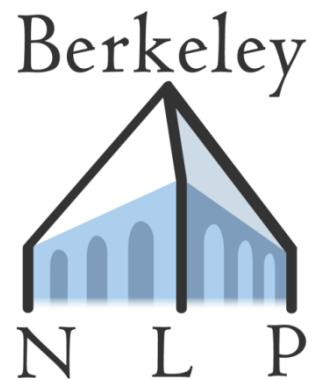


Natural Language Processing



Large Language Models

Language Modeling



Recap: What is a language model?

- Language models assign a probability to a sequence of words
- We can decompose this probability using the chain rule
- We can autoregressively generate sequences from the language model by sampling from its token-level probability
- We can condition on our language distribution on something else

$$p(\bar{y})$$

$$p(\bar{y}) = \prod_{i=1}^T p(y_i | y_{0:i-1})$$

$$p(y_i | y_{0:i-1})$$

$$p(y_i | y_{0:i-1}; \bar{x})$$



What can we do with language models?

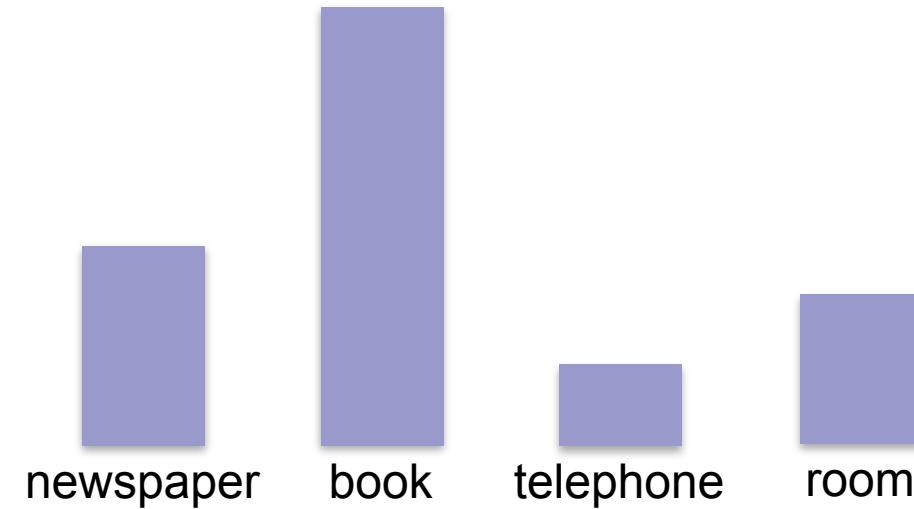
- Computing probabilities of a sequence
- Autoregressive sequence generation



Decoding strategies

- Argmax (greedy decoding)

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$





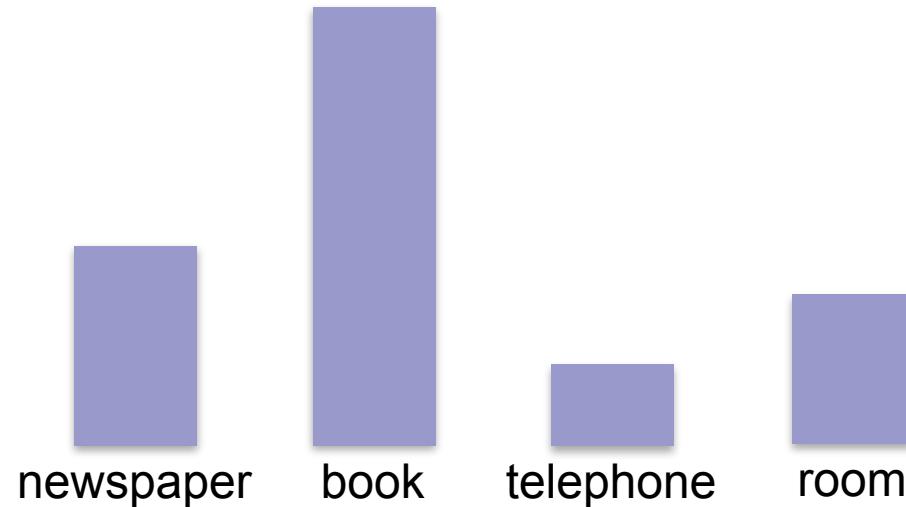
Decoding strategies

- Argmax (greedy decoding)

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

- Sampling from language model directly

$$y_T \sim p(\cdot \mid y_{0:t-1})$$





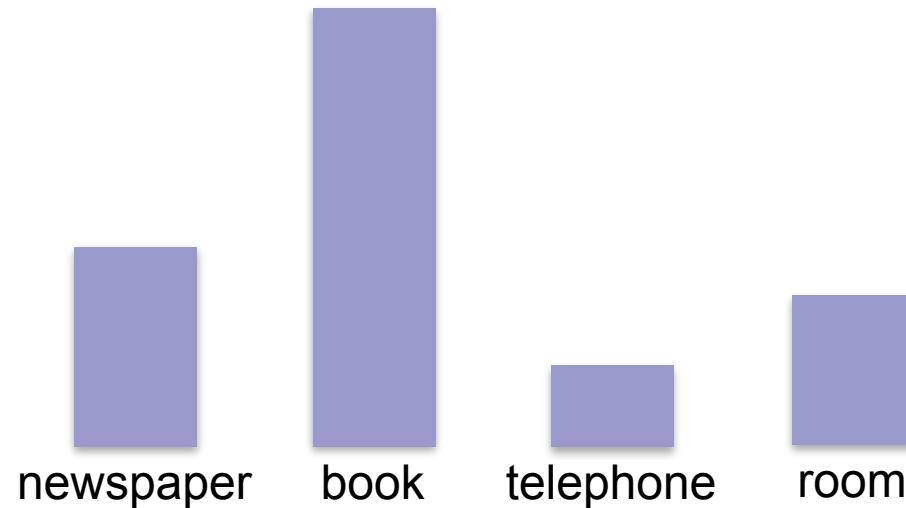
Decoding strategies

- Argmax (greedy decoding)
- Sampling from language model directly
- Adjusting temperature of distribution

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

$$y_T \sim p(\cdot \mid y_{0:t-1})$$

$$p'(y_T = y) = \frac{\exp(z_y/T)}{\sum_{y' \in \mathcal{V}} (z_{y'}/T)}$$





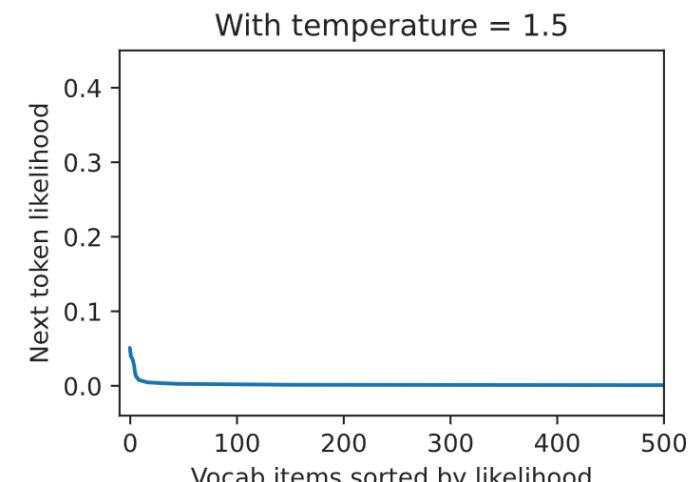
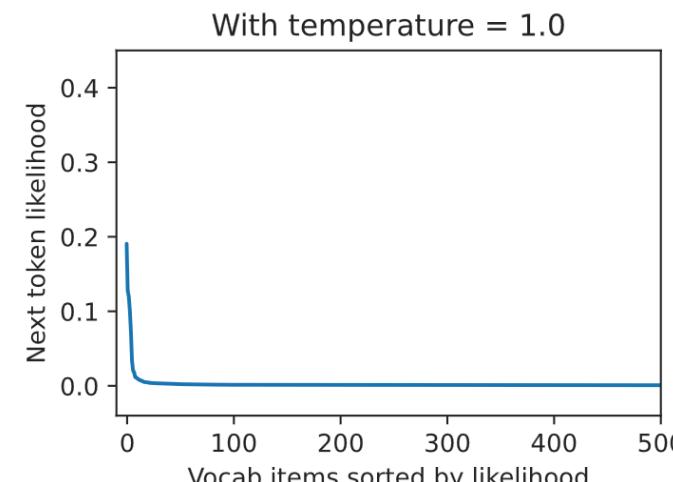
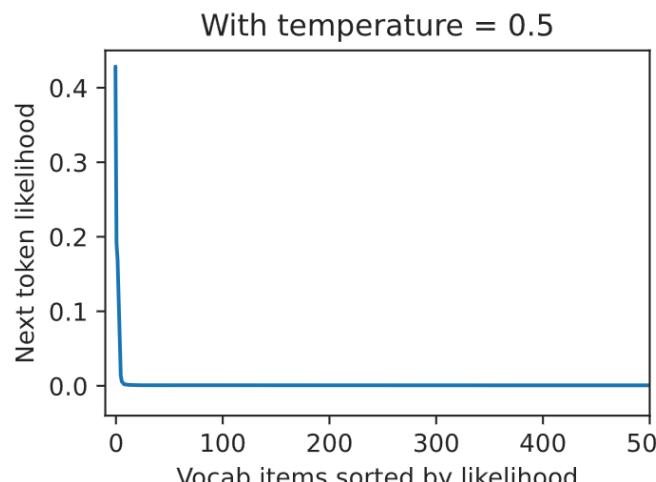
Decoding strategies

- Argmax (greedy decoding)
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$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

$$y_T \sim p(\cdot \mid y_{0:t-1})$$

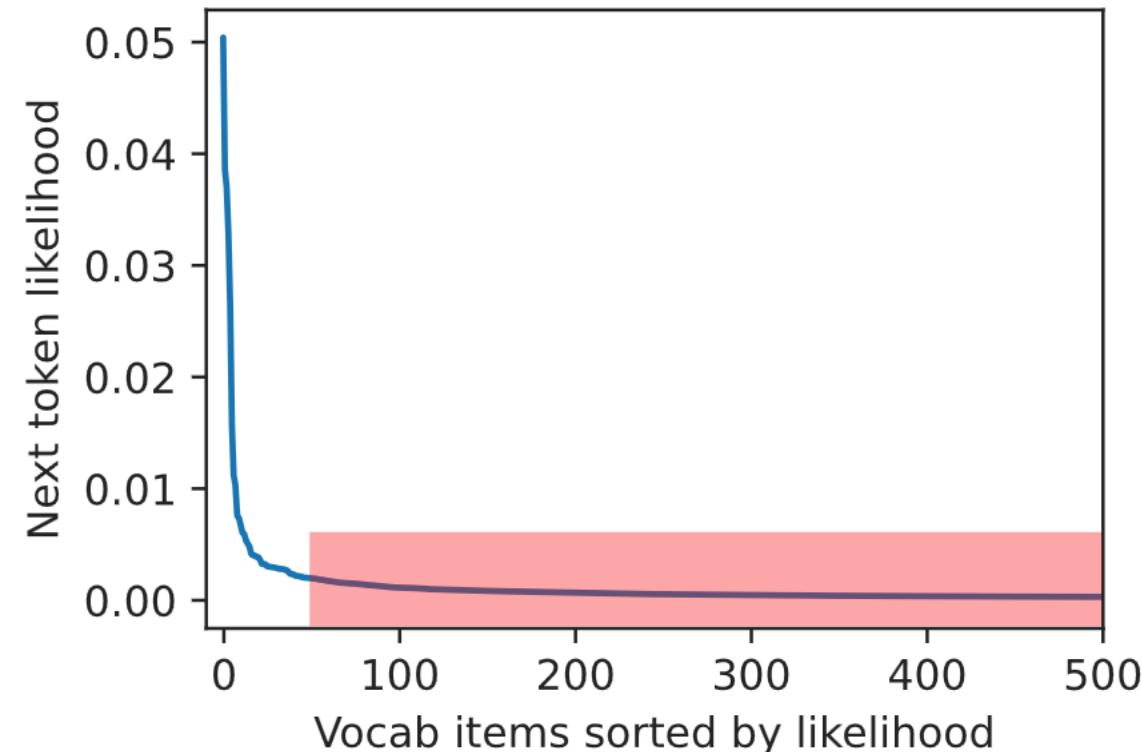
$$p'(y_T = y) = \frac{\exp(z_y/T)}{\sum_{y' \in \mathcal{V}} (\exp(z_{y'}/T))}$$





Decoding strategies

- Top- k sampling: reassign probability mass from all but the top k tokens to the top k tokens





Decoding strategies

- Nucleus sampling: reassign probability mass to the most probable tokens whose cumulative probability is at least p

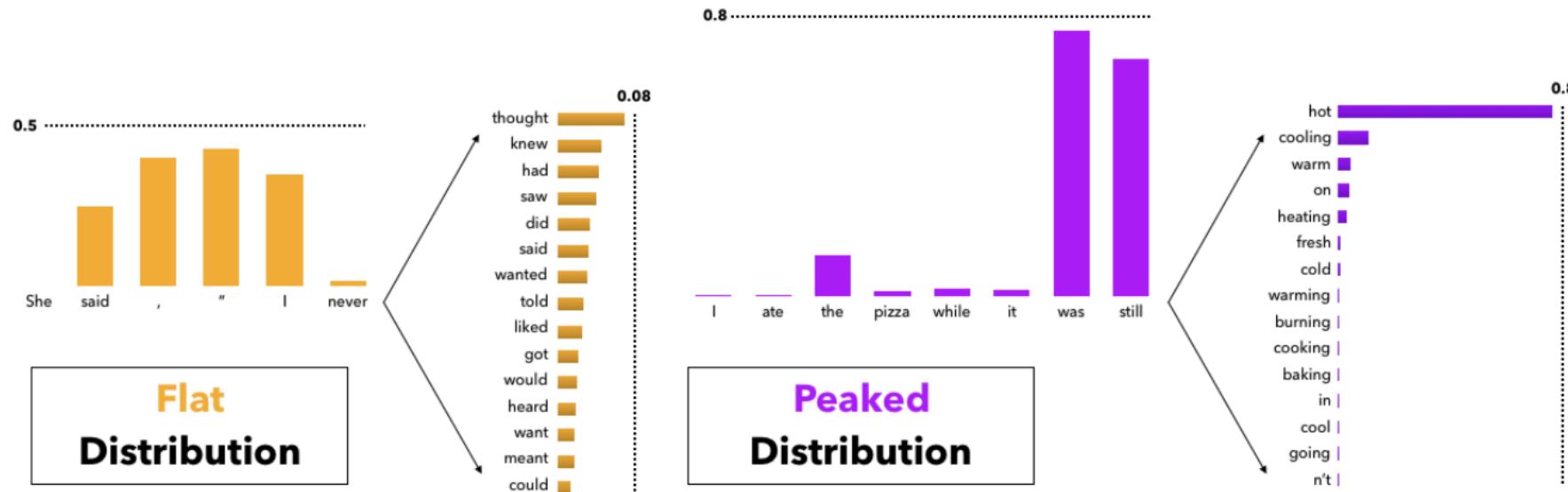


Figure 5: The probability mass assigned to partial human sentences. Flat distributions lead to many moderately probable tokens, while peaked distributions concentrate most probability mass into just a few tokens. The presence of flat distributions makes the use of a small k in top- k sampling problematic, while the presence of peaked distributions makes large k 's problematic.



Beam search

- It's intractable to find the *most probable sequence* according to a language model
- Greedy search doesn't yield the most probably sequence
- Instead: beam search
 - Approximate the search by keeping around candidate continuations
 - At the end, choose the highest probability sequence in the beam

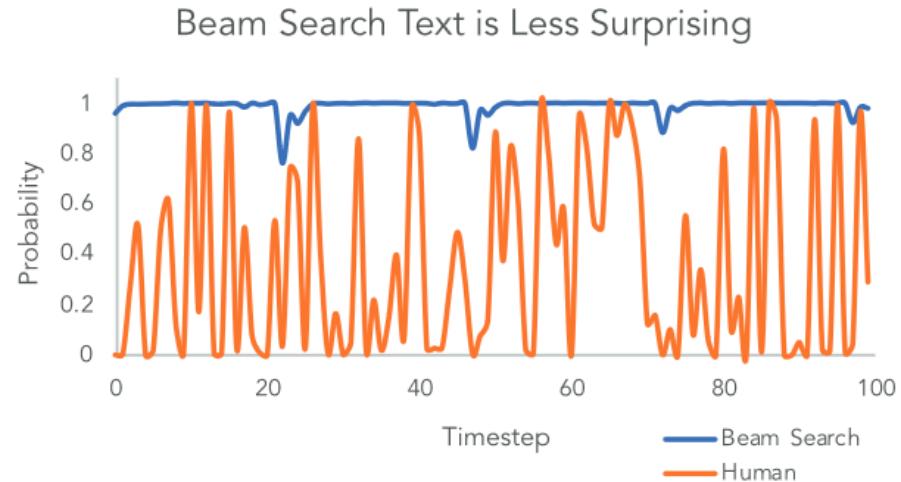
$$\bar{y}^* = \arg \max_{\bar{y} \in \mathcal{V}^*} p(\bar{y})$$

$$y_t = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$



Beam search

- But do we even want to find the highest-probability sequence according to a LM?
- Human language is noisy and surprising
- Optimizing for LM probability leads to repetitive and uninteresting text



Beam Search

...to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and...

Human

...which grant increased life span and three years warranty. The Antec HCG series consists of five models with capacities spanning from 400W to 900W. Here we should note that we have already tested the HCG-620 in a previous review and were quite satisfied With its performance. In today's review we will rigorously test the Antec HCG-520, which as its model number implies, has 520W capacity and contrary to Antec's strong beliefs in multi-rail PSUs is equipped...



