CS 184 HW3.2 Report

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2022 Spring

Webpage Link

`https://github.com/cal-cs184-student/sp22-project-webpages-Wenhan0112/blob/master/proj3-2/index.html`

Introduction

In this homework, the bidirectional scattering distribution function are implemented for some of the materials:

- Mirror (Part 1): material where only specular reflection occurs.
- Pure refraction (Part 1): material where only reflection occurs.
- Glass (Part 1): dielectric material where refraction and reflection occur.
- Metal (Part 2): microfacet conductive materials where only reflection occurs.

NOTE: Part 1 and Part 2 are chosen!

Part 1: Mirror and Glass Materials

Task 1: Reflect

Given an incoming direction $\mathbf{e}_i \in \partial B(\mathbf{0}, 1) \subseteq \mathbb{R}^3$ and a normal direction of the surface $\mathbf{n} \in \partial B(\mathbf{0}, 1)$, the outgoing direction of the light $\mathbf{e}_r \in \partial B(\mathbf{0}, 1)$ is given by:

$$\mathbf{e}_r = -\mathbf{e}_i + 2(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}$$

Moreover, Suppose $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ is a positively oriented orthonormal basis of $\mathbb{R}^3$ where $\mathbf{n} = \hat{\mathbf{z}}$. Suppose

$$\mathbf{e}_i = e_{i,x}\hat{\mathbf{x}} + e_{i,y}\hat{\mathbf{y}} + e_{i,z}\hat{\mathbf{z}}$$

Then

$$\begin{aligned}\mathbf{e}_r &= -\mathbf{e}_i + 2(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n} = -(e_{i,x}\hat{\mathbf{x}} + e_{i,y}\hat{\mathbf{y}} + e_{i,z}\hat{\mathbf{z}}) + 2((e_{i,x}\hat{\mathbf{x}} + e_{i,y}\hat{\mathbf{y}} + e_{i,z}\hat{\mathbf{z}}) \cdot \hat{\mathbf{z}})\hat{\mathbf{z}} \\ &= -e_{i,x}\hat{\mathbf{x}} - e_{i,y}\hat{\mathbf{y}} + e_{i,z}\hat{\mathbf{z}}\end{aligned}$$

Suppose

$$\mathbf{e}_r = e_{r,x}\hat{\mathbf{x}} + e_{r,y}\hat{\mathbf{y}} + e_{r,z}\hat{\mathbf{z}}$$

Then by reversibility of light path,

$$\begin{aligned}\mathbf{e}_i &= -\mathbf{e}_r + 2(\mathbf{e}_r \cdot \mathbf{n})\mathbf{n} \\ &= e_{r,x}\hat{\mathbf{x}} + e_{r,y}\hat{\mathbf{y}} + e_{r,z}\hat{\mathbf{z}}\end{aligned}$$

Task 2: Mirror Material

The bidirectional scattering distribution function of a mirror material is given by:

$$(\mathbf{e}_i, \mathbf{e}_r) \mapsto \frac{R}{\mathbf{e}_i \cdot \mathbf{n}}\delta(\mathbf{e}_r, -\mathbf{e}_i + 2(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n})$$

where R is the reflectance, δ is the Dirac delta function defined in $\partial B(\mathbf{0}, 1)$. This delta function is given upon the fact that incident light from $\mathbf{e}_i$ must be reflect to $\mathbf{e}_r = -\mathbf{e}_i + 2(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}$ direction.

In the implementation, when the incoming direction is sampled, importance sampling is done with probability 1 that the sampled incident direction corresponds to the specular reflection direction of the output direction. This yields a delta function in the probability density function. However, in the implementation, it is checked if the bounce is specular beforehand. Therefore, the delta function is reduced to 1 in the implementation. Moreover, the implementation bidirectional scattering distribution function is implemented to be 0 because the set where it takes nonzero value is of measure 0.

Task 3: Refraction

Given an incoming direction $\mathbf{e}_i \in \partial B(\mathbf{0}, 1) \subseteq \mathbb{R}^3$, a normal direction of the surface $\mathbf{n} \in \partial B(\mathbf{0}, 1)$, a relative index of refraction of the outgoing to the incoming material n_r, suppose the refracted direction is $\mathbf{e}_r \in \partial B(\mathbf{0}, 1)$. Suppose

$$\begin{cases}\mathbf{e}_r = \mathbf{e}_{r,||} + (\mathbf{e}_r \cdot \mathbf{n})\mathbf{n} \\ \mathbf{e}_i = \mathbf{e}_{i,||} + (\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}\end{cases}$$

where $\mathbf{e}_{r,||}, \mathbf{e}_{i,||} \in \operatorname{span}(\mathbf{n})^\perp$

Then, by boundary condition of electromagnetic field in a dielectric material (invariance of phase on the boundary, detail in Physics 5B/110A), if refraction occurs,

$$n_r\mathbf{e}_{r,||} = -\mathbf{e}_{i,||}$$

However, if

$$\left|-\frac{1}{n_r}\mathbf{e}_{i,||}\right| = \frac{|\mathbf{e}_{i,||}|}{n_r} = \frac{1}{n_r}\sqrt{1 - (\mathbf{e}_i \cdot \mathbf{n})^2} > 1$$

then there does not exists $\mathbf{e}_r \in \partial B(\mathbf{0}, 1)$ that satisfies the condition. Therefore, in this case, no refraction occurs. This phenomenon is defined as total internal reflection.

If refraction occurs, then

$$|\mathbf{e}_r \cdot \mathbf{n}| = \sqrt{1 - |\mathbf{e}_{r,||}|^2} = \sqrt{1 - \frac{1}{n_r^2}\left(1 - (\mathbf{e}_i \cdot \mathbf{n})^2\right)}$$

Because refraction occurs in two sides of the surface, $\mathbf{e}_r \cdot \mathbf{n}$ should has a different sign of $\mathbf{e}_i \cdot \mathbf{n}$. Therefore,

$$\mathbf{e}_r \cdot \mathbf{n} = -\sqrt{1 - \frac{1}{n_r^2}\left(1 - (\mathbf{e}_i \cdot \mathbf{n})^2\right)}\mathrm{sgn}(\mathbf{e}_i \cdot \mathbf{n})$$

Therefore,

$$\mathbf{e}_r = -\frac{\mathbf{e}_{i,||}}{n_r} - \sqrt{1 - \frac{1}{n_r^2}\left(1 - (\mathbf{e}_i \cdot \mathbf{n})^2\right)}\mathrm{sgn}(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}$$

$$\mathbf{e}_r = -\frac{\mathbf{e}_i - (\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}}{n_r} - \sqrt{1 - \frac{1}{n_r^2}\left(1 - (\mathbf{e}_i \cdot \mathbf{n})^2\right)}\mathrm{sgn}(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}$$

Moreover, Suppose $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ is a positively oriented orthonormal basis of $\mathbb{R}^3$ where $\mathbf{n} = \hat{\mathbf{z}}$. Suppose

$$\mathbf{e}_i = e_{i,x}\hat{\mathbf{x}} + e_{i,y}\hat{\mathbf{y}} + e_{i,z}\hat{\mathbf{z}}$$

