p'_i = \text{lerp}(p_i, p_{i+1}, t) = (1 - t)p_i + tp_{i+1}$ . After each iteration, there will be one fewer control point than the previous iteration. This process can be repeated until there is only one point left, which would be the final evaluated point.

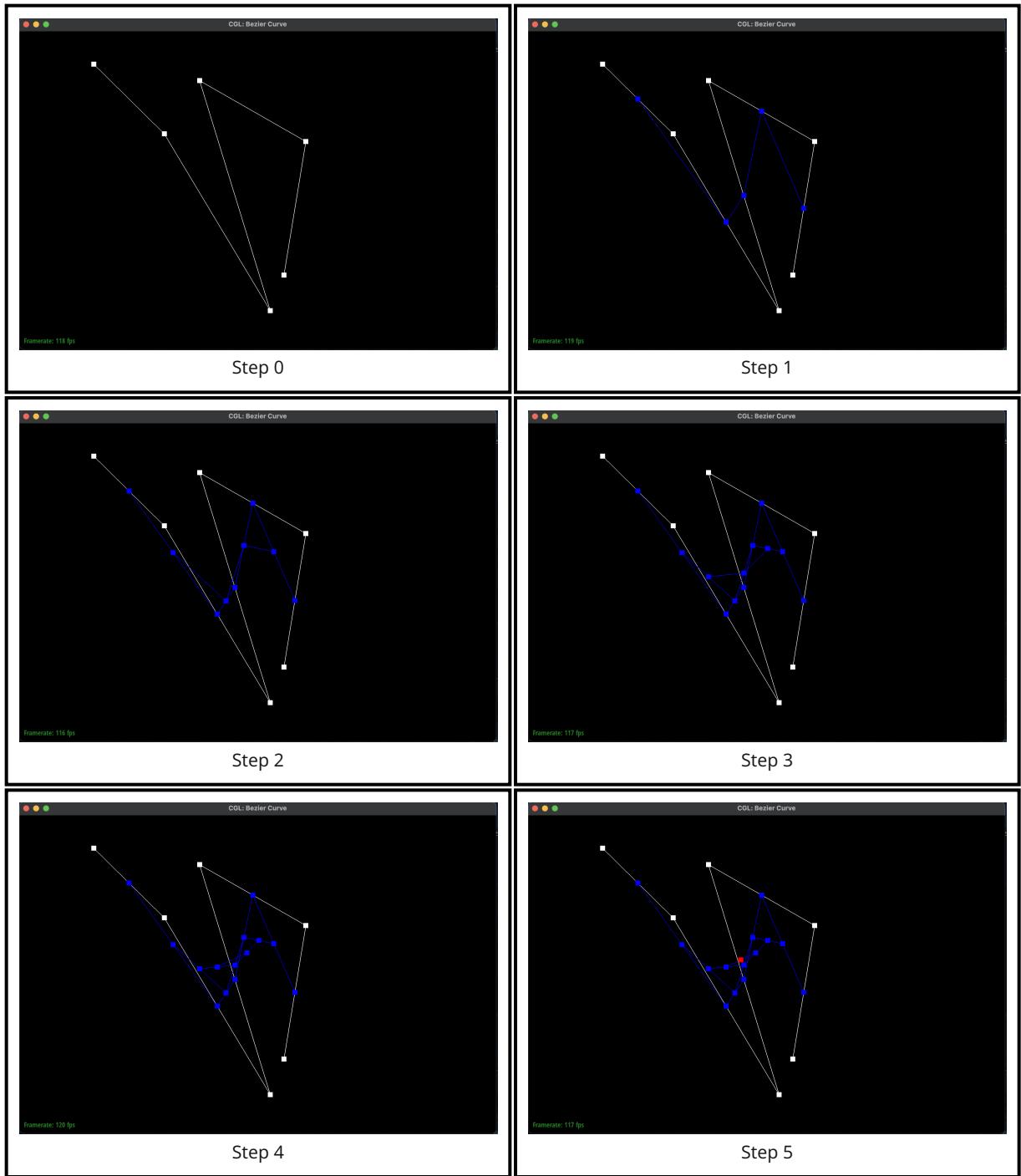
Take a look at the provided .bzc files and create your own Bezier curve with 6 control points of your choosing. Use this Bezier curve for your screenshots below.

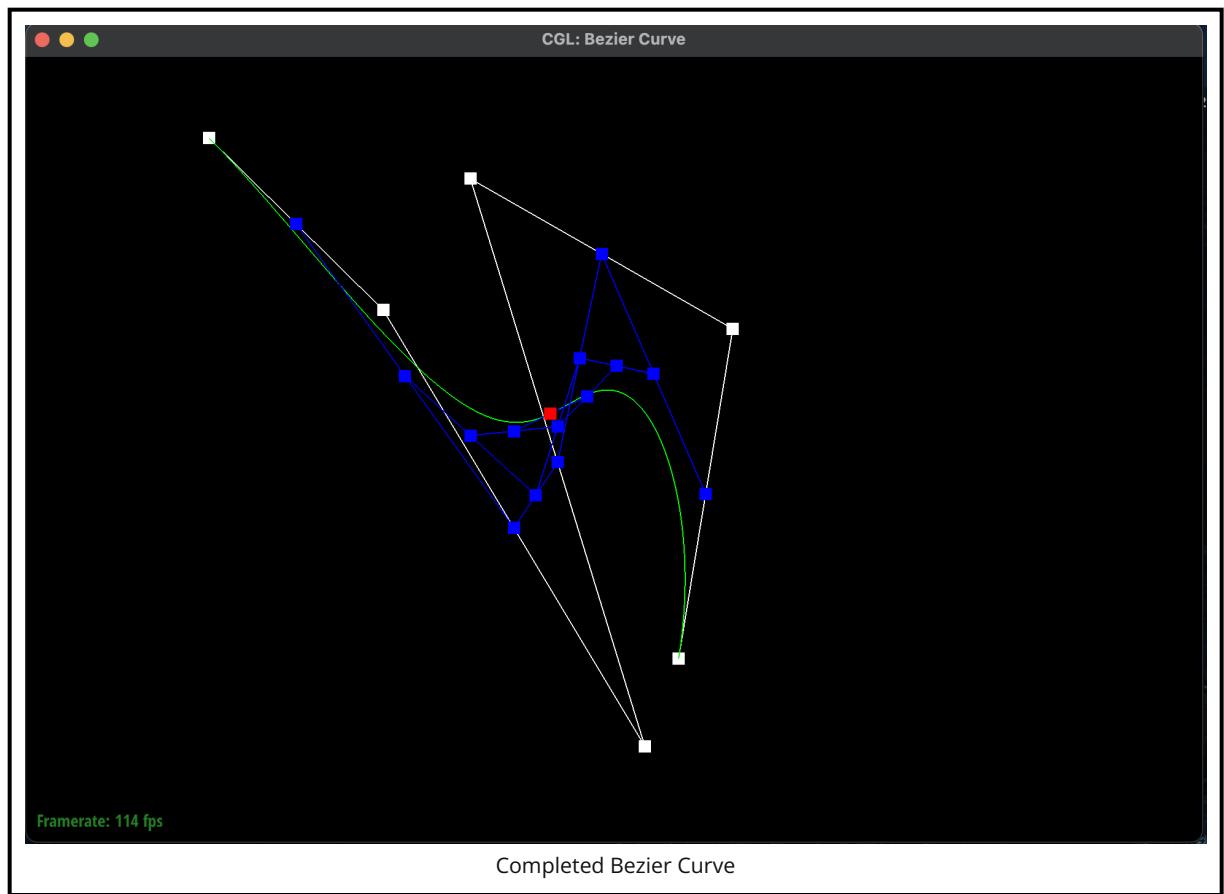
- Here is a Bezier curve with 6 control points of my choosing:



Show screenshots of each step / level of the evaluation from the original control points down to the final evaluated point. Press E to step through. Toggle c to show the completed Bezier curve as well.

