CS 184/284A Spring 2023 **Name:** ____________________

Exam 2 (Online) **SID Number:** ____________________

April 20, 2023

Time Limit: 120 minutes

- ***NEW*** As described in Ed, for this exam we are using a single answer booklet for you to write answers to all questions. Printed copies of the booklet were previously distributed; you can also print your own copy. Please write all your answers into this booklet and upload into Gradescope at the end by taking photos of each page or uploading a single PDF (either method is acceptable).
- Please carefully read, sign and submit the Honor Code described on the next page.
- You will need your own blank paper, a pen, a web browser, your class Gradescope account, and an internet-connected device capable of taking and uploading photos (a smartphone is ideal).
- During the 24 hour period that the exam is running, you must not communicate with anyone about the exam, including posting to or reading from the internet any information about the exam questions. This is a strict requirement.
- This exam is open book, and open internet, subject to the limitations described above.
- This exam is open for **120 minutes**, including 10 minutes to scan and upload your answers. It has a total of 107 points. There are 15 pages (including this cover page) and 8 parts. Problem difficulty varies throughout the exam, so move on if you find yourself stuck. You are encouraged to show your work and what you know for partial credit.
- If you encounter uploading difficulties you may make a private Ed post with subject 'CS184 MT2'. Please do so before your Gradescope exam expires.
- For fairness to students taking the exam at different times, the teaching staff will not answer any questions or post any corrections about the exam during the 24 hour period.

Problem	Points	Score
1	0	
2	20	
3	17	
4	10	
5	18	
6	14	
7	12	
8	16	
Total:	107	

1. Honor Code and Academic Integrity Certification

The Honor Code is the commitment and work of students individually and as a community to uphold honesty and integrity. This commitment is comprehensive. Specifically on exams, it means not giving or receiving any help, or using any resources that are not permitted. The Honor Code requires that students take an active role in seeing to it that others as well as themselves uphold the letter and spirit of the Honor Code.

By uploading my signature I certify that:

- All the work submitted in my name for this exam is my work alone.
- I have not given or received any help during this exam.
- I have not used any un-permitted resources during this exam (see cover page for permitted resources).
- I have timed myself and used no more than the allotted time to write my answers for this exam.
- If I become aware of any Honor Code violations related to this exam, I will inform the course staff immediately.

I understand that a false statement on this declaration is a violation of the Honor Code, and would be subject to the highest level of disciplinary sanction under the University Code of Conduct.

1.i. Certification

Signature: ______________________________

Name: __________________________________

SID Number: ____________________

Date: ____________________

2. (Total : 20 points) True / False

   Mark each statement true or false. (1 point each)

   (2a) (1 point) ___ A z-buffer is a 2D array of z values that is commonly used for hidden surface removal.

   (2b) (1 point) ___ A directional light can be thought of as the limit of shrinking an area light down to an infinitesimal point.

   (2c) (1 point) ___ The Painter's Algorithm requires polygons that are sorted in front-to-back or back-to-front order.

   (2d) (1 point) ___ If a ball illuminated by a point light source has a specular highlight in the shape of an elongated curve, the ball has an anisotropic BRDF.

   (2e) (1 point) ___ In the subsurface scattering reflection model, the amount of outgoing light scattered at one point on a surface may depend on the light incident at a different point.

   (2f) (1 point) ___ In the path tracing algorithm, Russian Roulette enables an unbiased estimate of scene radiance without infinite recursion.

   (2g) (1 point) ___ These 2 photographic exposure levels are the same: 1/400th of a second at F2, compared to 1/100th of a second at F4.

   (2h) (1 point) ___ In Gauss' ray tracing diagram for a thin lens, all rays that pass through the focal point on one side of the lens will refract through the lens to pass through the focal point on the other side of the lens.

   (2i) (1 point) ___ The depth of field increases as the focus of a lens is pulled inward from the hyperfocal distance to nearer focus distances.

   (2j) (1 point) ___ In portrait photography, you can reduce the amount of visible background while keeping the subject the same visual size, by stepping away from the subject and increasing the focal length.