Beam search

- But do we even want to find the highest-probability sequence according to a LM?
- Human language is noisy and surprising
- Optimizing for LM probability leads to repetitive and uninteresting text

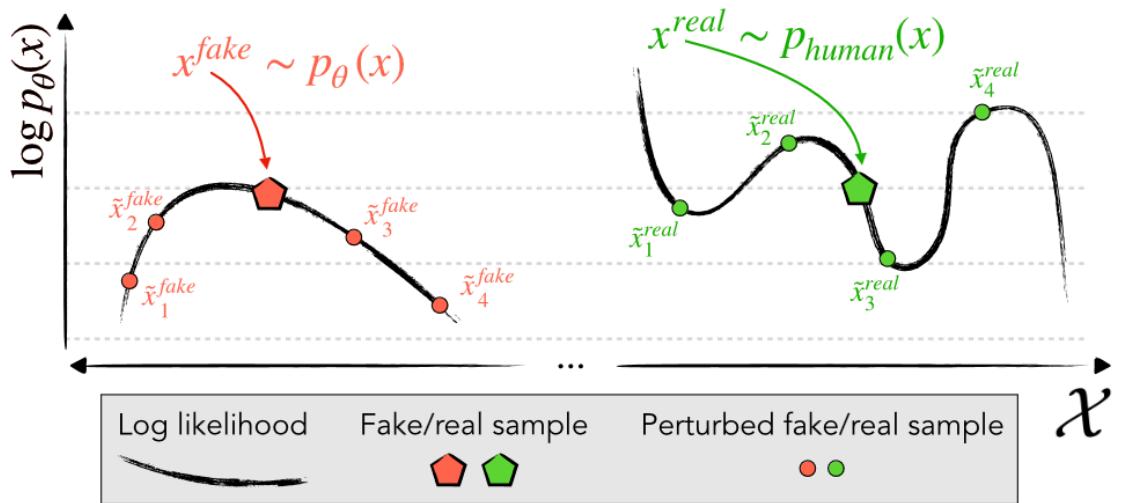


Figure 2. We identify and exploit the tendency of machine-generated passages $x \sim p_\theta(\cdot)$ (**left**) to lie in negative curvature regions of $\log p(x)$, where nearby samples have lower model log probability on average. In contrast, human-written text $x \sim p_{real}(\cdot)$ (**right**) tends not to occupy regions with clear negative log probability curvature; nearby samples may have higher or lower log probability.



Recap: Feedforward Networks

- Tokenize
- Embed
- Concatenate
- Linear layer
- Softmax
- Fixed window?
- Word averaging?

output distribution

$$\hat{y} = \text{softmax}(\mathbf{U}\mathbf{h} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

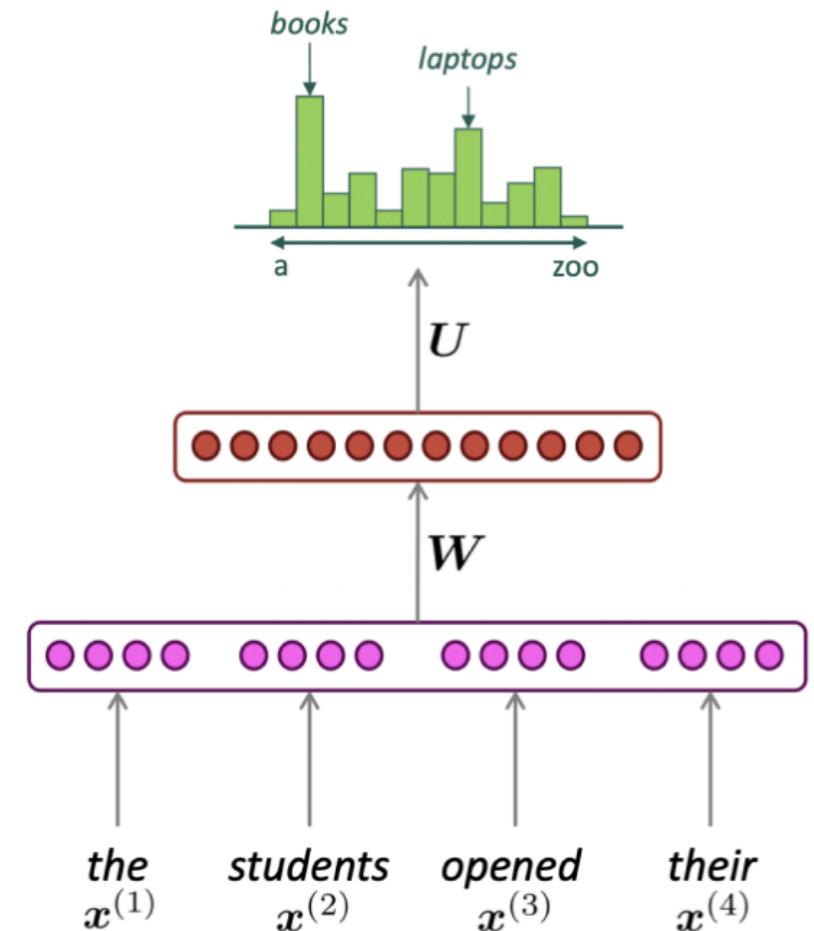
$$\mathbf{h} = f(\mathbf{W}\mathbf{e} + \mathbf{b}_1)$$

concatenated word embeddings

$$\mathbf{e} = [\mathbf{e}^{(1)}; \mathbf{e}^{(2)}; \mathbf{e}^{(3)}; \mathbf{e}^{(4)}]$$

words / one-hot vectors

$$\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}$$





Recap: Recurrence

output distribution

$$\hat{y}^{(t)} = \text{softmax}(\mathbf{U}\mathbf{h}^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden states

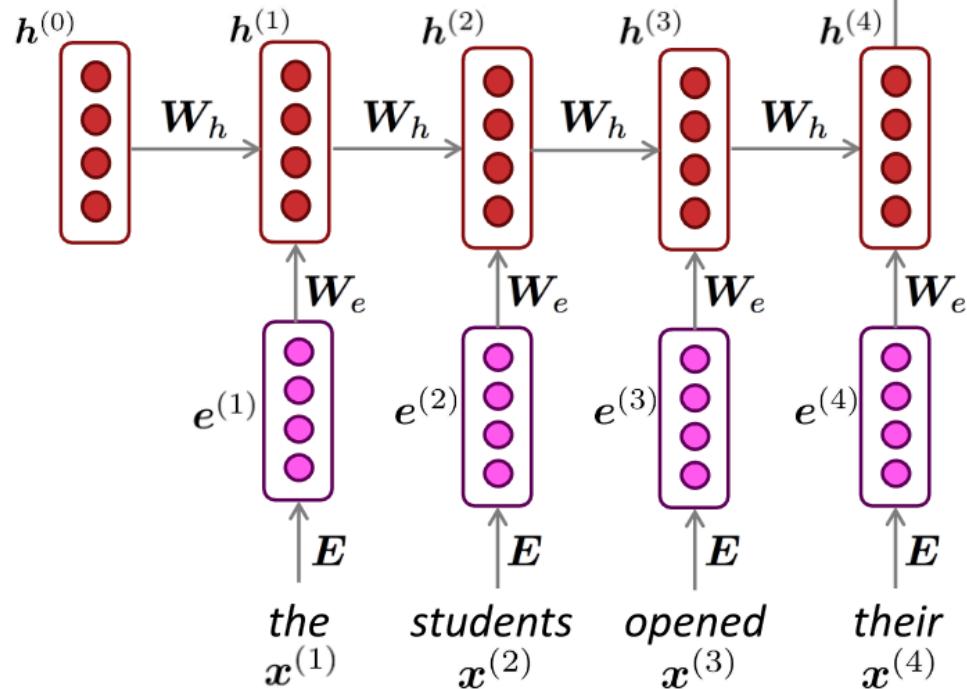
$$\mathbf{h}^{(t)} = \sigma(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_e \mathbf{e}^{(t)} + \mathbf{b}_1)$$

$\mathbf{h}^{(0)}$ is the initial hidden state

word embeddings

$$\mathbf{e}^{(t)} = \mathbf{E}\mathbf{x}^{(t)}$$

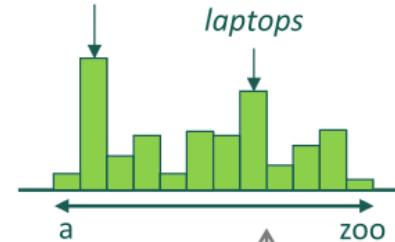
words / one-hot vectors
 $\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$



$$\hat{y}^{(4)} = P(x^{(5)} | \text{the students opened their})$$

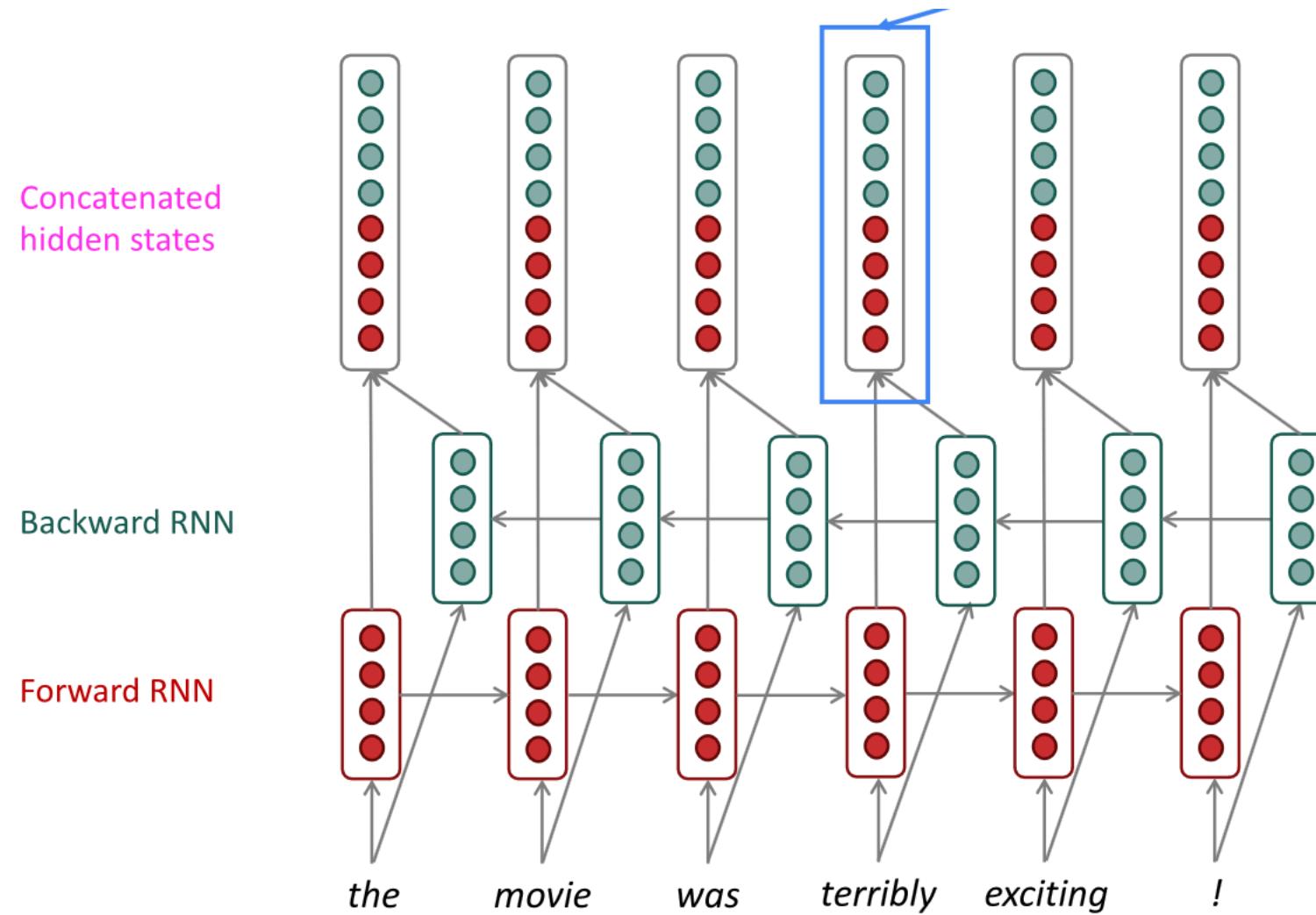
books

laptops



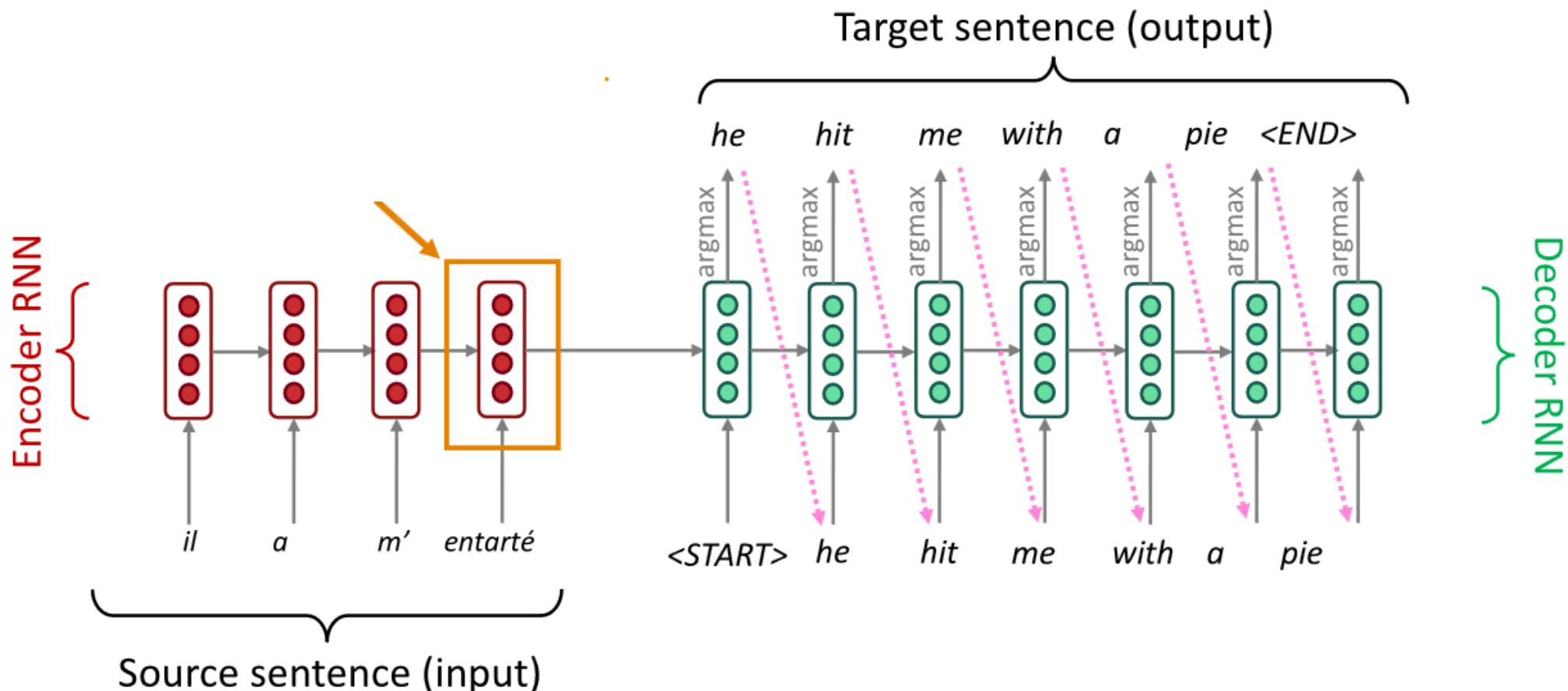


Recap: Recurrence



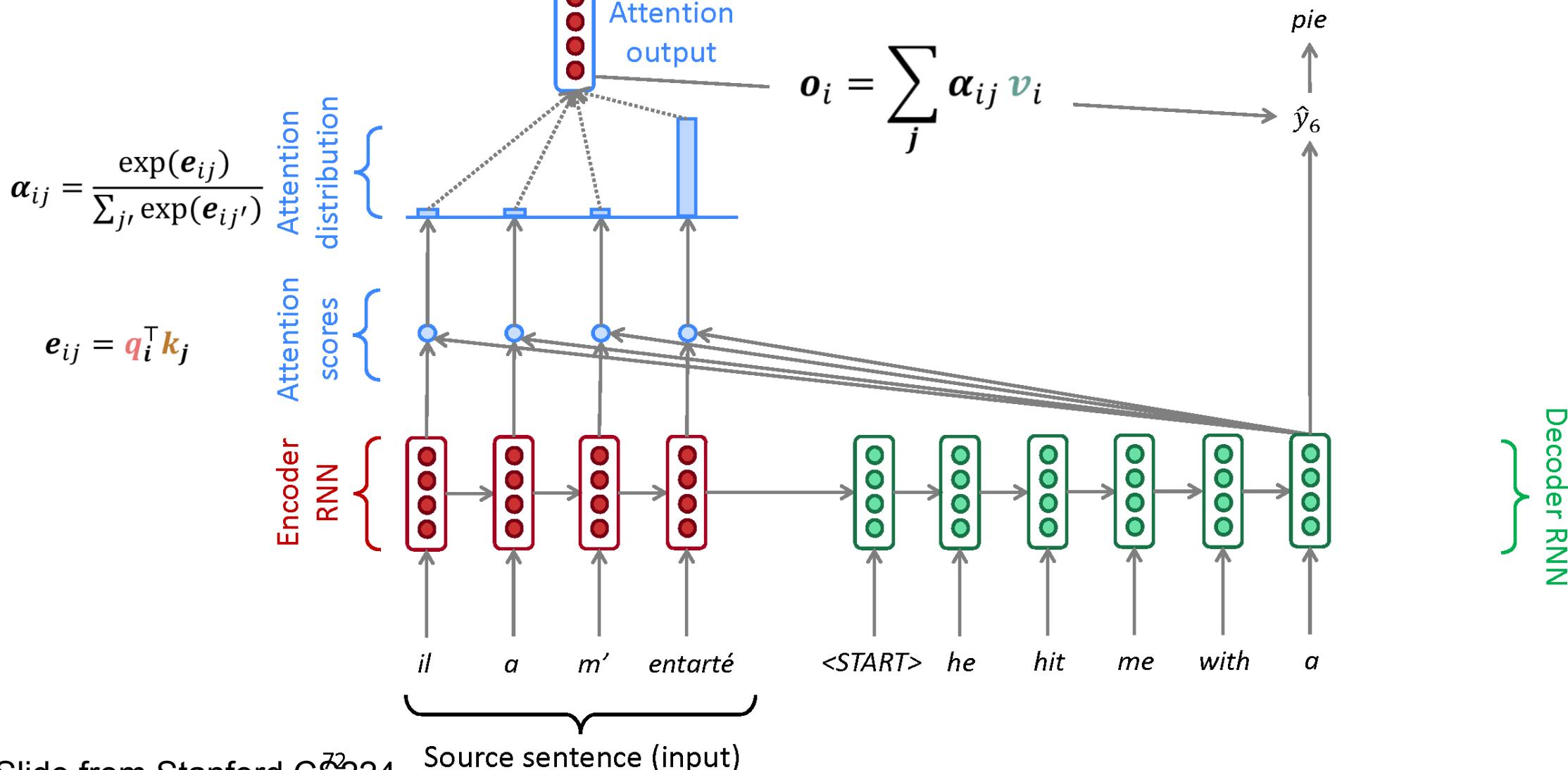


Recap: Recurrence





Recap: Attention





Recap: Attention

- Generic dot-product attention:

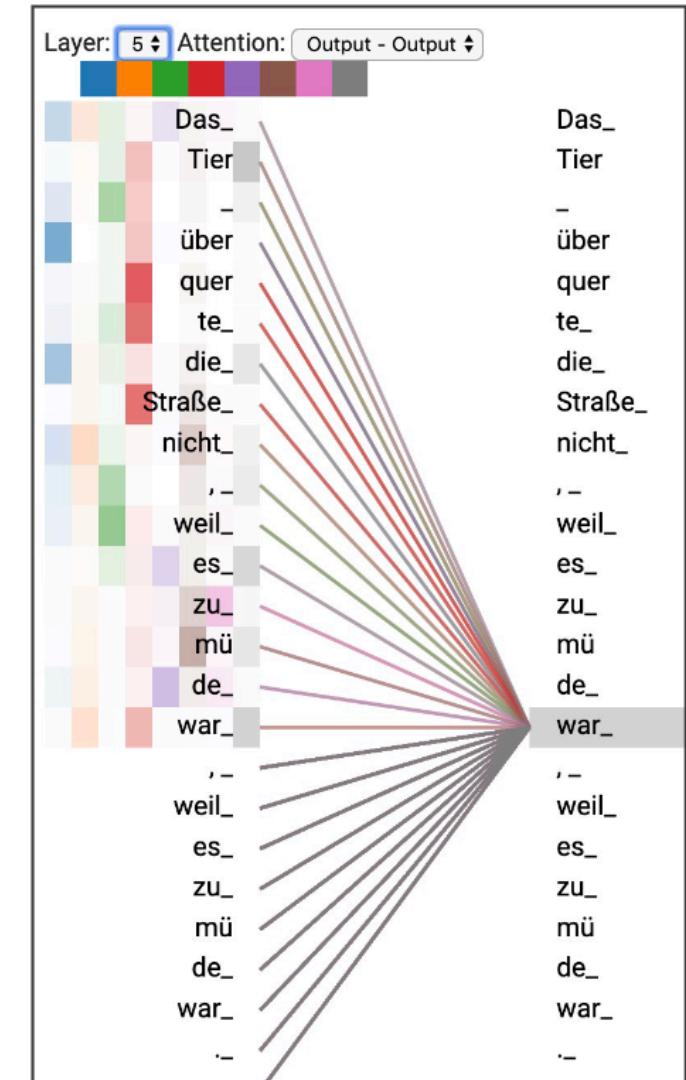
$$\mathbf{e}_{ij} = \mathbf{q}_i^\top \mathbf{k}_j \quad \alpha_{ij} = \frac{\exp(\mathbf{e}_{ij})}{\sum_j \exp(\mathbf{e}_{ij'})} \quad \mathbf{o}_i = \sum_j \alpha_{ij} \mathbf{v}_i$$

- Self-attention: queries, keys, and values are all different transformations of the same item-level representation of some sequence:

$$\mathbf{q}_i = Q\mathbf{x}_i \text{ (queries)}$$

$$\mathbf{k}_i = K\mathbf{x}_i \text{ (keys)}$$

$$\mathbf{v}_i = V\mathbf{x}_i \text{ (values)}$$



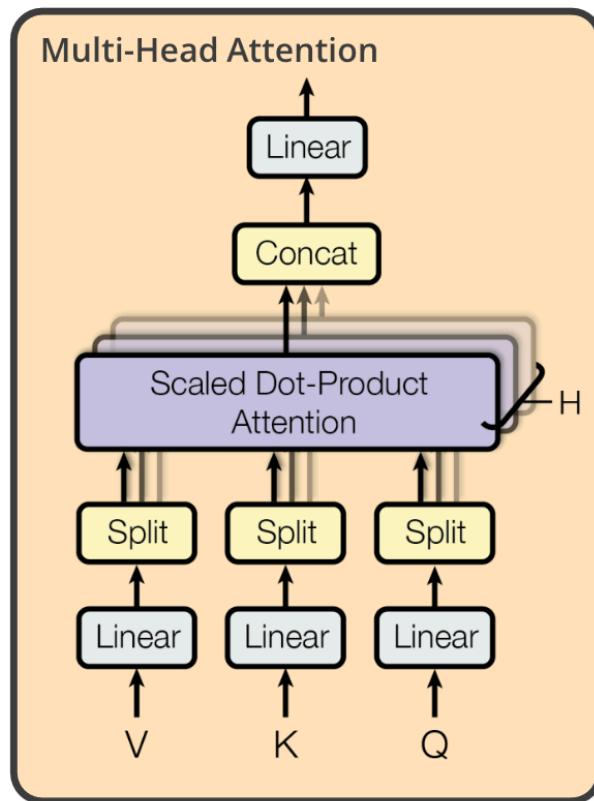


Multi-Head Attention

$$\mathbf{q}_i = Q\mathbf{x}_i \text{ (queries)}$$

$$\mathbf{k}_i = K\mathbf{x}_i \text{ (keys)}$$

$$\mathbf{v}_i = V\mathbf{x}_i \text{ (values)}$$



$$\text{head}_1 = \text{Attention}(\mathbf{QW}_1^Q, \mathbf{KW}_1^K, \mathbf{VW}_1^V)$$

:

$$\text{head}_H = \text{Attention}(\mathbf{QW}_H^Q, \mathbf{KW}_H^K, \mathbf{VW}_H^V)$$

$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$

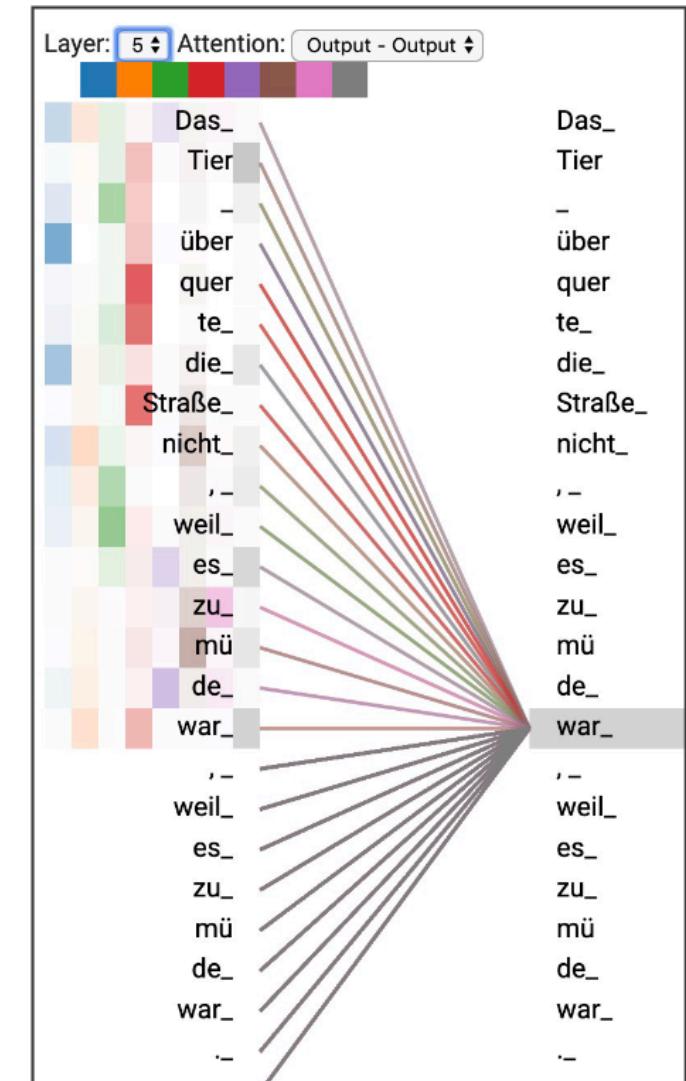
Inputs and outputs of each layer are the same dimensions:

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\mathbf{K} \in \mathbb{R}^{T \times d_{\text{model}}}$$

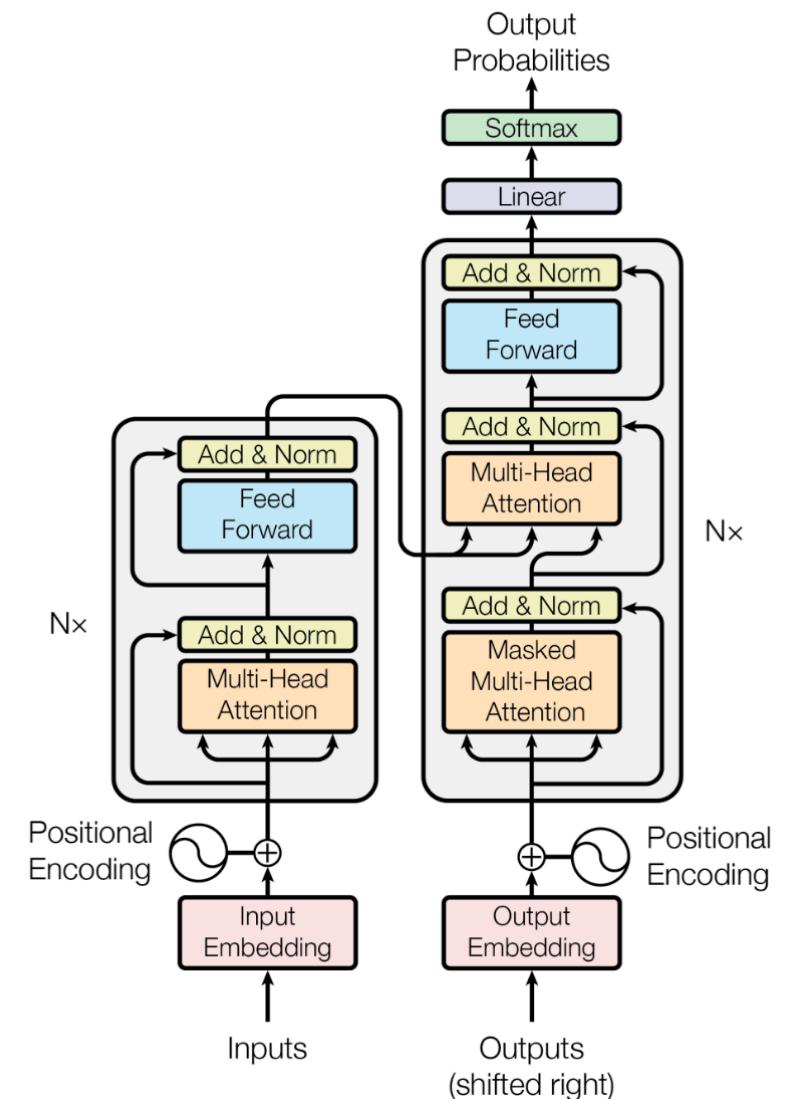
$$\mathbf{V} \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$\text{MultiHeadAtt}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) \in \mathbb{R}^{T \times d_{\text{model}}}$$

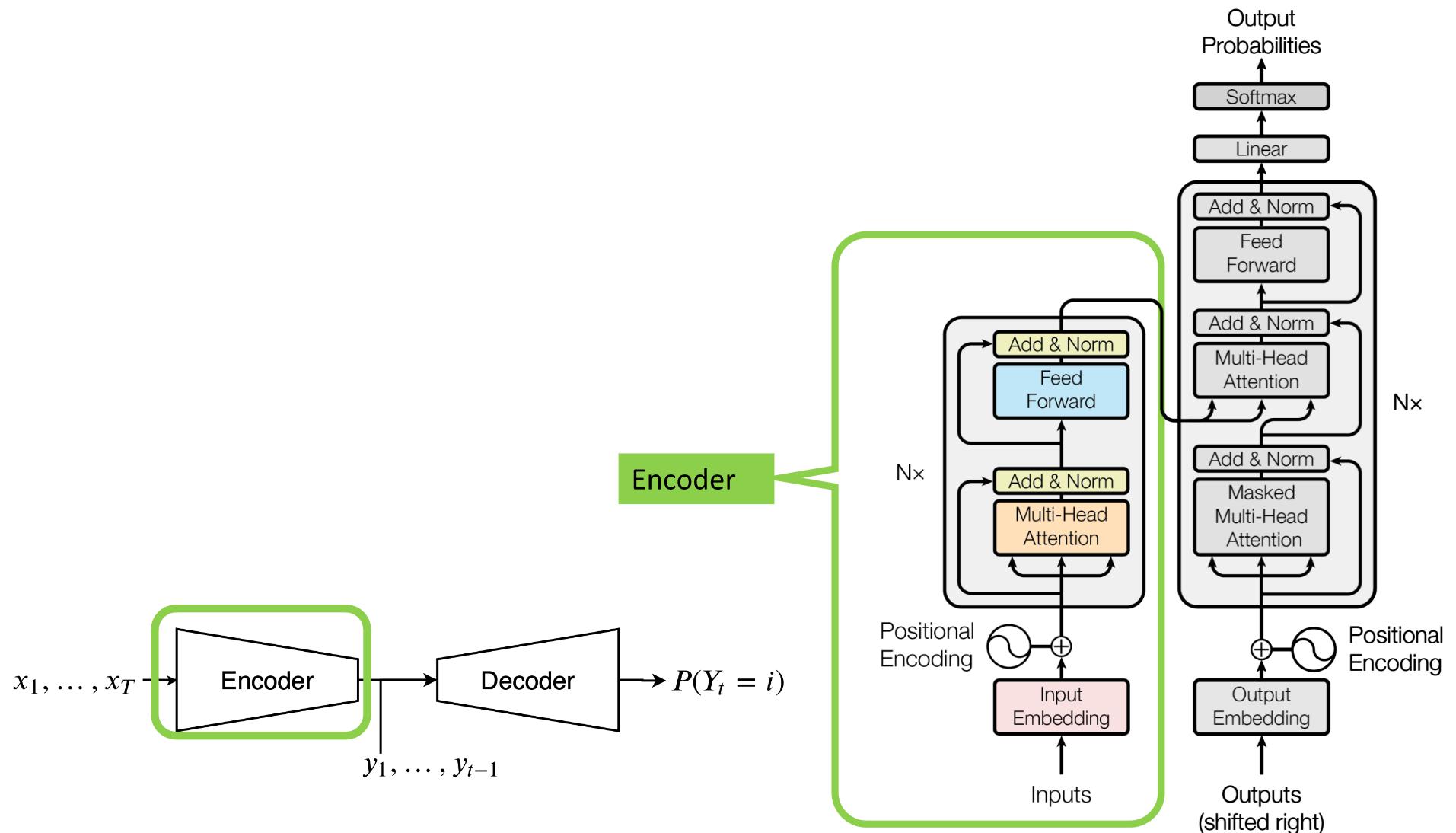




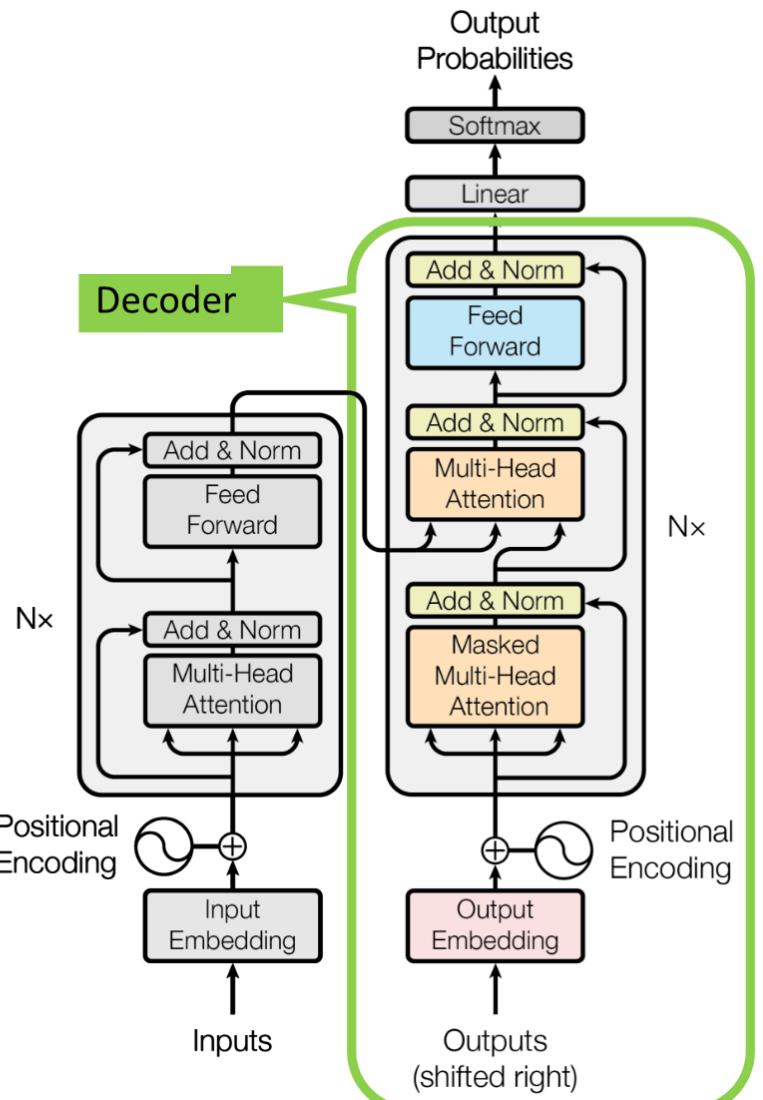
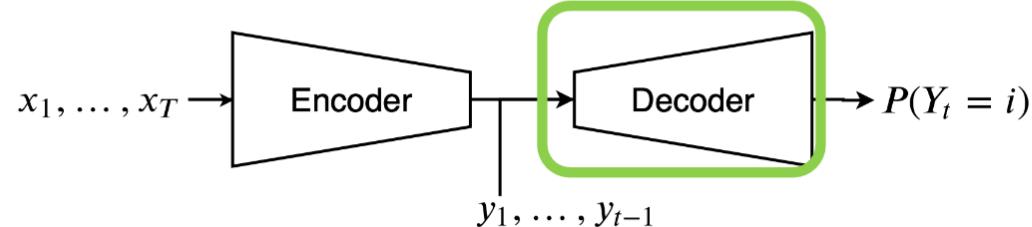
Transformer