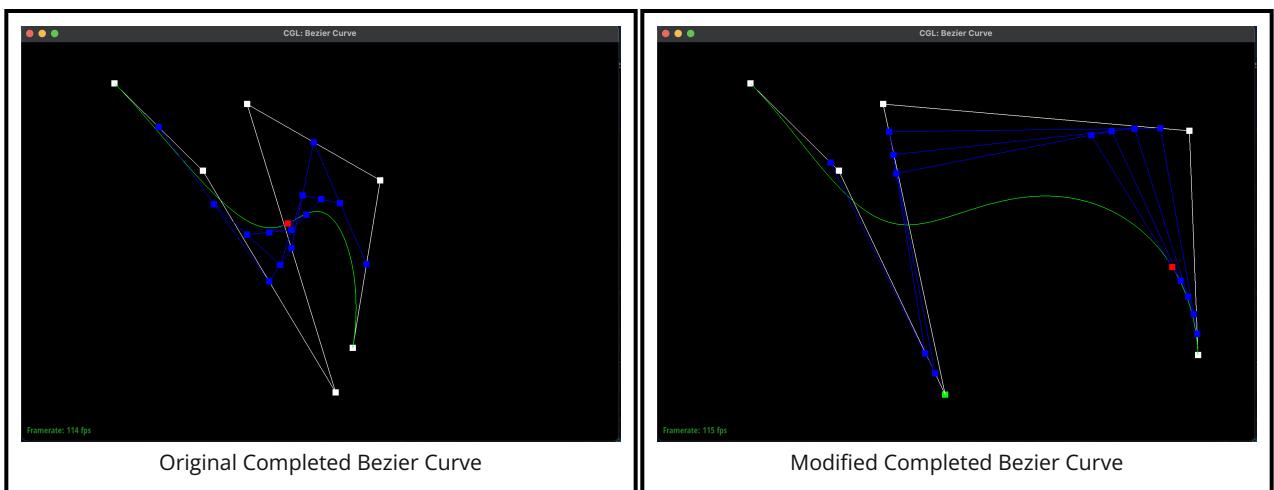
- Here are the screenshots of each step of the evaluation from the original control points down to the final evaluated point as well as the completed Bezier curve:





Show a screenshot of a slightly different Bezier curve by moving the original control points around and modifying the parameter  $t$  via mouse scrolling.

- I had shifted  $t$  to a higher value which meant that the curve was more towards the right. I also moved the control points around to create a different curve. Here is a screenshot of a slightly different Bezier curve by moving the original control points around and modifying the parameter  $t$  via mouse scrolling:



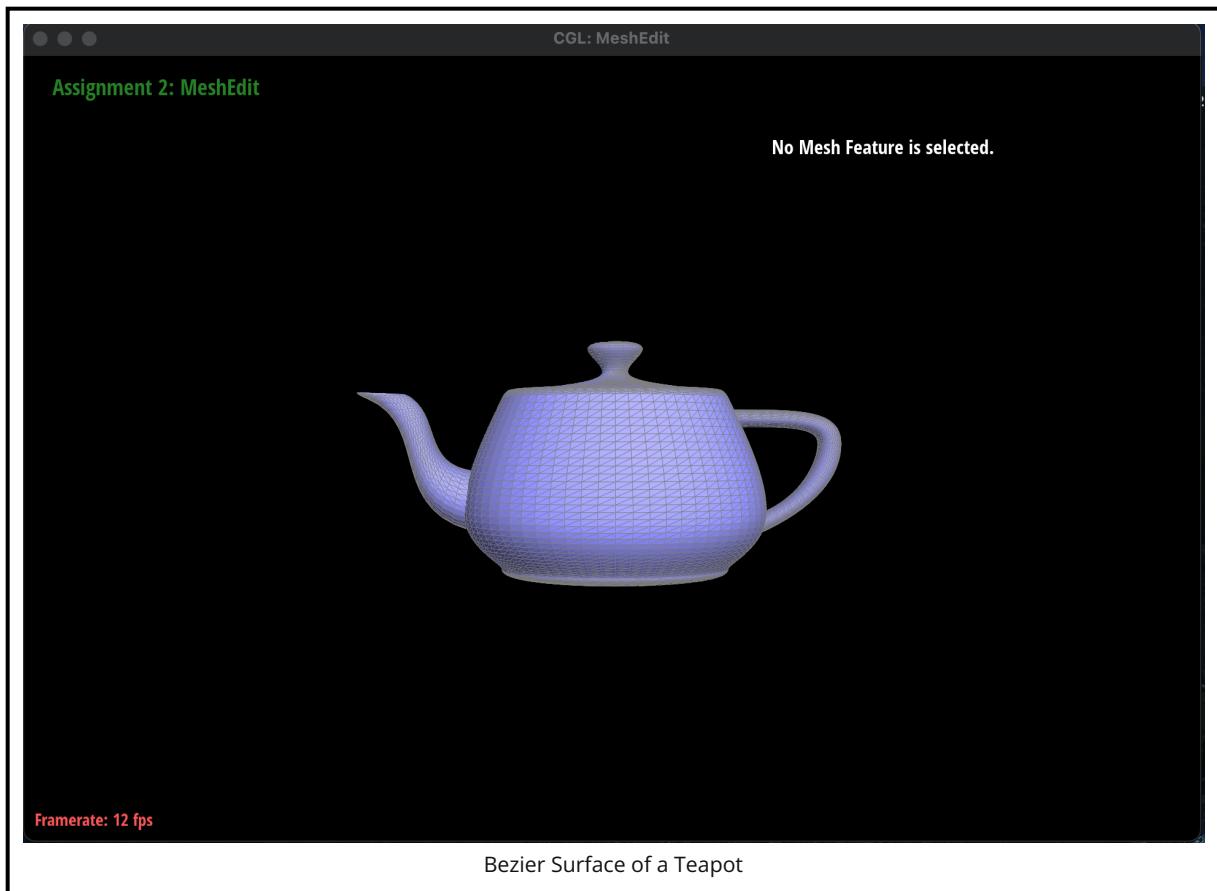
## Part 2: Bezier Surfaces with Separable 1D de Casteljau

Briefly explain how de Casteljau algorithm extends to Bezier surfaces and how you implemented it in order to evaluate Bezier surfaces.

- A 3D Bezier surface is an  $n \times n$  grid of control points where there are  $n$  parallel Bezier curves in  $u$ . The separable 1D de Casteljau's algorithm can evaluate the surface position corresponding to  $u, v$  along an axis  $x$  and an orthogonal axis  $y$ . This algorithm extends by first finding the final interpolated point  $u$  at each of these  $n$  Bezier curves. Each of these points combined will help make up a new set of  $n$  control points for the "moving" Bezier curve. Finally, the 1D de Casteljau's algorithm can evaluate  $v$  on this final curve. I implemented this algorithm by first evaluating the  $n$  parallel Bezier curves in  $u$  and storing them into a new vector. The resulting  $n$  points became my next set of control pointers for another Bezier curve in  $v$ . This process repeats until the final point is evaluated.