   (2k) (1 point) ___ Forward Euler integration is a commonly used algorithm for computing time steps in computer simulations that is unconditionally stable for spring-and-mass systems where the masses of the points are all the same.

   (2l) (1 point) ___ A volume that is occupied by an incompressible fluid of constant density must have positive divergence.

   (2m) (1 point) ___ The reason to use SVD for inverse kinematics is that it produces a unique solution configuration.

   (2n) (1 point) ___ Although physics simulations, such as of cloth or smoke, are typically computed by integrating the simulated system forward in time, optimization methods allow solutions that propagate constraints both forward and backward through simulated time.

   (2o) (1 point) ___ When rendering motion sequences, motion blur is the temporal equivalent of anti-aliasing.

   (2p) (1 point) ___ Some motion capture systems use multiple cameras and markers that reflect gamma radiation so that the 3D location of each marker can be computed using the video from the cameras.

   (2q) (1 point) ___ Many VR systems reduce judder by reading the head tracker at the beginning of the frame interval and saving the head pose for rendering future frames.

   (2r) (1 point) ___ Two sheets of paper in different environments may appear white, even if they cause different tristimulus responses (i.e. the responses of S, M and L cone cells are different).

(2s) (1 point) ___ The S cone cell has a peak response at a wavelength that is shorter than the L cone cell.

(2t) (1 point) ___ The Euclidean distance between the color coordinates for two colors is a color difference metric that is more perceptually uniform when calculated in CIELAB coordinates than RGB or CIEXYZ.

3. (Total : 17 points) Graphics Pipeline and Geometry

(3a) Transforms

Imagine that you had a 3×3 matrix $\mathbf{M}$ and vector $\mathbf{y}$ in R3 that transforms a 3D object according to: $\mathbf{x}' = \mathbf{M} \cdot \mathbf{x} + \mathbf{y}$. Where $\mathbf{x}$ is a point in 3D space.

3a.i. (1 point) What matrix decomposition technique could you use to express this transformation as a series of translations, rotations, and axis-aligned scales?

3a.ii. (1 point) What would the matrix decomposition be applied to?

3a.iii. (1 point) What special properties do each of the output from the decomposition have? Using variables that represent the outputs of your numerical algorithm, indicate what are the rotations, translations, and axis-aligned scales? Be clear!

3a.iv. (1 point) Translation part(s):

3a.v. (1 point) Rotation part(s):

3a.vi. (1 point) Axis-aligned scale part(s):

(3b) Consider the following code:

```
Vec3 DoSomething(const Triangle t, const Vec3 p) {
    Vec3 a = t.verts[0].position;
    Vec3 b = t.verts[1].position;
    Vec3 c = t.verts[2].position;

    Vec3 n = CrossProduct(b-a,c-a);
    n = Normalize(n);

    float area_a = DotProduct(CrossProduct(b-a,p-a), n);
    float area_b = DotProduct(CrossProduct(c-b,p-b), n);
    float area_c = DotProduct(CrossProduct(a-c,p-c), n);

    float area_sum = area_a + area_b + area_c;

    Vec3 tmp;
    tmp[0] = area_a / area_sum;
    tmp[1] = area_b / area_sum;
    tmp[2] = area_c / area_sum;

    return tmp;
}
```

3b.i. (1 point) What is being computed and then returned by this code?

3b.ii. (2 points) Ignoring the issue of rounding / floating point error, would it be possible to remove part of the code without changing the output? If so which part and why?

3b.iii. (1 point) If the point p is not in the plane of the triangle, then what do the resulting return values correspond to?

(3c) Consider the following code:

```
Vec3 DoSomething(const Vec3 cp[4], float t) {
    const Vec4 basis_0 = Vec4( 1, 0, -3,  2)'
    const Vec4 basis_1 = Vec4( 0, 0,  3, -2)'
    const Vec4 basis_2 = Vec4( 0, 1, -2,  1)'
    const Vec4 basis_3 = Vec4( 0, 0, -1,  1)'

    Vec4 pow_basis = Vec4(1,t, t*t, t*t*t);

    Vec3 eval_0 = Dot(basis_0, pow_basis) * cp[0];
    Vec3 eval_1 = Dot(basis_1, pow_basis) * cp[1];
    Vec3 eval_2 = Dot(basis_2, pow_basis) * cp[2];
    Vec3 eval_3 = Dot(basis_3, pow_basis) * cp[3];

    return eval_0 + eval_1 + eval_2 + eval_3;
}
```

3c.i. (2 points) What is this code doing?

