CS184/284A Spring 2025 Homework 2 Write-Up

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Link to webpage: https://github.com/cal-cs184-student/hw-webpages-zhangnd16
Link to GitHub repository: https://github.com/cal-cs184-student/sp25-hw2-mondrian

Overview

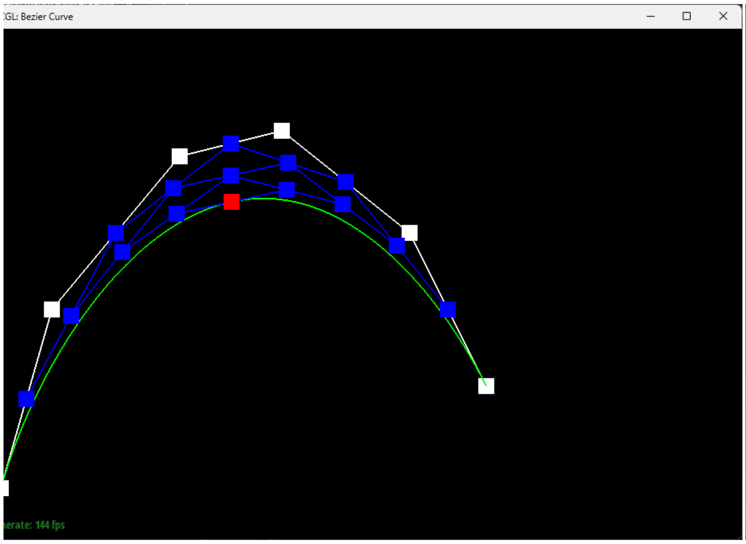
In this project, in the first part, I implemented methods to draw Bezier curves and Bezier surfaces; in the second part, I gained some experiences working with the halfedge data structure by implementing the calculation of vertices normals, edge flips, edge splits, which are very useful in some mesh processing. I also implemented the loop subdivision upsampling method, which is very useful in increasing the resolution of a mesh.

Section I: Bezier Curves and Surfaces

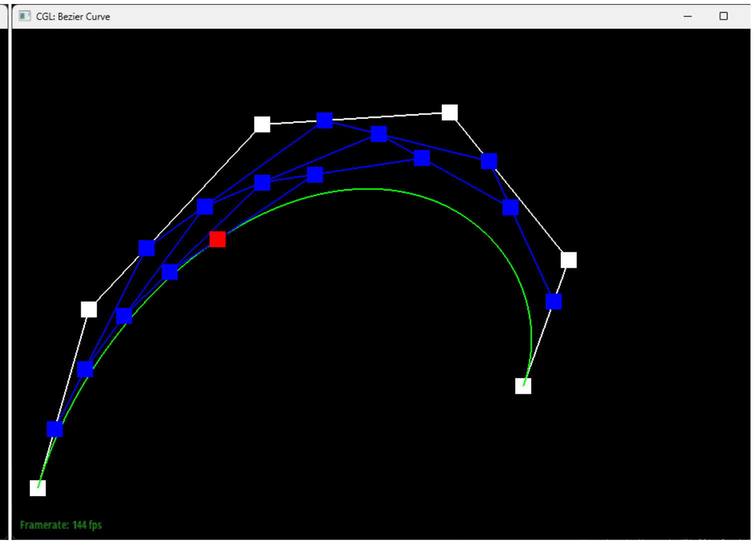
Part 1: Bezier curves with 1D de Casteljau subdivision

De Casteljau's algorithm is an iterative method to generate Bezier curves. Given a list of n vertices and a lerp value $0 < t < 1$, for each pair of adjacent vertices, generate n-1 intermediate lerp vertices. Iteratively perform this step until n=1, where there is the final point on the Bezier curve. Repeat the process for all t values between 0 and 1, and the set of all final points consist to the desired Bezier curve.

Below is a Bezier curve with 6 control points and another slightly different modified one.



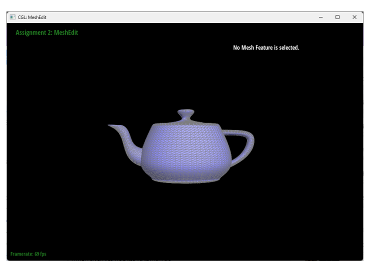
A Bezier curve with 6 control points



A slightly adjusted Bezier curve with modified t valu

Part 2: Bezier surfaces with separable 1D de Casteljau

To generate a Bezier surface on a set of n * n control points, consider n Bezier curve generated with n rows, each row with n control points. Taking the point on each Bezier curve with parameter u, one can generate another Bezier curve using these points as control points. With a parameter v, one can calculate a point on the "second-order" Bezier curve, which is on the Bezier surface. With all possible (u, v), both between 0 and 1, one can generate all points to form the Bezier surface.



