Hw1

Task1

1.1 How to rasterize triangles in your own words.

1.1.1 Compute the axis-aligned bounding box (AABB).

I take min/max of the three vertices in screen space, then shift by **-0.5 px** so that pixel indices represent **pixel centers**. Finally, I clamp the range to the framebuffer.

1.1.2 Edge-Function Pre-computation

Each directed edge $\mathbf{e}_i = (v_i, v_j)$ is represented by a linear function

$$e(x, y) = ax + by + c$$

with

$$a = y_i - y_j, \quad b = x_j - x_i, \quad c = x_i y_j - x_j y_i.$$

1.1.3 Consistent Orientation

The signed doubled area

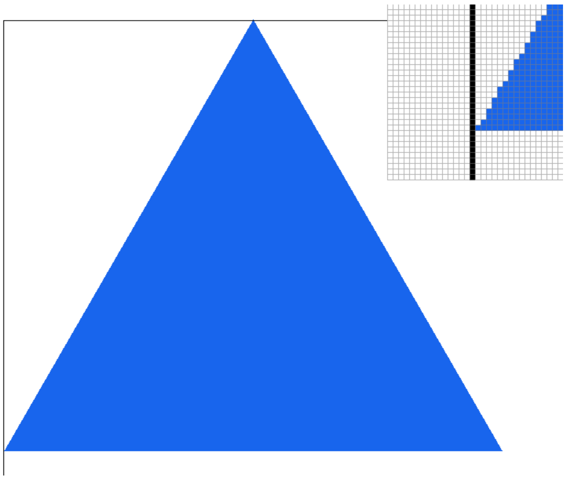
$$A_2 = (x_1 - x_0)(y_2 - y_0) - (y_1 - y_0)(x_2 - x_0)$$

1.1.4 Scanning the Bounding Box

The code iterates row by row through every pixel whose centre lies inside the bounding box. For each (p_x, p_y), the centre coordinates are (x_s, y_s) = (p_x + 0.5, p_y + 0.5). Computing e_0, e_1, e_2 requires three multiply-add operations per edge—nine FLOPs total—followed by three comparisons.
If the point passes the edge test, fill_pixel writes the colour to every subsample of that pixel.

1.2 Why the Algorithm Is No Worse than a Full AABB Scan

- I iterate over **exactly** the pixels inside the AABB, which is the same set examined by a naïve "sample-every-point-in-bbox" approach.
- Each pixel entails only **three multiply-adds and three comparisons**. No costly operations (div, sqrt) are used.
- Therefore the complexity is **O(bbox pixels)** in time and **O(1)** in extra memory.



About Extra Credit

I tried to complete the extra credit for task 1, but ultimately was not able to finish it. To measure the time taken, I added the following code at line 307 in src/drawrend.cpp. However, I found that the time required to render the same image was unstable with each run. Can this kind of timing be used for comparison?

```
auto t0 = std::chrono::high_resolution_clock::now();
svg.draw(software_rasterizer, ndc_to_screen * svg_to_ndc[current_svg]);
auto t1 = std::chrono::high_resolution_clock::now();
double ms = std::chrono::duration<double, std::milli>(t1 - t0).count();
std::cout << ms << " ms\n";
```

Task2

2.1Supersampling algorithm & data structures

2.1.1. sample_rate

This integer controls how many sub-samples live inside each pixel—1, 4, 9, 16, and so on. You can change it at run time via set_sample_rate(), letting you trade rendering speed for image quality. Every buffer size allocation and every loop that iterates over sub-samples is driven by this value.

2.1.2. sample_buffer

sample_buffer is the central data structure that stores the intermediate colors of all sub-samples. It is a vector<Color> whose length equals width × height × sample_rate, reserving sample_rate color slots for every pixel on the screen. The index scheme is simple: locate the pixel first, then offset by the sub-sample index—pixel_id * sample_rate + sub_id. During rasterization, we write directly into this buffer; we do **not** touch the 8-bit RGB framebuffer until the very end.

2.1.3 Pipeline for a triangle

a) **Bounding box clipping**.
b) **Per-sub-sample test**.
    1. Iterate over all sub-samples inside each pixel.
    2. Evaluate edge functions at the floating-point positions.
    3. If inside, write the color directly to sample_buffer.
c) **Resolve**. Average the n samples back to the 8-bit RGB framebuffer in resolve_to_framebuffer().

2.2 Why supersampling is useful

- **Aliasing**: Single-point sampling cannot estimate partial coverage → "jaggies".
- **Coverage approximation**: With ≥4 samples, we can approximate the true area coverage by k/n.
- **Renderer-agnostic**: Pure geometry approach, independent of texture filters.