3c.ii. (1 point) What is the simplest change you could make to this code so that it would compute Bézier interpolation?

(3d) Consider the following code:

```
float SlowFunction( const List_of_Vec3 normals,
                    const List_of_float offsets, Vec3 point) {
    float sum = 0.0;
    int N = normals.length();
    assert(N == offsets.length());
    for (int i = 0; i < N; i++) {
        sum += pow(Dot_Product(normals[i],point) + offsets[i],2);
    }
    return sum;
}
```

3d.i. (2 points) In the context of mesh processing, what is this function computing?
Now consider the following code:

```
float FastFunction( const List_of_Vec3 normals,
                    const List_of_float offsets, Vec3 point) {
    Mat4x4 matrix = ComputeMatrix(normals, offsets);
    Vec4 pointH(point[0],point[1],point[2],1);
    Vec4 tmp = Mat_Vec_Multiply(matrix,pointH);
    return Dot_Product(tmp,pointH);
}

Mat4x4 ComputeMatrix(   const List_of_Vec3 normals,
                        const List_of_float offsets {

    Mat4x4 matrix = 0.0; // Initalize to matrix of zeros.
    int N = normals.length();
    assert(N == offsets.length());
    for (int i = 0; i < N; i++) {
        ??????
    }
    return matrix;
}
```

3d.ii. (2 points) Write 1-3 lines of code that if placed where ?????? is indicated would compute `matrix` such that `FastFunction` would return the same result as `SlowFunction`.

4. (Total : 10 points) Rendering

(4a) (4 points) Your friend renders an image of a ball with a microfacet BRDF, illuminated by an area light source. The light source appears as a small specular highlight on the ball in the rendering, but you want the size of the specular highlight to appear larger. Describe two things that you could change in this scene that would have the desired effect.

(4b) (6 points) Debugging A Path Tracing Implementation

Your friend is trying to debug his physically-based raytracing code, but something strange is happening. He is doing path tracing, except that rather than using Russian Roulette to terminate paths, he is cutting off the recursion after N bounces. Specifically, he calculates the total radiance by summing ZeroBounceRadiance + AtLeastOneBounceRadiance as described in lecture, and terminating the recursion at depth N. The strange thing is that as he increases N, the image keeps getting brighter, more quickly as N gets larger. For each of the following programming bugs, state whether it could be the source of the problem (Yes or No), and give a brief 1-2 sentence explanation.

(i) Failing to cast shadow rays to check whether the light source is blocked.

(ii) Dividing by the square of the PDF value when importance sampling the lights.

(iii) Dividing by the square of the PDF value when importance sampling the BRDF.

5. (Total : 18 points) Cameras and Lenses

(5a) (2 points) Field of View

On a boat tour of San Francisco Bay, you take a picture of the Golden Gate Bridge. The bridge appears half the height of the full photo. You take the picture with a lens that has a focal length of 50mm on a camera with a 25x25mm image sensor. The bridge is 227m tall. How far away from the bridge are you located, in meters? Hint: for this part of the question you can safely assume that the camera is focused at infinity.

(5b) (6 points) Focus and Magnification

Later that day you are at the Conservatory of Flowers in Golden Gate Park. Using the same camera, you take a picture focused on a small rare orchid. The flower also appears half the height of the full photo. If the orchid is 25mm tall, how far away from the lens was the flower when you took the photo? Assume an ideal thin lens. Show your work. Hint: for this part of the question you cannot assume the camera will be focused at infinity.