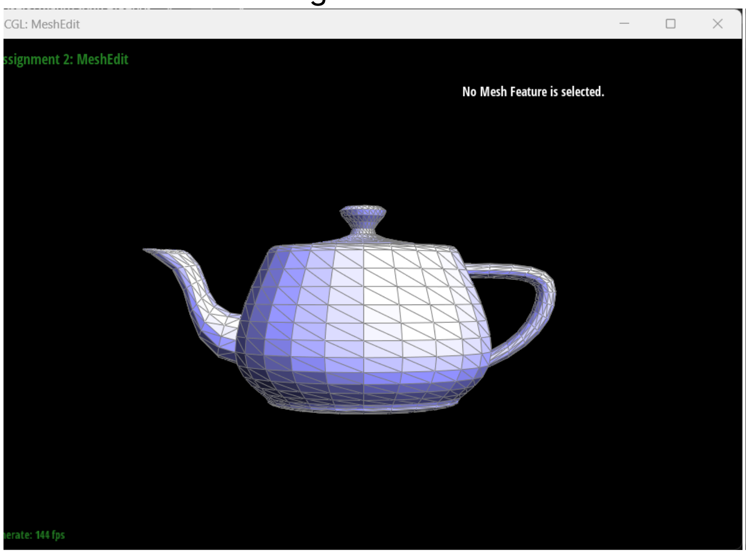
A teapot formed with Bezier surfaces

Section II: Triangle Meshes and Half-Edge Data Structure

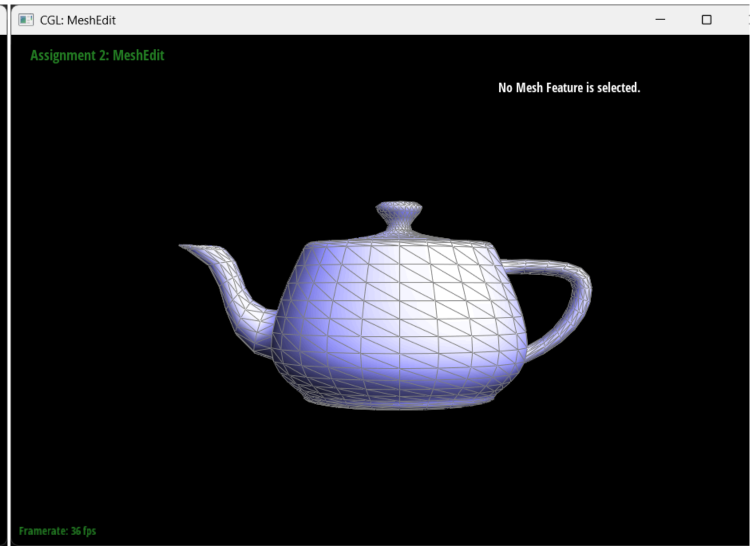
Part 3: Area-weighted vertex normals

To calculate the area-weighted vertex normals for a vertex, one should sum up all normal vectors of the faces around the vertex. To calculate the normal vector of one of the faces, start from one of the halfedge of the vertex, take the vertex of the halfedge, the vertex of the next halfedge (halfedge→next()), and the vertex of the second next halfedge (halfedge→next()→next()). These three vertices consist to the triangle face, and one can calculate the normal vector by taking cross product of the differences of these vectors.

To traverse to the next face, take next halfedge of the halfedge's twin component (halfedge→twin()→next()). Repeat the process above for each face around the vertex and sum up the normal vectors, and normalize the sum to get the desired result.