2. 3 Modifications to the rasterization pipeline

Location	Change
fill_pixel	For points/lines, write the same color for every sub-sample.
rasterize_triangle	Added nested loops over sub-samples; kept fast path when n==1.
set_sample_rate & set_framebuffer_target	Re-allocate sample_buffer.
resolve_to_framebuffer	Average n sub-samples and write back to the RGB framebuffer.

2.4. Using supersampling to antialias triangles

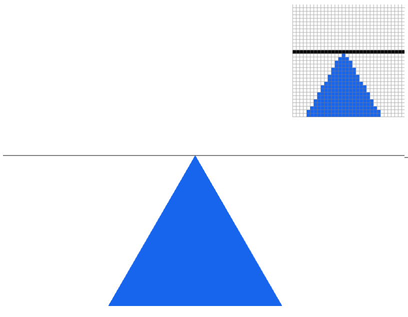
For each pixel we count hit samples k.

- Completely inside: k=n.
- Completely outside: k=0.
- **Edge pixels**: $0 < k < n$. Final $color = triangle\_color \times k/n + background \times (n-k)/n$.
  When n increases 1 → 4 → 16, the coverage levels become finer (25 %, 50 %, 75 %, …) and the staircase edge progressively turns into a smooth gradient.

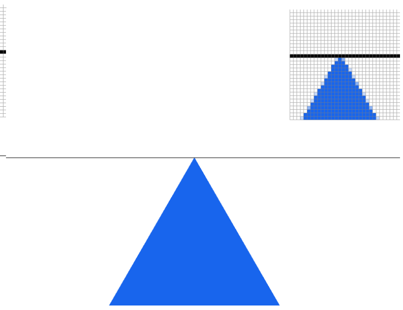
2.5. Results & explanation

The pixel inspector is placed over the skinny tip of the blue triangle:

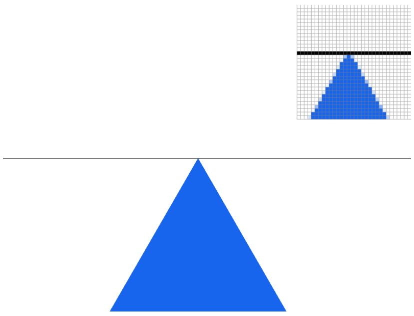
- **1×** Only two colors (blue/white) → harsh jaggies.
- **4×** Mixed shades appear; 25/50/75 % coverage soften the edge.
- **16×** Even finer coverage steps; edge looks virtually smooth at normal viewing distance. Frame-rate drops because the inner loops run 16× more often.



1X



4X



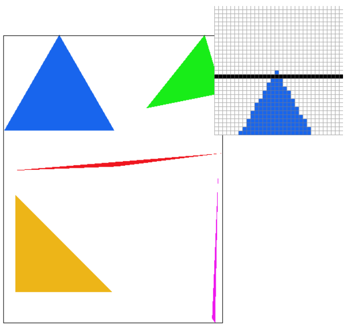
16X

Additional Notes:

I completed the entire assignment under WSL. When I open the file with the default viewing parameters using

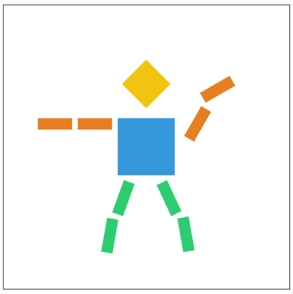
```
./draw ../svg/basic/test4.svg
```

I see a single blue pixel spilling outside the black frame. However, after I zoom in or out with the mouse wheel, all blue pixels are confined strictly within the frame.



Task3

This picture looks like a robot dancing, though it could also be seen as throwing a softball.



For extra Credit

Store the Rotation Angle

Add a float view_rotation field to the DrawRend class to store the current viewport rotation angle (in degrees). Initialize it to 0 in the constructor, init(), and view_init().

Listen for Keyboard Events

In the switch statement inside keyboard_event, add:

```
case 'E': view_rotation -= 5.f; redraw(); break;    // clockwise
case 'Q': view_rotation += 5.f; redraw(); break;    // counterclockwise
```

Each key press updates the rotation angle and triggers a redraw.

Construct the Rotation Matrix

At the beginning of redraw(), create:

```
Matrix3x3 R = translate(0.5, 0.5) *
              rotate(view_rotation) *
              translate(-0.5, -0.5);
```

This applies a rotation around the center of the NDC (Normalized Device Coordinates) space, which is at (0.5, 0.5).