(5c) Defocus

You take an evening photo of your friend, who is standing 2 meters away. Your camera has an ideal thin lens with 100 mm focal length, with an f/2.0 aperture, and is focused on your friend. The image sensor in your camera has a physical size of 25x25 mm. In the background of the picture behind your friend, there are lights in the courtyard, at a distance of 20 meters. You can assume that these lights are far enough away to be considered point lights.

In the following questions, you may use the following variables:

- w and h for the width and height of the image sensor, in millimeters. As mentioned, in this problem $w = h = 25$mm.
- f – the focal length of the lens. As mentioned, $f = 100$ mm.
- N – the f-number of the lens. As mentioned, the f-number is F2.0 (i.e. $N = 2$).
- A – the diameter of the lens.
- z_f – the distance between the lens and your friend when taking the photo. $z_f = 2$ meters.
- z_l – the distance between the lens and the background lights. $z_l = 20$ meters.
- z_s – the distance between the lens and sensor when taking the photo.
- z_l' – the distance between the lens and the plane inside the camera where the background lights would come to focus.
- C – the diameter of the circle of confusion of the background lights as they appear on the image sensor.

5c.i. (5 points) Draw a ray diagram representing this problem that includes the following items and labels: background lights, friend, lens, A, lens focal points, focal length f, sensor, the rays coming from the lights and refracting through the periphery of the lens onto the sensor, circle of confusion, C, and distances z_f, z_l, z_s and z_l'. Your drawing should be neat, clear and represent the geometric relationships needed to solve the next problem correctly, but it does not need to be drawn to scale.

5c.ii. (5 points) Derive the diameter of the circle of confusion that the background lights make on the sensor surface. Give your final answer in millimeters to two decimal places.

6. (Total : 14 points) Animation: Kinematics, Motion Capture, Physical Simulation

(6a) (5 points) Imagine you have a one-dimensional system consorting of a single spring and single mass point. One end of the spring is attached to "ground" at $x = 0$ and the other end is attached to the mass point. The mass of the point is $m = 0.5$Kg, and the spring follows Hooke's law with $k = 4.0N/m$. The rest length of the spring is zero.

If the system initial starts with the mass at rest at $x = 2.0$ and the simulation takes steps of $\Delta t = 0.5$ using Forward Euler integration, then write the resulting position, velocity, and acceleration for the first 8 time steps.

Hint: The numbers are such that computing by hand is reasonable, but free to write a little program/script or set up spreadsheet to compute it for you. Also, there is no gravity in this problem.

$x_0 =$ 2.0	$v_0 =$ 0.0	$a_0 =$
$x_1 =$	$v_1 =$	$a_1 =$
$x_2 =$	$v_3 =$	$a_2 =$
$x_3 =$	$v_3 =$	$a_3 =$
$x_4 =$	$v_4 =$	$a_4 =$
$x_5 =$	$v_5 =$	$a_5 =$
$x_6 =$	$v_6 =$	$a_6 =$
$x_7 =$	$v_7 =$	$a_7 =$
$x_8 =$	$v_8 =$	$a_8 =$

(6b) (1 point) What problem do these numbers demonstrate?

(6c) (1 point) How could you fix the problem?

(6d) (3 points) Fluid Simulation

Imagine that you have a job at a company that designs acoustic devices. Your team lead wants to build a computer simulation to model sound propagation through the air as a way of testing different device designs. They suggest using the ”Stable Fluids” fluid simulation method discussed in class.

Do you think the Stable Fluids method is appropriate for this situation? Why or why not? (Hint: This question can be answered in one or two sentences. If you are writing an long essay then you are probably on the wrong track.)