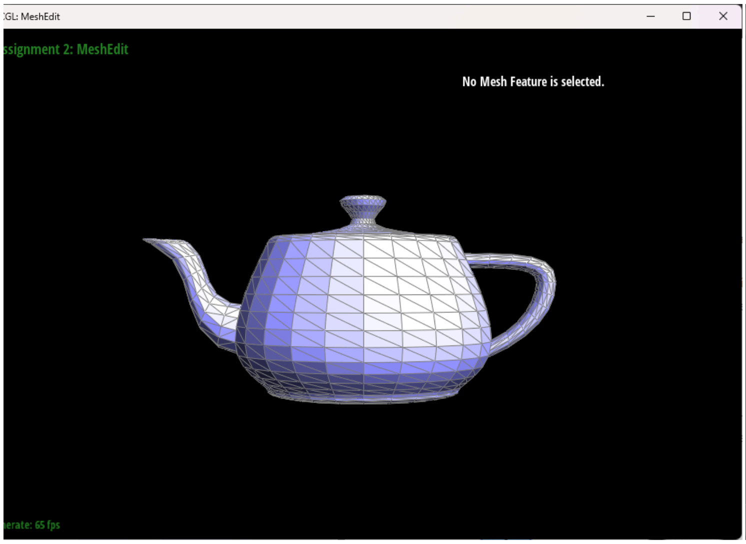
Teapot with flat shading



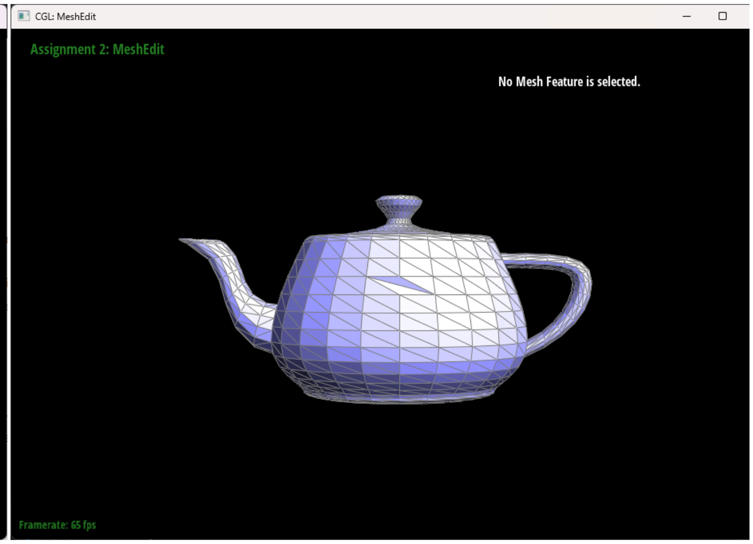
Teapot with Phong shading

Part 4: Edge flip

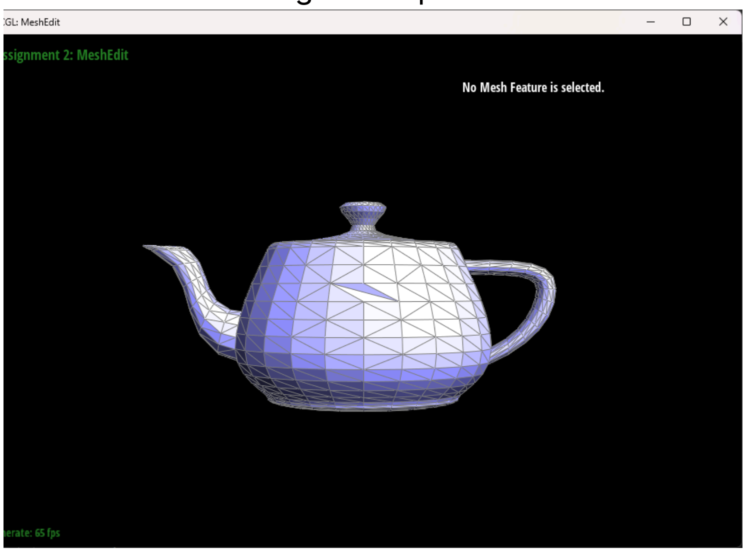
To perform an edge flip, notice that no new edges are created, therefore the assignments between halfedges and edges, also halfedges and their twin complements are unchanged after a edge flip. Therefore, we can just worry about next, vertex, face of halfedges, and the halfedge assignments of vertex and face. One should be very careful about the next component of the halfedge, and make sure that they form a proper loop within a face. I encountered a bug when I tried to flip an edge, the program just froze. It turns out to be that I was not following the halfedge's next assignments, and I did not the proper halfedge's next pointer to the flipped halfedge, therefore causing an infinite loop.