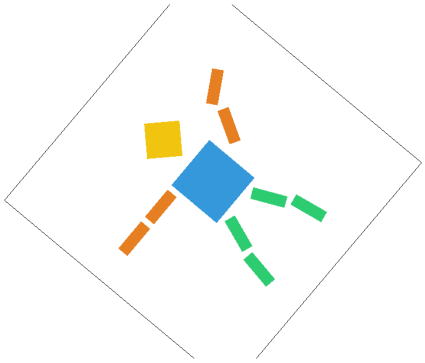
Insert Into the Matrix Stack

Replace the original drawing transformation, ndc_to_screen * svg_to_ndc with:

```
Matrix3x3 total = ndc_to_screen * R * svg_to_ndc[current_svg];
```

Apply this total matrix to all vertices (both the SVG content and the border lines), so the entire canvas rotates as the angle changes.

Reset on View Initialization
In view_init(), reset view_rotation to 0 to ensure that pressing the space bar not only resets the view but also clears any rotation.

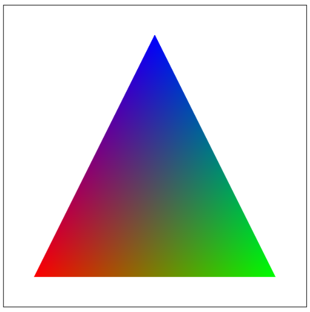


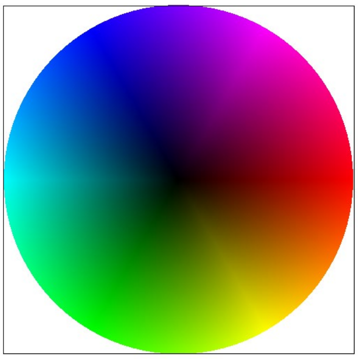
Task 4

Inside a triangle, any point can be thought of as a blend of the three vertices in certain "proportions"; these three proportions are the point's **barycentric coordinates**, denoted (w_0, w_1, w_2).

As shown in the figure below, you can view it by running:

```
./draw ../docs/task4.svg
```





screenshot of *svg/basic/test7.svg*

Task5

5.1 What is Pixel Sampling?

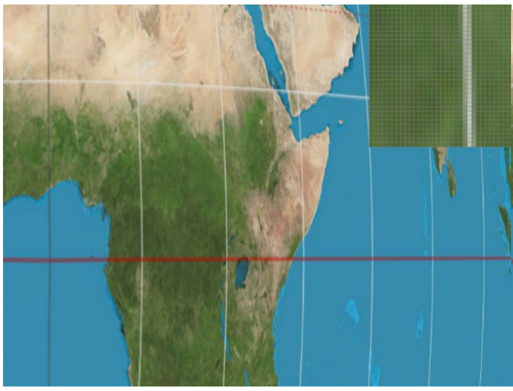
During texture mapping, we need to determine the color of each pixel on the screen based on a continuous texture coordinate $(\boldsymbol{u}, \boldsymbol{v})$. Since the texture is stored as a discrete grid of texels, we must "sample" from this grid—this process is called **pixel sampling**. The choice of sampling method directly affects image quality, influencing sharpness, smoothness, and the presence of artifacts such as jaggies or moiré patterns.

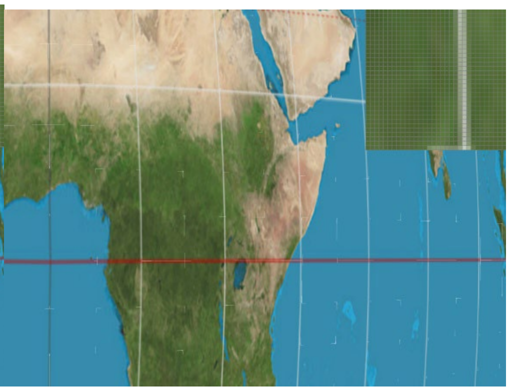
5.2 Implementation Overview

- In rasterize_textured_triangle, barycentric weights are computed and used to interpolate vertex UVs for every subsample, resulting in the target texture coordinate (u, v).
- A SampleParams structure is filled and passed to the Texture at mip level 0.
- The Texture::sample function dispatches to either sample_nearest or sample_bilinear based on the psm parameter

Nearest: Converts (u, v) to texel space, rounds to the nearest integer (t_x, t_y) , and fetches the single texel using get_texel.