(6e) (4 points) Kinematics

Consider the following code:

```
VecN SimpleIK(const KinematicObject arm, const VecN startConfig, const Vec3 goal) {
    static const float TOLERANCE = 0.010;
    static const float MINSTEP   = 0.001;
    VecN config = startConfig;
    float dist = arm.forwardKinematics(config).distance_from(goal);
    bool giveup = false;
    while ( (dist > TOLERANCE) && (!giveup) ) {
        giveup = true;
        VecN grad =
              normalize(arm.forwardKinematics(config).distance_gradient(goal));
        for ( float alpha = 1.0 ; alpha > MINSTEP ; alpha *= 0.5  ) {
            float testConfig = config - (M_PI/180.0) * alpha * grad;
            float testDist
                 = arm.forwardKinematics(testConfig).distance_from(goal);
            if ( testDist < dist ) {
                config = testConfig;
                dist   = testDist;
                alpha = 1.0;
                giveup = false;
            }
        }
    }
    return config;
}
```

This code has an error, something is either incorrect or missing. What is the problem and what a simple way to correct it?

7. (Total : 12 points) Virtual Reality

(7a) Imagine that you are working on a VR application where the user is supposed to hold a virtual tablet and watch a video that is displayed on the hand-held virtual tablet. After building a prototype of the application, you find out that users are reporting headaches and feeling nauseated.

7a.i. (2 points) Concisely provide a plausible explanation for why this is happening?

7a.ii. (2 points) Assuming that you can't change the hardware, how could you display the video so as to not have this problem?

(7b) (8 points) For each of the following terms, find a description that applies to that term. **Note that some of the descriptions are used more than once and some are not used at all. Every one of the terms has only one applicable description.**

1) ______ Foveated rendering
2) ______ Inside-out tracking
3) ______ Augmented reality
4) ______ Accommodation
5) ______ Asynchronous space warp
6) ______ A pancake lens
7) ______ Vergence
8) ______ Virtual reality

A) provides an immersive virtual experience.
B) requires large glass lenses.
C) uses cameras to track something.
D) requires a very bright screen.
E) provides depth cues.
F) reduces perceived latency.
H) is completely flat.
I) may be implemented with either optical or video pass-through.

8. (Total : 16 points) Color Reproduction Across Displays

You and your friend, who live across the country from one another, have each built your own color displays, and you want to perform color matching across the two displays. You can each set the color by directly choosing the intensity of your R, G and B pixels. However, your R, G and B pixels are not the same as hers, so colors do NOT look the same on the two displays when we set the same R, G and B intensities for both.

It turns out that you have a spectrometer, which allows you to determine the emission spectra of the R, G and B pixels in your display. Assume that s_{R1}, s_{G1}, s_{B1} are row vectors representing these emission spectra.

Unfortunately, your friend doesn't have a spectrometer, but she does have a monochromator. She uses it to determine the color matching functions for her display, obtaining row vectors $\overline{r_2}$, $\overline{g_2}$ and $\overline{b_2}$ that represent the color matching functions for the R, G and B pixels in her display.

Using these 6 functions, you will derive equations for setting the color on your friend's display to match the appearance on your display, by solving for the necessary scalar intensity values R_2, G_2, B_2 for her display, as a function of the values R_1, G_1, B_1 used on your display.

(8a) (3 points) Describe a procedure using the spectrometer that you perform to determine s_{R1}, s_{G1}, s_{B1} for your display.

(8b) (3 points) Describe a procedure using the monochromator that your friend uses to determine $\overline{r_2}$, $\overline{g_2}$ and $\overline{b_2}$ for her display.

(8c) (3 points) As a step towards color matching equations across your displays, write down expressions for the spectral power distribution emitted by your display, as a function of R_1, G_1, B_1. You may use any of the variables defined above. Explain your work for partial credit.

(8d) (5 points) Now, derive a formula for the intensities $\begin{bmatrix} R_2 & G_2 & B_2 \end{bmatrix}^T$ for your friend's display, as a function of $\begin{bmatrix} R_1 & G_1 & B_1 \end{bmatrix}^T$ for your display. Put your final answer in the form of a single matrix multiplied by column vector $\begin{bmatrix} R_1 & G_1 & B_1 \end{bmatrix}^T$. Show your work for partial credit.

$$\begin{bmatrix} R_2 \\ G_2 \\ B_2 \end{bmatrix} =$$

(8e) (2 points) The solution from the previous part works well for many colors, but there is a problem. For some input R_1, G_1, B_1 values, the calculated R_2, G_2, B_2 values contain a negative number. Explain what the problem is.