Original teapot



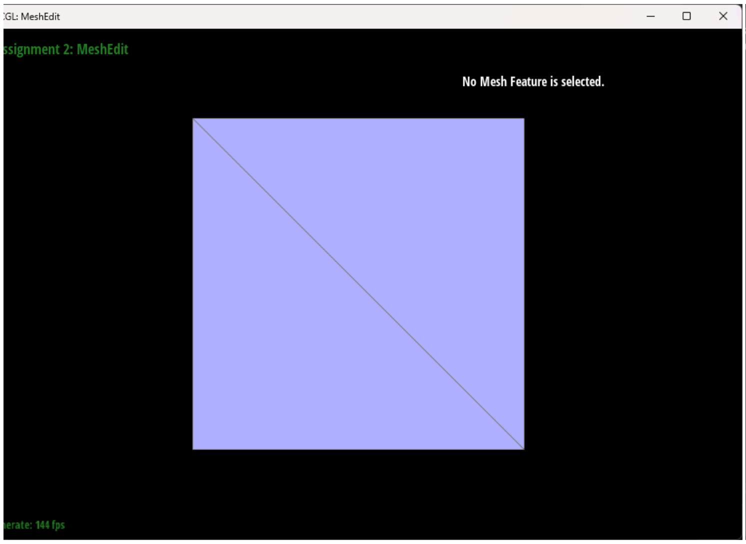
Teapot with a flip at the center



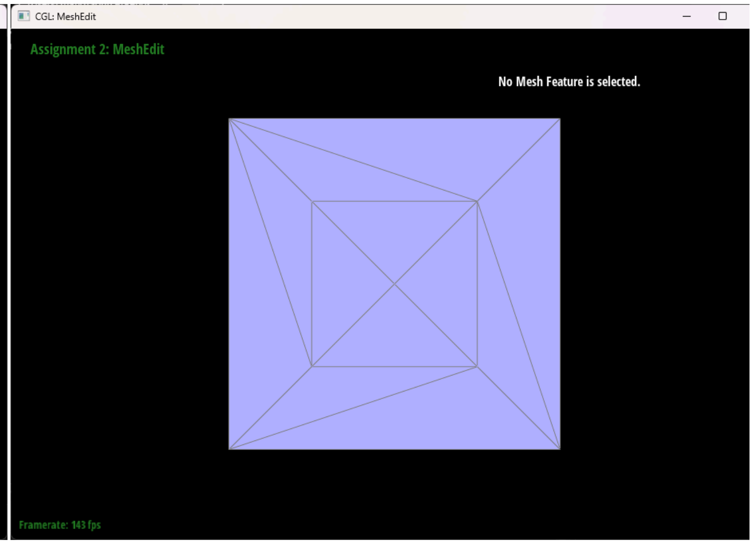
Teapot with more edge filps

Part 5: Edge split

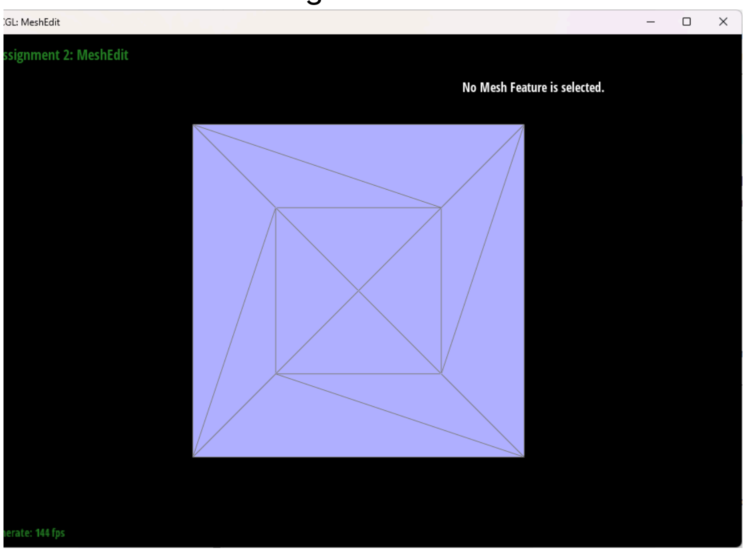
To perform an edge split is more complicated than an edge flip, as many new elements are created. A new vertex, three new edges, six new halfedges, and two new faces are created and needed to be assigned to the proper elements. For this part, as things become more complicated, I just wrote down all parts that are in the given mesh and assigned everything needed to each component to avoid any potential bugs. I wrote each variable name as the example picture in the instruction (e.g., halfedgebc). I also found a helper function to assign elements to halfedge is very helpful, as there are many repetitious similar assignments for halfedges.