Encoder



Decoder





Encoder Input

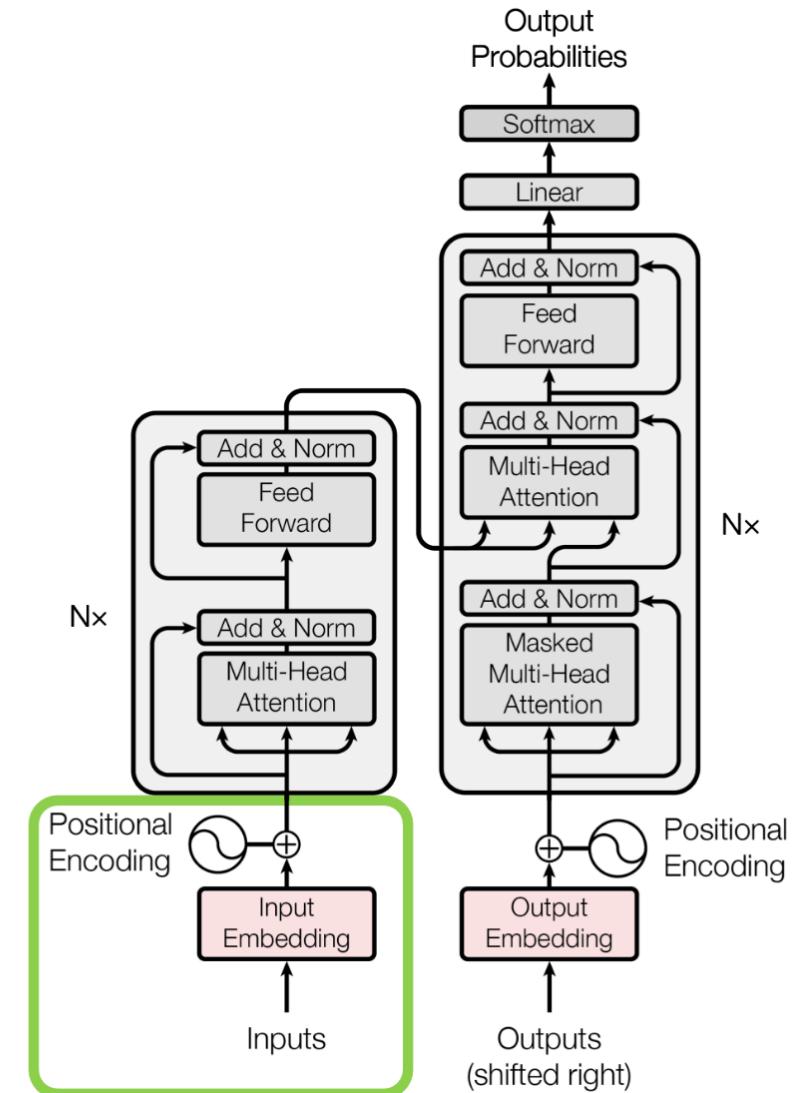
The input into the encoder looks like:

$$\mathbf{H}_0^{\text{enc}} = \begin{matrix} \text{Token Embeddings} \\ \mathbf{H}_0^{\text{enc}} = \left[\begin{array}{c|c} \hline \text{padding} & \text{embedding size} \\ \hline \text{maximum sequence length} & \\ \hline \end{array} \right] \end{matrix} + \text{Position Embeddings}$$

$\mathbf{p}_i = \begin{cases} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{cases}$

Dimension

Index in the sequence





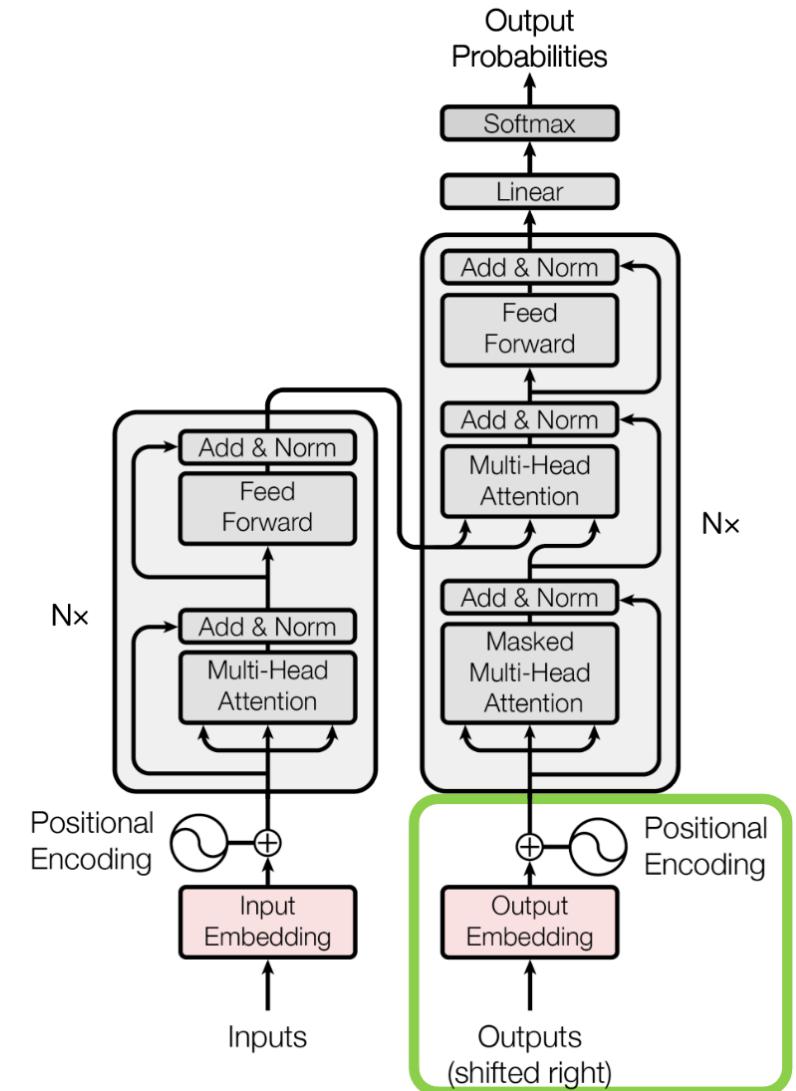
Decoder Input

The input into the encoder looks like:

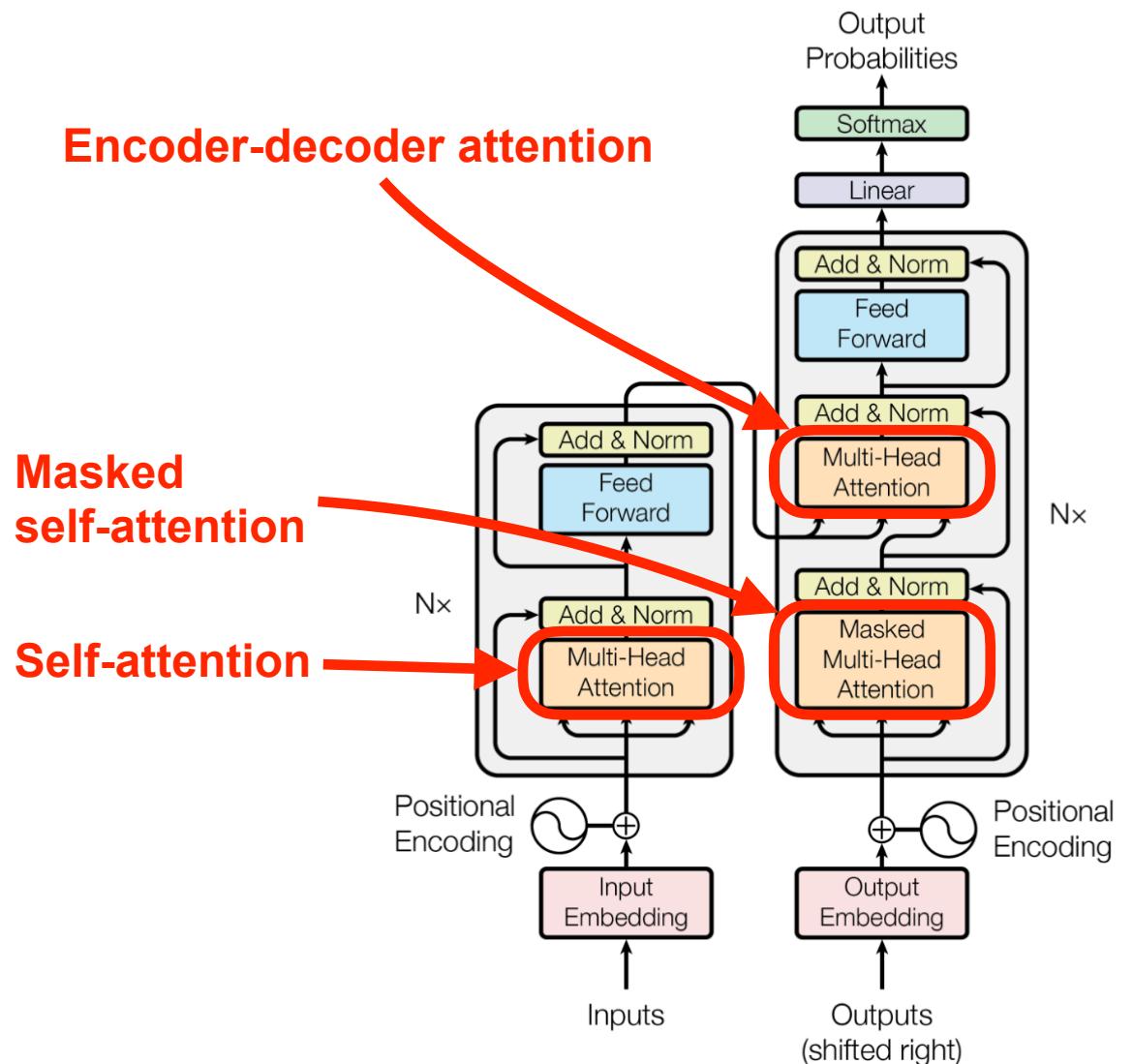
$$H_0^{\text{enc}} = \begin{matrix} \text{Token Embeddings} \\ \left[\begin{array}{c|c} \text{padding} & \end{array} \right] \end{matrix} + \begin{matrix} \text{Position Embeddings} \\ \left[\begin{array}{c} \text{embedding size} \\ \hline \text{maximum sequence length} \end{array} \right] \end{matrix}$$

The input to the decoder looks like:

$$H_0^{\text{dec}} = \begin{matrix} \text{Shifted Token Embeddings} \\ \left[\begin{array}{c|c} \text{padding} & \end{array} \right] \end{matrix} + \begin{matrix} \text{Position Embeddings} \\ \left[\begin{array}{c} \text{embedding size} \\ \hline \text{maximum sequence length} \end{array} \right] \end{matrix}$$



Attention

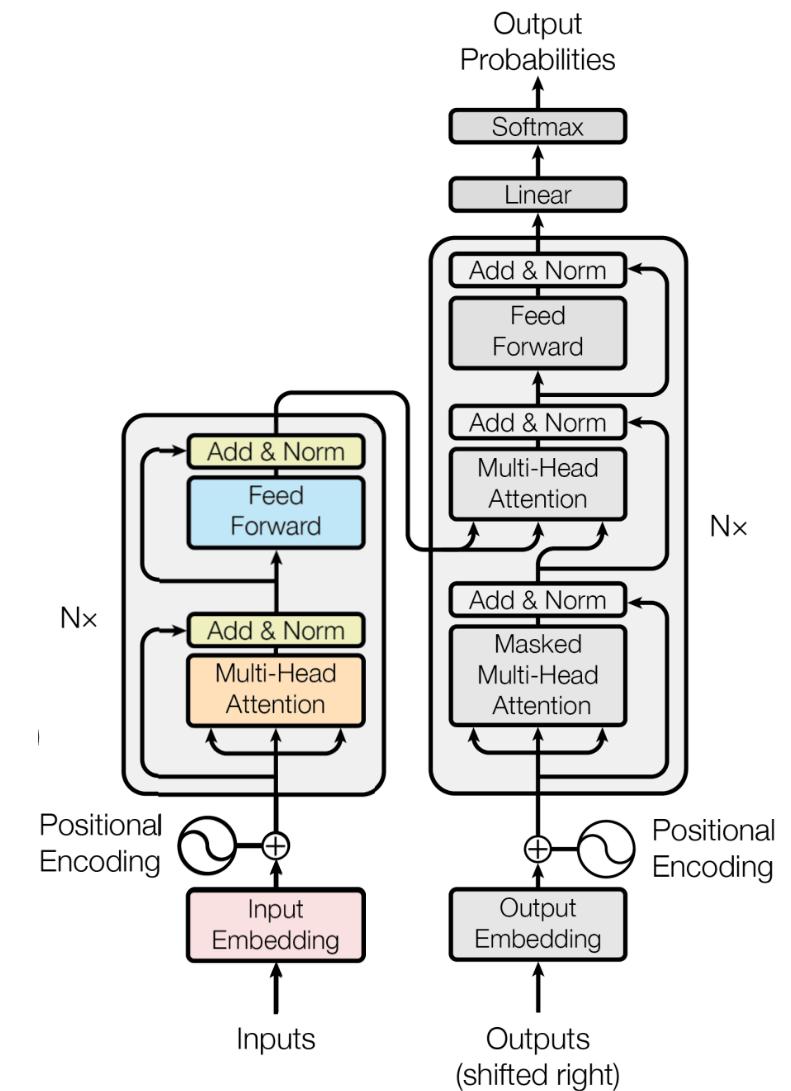




Encoder

Multi-Head
Attention

$$= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$$

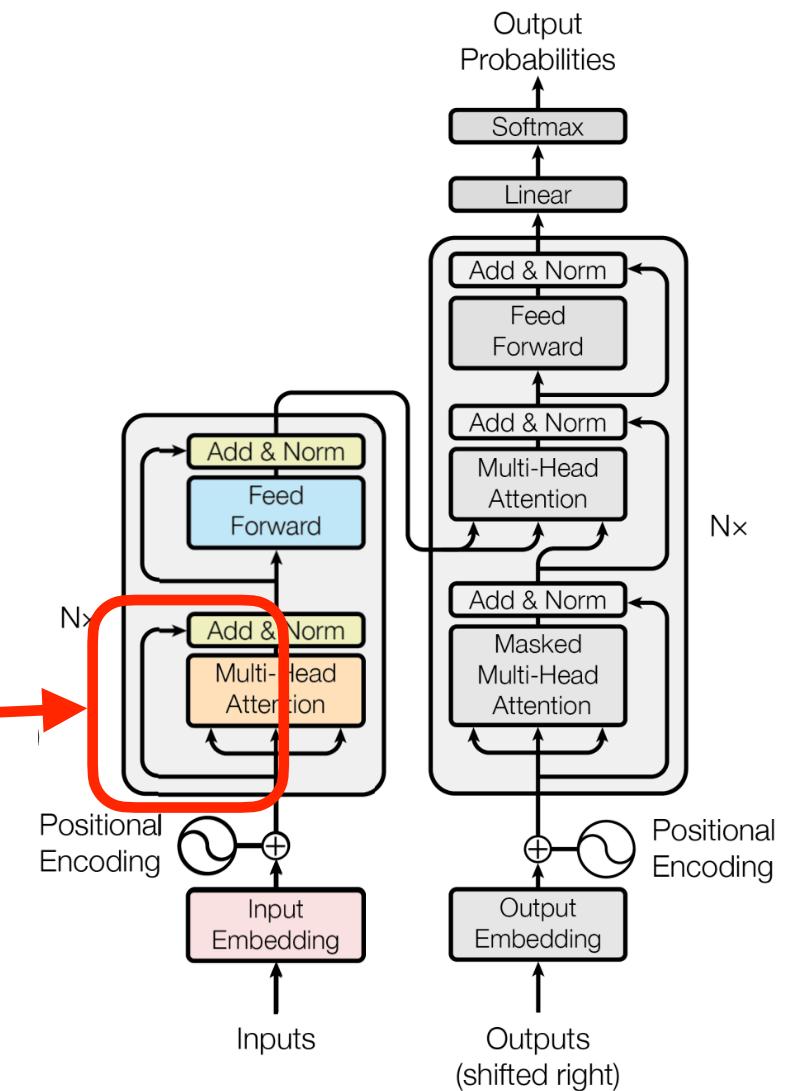


Encoder

$$\text{Multi-Head Attention} = \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$$

$$\text{Add & Norm} = \text{LayerNorm}(\text{Multi-Head Attention} + \mathbf{H}_i^{enc})$$

Residual connection



Encoder



Multi-Head Attention

$$= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$$

Add & Norm

$$= \text{LayerNorm}(\text{Multi-Head Attention} + \mathbf{H}_i^{enc})$$

Let $x \in \mathbb{R}^d$ be an individual (word) vector in the model.