Show a screenshot of `bez/teapot.bez` (not code) evaluated by your implementation.

- Here is a screenshot of `bez/teapot.bez` evaluated by my implementation of the Bezier surface:



## Section II: Triangle Meshes and Half-Edge Data Structure

### Part 3: Area-Weighted Vertex Normals

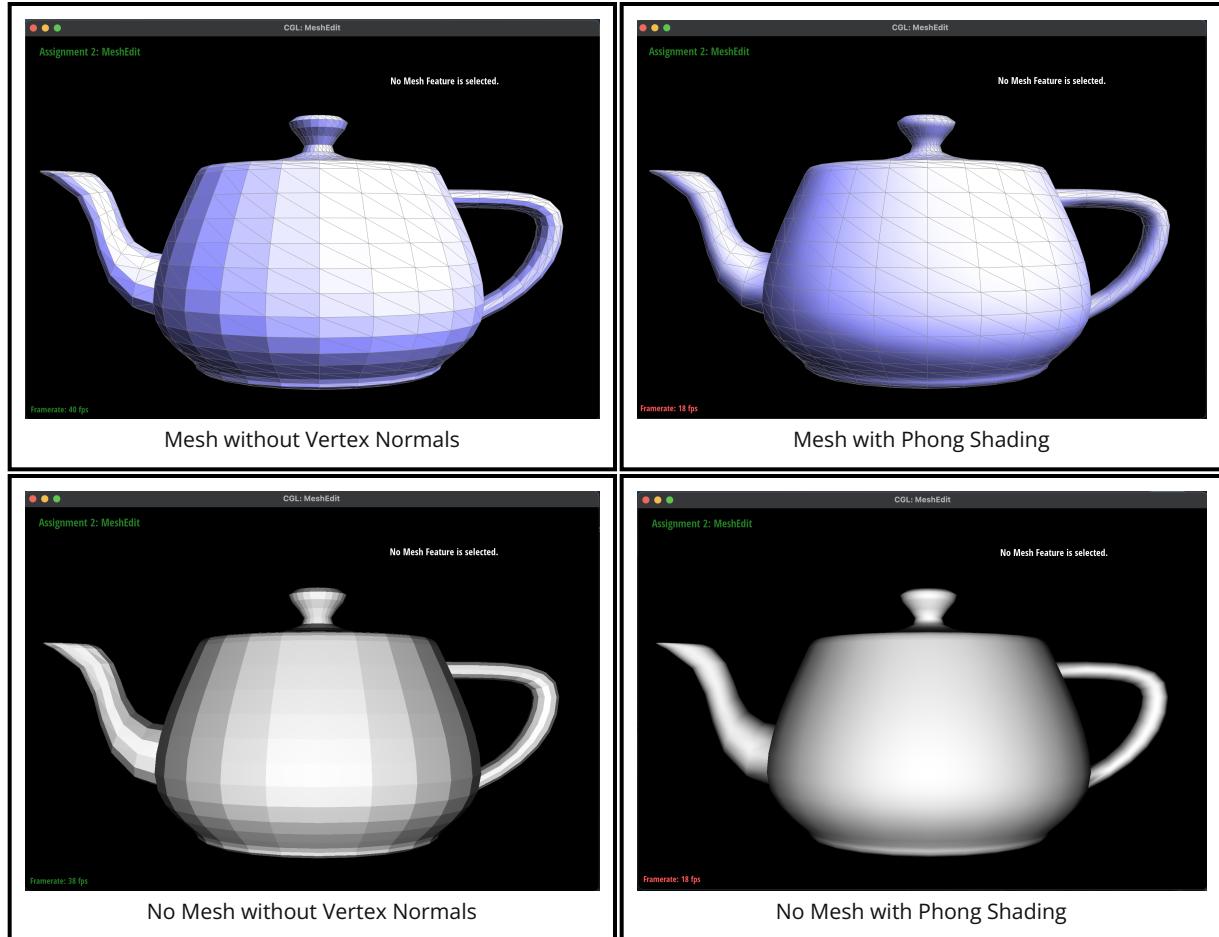
Briefly explain how you implemented the area-weighted vertex normals.

- I implemented the area-weighted vertex normals by making a constant iterator of the half-edge data structure to traverse over all of the neighboring triangles and weighting each one by its area. I defined a `find_area` function that used the cross product formula of the vertices to find the area of the triangle. Here are the formal steps I took to implement the area-weighted vertex normals:
  - I initialized an empty `Vertex3D` vertex to keep track of the weighted vertex.
  - I found the starting half-edge and used a `do-while` loop to traverse through all the triangles and stopping once we reached the original initial half-edge.

3. For each triangle, I calculated the area of the triangle using the cross product formula. This function found all three vertices of the triangle by using the next and vertex methods. I then found the difference vectors and took the cross product before normalizing the result and dividing by 2 because the area of a triangle is half the area of the parallelogram formed by the vectors.
4. I used this calculated area to weight the normal of the triangle and added it to the weighted vertex from earlier.
5. I called on the `twin().next()` to find the next half-edge and face.
6. Finally, once all of the half-edges have been traversed, I normalized the weighted vertex by calling `unit()` on it.

Show screenshots of dae/teapot.dae (not .bez) comparing teapot shading with and without vertex normals. Use Q to toggle default flat shading and Phong shading.

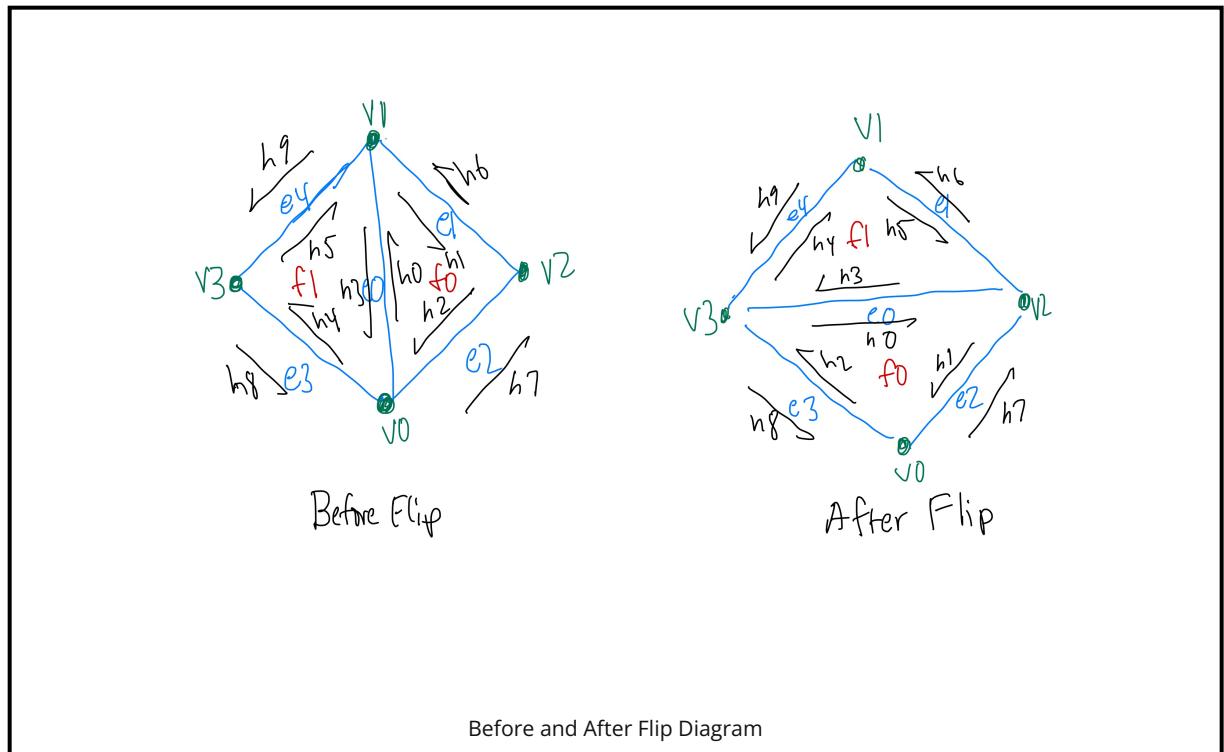
- Here are some screenshots of dae/teapot.dae shading with and without vertex normals:



#### Part 4: Edge Flip

Briefly explain how you implemented the edge flip operation and describe any interesting implementation / debugging tricks you have used.

- I first started by creating a diagram of each of the half-edges, edges, vertices and faces before and after the flip to ensure that the pointers would be correct. Here is the diagram shown below:

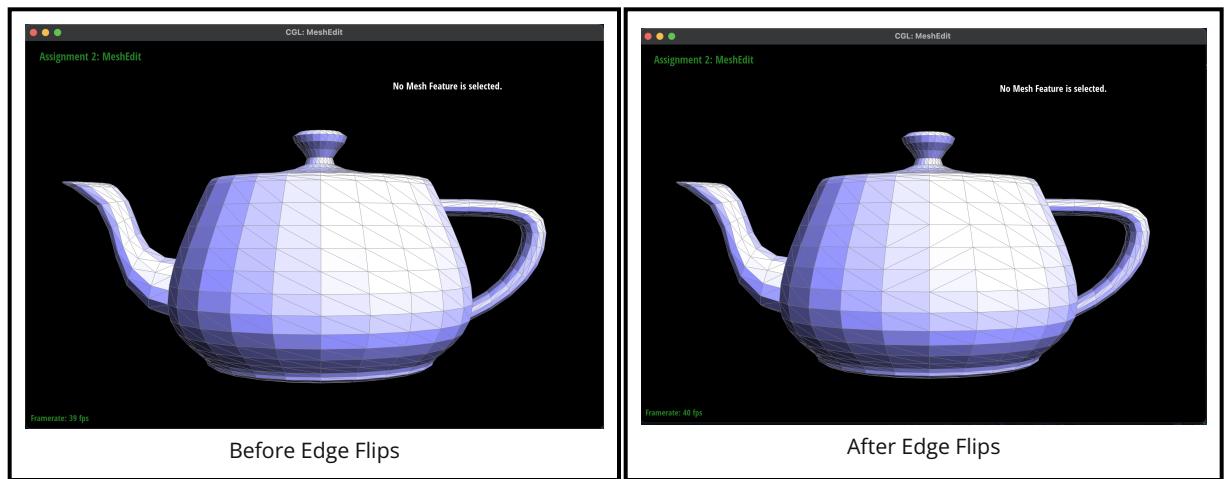


- Here are the formal steps I took to implement the edge flip operation:
  - First, I checked if  $e0 \rightarrow \text{isBoundary}()$  was true to make sure to never flip a boundary edge and simply returned if it was.
  - Then, I defined the inner and outer half-edges of the two triangles using the `twin()` and `next()` methods. Each of these half-edges corresponded to the 10 half-edges,  $h_0 \dots h_9$ , as shown in the diagram above.
  - Next, I defined the vertices of the two triangles using the `vertex()` method on the appropriate half-edge. Each of these vertices corresponded to the 4 vertices,  $v_0 \dots v_3$ , as shown in the diagram above.
  - Afterwards, I defined the edges and faces of the two triangles using the `edge()` and `face()` methods on the appropriate half-edge. Each of these edges and faces corresponded to the 5 edges,  $e_0 \dots e_4$ , and 2 faces,  $f_0$ ,  $f_1$ , as shown in the diagram above.
  - Then, I updated each of the 10 half-edge pointers using the `setNeighbors()` method according to the diagram above.
  - Finally, I reassigned the half-edge pointers for each of the 4 vertices, 5 edges, and 2 faces according to the diagram above and returned the newly updated  $e0$ .

Some tricks I used was to follow my diagram very closely and checking which pointers I was passing into my functions as well as using the additional debugging utilities provided in the spec.

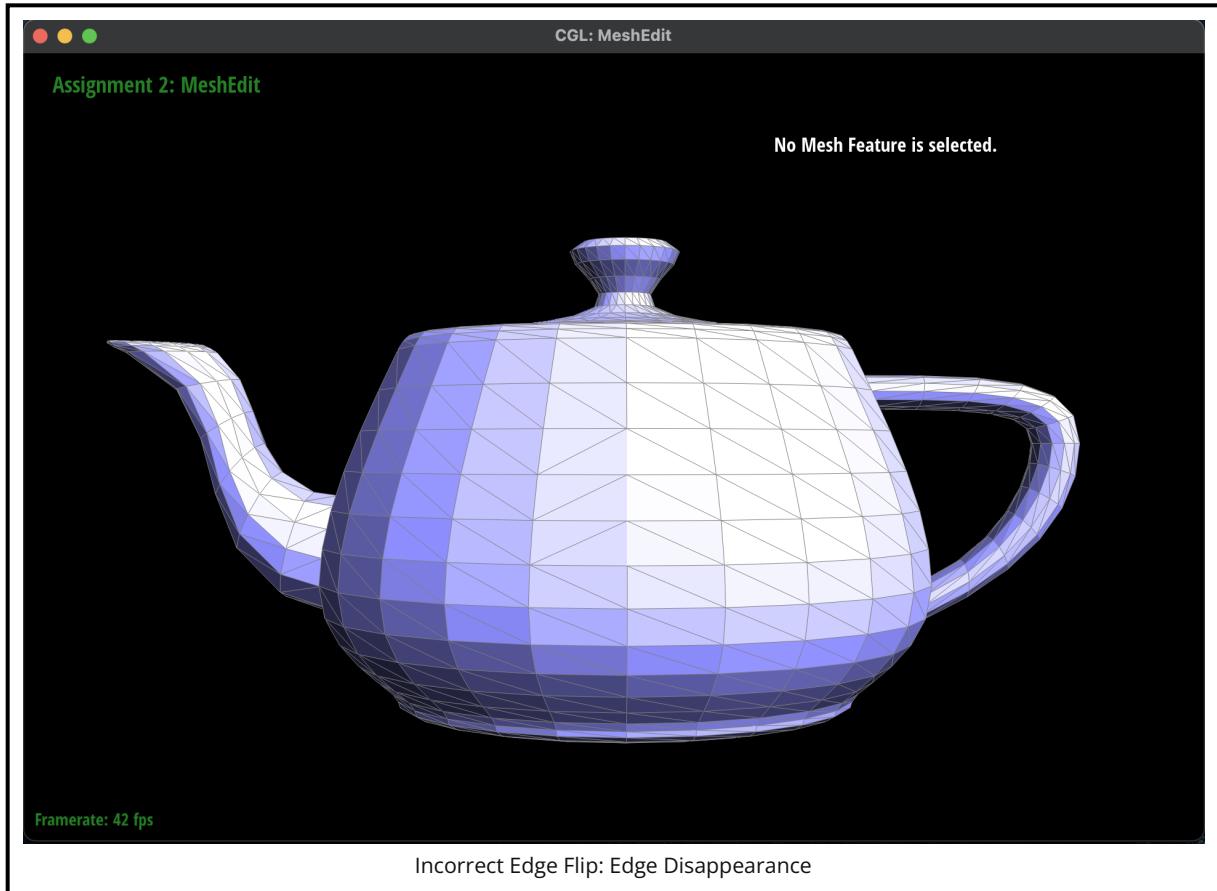
Show screenshots of the teapot before and after some edge flips.