Original cube



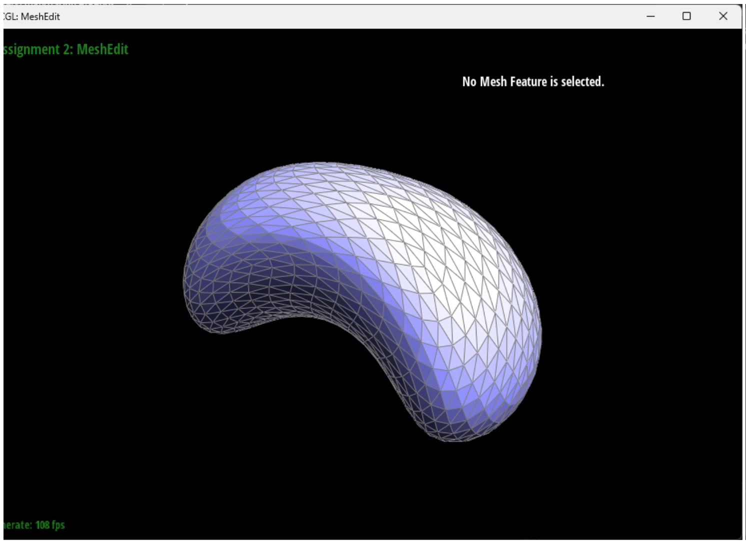
Cube with some edge splits



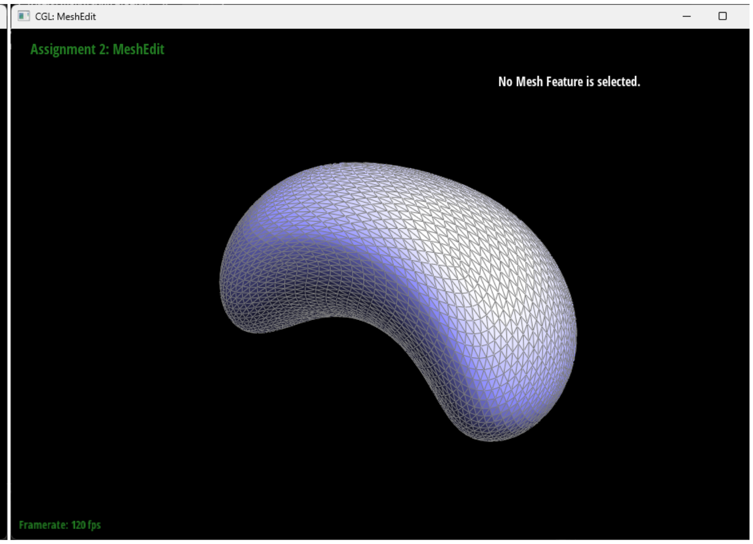
Cube with some edge splits and flips

Part 6: Loop subdivision for mesh upsampling

To perform loop subdivision, I follow the instructions to store the new positions into the old vertex and positions of new vertex into their corresponding edges and I found it much easier to traverse the mesh before the subdivision. Then, to split all edges in the original mesh and avoid splitting any new edges, I first record all edges into a vector and split all edges in that vector. To identify all new edges, after an edge splitting, find the edges that do not connect to an old vertex. After this is to flip all new edges connecting a new vertex and an old vertex. Last, update the positions of all vertices. I encountered a bug where all the new vertices ending up to have a position at the origin. It turns out that I assigned the new position to the position attribute of the new vertex, immediately after the edge flipping, which is overrided by the position updating later.