Let $\mu = \sum_{j=1}^d x_j$; this is the mean; $\mu \in \mathbb{R}$.

Let $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$; this is the standard deviation; $\sigma \in \mathbb{R}$.

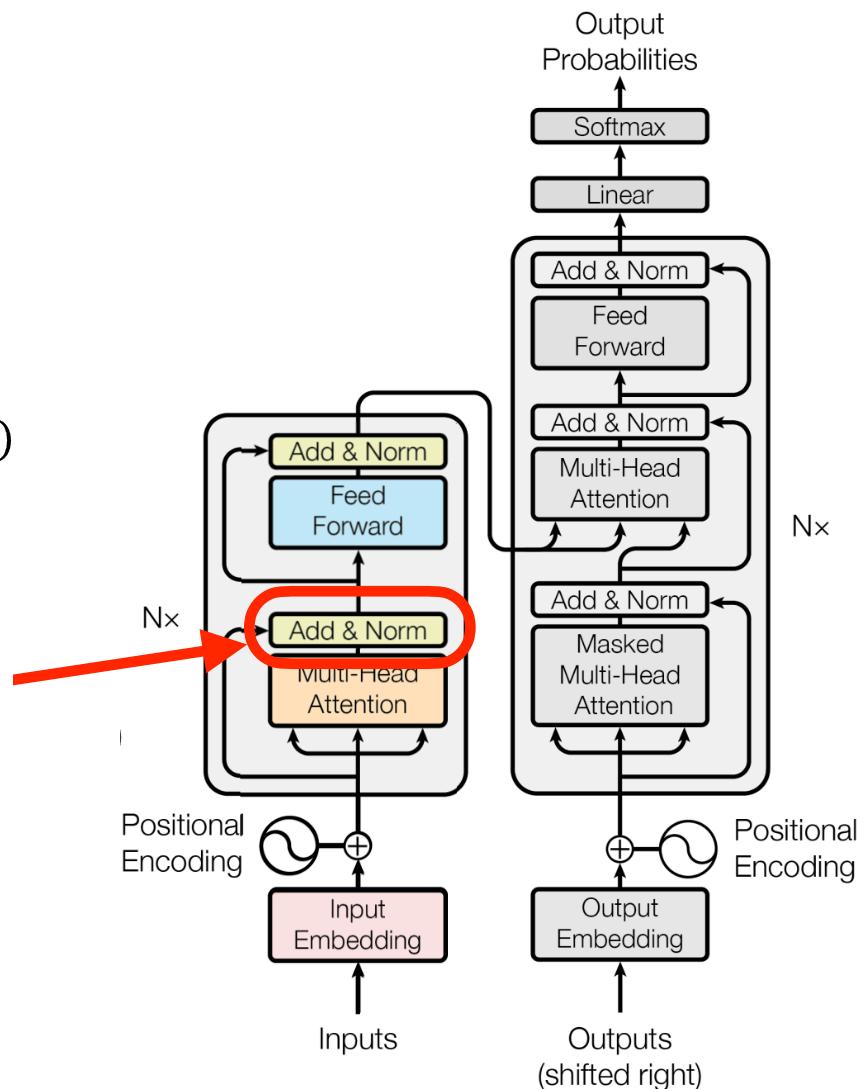
Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned “gain” and “bias” parameters. (Can omit!)

Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

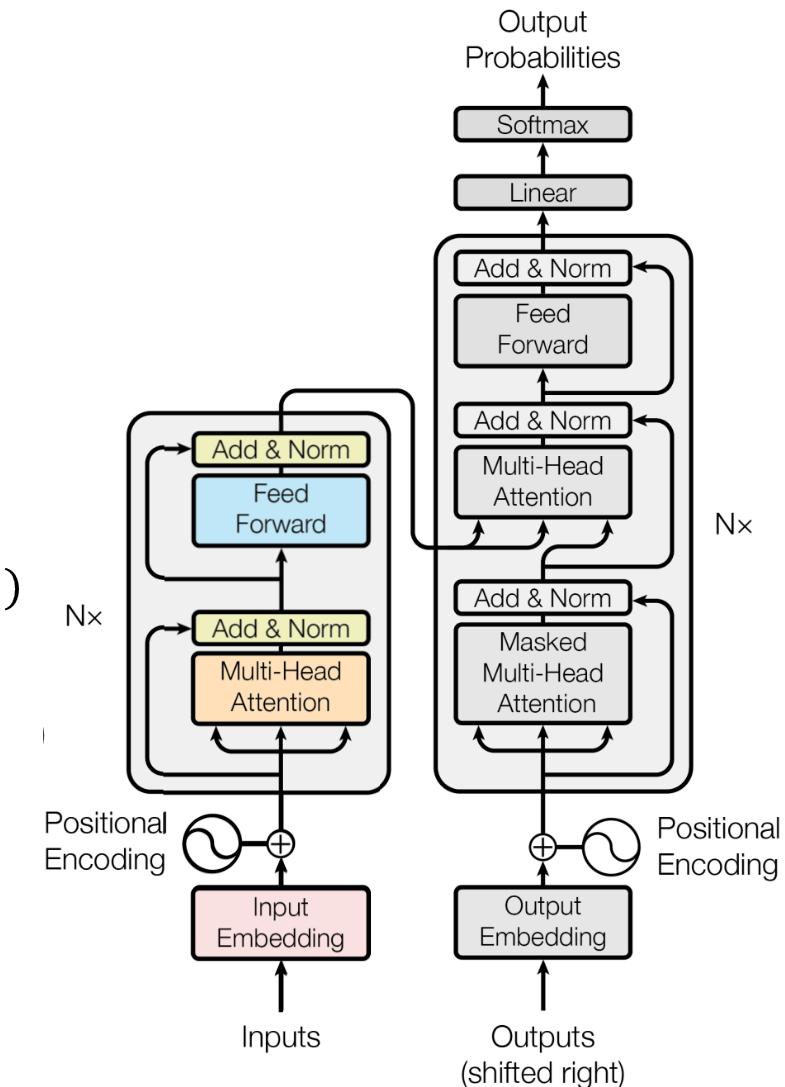
Modulate by learned elementwise gain and bias





Encoder

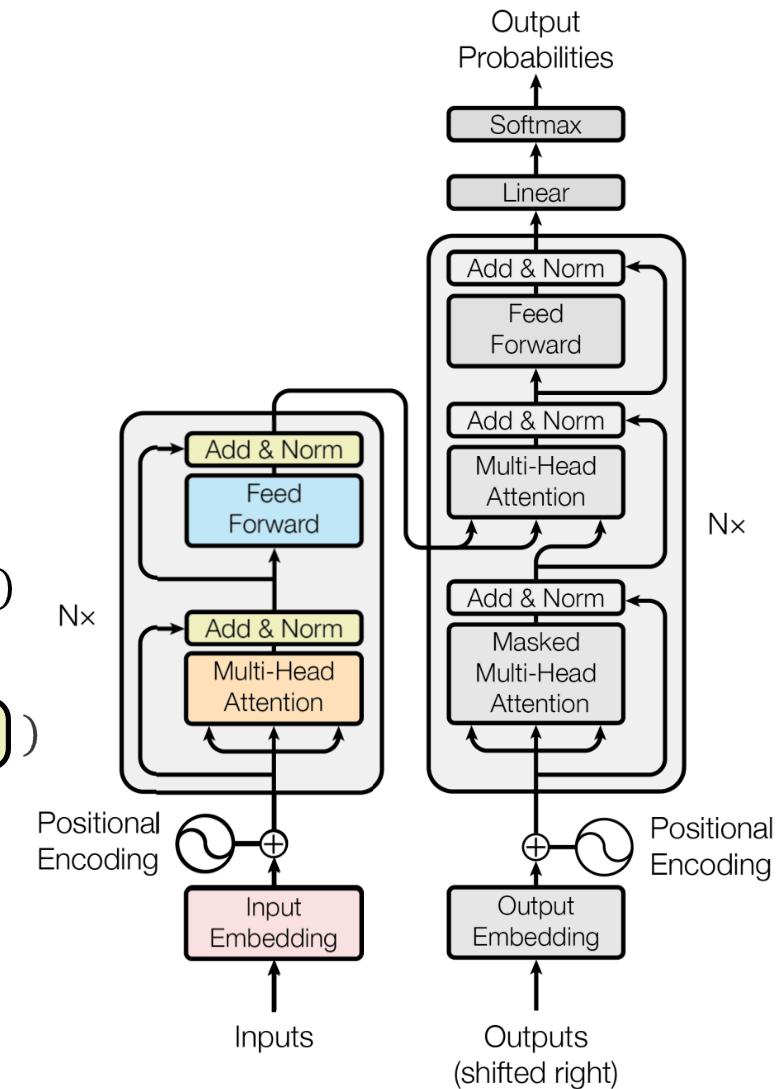
$$\begin{aligned}\text{Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}) \\ \text{Add & Norm} &= \text{LayerNorm}(\text{Multi-Head Attention} + \mathbf{H}_i^{enc}) \\ \text{Feed Forward} &= \max(0, \text{Add & Norm } \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2)\end{aligned}$$





Encoder

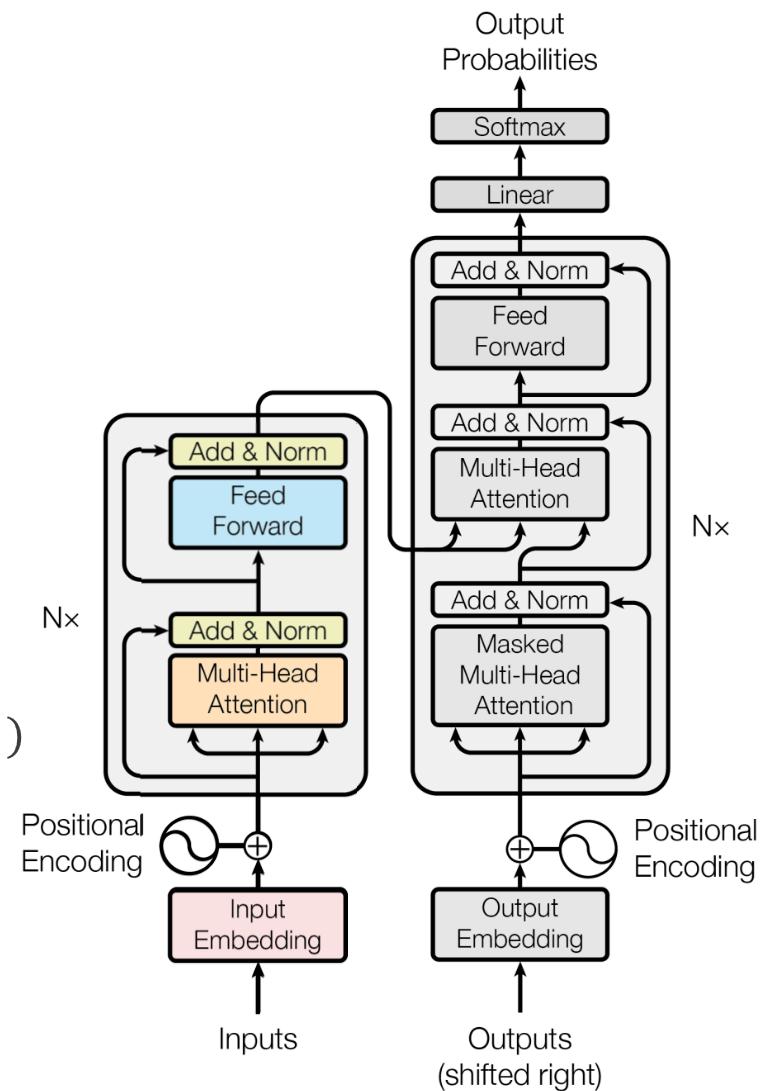
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Encoder



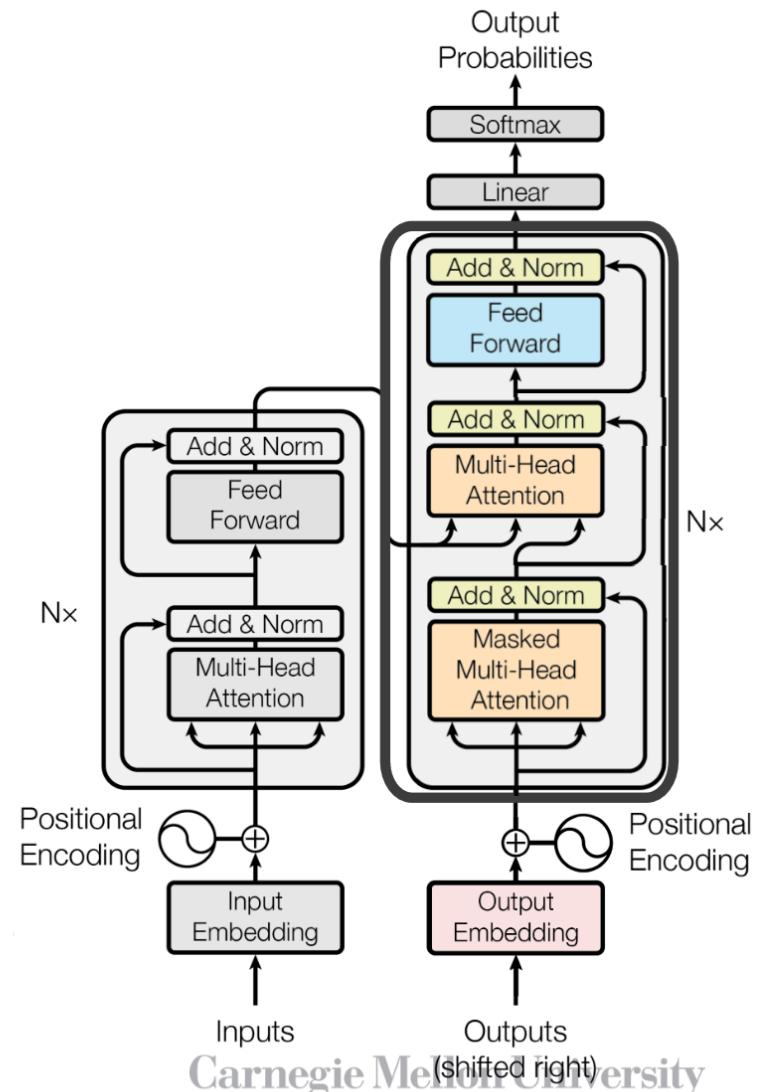
$$\begin{aligned}
 \text{Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}) \\
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 \text{Feed Forward} &= \max(0, \text{Add & Norm } (\mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2) \\
 \text{Add & Norm (2)} &= \text{LayerNorm}(\text{Feed Forward} + \text{Add & Norm }) \\
 \mathbf{H}_{i+1}^{enc} &= \text{Add & Norm}(2)
 \end{aligned}$$





Decoder

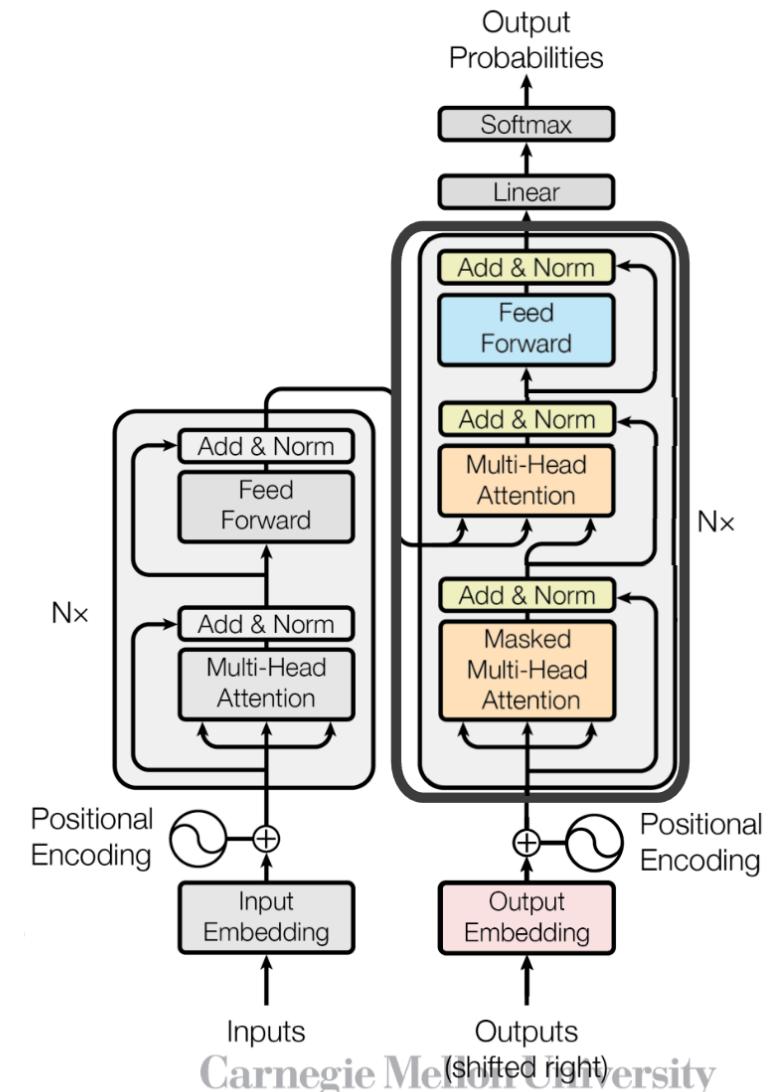
$$\text{Masked Multi-Head Attention} = \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec})$$