- Here are some screenshots of `dae/teapot.dae` before and after some edge flips.



Write about your eventful debugging journey, if you have experienced one.

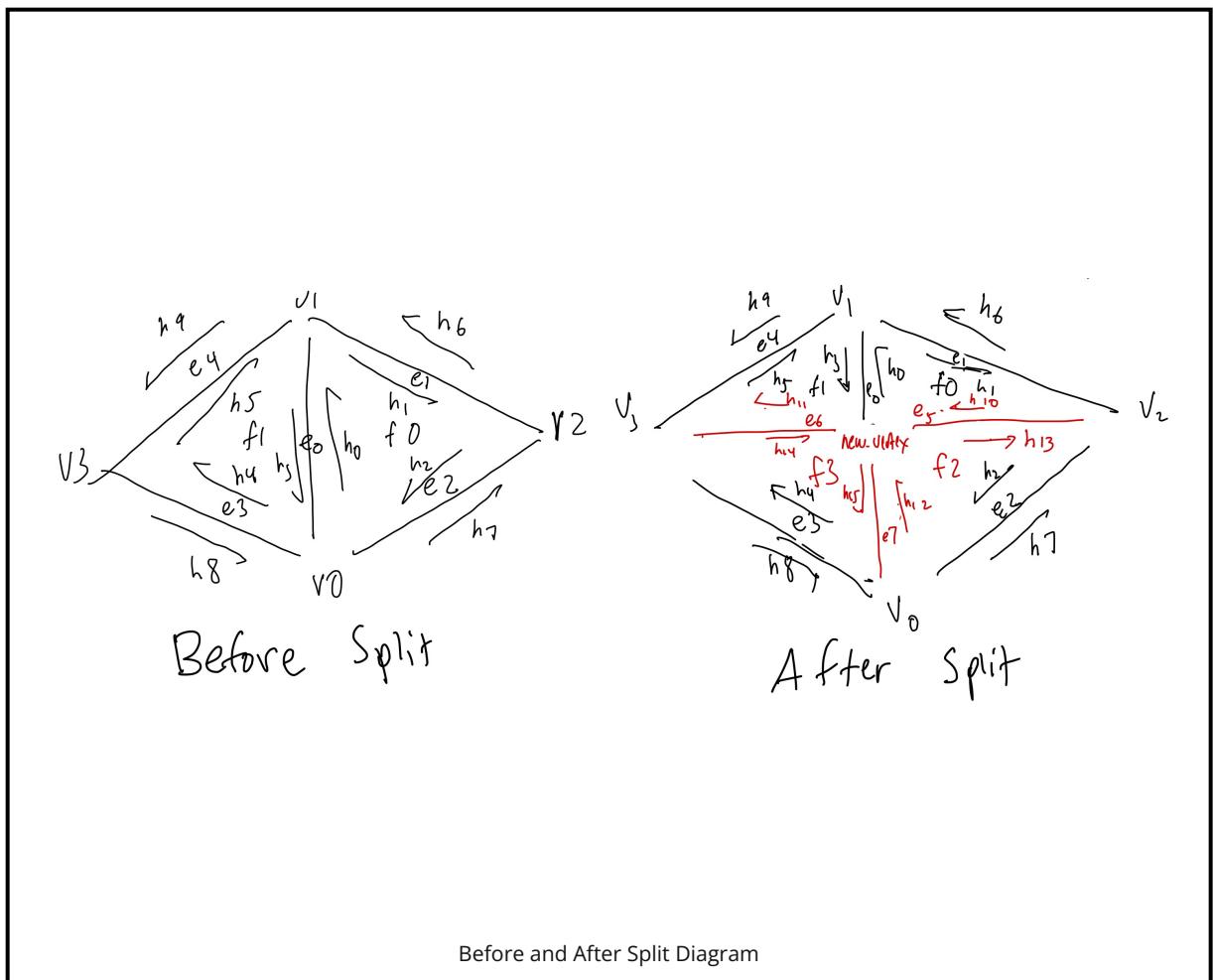
- In the process of implementing the edge flip operation, I ran into some issues where the mesh would look a bit distorted and that edges would disappear after the flip. I realized that in my diagram of the half-edge data structure, I had not taken into account the half-edge pointers for the edges on the outside of the current mesh element. As a result, I went back to my implementation and made sure to redraw the half-edge data structure to include the outer edges and vertices and correctly updated the pointers. Afterwards, the mesh looked much better after each of the edge flips.
- Here is the incorrect half-edge flip:



### Part 5: Edge Split

Briefly explain how you implemented the edge split operation and describe any interesting implementation / debugging tricks you have used.

- I first started by creating a diagram of each of the half-edges, edges, vertices and faces before and after the flip to ensure that the pointers would be correct. I color coded with red being the new half-edges, edges, vertices, and faces created and black being the older counterparts. Here is the diagram shown below:



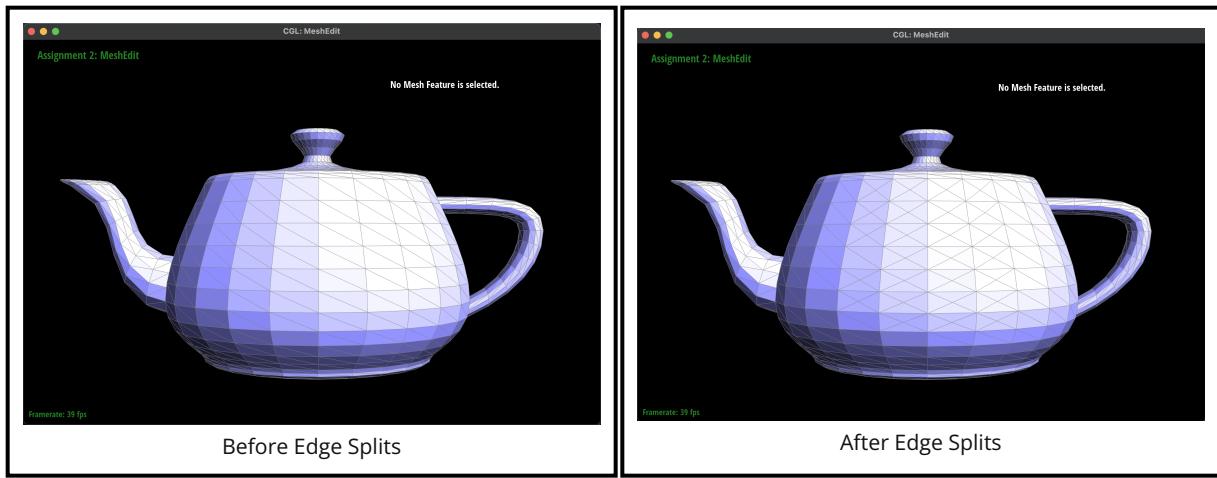
Before and After Split Diagram

- Here are the formal steps I took to implement the edge split operation.
  1. First, I checked if `e0->isBoundary()` was true to make sure to not split a boundary edge and simply returned if it was.
  2. Then, I defined the inner and outer half-edges of the two triangles using the `twin()` and `next()` methods. Each of these half-edges corresponded to the 10 half-edges, `h0` ... `h9`, as shown in the diagram above.
  3. Next, I defined the vertices of the two triangles using the `vertex()` method on the appropriate half-edge. Each of these vertices corresponded to the 4 vertices, `v0` ... `v3`, as shown in the diagram above.
  4. Afterwards, I defined the edges and faces of the two triangles using the `edge()` and `face()` methods on the appropriate half-edge. Each of these edges and faces corresponded to the 5 edges, `e0` ... `e4`, and 2 faces, `f0`, `f1`, as shown in the diagram above.
  5. In addition, I created 6 new half-edges, 3 new edges, 1 new vertex, and 2 new faces. The new vertex is defined as the center of the edge that is being split while the 2 new faces are the 2 bottom triangles that are created from the split.
  6. Then, I updated each of the 16 half-edge pointers using the `setNeighbors()` method according to the diagram above.
  7. Finally, I reassigned the half-edge pointers for each of the 5 vertices, 8 edges, and 4 faces according to the diagram above and returned the newly updated `e0`.

Some tricks I used was to follow my diagram very closely and checking which pointers I was passing into my functions as well as using the additional debugging utilities provided in the spec.

Show screenshots of a mesh before and after some edge splits.

- Here are some screenshots of `dae/teapot.dae` before and after some edge flips.



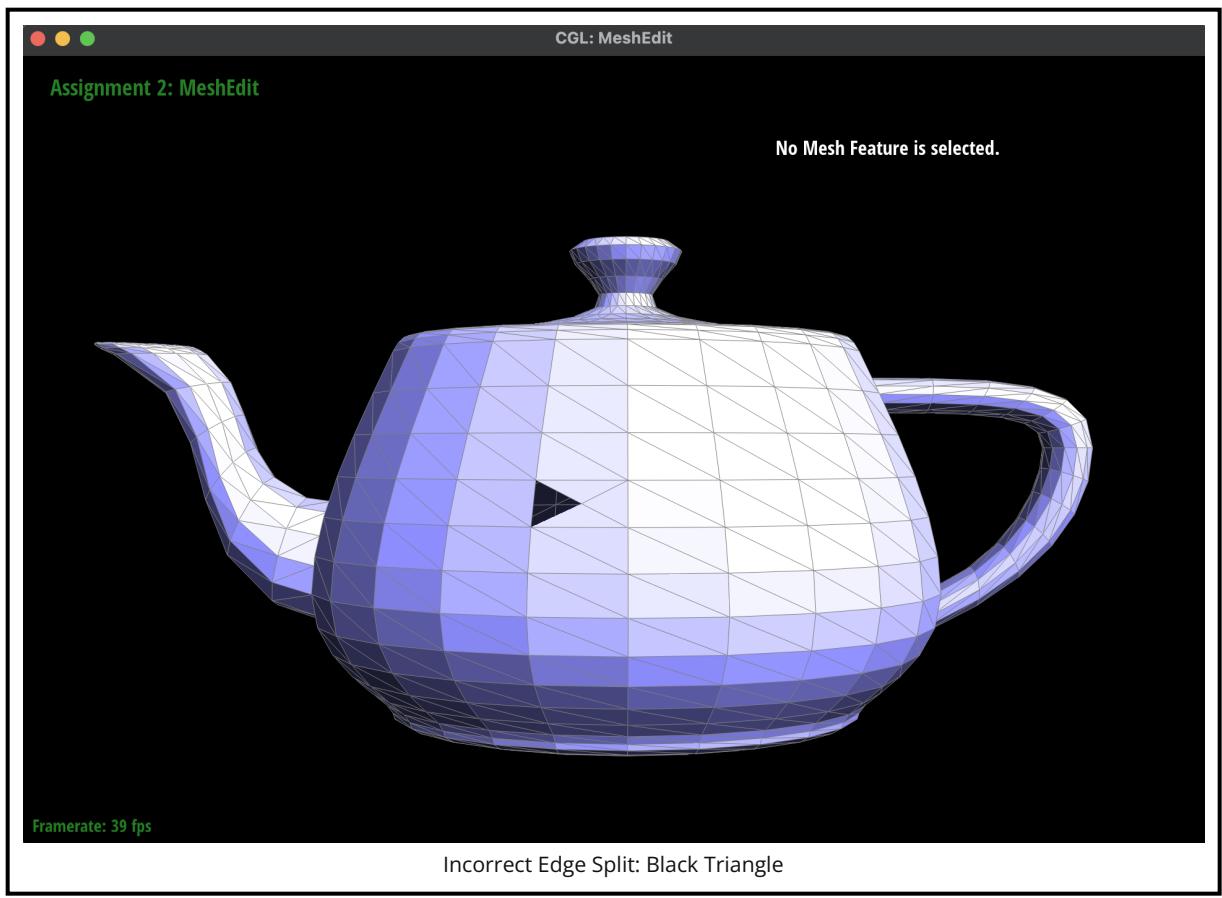
Show screenshots of a mesh before and after a combination of both edge splits and edge flips.

- Here are some screenshots of dae/teapot.dae before and after some edge flips.



Write about your eventful debugging journey, if you have experienced one.

- Learning from my mistakes, I made sure to draw the diagram correctly and to follow it very closely when I was assigning the pointers for each half-edge and the other edges, vertices, and faces. The only issue I ran into was that sometimes when I clicked on an edge or vertex the program would crash and this was because of a segmentation fault. After a while of debugging by rereading my code and my diagram, I realized I had forgot to set on the edge and vertex pointers for the newly updated ones. Then another issue occurred where one of the triangles would turn black and after debugging for a bit I realized I had set the incorrect vertex for one of the newly created half-edge.
- Here is the incorrect half-edge split:



If you have implemented support for boundary edges, show screenshots of your implementation properly handling split operations on boundary edges.