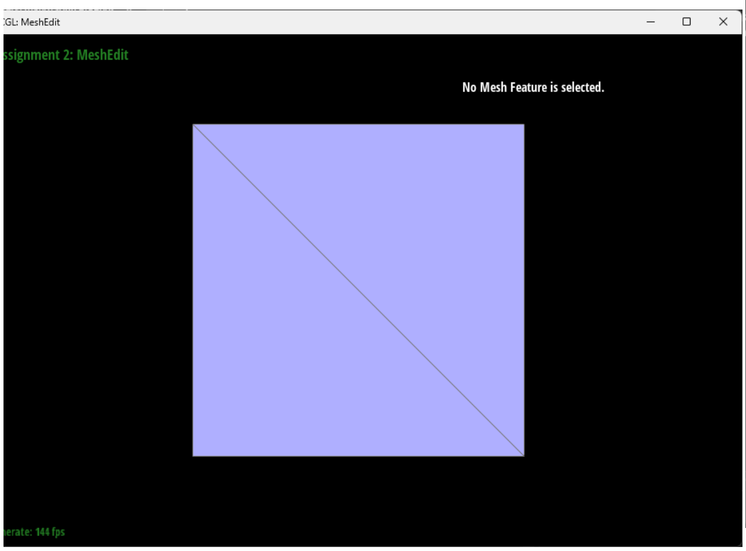


Original bean

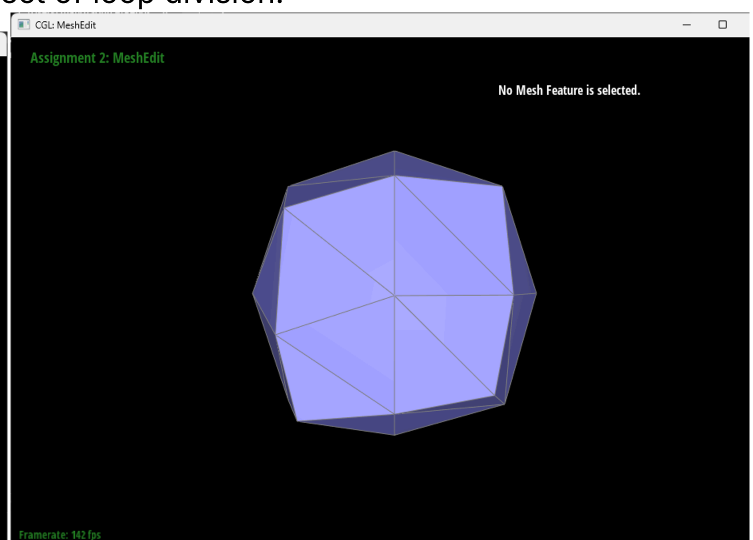


Bean after one loop subdivision

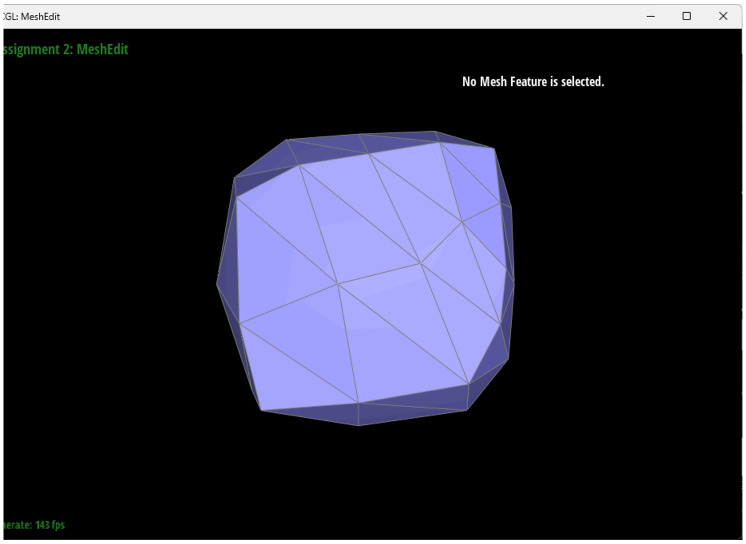
After loop subdivision, all sharp corners and edges become much smoother. This is because the position of the sharp vertices are taken average with less-sharp vertices adjacent to them. This effect can be reduced by pre-splitting all adjacent edges to sharp vertex. Below is an example of a loop subdivision on a cube,comparing with no pre-splitting, and with pre-splitting on each edge adjacent to the sharp vertex at the upper right. It is evident from the below that pre-splitting on edges near the sharp vertices can reduce the smoothing effect of loop division.



Original cube

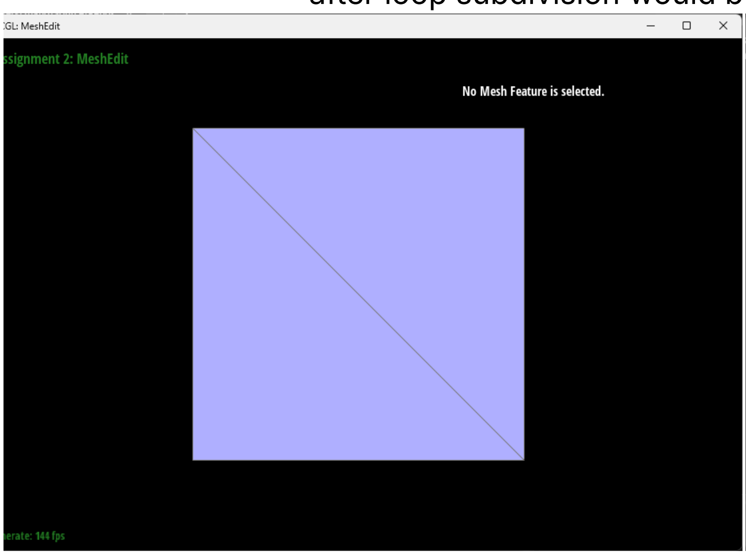


The cube after one loop subdivision, with sharp vertices on the upper right and bottom left