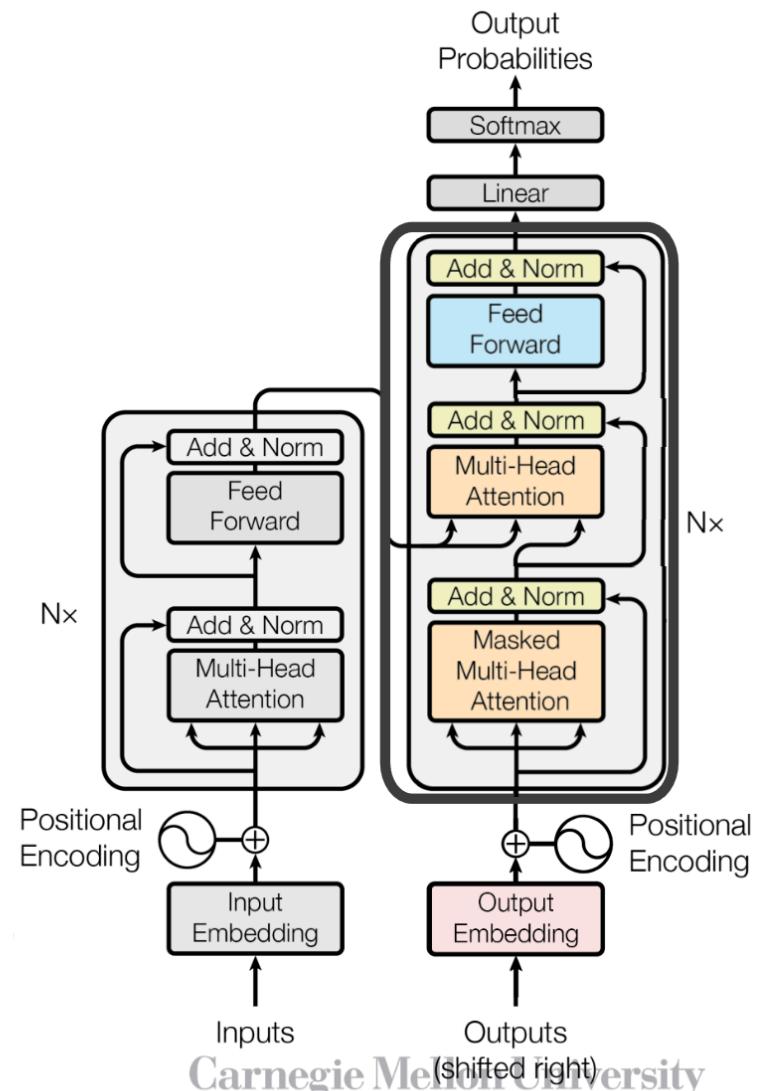
Decoder

$$\begin{aligned} \text{Masked Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}) \\ \text{Add \& Norm} &= \text{LayerNorm}(\text{Masked Multi-Head Attention} + \mathbf{H}_i^{dec}) \end{aligned}$$



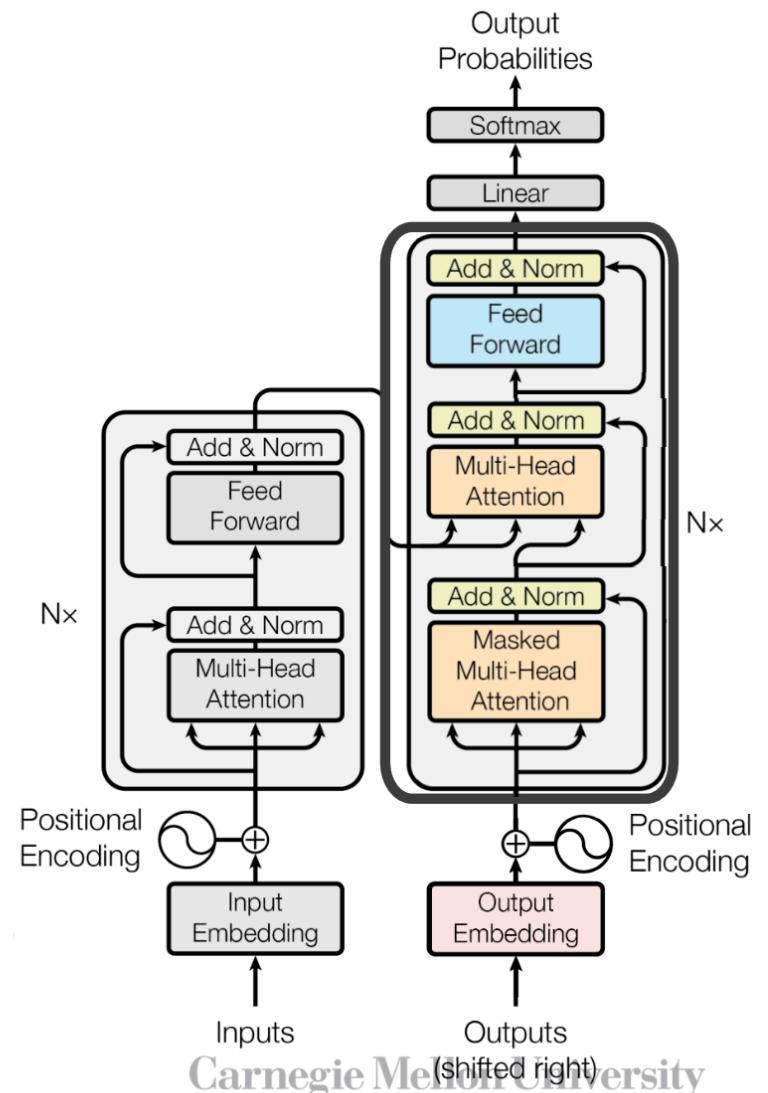
Decoder

$$\begin{aligned}
 \text{Masked Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}) \\
 \text{Add & Norm} &= \text{LayerNorm}(\text{Masked Multi-Head Attention} + \mathbf{H}_i^{dec}) \\
 \text{Enc-Dec Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \text{Add & Norm})
 \end{aligned}$$



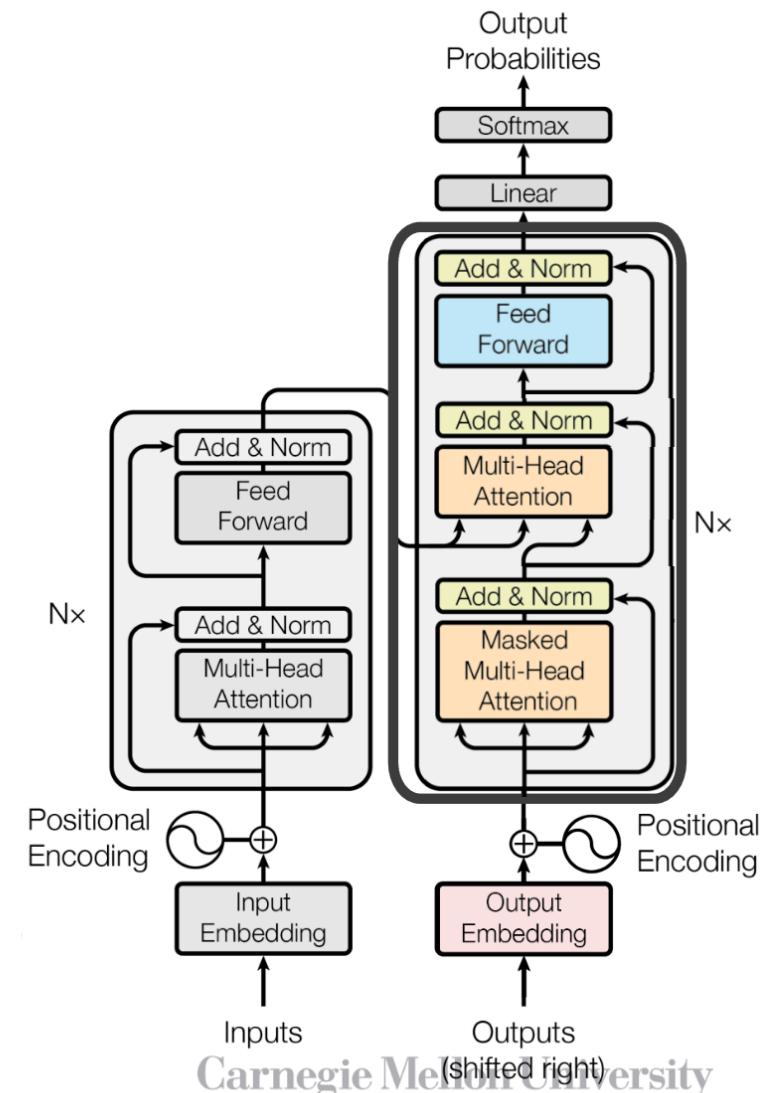
Decoder

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 \text{Enc-Dec Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \text{Add & Norm}) \\
 \text{Add & Norm (2)} &= \text{LayerNorm}(\text{Enc-Dec Multi-Head Attention} + \text{Add & Norm})
 \end{aligned}$$



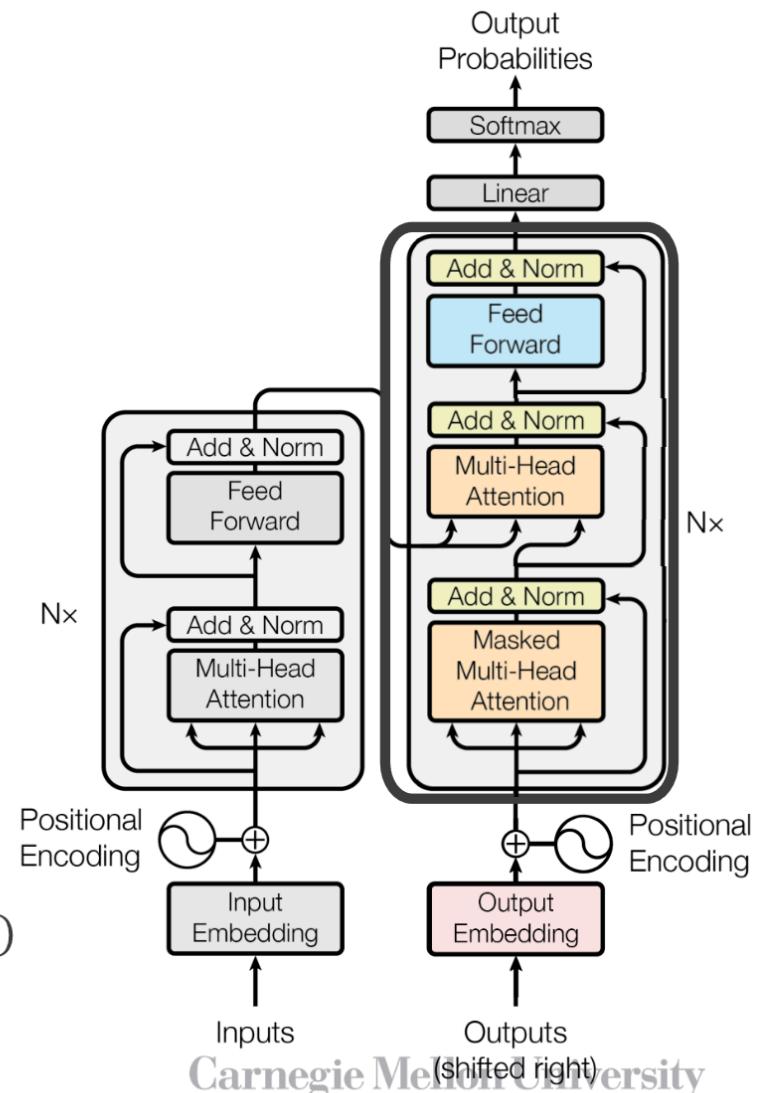
Decoder

$$\begin{aligned}
 \text{Masked Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}) \\
 \text{Add & Norm} &= \text{LayerNorm}(\text{Masked Multi-Head Attention} + \mathbf{H}_i^{dec}) \\
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 \text{Add & Norm (2)} &= \text{LayerNorm}(\text{Enc-Dec Multi-Head Attention} + \text{Add & Norm}) \\
 \text{Feed Forward} &= \max(0, \text{Add & Norm (2)} \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2)
 \end{aligned}$$



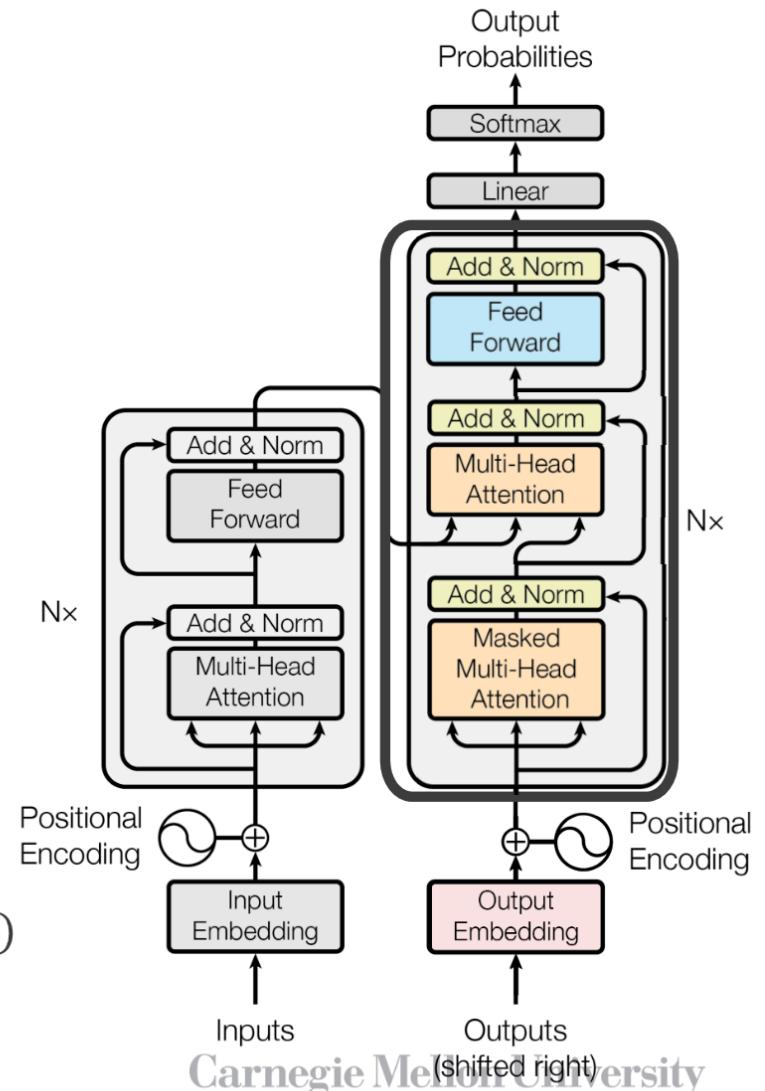
Decoder

$$\begin{aligned}
 \text{Masked Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}) \\
 \text{Add & Norm} &= \text{LayerNorm}(\text{Masked Multi-Head Attention} + \mathbf{H}_i^{dec}) \\
 \text{Enc-Dec Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \text{Add & Norm}) \\
 \text{Add & Norm (2)} &= \text{LayerNorm}(\text{Enc-Dec Multi-Head Attention} + \text{Add & Norm}) \\
 \text{Feed Forward} &= \max(0, \text{Add & Norm (2)} \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2 \\
 \text{Add & Norm (3)} &= \text{LayerNorm}(\text{Feed Forward} + \text{Add & Norm (2)})
 \end{aligned}$$



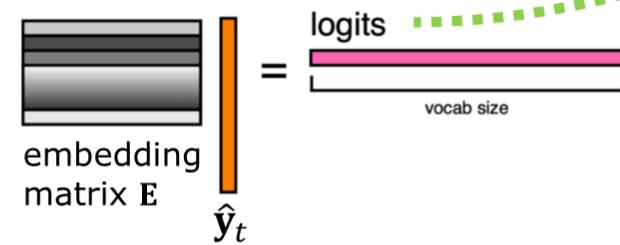
Decoder

$$\begin{aligned}
 \text{Masked Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}) \\
 \text{Add & Norm} &= \text{LayerNorm}(\text{Masked Multi-Head Attention} + \mathbf{H}_i^{dec}) \\
 \text{Enc-Dec Multi-Head Attention} &= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \text{Add & Norm}) \\
 \text{Add & Norm (2)} &= \text{LayerNorm}(\text{Enc-Dec Multi-Head Attention} + \text{Add & Norm}) \\
 \text{Feed Forward} &= \max(0, \text{Add & Norm (2)} \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2 \\
 \text{Add & Norm (3)} &= \text{LayerNorm}(\text{Feed Forward} + \text{Add & Norm (2)}) \\
 \mathbf{H}_{i+1}^{dec} &= \text{Add & Norm (3)}
 \end{aligned}$$