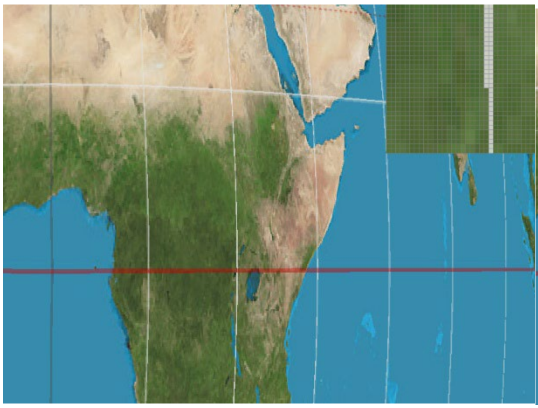
Bilinear: Fetches the four neighboring texels surrounding (x, y), computes the horizontal (s_x) and vertical (s_y) interpolation weights, and performs two linear interpolations to blend the colors

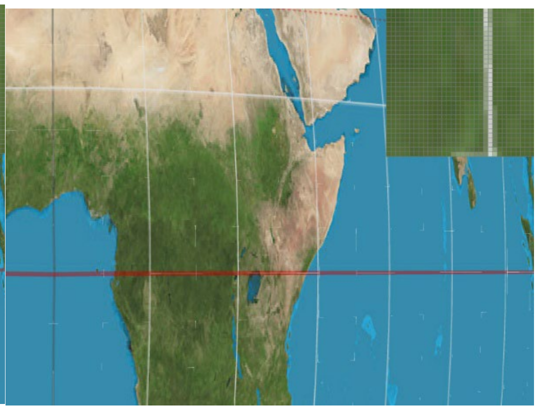




bilinear sampling at 1 sample

bilinear sampling at 16 sample





nearest sampling at 1 sample

nearest sampling at 16 sample

5.3 Discuss the difference

A large difference emerges when a screen pixel spans fractions of a texel—such as during strong magnification of high-frequency textures or thin diagonal edges—because nearest-neighbor snaps to one texel while bilinear interpolates among four. The resulting texel-to-texel discontinuities produce jagged aliasing and moiré in nearest sampling, whereas bilinear's low-pass blend smooths those high-frequency transitions.

Task6

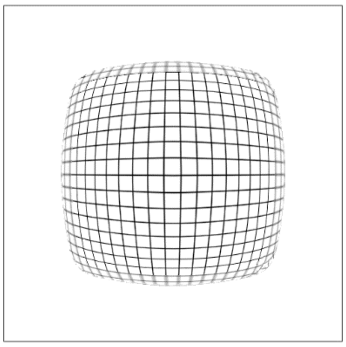
1 What "level sampling" means

When the same texture is projected onto screen-space, one screen pixel can cover anything from **less than one texel** (magnification) to **hundreds of texels** (strong minification, e.g. a checkerboard far in the distance).
Level sampling decides **which pre-filtered "mipmap" level** of the texture we should read so that the texel density roughly matches the pixel footprint:

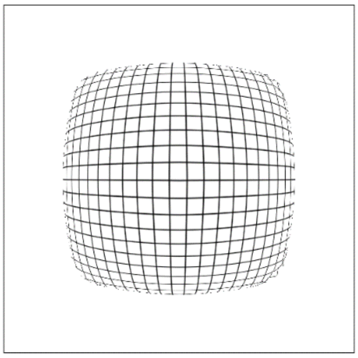
Comparison

Technique you turn up	Speed (per shaded pixel)	Extra memory	Antialiasing ability
Pixel sampling•	Nearest = 1 texel fetchBilinear = 4 fetches	none	Removes blockiness while magnifying, but cannot fight minification aliasing
Level sampling	L_NEAREST ≈ 1× pixel cost of chosen psmL_LINEAR ≈ 2× pixel cost of psm	All mip levels ≈ 33 % extra texture RAM	Eliminates high-frequency flicker when the texture shrinks; tri-linear further hides discontinuities between levels
Samples-per-pixel	× sample_rate	A larger multi-sampling cache	Kills jagged edges on geometry and picks up true pixel coverage; also improves *isotropic* texture aliasing, but less efficient for heavy minification

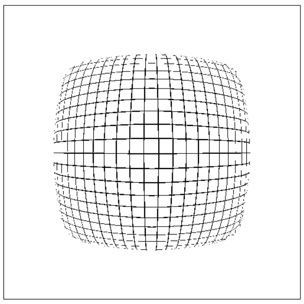
Screenshot



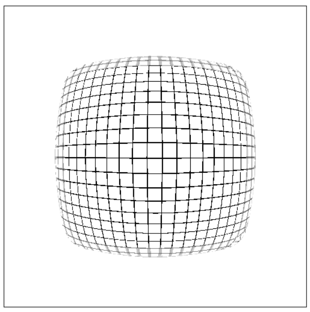
L_NEAREST and P_LINEAR



L_ZERO and P_LINEAR



L_ZERO and P_NEAREST



L_NEAREST and P_NEAREST