### Part 6: Loop Subdivision for Mesh Upsampling

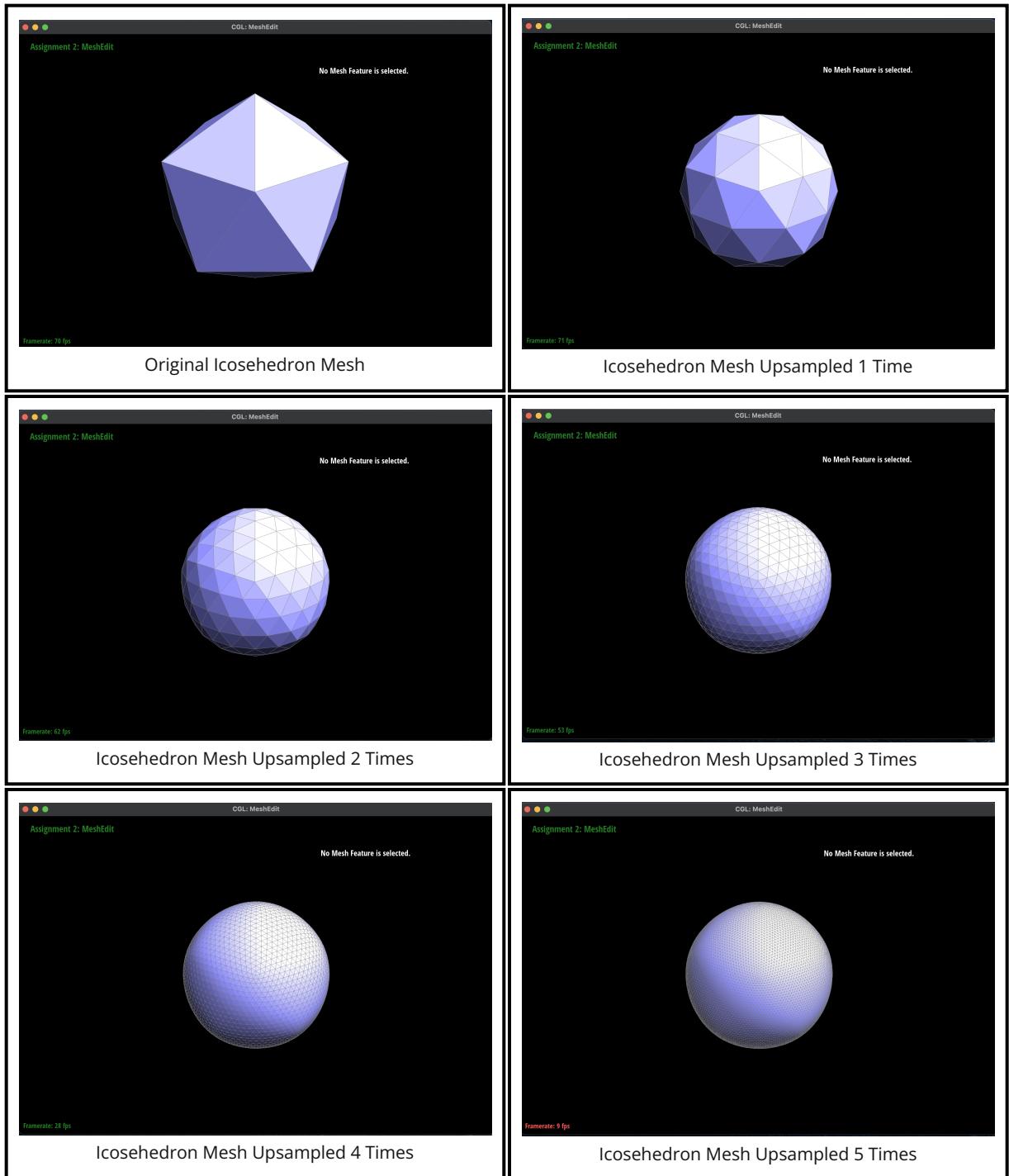
Briefly explain how you implemented the loop subdivision and describe any interesting implementation / debugging tricks you have used.

I decided to follow the order of operations as described in the spec. Here are the formal steps I took to implement the loop subdivision:

1. First, I iterated through all of the vertices in mesh using a `for` loop over the `mesh.verticesBegin()` and `mesh.verticesEnd()` iterators. For each vertex, I found all of the neighbors that were connected to it and then computed the new position by weighting it as the sum following the formula from lecture. I then set the `vertex->newPosition` to this weighted position and the `vertex->isNew` to `false` because this was not a newly created vertex.
2. Next, I iterated through all of the edges in the mesh using a `for` loop over the `mesh.EdgesBegin()` and `mesh.EdgesEnd()` iterators. For each edge, I found the vertex and face that it was connected to and then computed the new position by weighting it as the sum following the formula from lecture. I then set the `e->newPosition` to this weighted position and the `e->isNew` to `false` because this was not a newly created edge.
3. Then, I iterated through all of these edges in the original mesh again in order to split each of them and updated their position to be that of the previously computed new position stored in the edge.
4. Afterwards, I iterated through all the edges in the mesh and flipped any of the newly created edges that connected an old and new vertex.
5. Finally, I iterated through all of the vertices to set their position to the previously computed new position stored in the vertex.

Take some notes, as well as some screenshots, of your observations on how meshes behave after loop subdivision. What happens to sharp corners and edges? Can you reduce this effect by pre-splitting some edges?

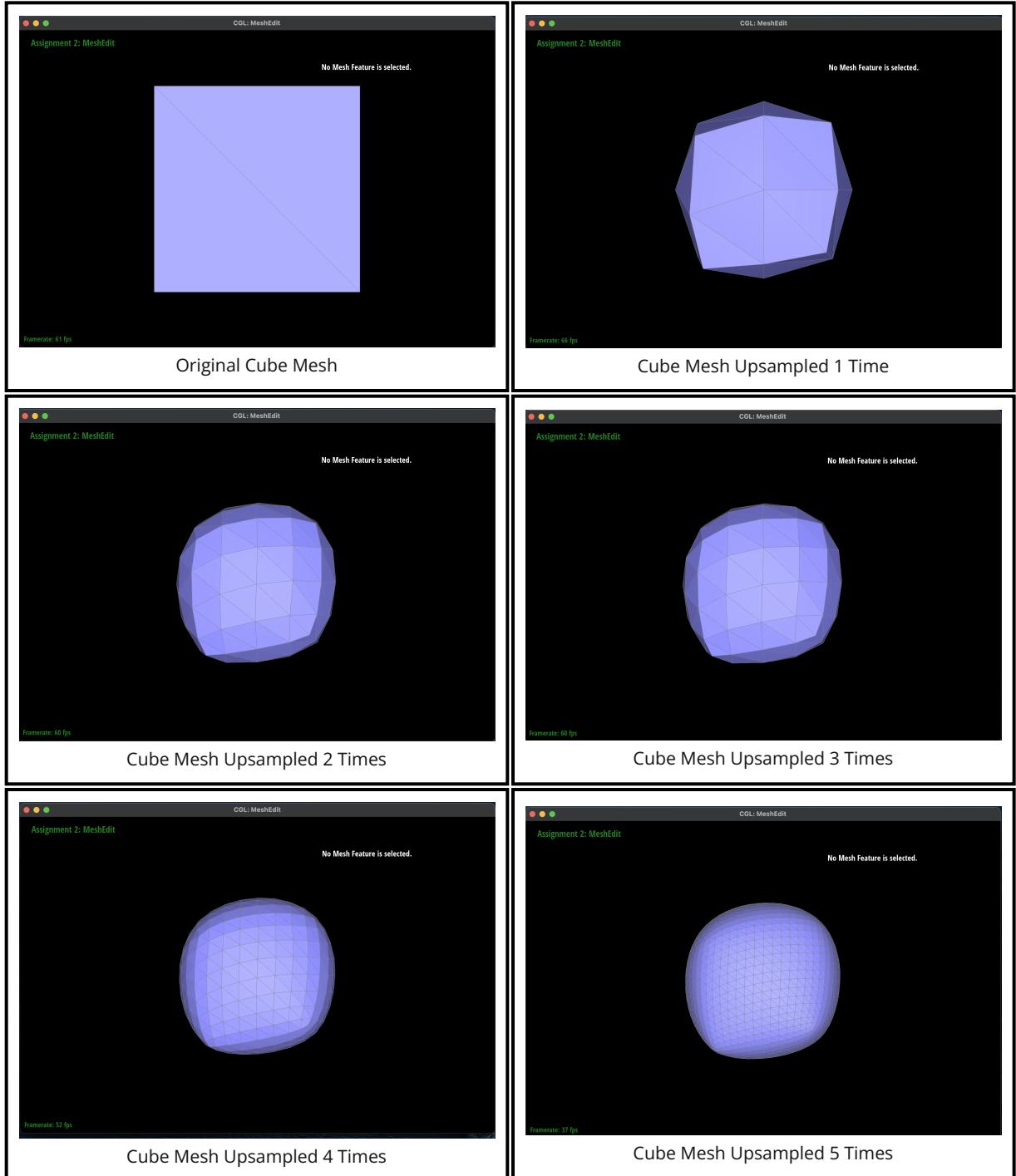
- Here are some screenshots of applying subdivision to the icosehedron mesh:



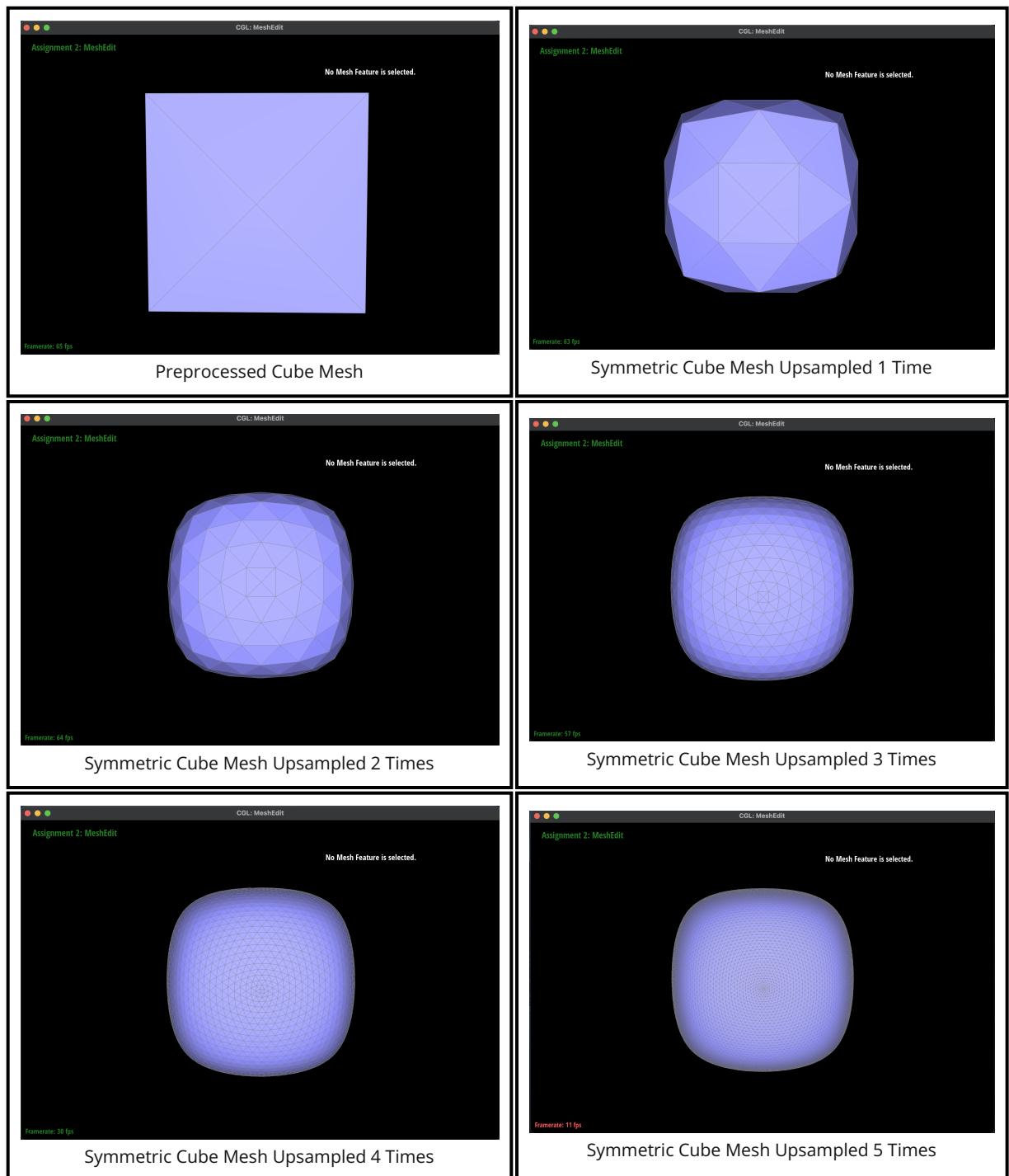
- I realized that the mesh subdivision was very similar the HW 1's supersampling to reduce aliasing and jaggies. The above meshes showed an analogous effect where after each subdivision performance the mesh would become smoother and more rounded. The sharp corners and edges would become less pronounced and the mesh would become more rounded. I realized that this effect could be reduced by pre-splitting some edges because the edge splits will create new vertices which will be used to update the positions of the existing vertices diagrammed in the implementation of the loop subdivision. Afterwards, each vertex has a higher degrees so the distorting effect when the mesh is upsampled will be reduced. Thus, the new vertices will be located at the corners of the mesh while the existing vertices will be located at the center of the mesh and will lead to a more rounded mesh but preserved corners and edges.

Load dae/cube.dae. Perform several iterations of loop subdivision on the cube. Notice that the cube becomes slightly asymmetric after repeated subdivisions. Can you pre-process the cube with edge flips and splits so that the cube subdivides symmetrically? Document these effects and explain why they occur. Also explain how your pre-processing helps alleviate the effects.

- Here are some screenshots of applying subdivision to the cube mesh:



- The asymmetry of the cube occurs because of the asymmetric original mesh. Before, the cube mesh has a single mesh edge that is shared by 2 of the vertices. Since the vertices are updated based on the weighting of the neighbors, the 2 connected ones will remain closer to their original positions and thus the cube looks stretched along the diagonal that is not originally connected.
- Here are some screenshots of applying subdivision to the cube mesh after pre-processing:



- Thus, to make sure that each vertex has the same degree, I preprocessed the cube by splitting the diagonal edge on every single face. This alleviates the asymmetric effect because each vertex will be weighted equally when it is calculating the new vertex position as the mesh is being upsampled. I did not need to flip nor split any edges after the mesh is upsampled and thus is a valid preprocessing.

If you have implemented any extra credit extensions, explain what you did and document how they work with screenshots.

I did not implement any extra credit extensions.

### Part 7 (Optional, Possible Extra Credit)