Then refraction occurs iff $\frac{1}{n_r}\sqrt{1 - e_{i,z}^2} \leq 1$, where the refracted direction is

$$\mathbf{e}_r = -\frac{e_{i,x}}{n_r}\hat{\mathbf{x}} - \frac{e_{i,y}}{n_r}\hat{\mathbf{y}} - \sqrt{1 - \frac{1}{n_r^2}(1 - e_{i,z})^2)}\mathrm{sgn}(\mathrm{e_{i,z}})\hat{\mathbf{z}}$$

Suppose

$$\mathbf{e}_r = e_{r,x}\hat{\mathbf{x}} + e_{r,y}\hat{\mathbf{y}} + e_{r,z}\hat{\mathbf{z}}$$

Then by reversibility of light path, refraction occurs iff

$$n_r\sqrt{1 - (\mathbf{e}_r \cdot \mathbf{n})^2} = n_r\sqrt{1 - e_{r,z}^2} \leq 1$$

where the incident direction is

$$\begin{aligned}\mathbf{e}_i &= -n_r\left(\mathbf{e}_i - (\mathbf{e}_i \cdot \mathbf{n})\mathbf{n}\right) - \sqrt{1 - n_r^2\left(1 - (\mathbf{e}_i \cdot \mathbf{n})^2\right)}\mathrm{sgn}(\mathbf{e}_i \cdot \mathbf{n})\mathbf{n} \\ &= -n_r e_{r,x}\hat{\mathbf{x}} - n_r e_{r,y}\hat{\mathbf{y}} - \sqrt{1 - n_r^2(1 - e_{r,z})^2)}\mathrm{sgn}(\mathrm{e_{r,z}})\hat{\mathbf{z}}\end{aligned}$$

Task 4: Glass Material

The bidirectional scattering distribution function of a glass material with normal $\mathbf{n} \in \partial B(\mathbf{0}, 1)$ is given by:

$$(\mathbf{e}_i, \mathbf{e}_r) \mapsto \begin{cases} \dfrac{R}{\mathbf{e}_i \cdot \mathbf{n}}\delta(\mathbf{e}_r - d_r(\mathbf{e}_i, \mathbf{n})) & , \mathbf{e}_i \notin D_t \\ \dfrac{r(d_r^{-1}(\mathbf{e}_r))^2 R}{\mathbf{e}_i \cdot \mathbf{n}}\delta(\mathbf{e}_r - d_r(\mathbf{e}_i)) + \dfrac{(1 - r(d_r^{-1}(\mathbf{e}_r))^2)T}{n_r(\mathbf{e}_i)^2(\mathbf{e}_i \cdot \mathbf{n})}\delta(\mathbf{e}_r - d_t(\mathbf{e}_i)) & , \mathbf{e}_i \in D_t \end{cases}$$

where:

- $D_t = \left\{ \mathbf{e}_i \in \partial B(\mathbf{0}, 1) \middle| \frac{1}{n_r(\mathbf{e}_i)} \sqrt{1 - (\mathbf{e}_i \cdot \mathbf{n})^2} > 1 \right\}$, that is, the set of incident directions where refraction occurs.
- R: The reflectance of the material
- T: The transmittance of the material.
- r: The amplitude reflection coefficient of the material as a function of incident direction.
- d_r: The reflection direction as a function of incident direction. Note that this function is invertible, and $d_r^{-1} = d_r$ by reversibility of light path.
- d_t: The refraction direction as a function of incident direction defined in D_t. Note that this function is invertible constrained on its range, and $d_t^{-1} = d_t$ by reversibility of light path.
- n_r: The relative index of refraction of the outgoing material to the incident material as a function of the incident direction.
- δ: The Dirac delta function defined in $\partial B(\mathbf{0}, 1)$

r is given by the Fresnel equation, weighted by polarization. However, the Schlick approximation is used: $\forall \mathbf{e}_i \in D_t$:

$$r(\mathbf{e}_i)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 + \frac{4 n_1 n_2}{(n_1 + n_2)^2} (1 - (\mathbf{e}_i \cdot \mathbf{n})^5)$$

. where n_1, n_2 are the index of refraction of the two materials. Note that r is symmetric under exchange of materials. Note that $\forall \mathbf{e}_r \in \partial B(\mathbf{0}, 1)$, $\mathbf{e}_r \cdot \mathbf{n} = d_r^{-1}(\mathbf{e}_r) \cdot \mathbf{n}$, so $r(\mathbf{e}_r)^2 = r(d_r^{-1}(\mathbf{e}_r))^2$.

To sample the incoming direction from the outgoing direction $\mathbf{e}_r$, there are two occasions to consider:

- Refraction does not occur: The only incoming direction is coming from the reflection. Therefore, the only direction that can be sampled is $d_r^{-1}(\mathbf{e}_r)$.
- Refraction occurs: The two incoming direction are due to reflection and refraction. To do importance sampling, they are weighted approximately by their intensity. That is, the probability of sampling on the specular direction $d_r^{-1}(\mathbf{e}_r)$ is $r(d_r^{-1}(\mathbf{e}_r))^2 = r(\mathbf{e}_r)^2$, and the probability of sampling on the specular direction $d_t^{-1}(\mathbf{e}_r)$ is $1 - r(d_r^{-1}(\mathbf{e}_r))^2 = 1 - r(\mathbf{e}_r)^2$.

Max Ray Depth Comparison

The renderings of `sky/CBspheres.dae` of various maximum ray depths are included as Fig.1. Following flags are used:

Maximum ray depth 0: `-t 8 -s 64 -l 4 -m 0 -r 480 360`
Maximum ray depth 1: `-t 8 -s 64 -l 4 -m 1 -r 480 360`
Maximum ray depth 2: `-t 8 -s 64 -l 4 -m 2 -r 480 360`
Maximum ray depth 3: `-t 8 -s 64 -l 4 -m 3 -r 480 360`
Maximum ray depth 4: `-t 8 -s 64 -l 4 -m 4 -r 480 360`

Maximum ray depth 5: `-t 8 -s 64 -l 4 -m 5 -r 480 360`

Maximum ray depth 100: `-t 8 -s 64 -l 4 -m 100 -r 480 360`

Differences between consecutive maximum ray depths and their explanations:

- None→ 0: Only the area light is rendered.
- 0 → 1:
  - Increase in overall lighting intensity.
  - The floor and walls are rendered. The light path is:
    Light source → Walls/floor → Observer.
  - The specular reflection of the light source on the mirror and glass ball are rendered. The light path is:
    Light source → Mirror ball → Observer.
- 1 → 2:
  - Increase in overall lighting intensity.
  - The ceiling is rendered. The light path is:
    Light source → Walls/floor → Ceiling → Observer.
  - The reflection of the floor and walls on the mirror ball are rendered. The light path is:
    Light source → Walls/floor → Mirror ball → Observer.
  - The specular reflection from the light source on the glass ball and then the mirror ball is rendered. The light path is:
    Light source → Outer reflect glass ball → Mirror ball → Observer.
  - The faint reflection of the floor and walls on the glass ball is rendered. The light path is:
    Light source → Walls/floor → Outer reflect glass ball → Observer.
  - The faint white spot on the left of the glass ball. It is the reflection of reflected light source on the mirror ball. The light path is:
    Light source → Mirror ball → Outer reflect glass ball → Observer.
- 2 → 3:
  - Increase in overall lighting intensity.
  - The reflection of the ceiling on the mirror ball is rendered. The light path is:
    Light source → Walls/floor → Ceiling → Mirror ball → Observer.
  - The refraction of the floor and walls through the glass ball is rendered. The light path is:
    Light source → Walls/floor → Enter glass ball → Exit glass ball → Observer.
  - The big white spot beneath the glass ball on the floor is rendered. The light path is:
    Light source → Enter glass ball → Exit glass ball → Floor → Observer.