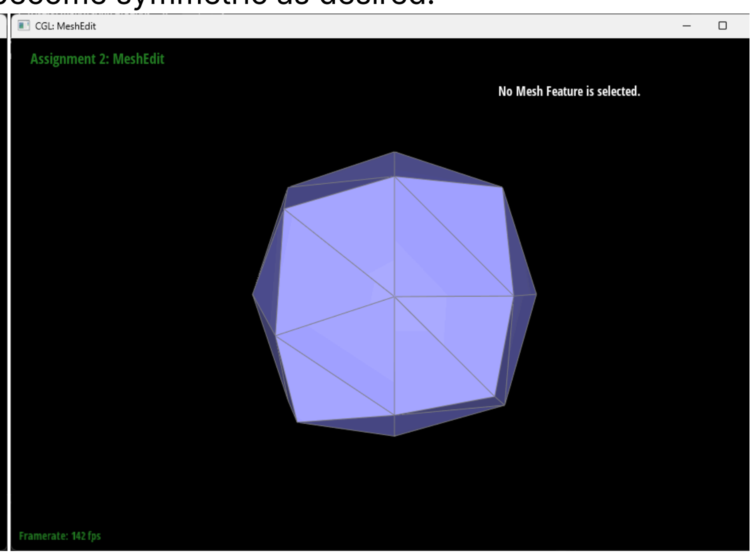


The cube with pre-splitting around the upper right vertex, then loop subdivision

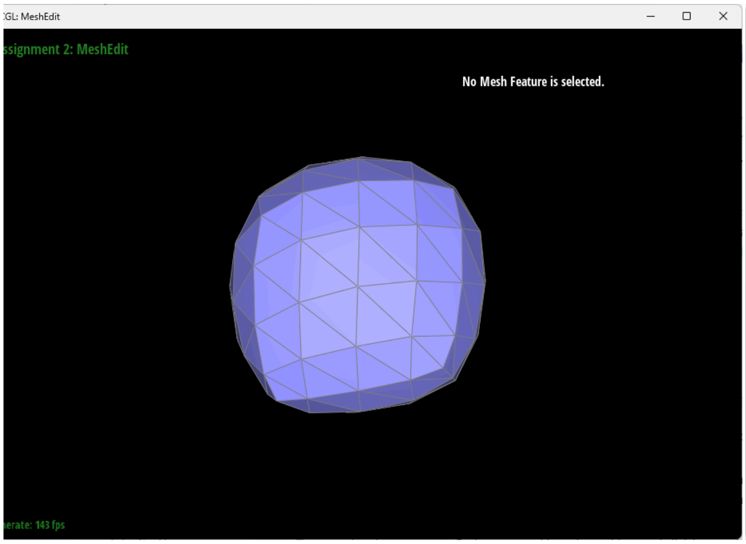
From the above example, one can also notice that the cube becomes asymmetric after the loop division, despite the cube itself is symmetric around the origin. This is because the vertices-edges topology is not symmetric. One can preprocess the cube to make the vertices-edges topology symmetric with some edge splitting, and the resulting mesh after loop subdivision would become symmetric as desired.



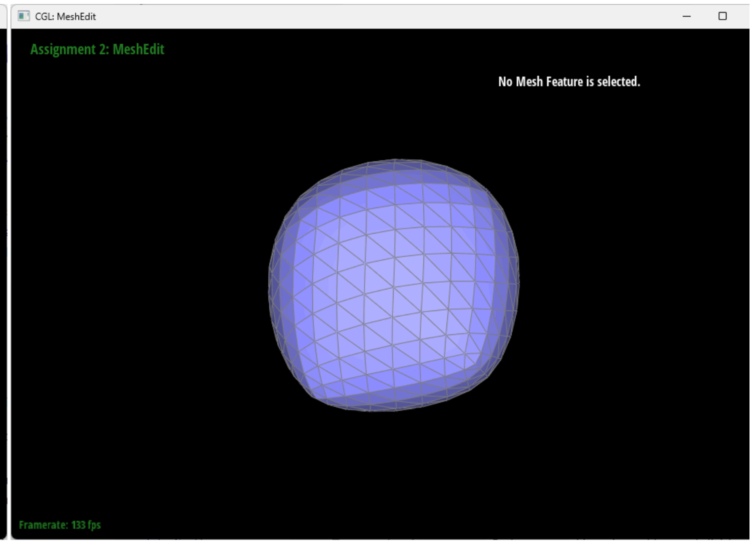
Original cube



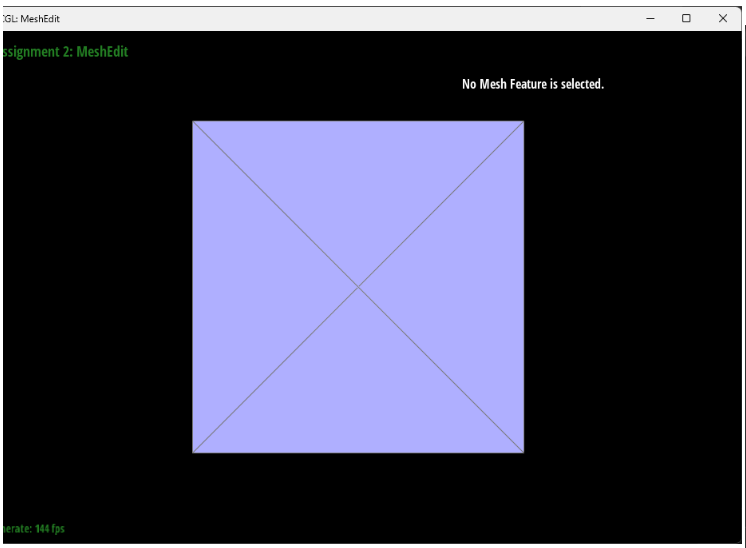
Cube after one loop subdivision, asymmetric