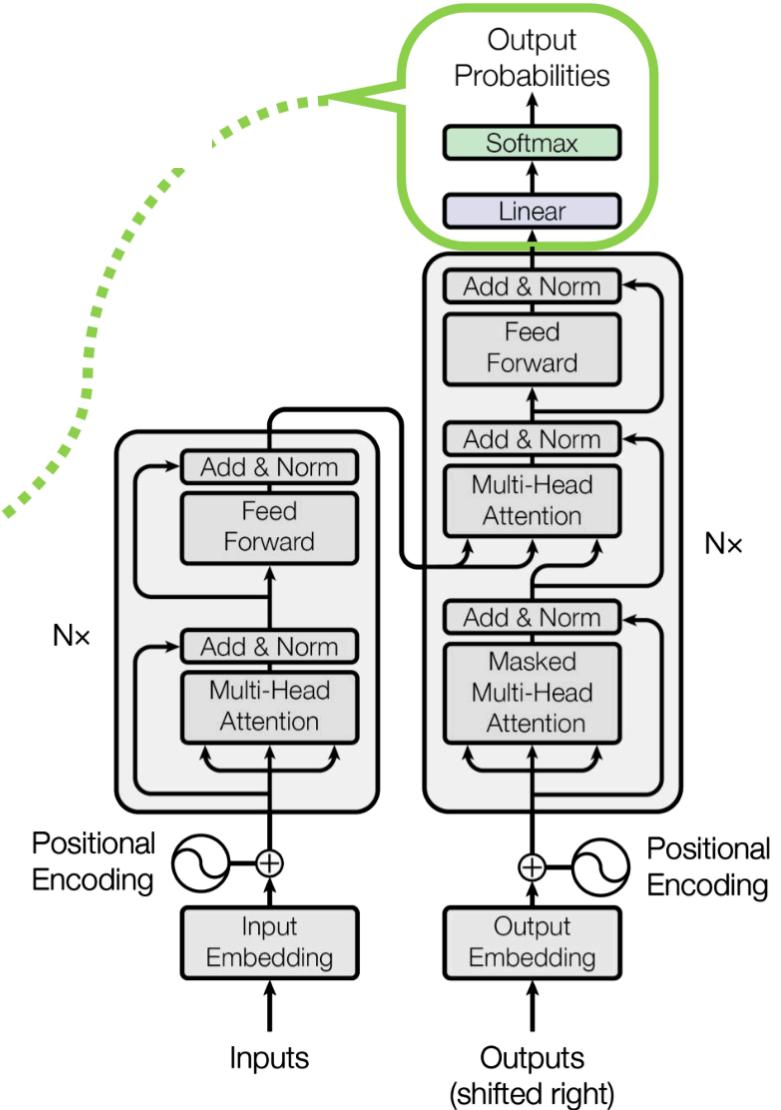




Output Probabilities



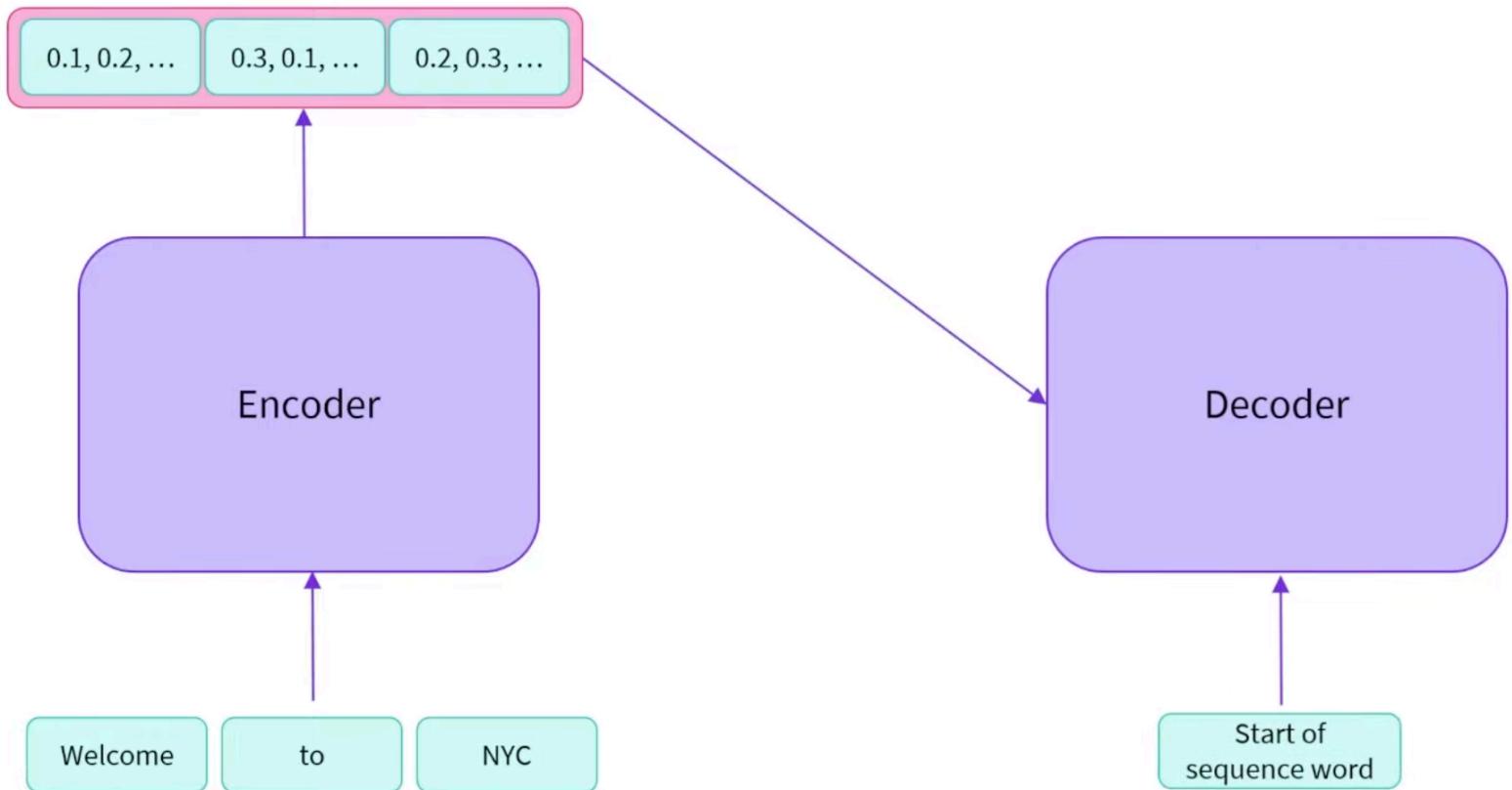
$$P(Y_t = i | \mathbf{x}_{1:T}, \mathbf{y}_{1:t-1}) = \frac{\exp(\mathbf{E}\hat{\mathbf{y}}_t[i])}{\sum_j \exp(\mathbf{E}\hat{\mathbf{y}}_t[j])}$$





Encoder-Decoder Inference

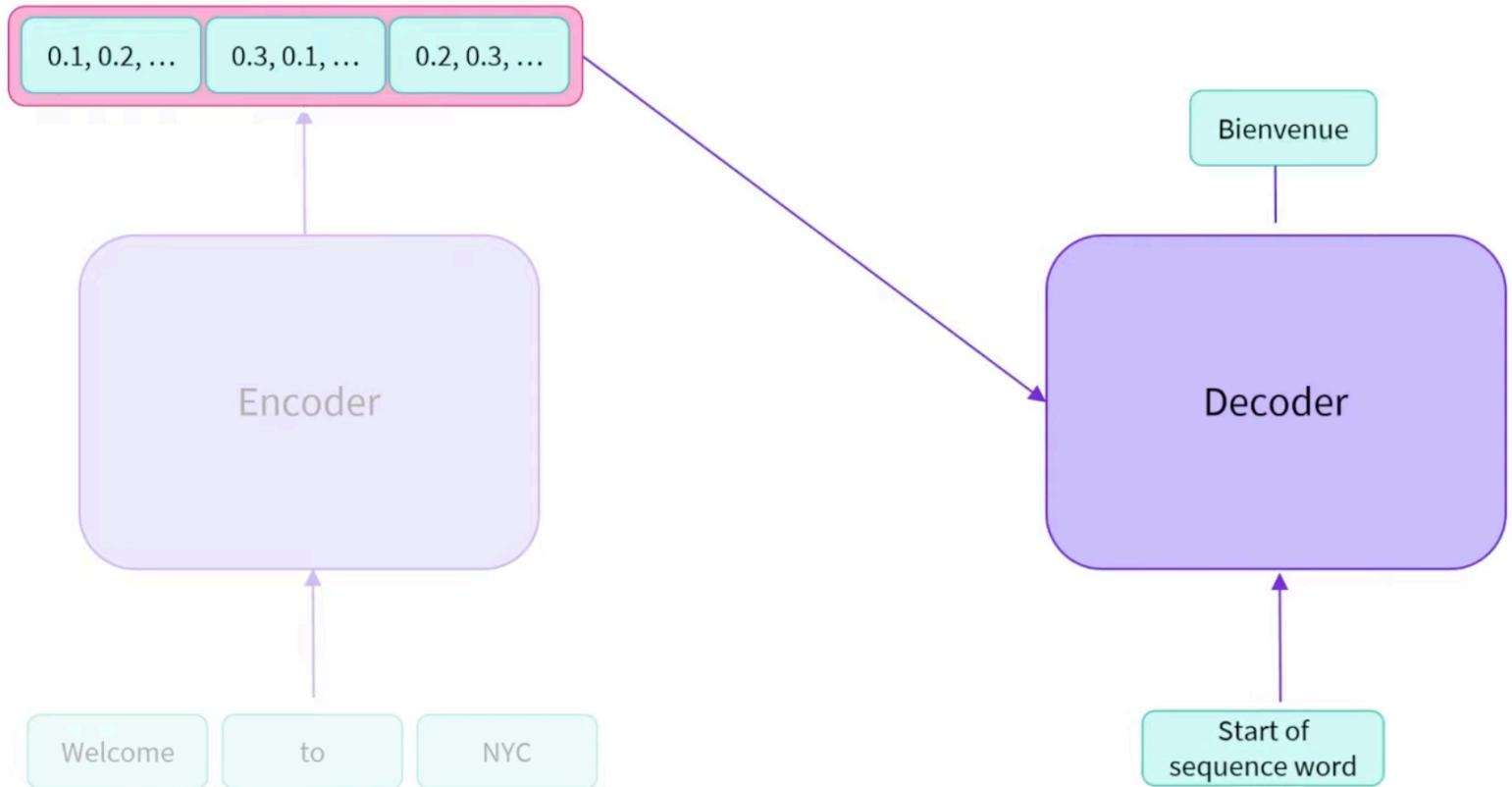
- Encode input sequence





Encoder-Decoder Inference

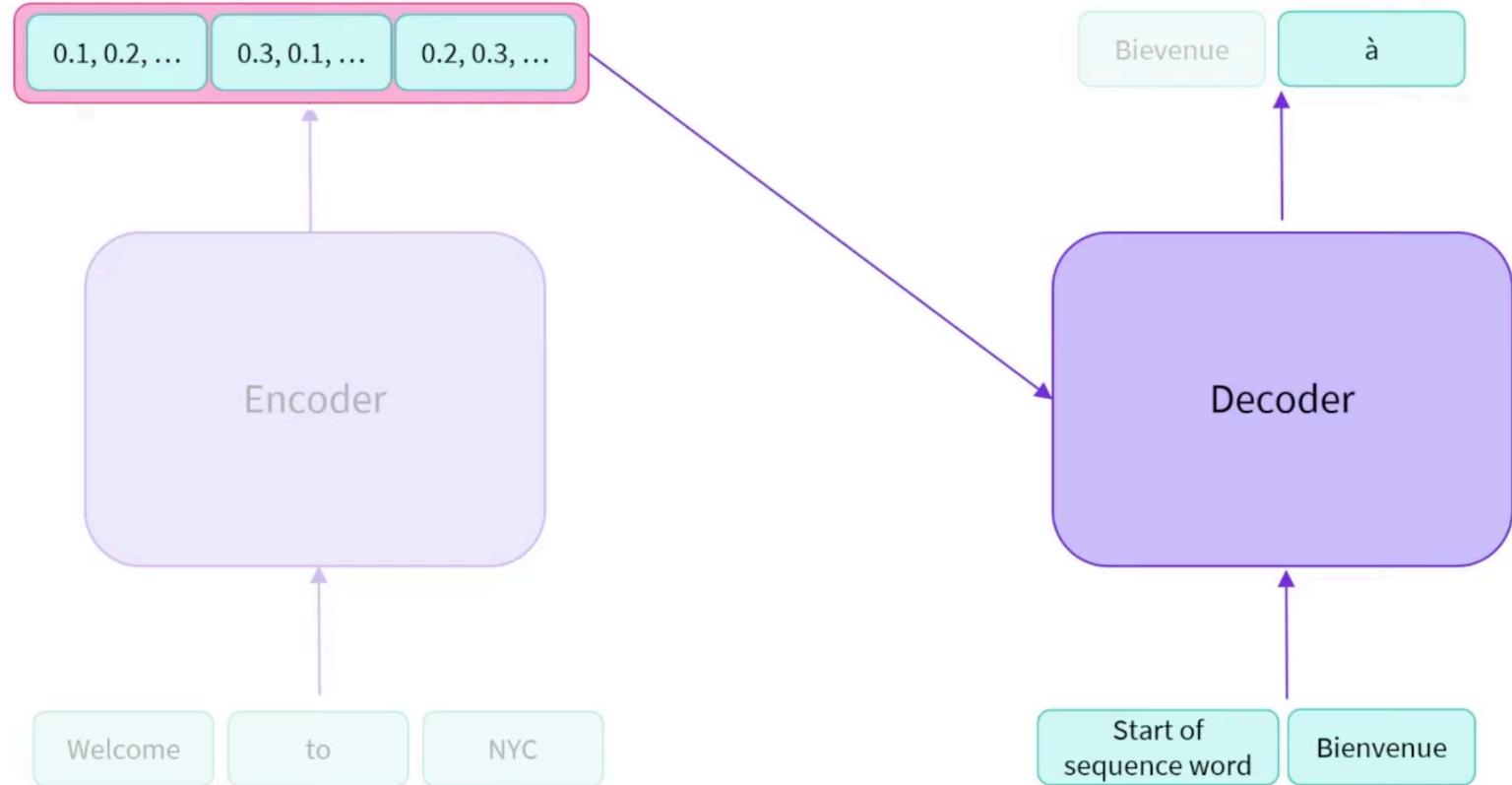
- Encode input sequence
- Attention over input token representations and <start>





Encoder-Decoder Inference

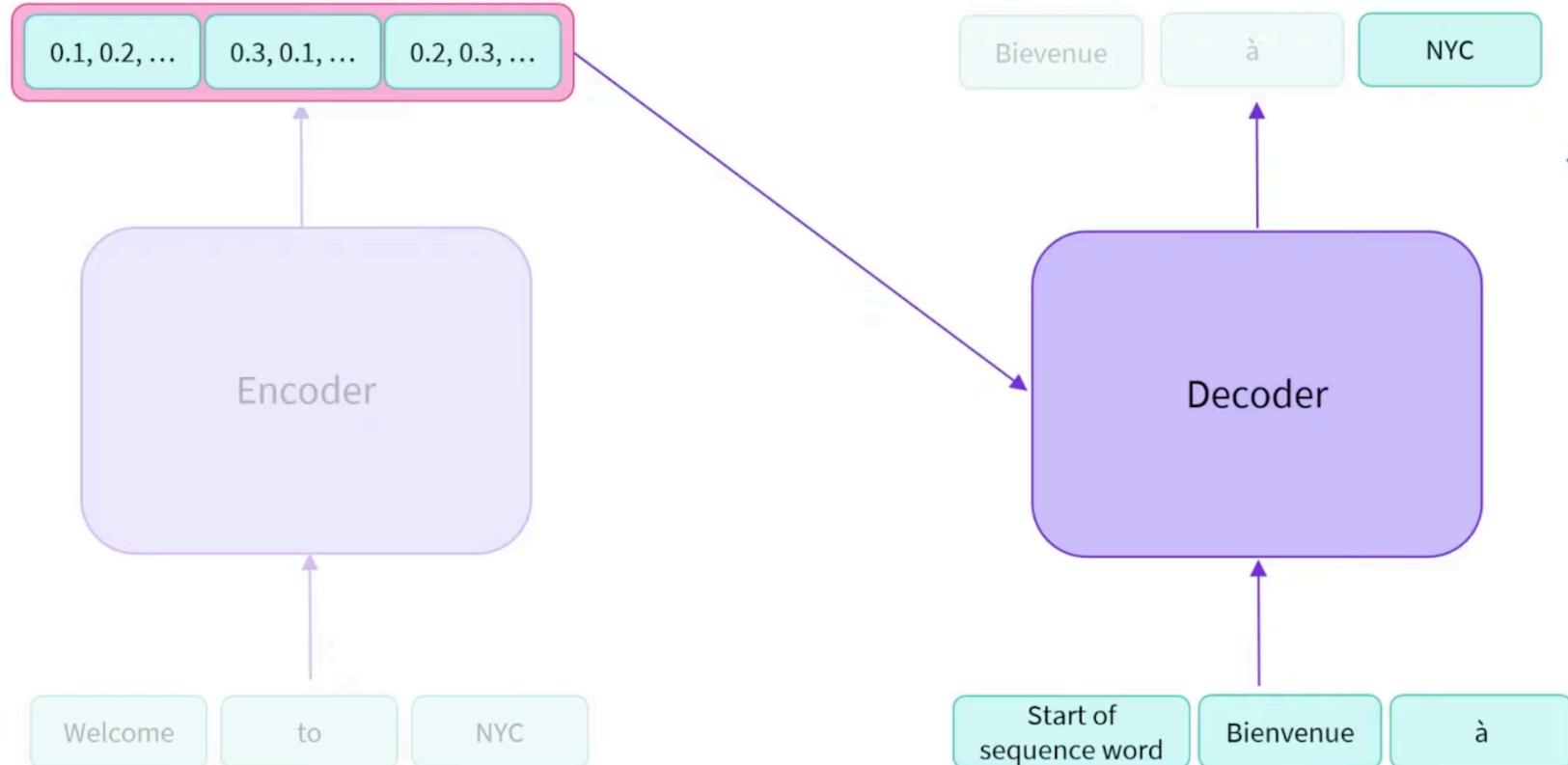
- Encode input sequence
- Attention over input token representations and <start>
- Self-attention