  - The faint reflection of the floor and walls on the glass ball is gone. The refraction intensity is much larger than the reflection intensity on the glass ball.

- $3 \to 4$:
  - Increase in overall lighting intensity.
  - The reflection on the mirror ball of the refracted floor and walls through the glass ball. The light path is:

    Light source $\to$ Walls/floor $\to$ Enter glass ball $\to$ Exit glass ball $\to$ Mirror ball $\to$ Observer.
  - The reflection on the mirror ball of the big white spot beneath the glass ball. The light path is:

    Light source $\to$ Enter glass ball $\to$ Exit glass ball $\to$ Floor $\to$ Mirror ball $\to$ Observer.
  - The light spot at the bottom of the blue wall. The light path is:

    Light source $\to$ Mirror ball $\to$ Enter glass ball $\to$ Exit glass ball $\to$ Blue wall $\to$ Observer.
  - The faint white spot on the bottom the glass ball. It is the glass reflection of the big white spot beneath the glass ball. The light path is:

    Light source $\to$ Enter glass ball $\to$ Exit glass ball $\to$ Floor $\to$ Outer reflect glass ball $\to$ Observer.

- $4 \to 5$:
  - Increase in overall lighting intensity.
  - No other significant feature observed.

- $5 \to 100$:
  - Increase in overall lighting intensity.
  - Glazing on the top of the glass ball. It is the outer reflection on the top of the glass ball, where the incident angle is almost $\frac{\pi}{2}$ and the reflection coefficient is almost 1.

Part 2: Microfacet Material

Task 1: Microfacet Bidirectional Scattering distribution Function

The microfacet bidirectional scattering distribution function of a conduction material with macro surface normal direction $\mathbf{n} \in \partial B(\mathbf{0}, \mathbf{1})$ is given by:

$$(\mathbf{e}_i, \mathbf{e}_r) \mapsto \frac{F(\mathbf{e}_i)G(\mathbf{e}_i, \mathbf{e}_r)D\left(\frac{\mathbf{e}_i+\mathbf{e}_r}{\|\mathbf{e}_i+\mathbf{e}_r\|}\right)}{4(\mathbf{e}_i \cdot \mathbf{n})(\mathbf{e}_r \cdot \mathbf{n})}$$

where

- F: the intensity reflection coefficient of the conducting material as a function of incident direction.
- G: the shadow-masking function of the surface.
- D: the normal distribution probability density function of the surface.

Task 2: Normal Distribution Function

The normal distribution probability density function is assumed to be the Beckmann distribution: given the macro surface normal direction $\mathbf{n} \in \partial B(\mathbf{0}, \mathbf{1})$,

$$\mathbf{h} \mapsto \frac{\mathrm{e}^{-\frac{(\mathbf{h}\cdot\mathbf{n})^{-2}-1}{\alpha^2}}}{\pi\alpha^2(\mathbf{h}\cdot\mathbf{n})^4}$$

where $\alpha \in \mathbb{R}_+$ represents the roughness of the surface. α is positively related with the roughness of the surface.

Task 3: Fresnel Term

It is assumed that the incoming light is natural light, with equal p and s polarization intensity. Then the overall intensity reflection coefficient is the average of the two.

Given an incident direction $\mathbf{e}_i$, a macro surface normal direction $\mathbf{n}$, and a complex index of refraction n, the s-polarization intensity reflection coefficient is given by:

$$\frac{|n|^2 - 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + (\mathbf{e}_i\cdot\mathbf{n})^2}{|n|^2 + 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + (\mathbf{e}_i\cdot\mathbf{n})^2}$$

while the p-polarization intensity reflection coefficient is given by:

$$\frac{|n|^2(\mathbf{e}_i\cdot\mathbf{n})^2 - 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + 1}{|n|^2(\mathbf{e}_i\cdot\mathbf{n})^2 + 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + 1}$$

Therefore, the overall intensity reflection coefficient evaluated at incident direction $\mathbf{e}_i$ is:

$$F(\mathbf{e}_i) = \frac{1}{2}\left(\frac{|n|^2 - 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + (\mathbf{e}_i\cdot\mathbf{n})^2}{|n|^2 + 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + (\mathbf{e}_i\cdot\mathbf{n})^2} + \frac{|n|^2(\mathbf{e}_i\cdot\mathbf{n})^2 - 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + 1}{|n|^2(\mathbf{e}_i\cdot\mathbf{n})^2 + 2\mathrm{Re}(n)(\mathbf{e}_i\cdot\mathbf{n}) + 1}\right)$$

Note that the light is not monochromatic, so the intensity of each wavelength/color is given by their different intensity reflection coefficient due to the difference of the corresponding index of refraction.

Task 4: Importance Sampling

Given a microfacet material surface with macro surface normal $\mathbf{n} \in \partial B(\mathbf{0}, 1)$ and outgoing direction $\mathbf{e}_r \in \partial B(\mathbf{0}, 1)$, the incident direction is wanted to be sampled. Suppose the incident direction $\mathbf{e}_i$ is a random variable in $\partial B(\mathbf{0}, 1)$ such that $\mathbf{h} = \frac{\mathbf{e}_r+\mathbf{e}_i}{\|\mathbf{e}_r+\mathbf{e}_i\|}$ satisfies the Beckmann distribution of roughness α. Then $\mathbf{e}_i = -\mathbf{e}_r + 2(\mathbf{e}_r\cdot\mathbf{h})\mathbf{h}$. Therefore, suppose $\mathbf{h}_0$ is sampled from the

Beckmann distribution with probability density function D. Then the sampled incoming direction is $-\mathbf{e}_r + 2(\mathbf{e}_r \cdot \mathbf{h}_0)\mathbf{h}_0$. The probability density function of the random variable $\mathbf{e}_i$ evaluated at that point is given by:

$$D(\mathbf{h}_0)\det(\partial(\mathbf{h} \mapsto -\mathbf{e}_r + 2(\mathbf{e}_r \cdot \mathbf{h})\mathbf{h})(\mathbf{h}_0)) = \frac{D(\mathbf{h}_0)}{4(\mathbf{e}_r \cdot \mathbf{h}_0)}$$