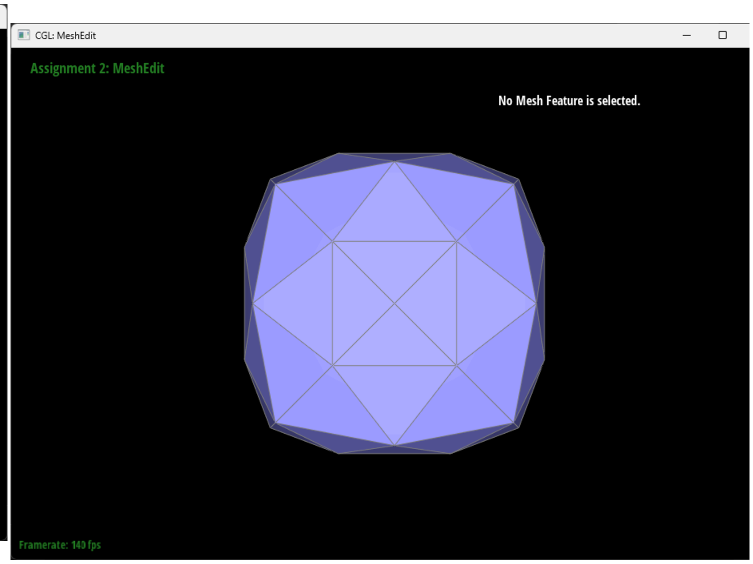
Cube after two loop subdivision, asymmetric



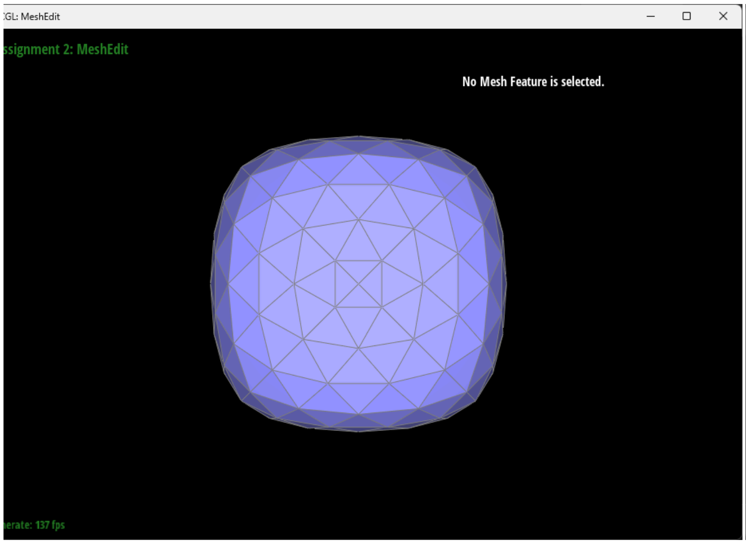
Cube after three loop subdivision, asymmetric



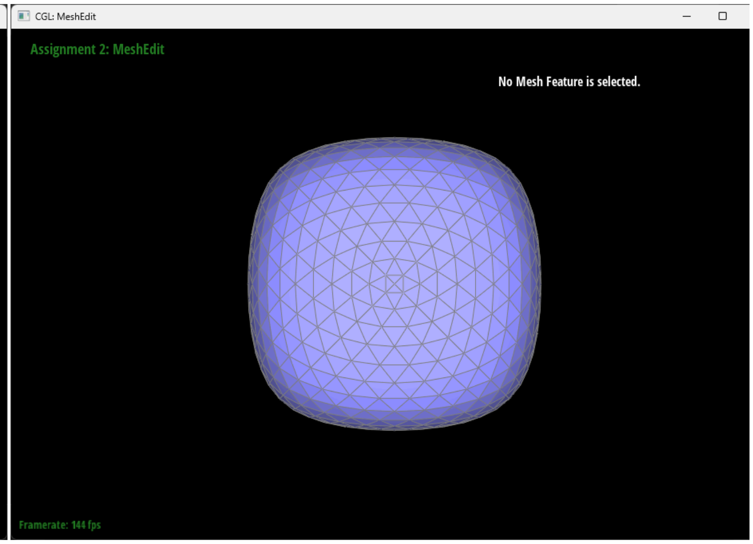
Cube with some edge spliting and with symmetric graph topology



Adjusted cube after one loop subdivision



Adjusted cube after two loop subdivision



Adjusted cube after three loop subdivision