Therefore, in summary, the incident direction is sampled from a distribution of probability density function:

$$\mathbf{e}_i \mapsto \frac{D\left(\frac{\mathbf{e}_r+\mathbf{e}_i}{\|\mathbf{e}_r+\mathbf{e}_i\|}\right)}{4\left(\mathbf{e}_r \cdot \frac{\mathbf{e}_r+\mathbf{e}_i}{\|\mathbf{e}_r+\mathbf{e}_i\|}\right)} = \frac{D\left(\frac{\mathbf{e}_r+\mathbf{e}_i}{\|\mathbf{e}_r+\mathbf{e}_i\|}\right)}{2\|\mathbf{e}_r + \mathbf{e}_i\|}$$

To sample the random variable $\mathbf{h}$ that satisfies the Beckmann distribution, the following inversion method could be used: suppose u_1, u_2 are independent standard uniform random variables. Suppose $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{n})$ is a positively oriented orthonormal basis in $\mathbb{R}^3$, then the following random variable follows the Beckmann distribution of roughness α around $\mathbf{n}$:

$$\cos(\phi)\sin(\theta)\hat{\mathbf{x}} + \sin(\phi)\sin(\theta)\hat{\mathbf{y}} + \cos(\theta)\mathbf{n}$$

where

$$\theta = \arctan(\sqrt{-\alpha^2 \ln(1-u_1)})$$

$$\phi = 2\pi u_2$$

Roughness Comparison

The renderings of `sky/CBdragon_microfacet_au.dae` of various roughness are included as Fig.2.

Following flags are used:

```
-t 8 -s 128 -l 1 -m 5 -r 480 360
```

As the figure shows, the higher the roughness, the more diffuse the surface is. The lower the roughness, the surface behave more like a mirror, and the color of the surface represents more of the reflected light. Therefore, in a low roughness image, there are black regions (reflecting light from the observer side, which is 0), and blue regions (reflecting light from the blue wall). In a high roughness image, the color is almost gold as a diffuse material.

Sampling Comparison

The renderings of `sky/CBbunny_microfacet_cu.dae` of importance sampling and uniform hemisphere sampling are included as Fig.3.

Following flags are used:

```
-t 8 -s 64 -l 1 -m 5 -r 480 360
```

In the figure with uniform hemisphere sampling, there is a black line around the bunny. This is because at those point, the reflected angle is almost $\frac{\pi}{2}$. If the incoming direction to those point are sampled uniformly in the hemisphere, then the micro surface normal between them would deviates significant from the macro surface normal (micro surface normal and macro surface normal has approximately $\frac{\pi}{4}$ angle between them). This would yield an exponentially low normal density term and thus, low bidirectional scattering distribution function. It would be very hard to sample close to the specular reflection direction.

In addition, noise is reduced in the overall image due to the importance sampling on the bidirectional scattering distribution function.

Additional Material

The rendering of `sky/CBdragon_microfacet_fe.dae` is included as Fig.4.

Following flags are used:

```
-t 8 -s 128 -l 1 -m 5 -r 480 360
```

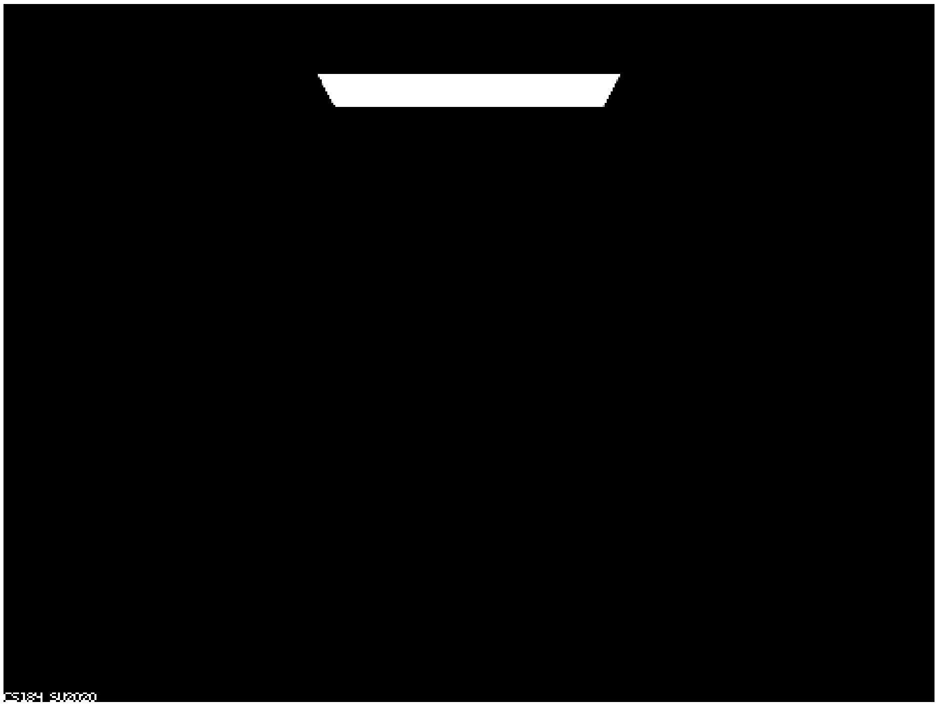
The material is iron, Fe, with roughness 0.5. The complex index of refraction at RGB wavelengths are given as Tab.1

Color	Wavelength	Complex index of refraction
Red	614nm	2.8851+3.0449i
Green	549nm	2.9500+2.9300i
Blue	466nm	2.6500+2.8095i

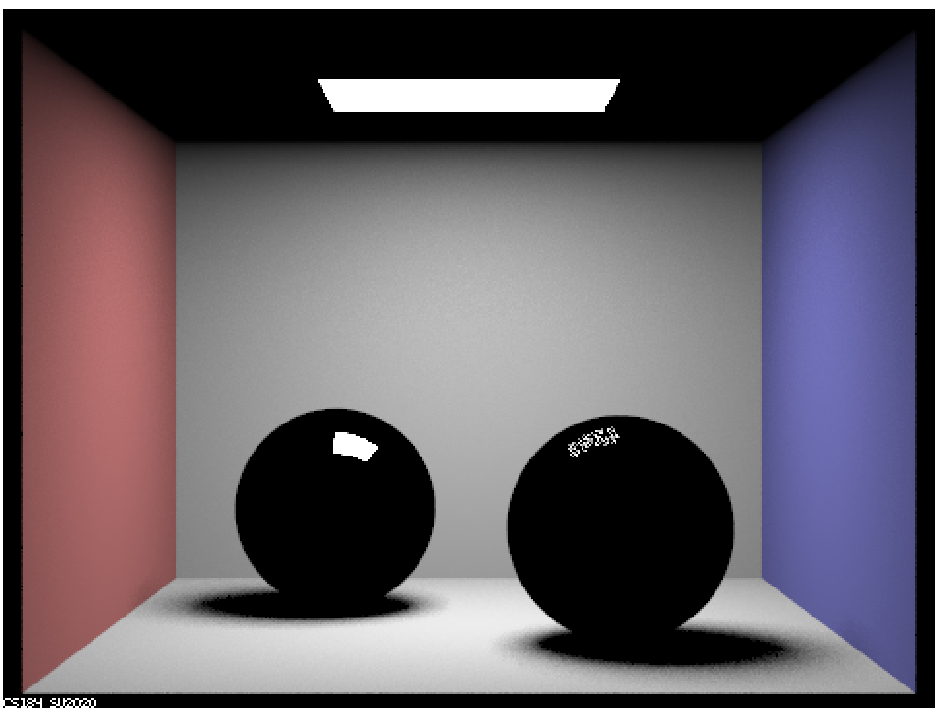
Table 1: Complex index of refraction at RGB wavelengths of iron, Fe.

Collaboration

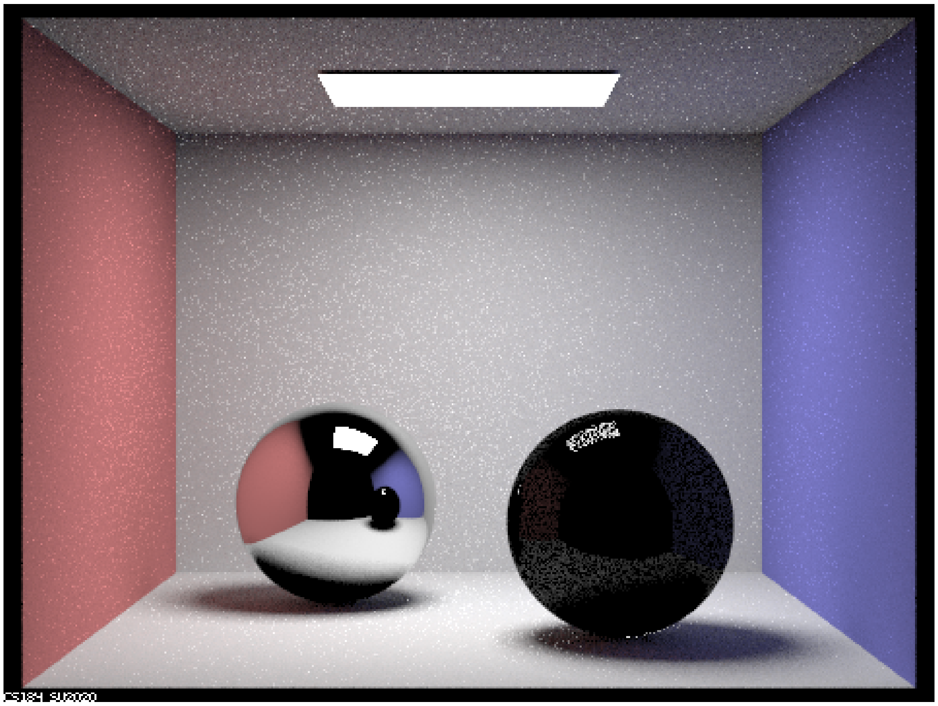
Wenhan Sun and Catherine Gai worked through all the project details together.



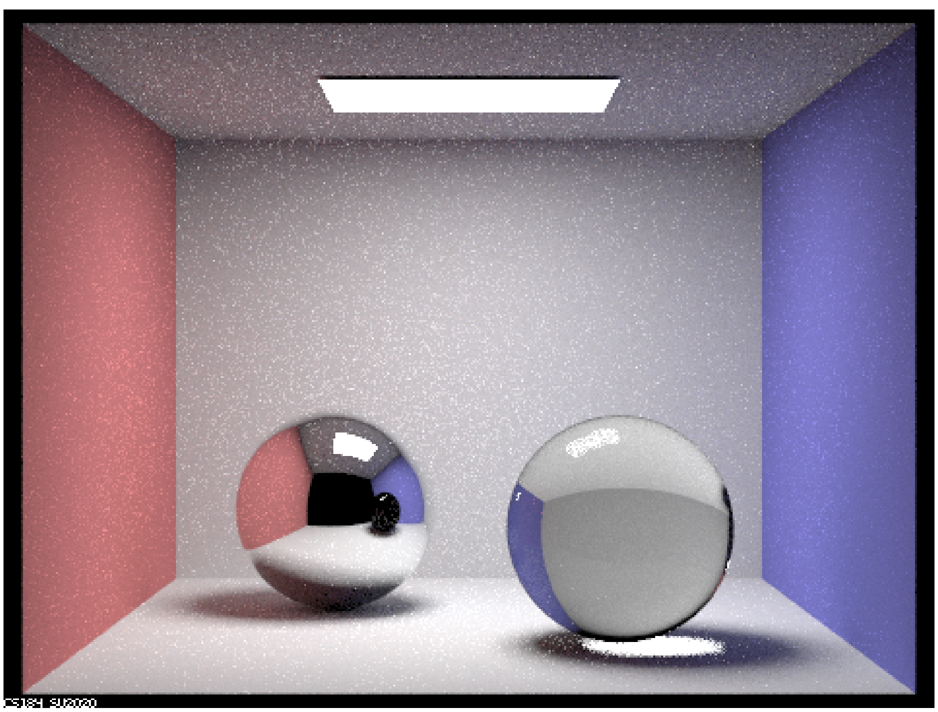
(a) Maximum ray depth 0



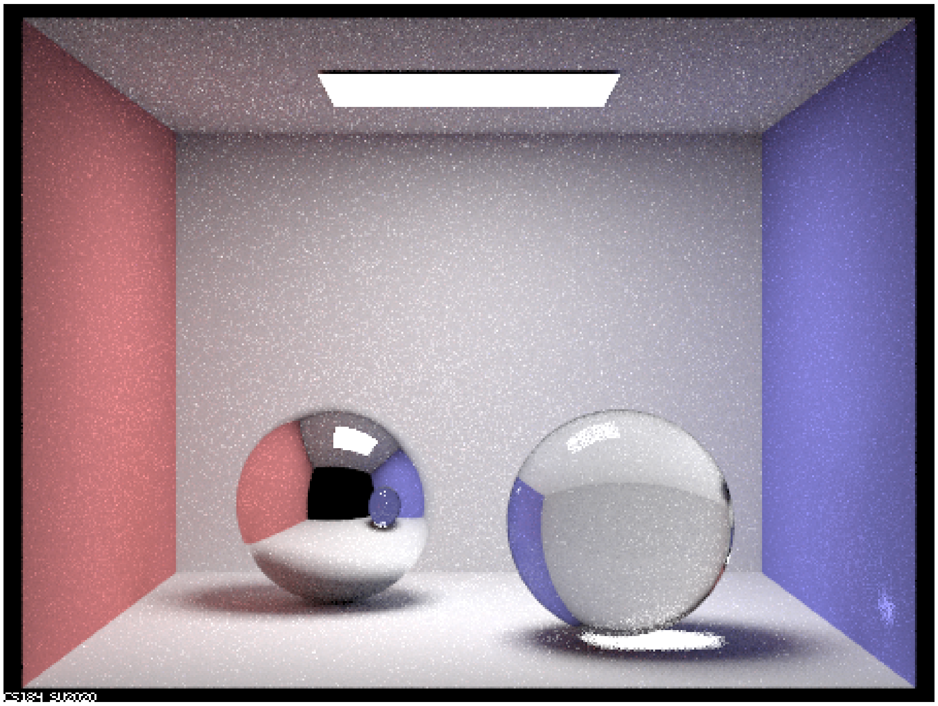
(b) Maximum ray depth 1



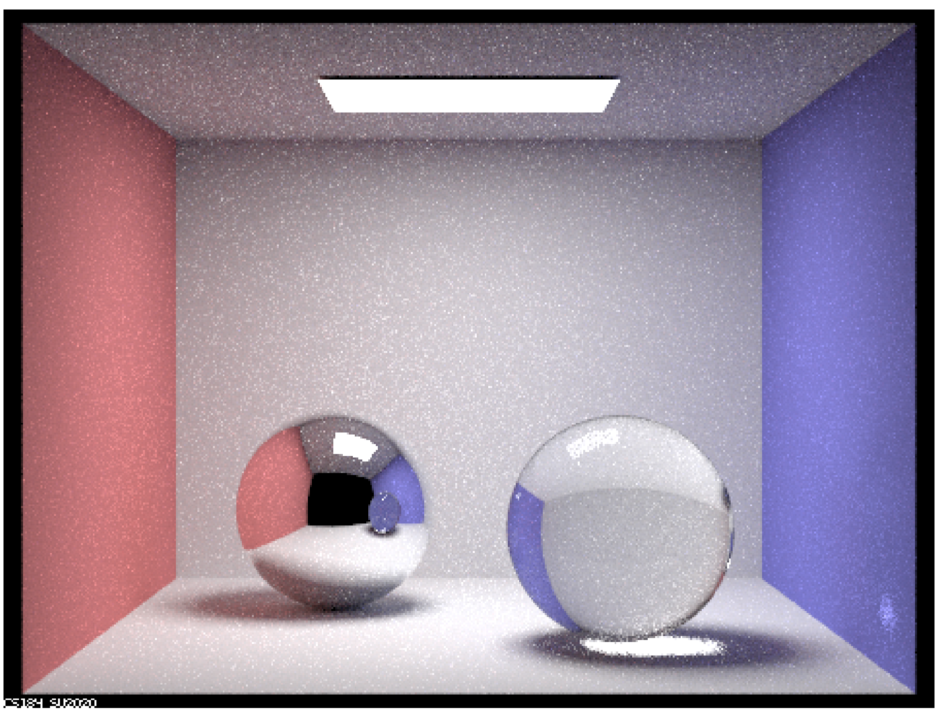
(c) Maximum ray depth 2



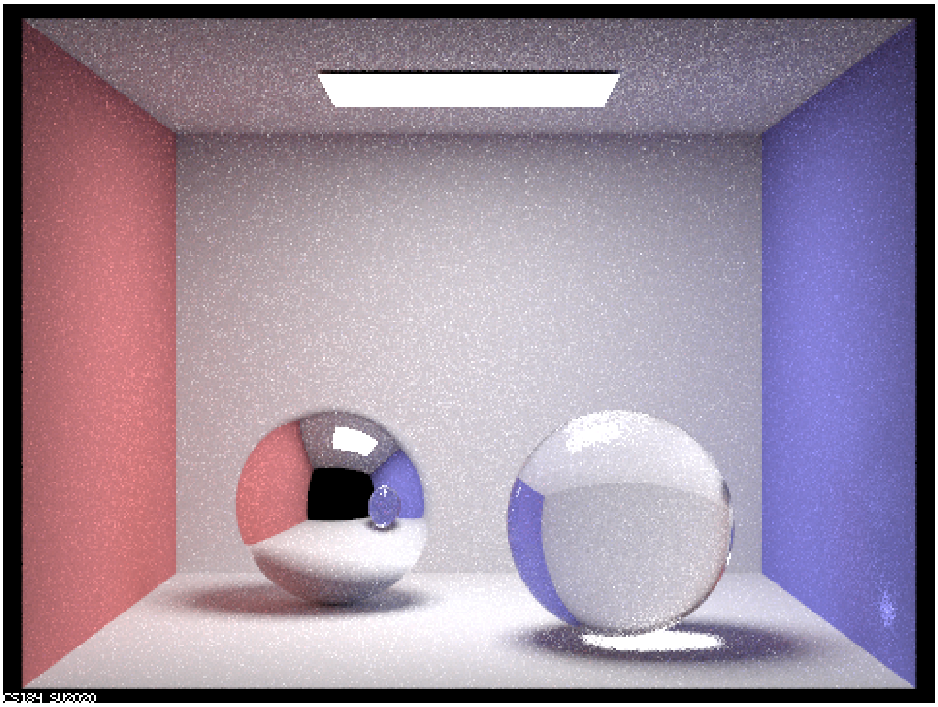
(d) Maximum ray depth 3



(e) Maximum ray depth 4

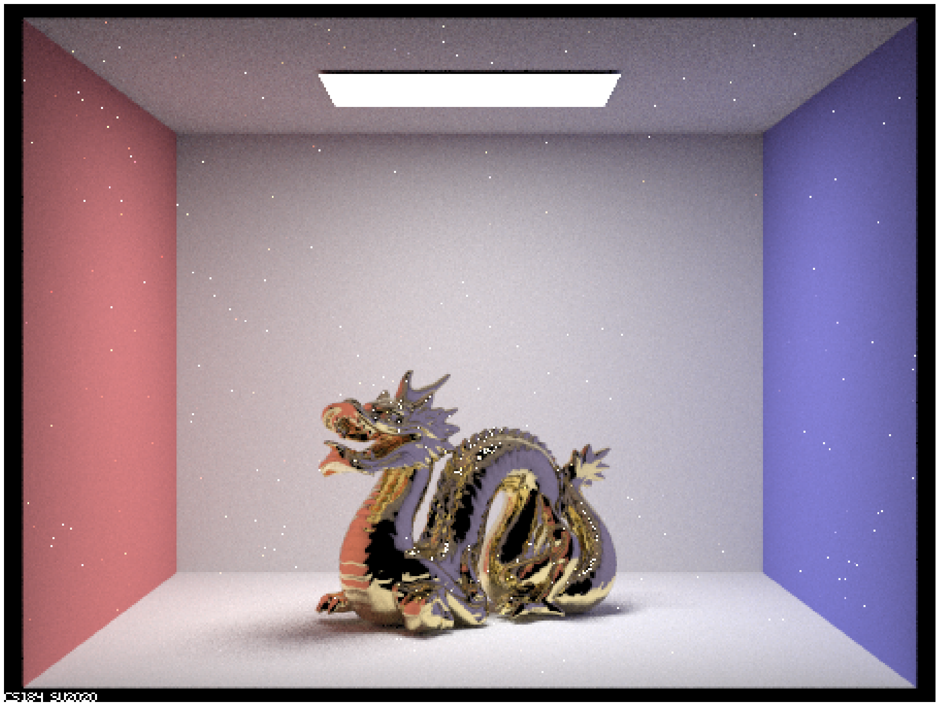


(f) Maximum ray depth 5

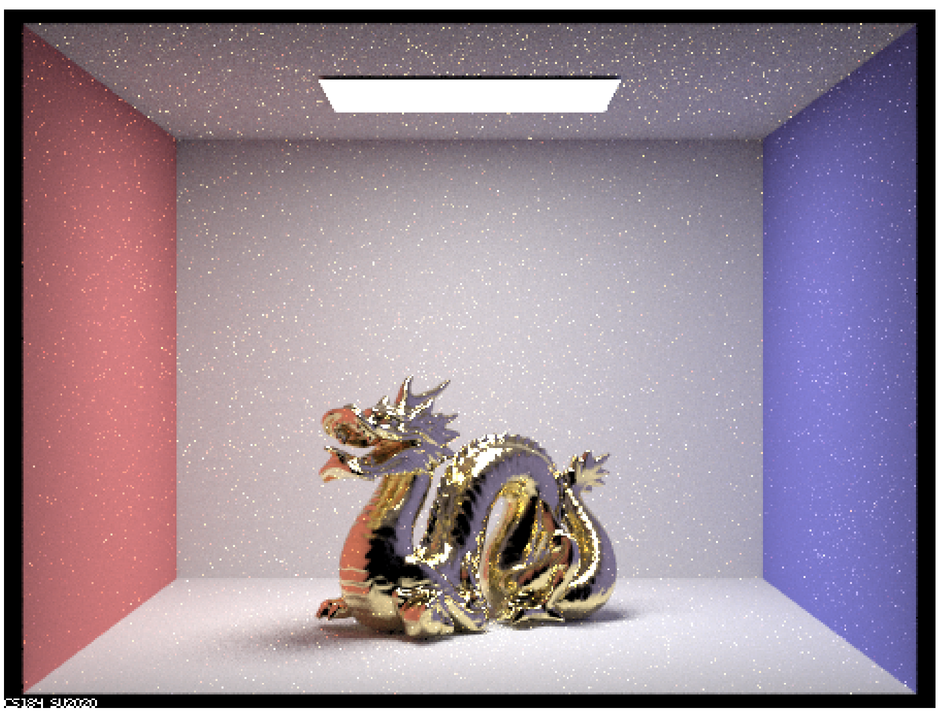


(g) Maximum ray depth 100

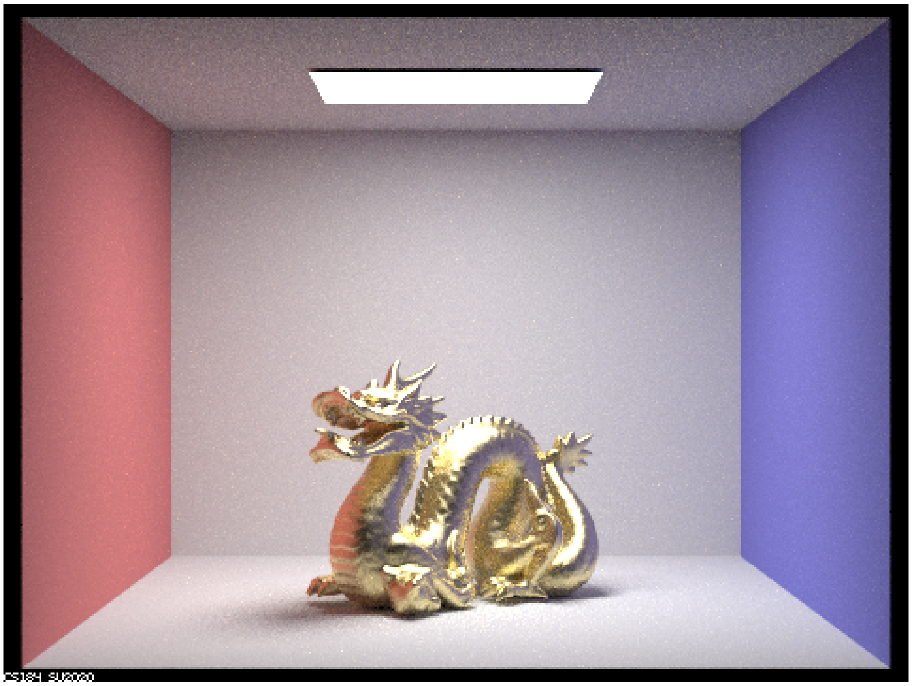
Figure 1: Renderings of `sky/CBspheres.dae` of various maximum ray depths. The following flags are used:
Maximum ray depth 0: `-t 8 -s 64 -l 4 -m 0 -r 480 360`
Maximum ray depth 1: `-t 8 -s 64 -l 4 -m 1 -r 480 360`
Maximum ray depth 2: `-t 8 -s 64 -l 4 -m 2 -r 480 360`
Maximum ray depth 3: `-t 8 -s 64 -l 4 -m 3 -r 480 360`
Maximum ray depth 4: `-t 8 -s 64 -l 4 -m 4 -r 480 360`
Maximum ray depth 5: `-t 8 -s 64 -l 4 -m 5 -r 480 360`
Maximum ray depth 100: `-t 8 -s 64 -l 4 -m 100 -r 480 360`



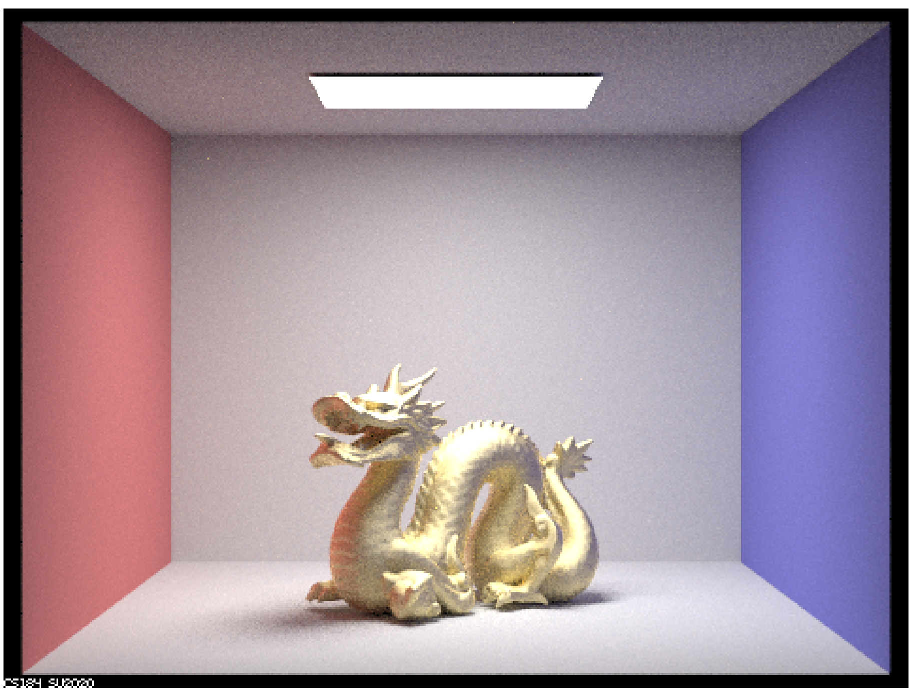
(a) Roughness 0.005



(b) Roughness 0.05

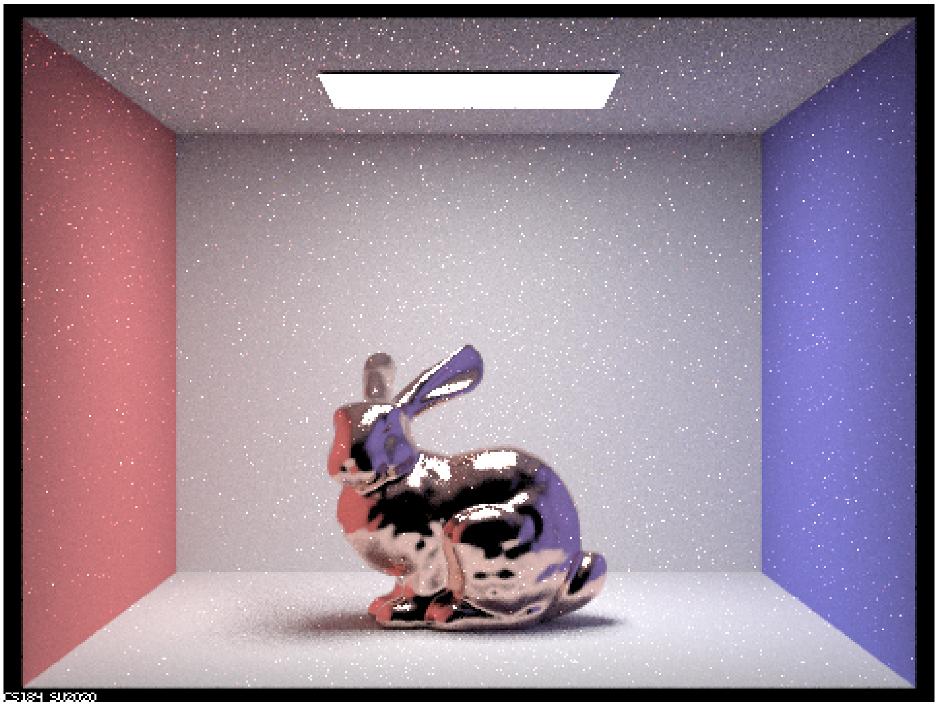


(c) Roughness 0.25

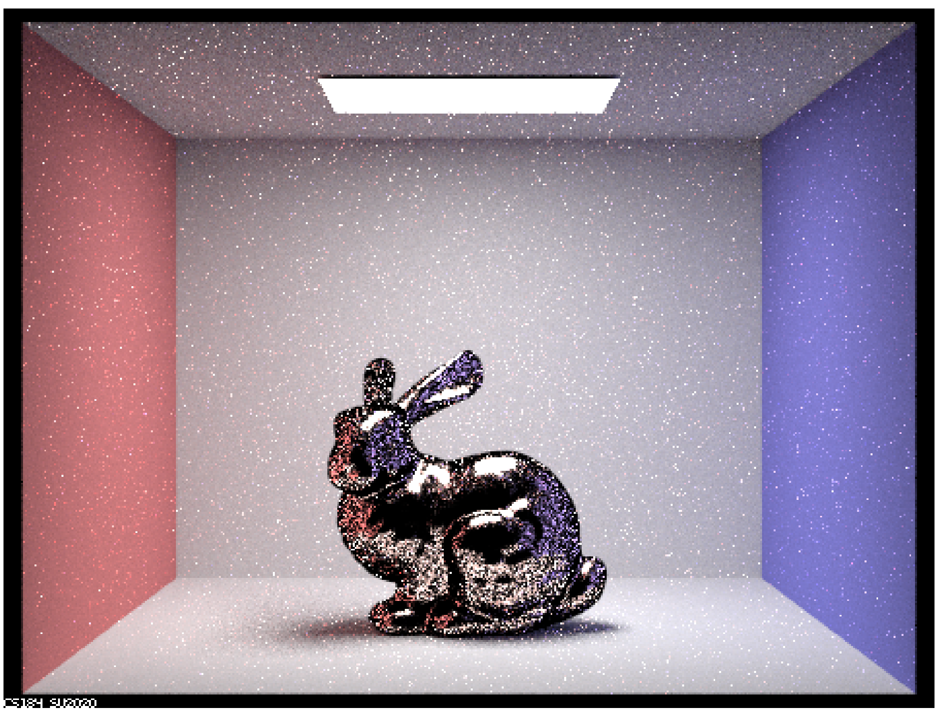


(d) Roughness 0.5

Figure 2: Renderings of `sky/CBdragon_microfacet_au.dae` of various roughness. The following flags are used:
`-t 8 -s 128 -l 1 -m 5 -r 480 360`



(a) Importance sampling



(b) Uniform hemisphere sampling

Figure 3: Renderings of `sky/CBbunny_microfacet_cu.dae` of importance sampling and uniform hemisphere sampling. The following flags are used:
`-t 8 -s 64 -l 1 -m 5 -r 480 360`

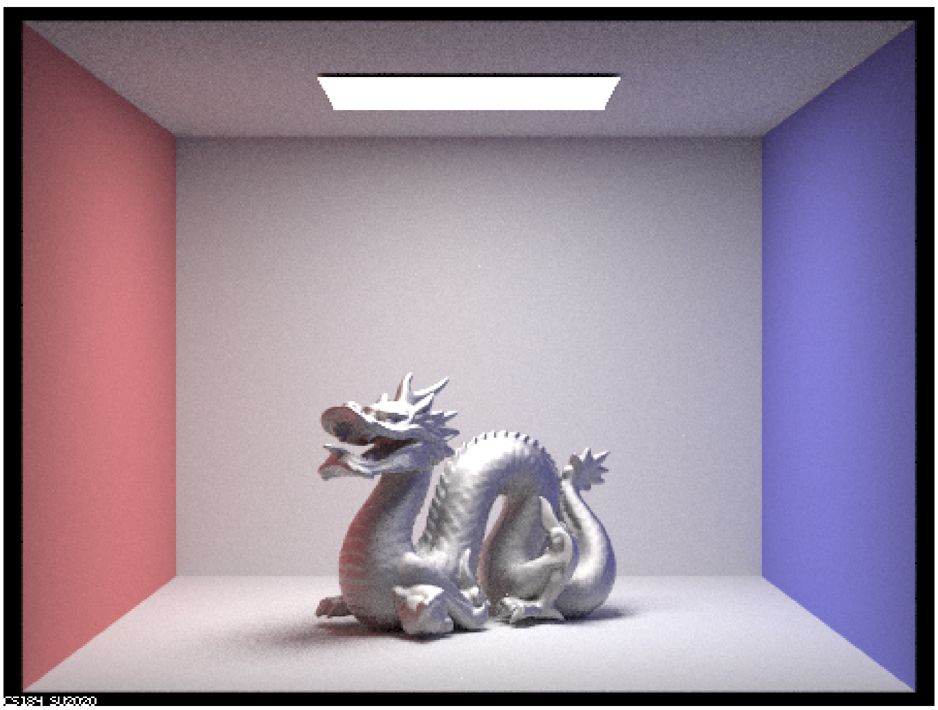


Figure 4: Rendering of `sky/CBdragon_microfacet_fe.dae` with material iron as specified in Tab.1.