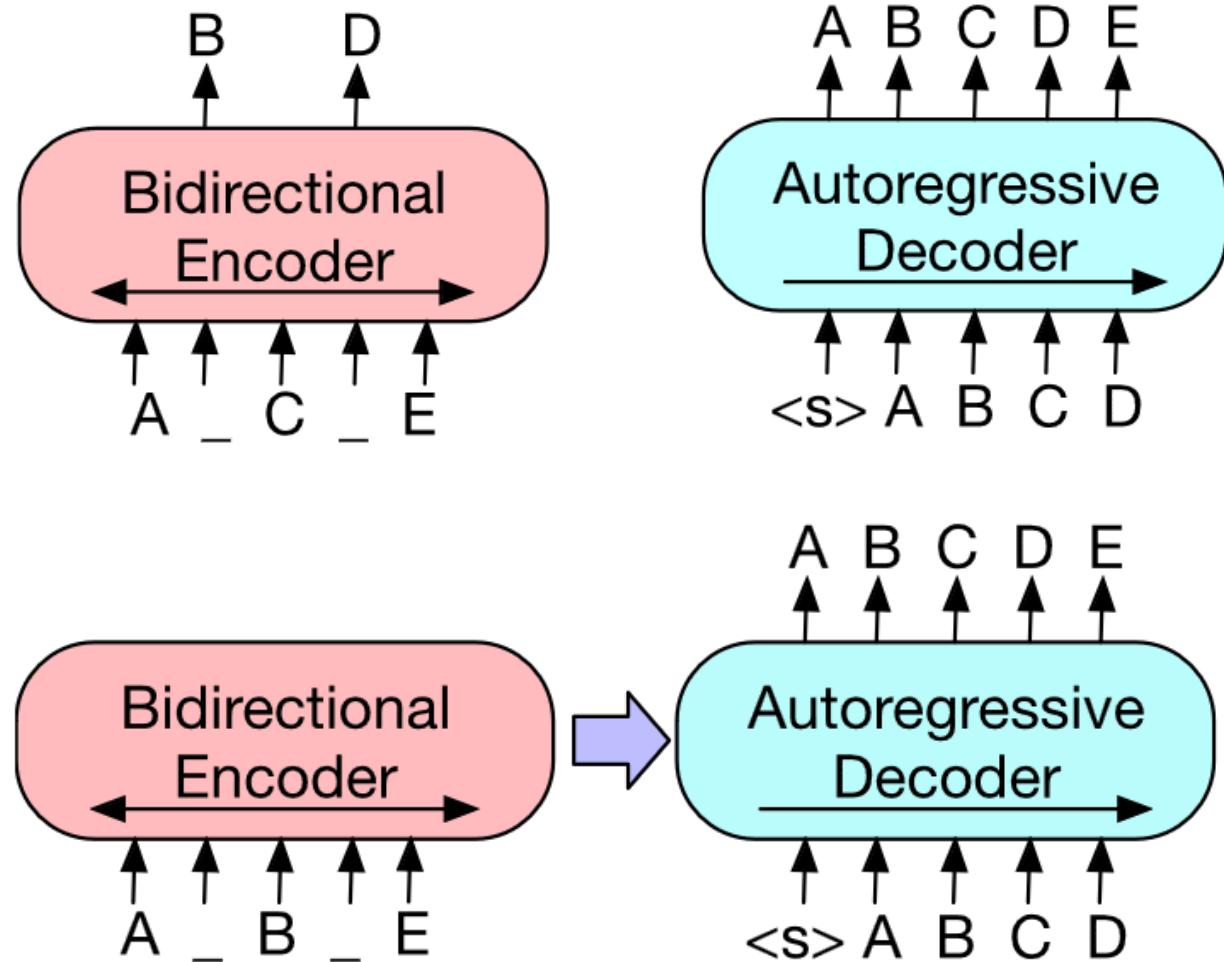
Encoder-Decoder Inference

- Encode input sequence
- Attention over input token representations and <start>
- Self-attention





Encoder, Decoder, Encoder-Decoder





Problems with the Transformer?

- Fixed context lengths “solved” with position embeddings
- Self-attention has quadratic cost $O(n^2d)$
- Plug: Annotated Transformer (Sasha Rush):
<http://nlp.seas.harvard.edu/annotated-transformer/>

Training Language Models



Recap: Language Modeling Objective

- Assume we have training data $\langle x_0 \dots x_T \rangle$
- Use current LM parameters to compute probability distributions over each token independently, conditioned on the prefix:

$$P(X_i) = p(\cdot | \langle x_0 \dots x_{i-1} \rangle; \theta)$$

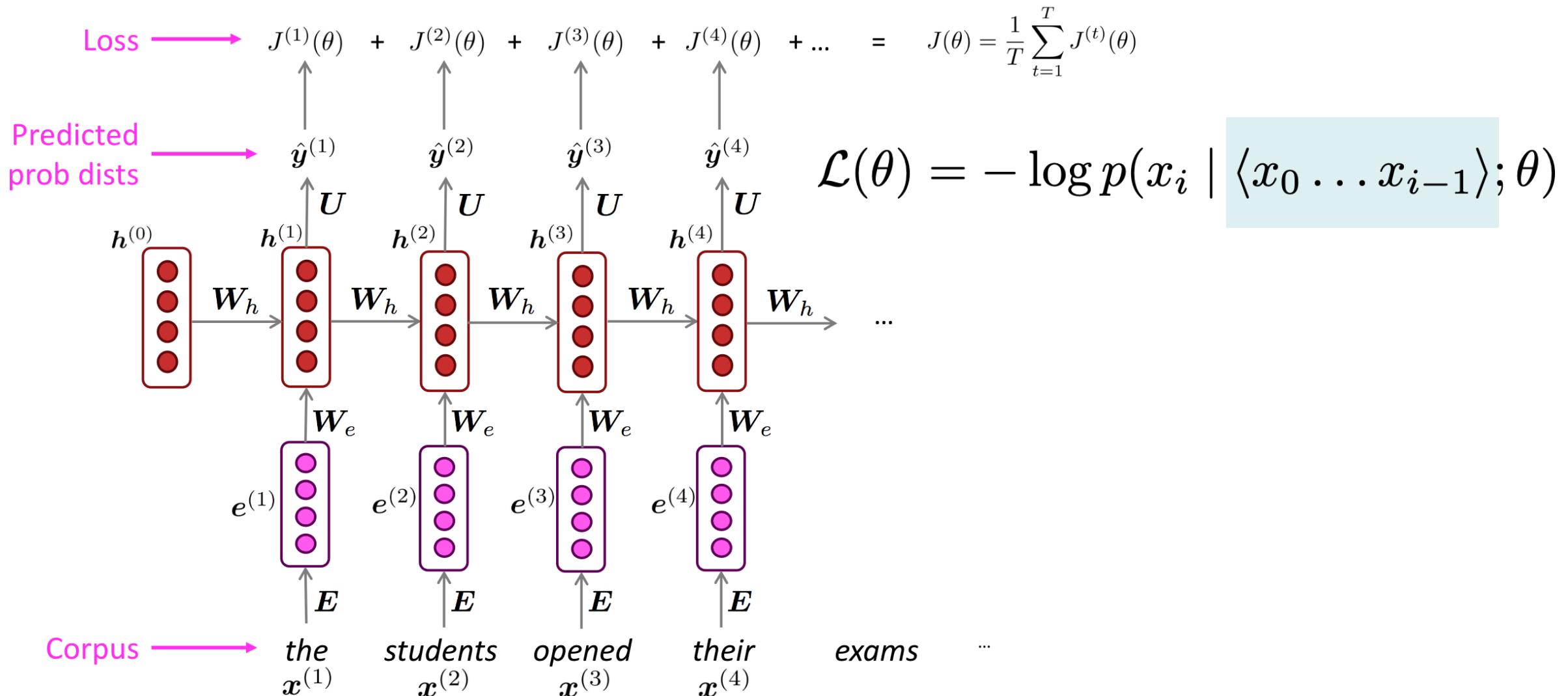
- Loss for step i is cross-entropy between true distribution p^* (i.e., one-hot) and predicted distribution:

$$\mathcal{L}(\theta) = - \sum_{x \in \mathcal{V}} p^*(x_i = x | \langle x_0 \dots x_{i-1} \rangle) \log p(x_i = x | \langle x_0 \dots x_{i-1} \rangle; \theta)$$

$$\mathcal{L}(\theta) = - \log p(x_i | \langle x_0 \dots x_{i-1} \rangle; \theta)$$

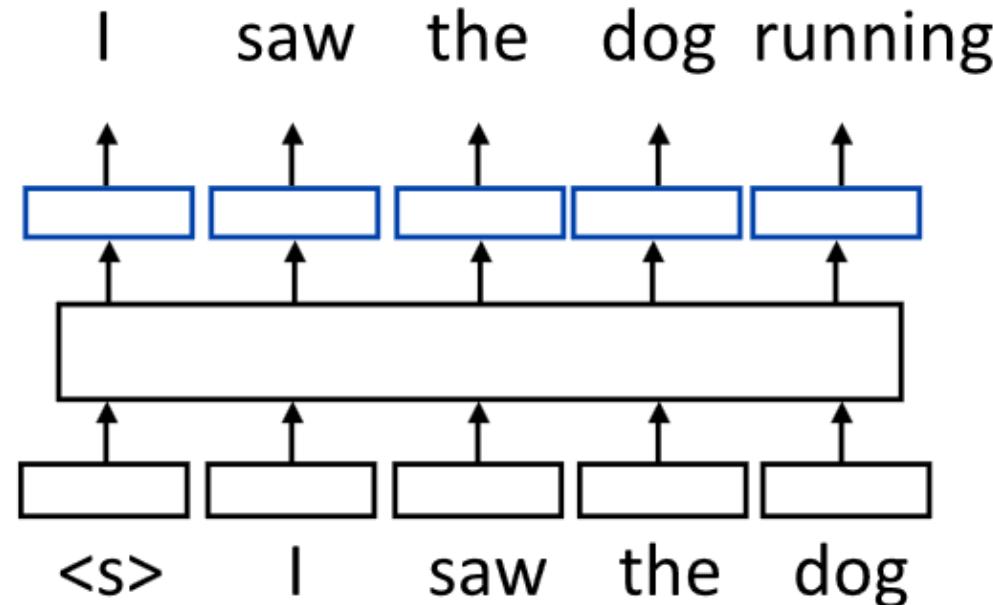


Next token prediction



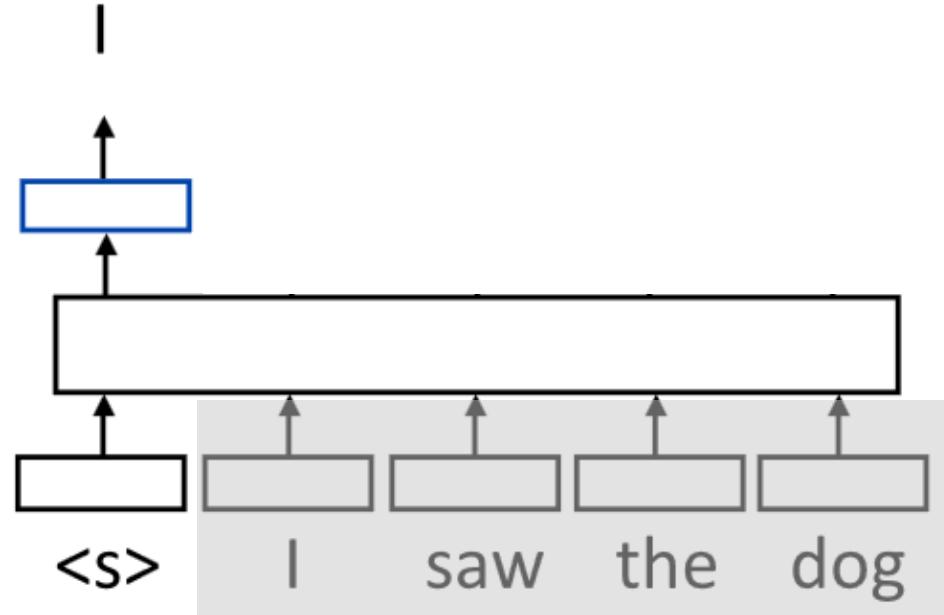


Next token prediction in Transformers



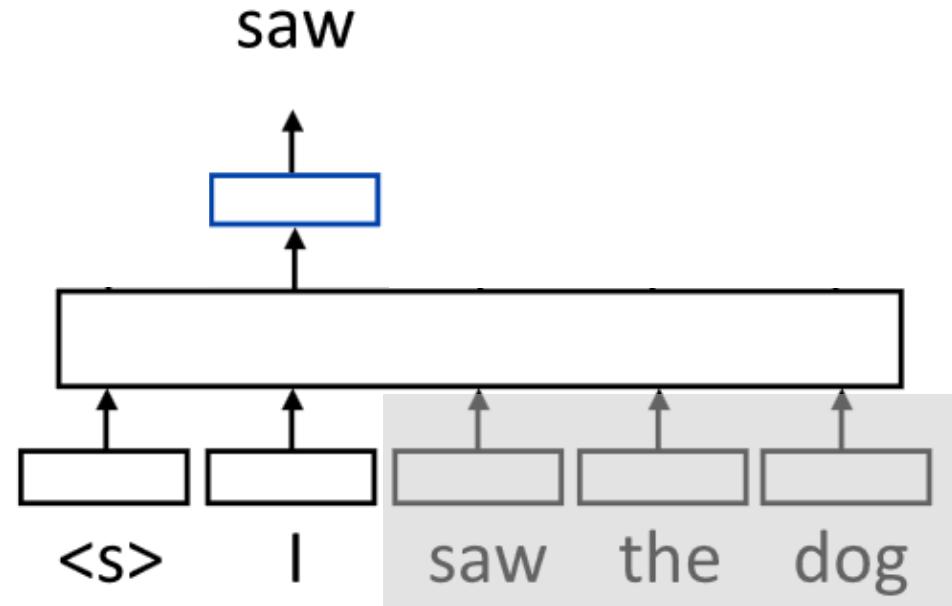


Next token prediction in Transformers



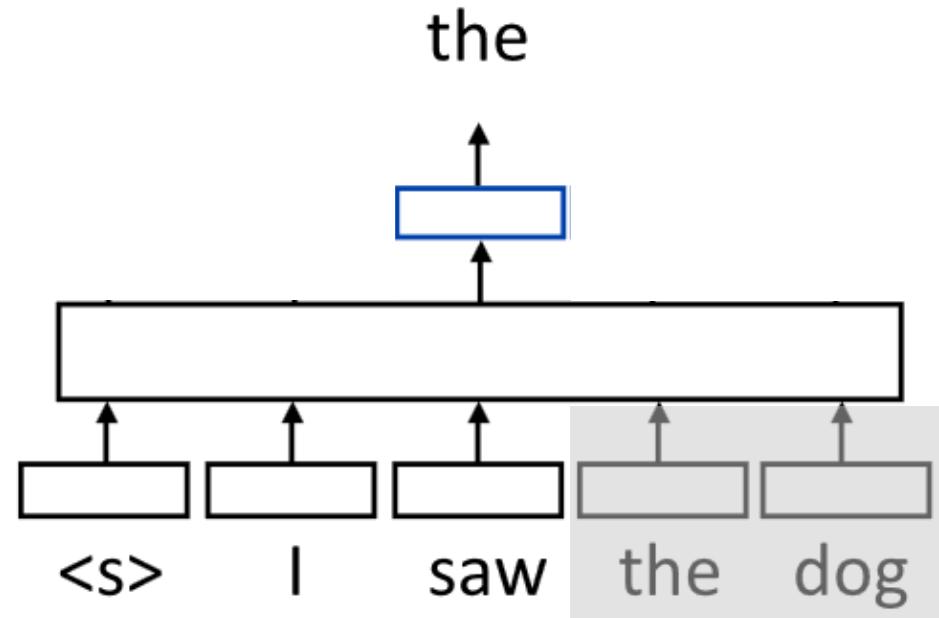


Next token prediction in Transformers



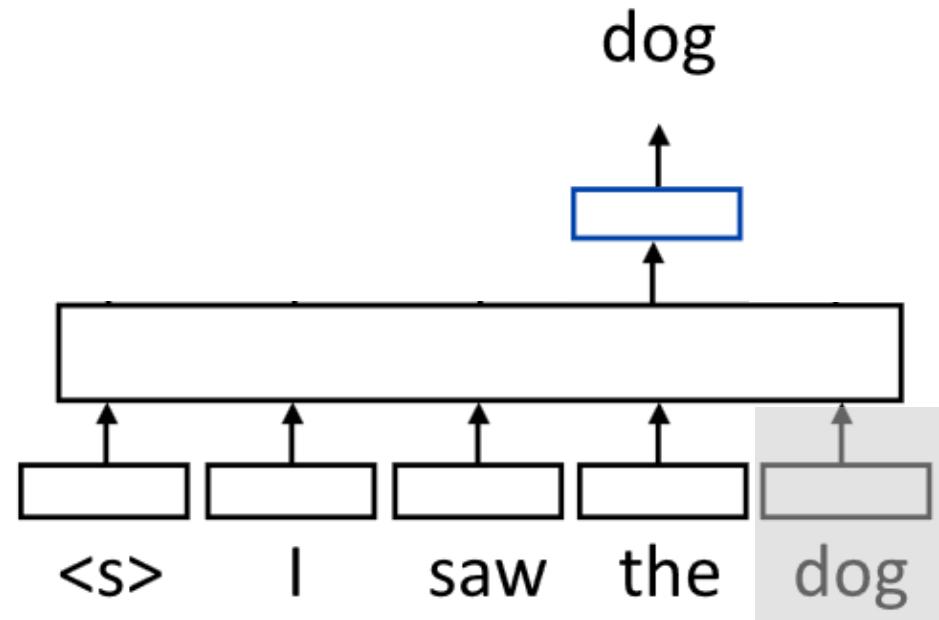


Next token prediction in Transformers



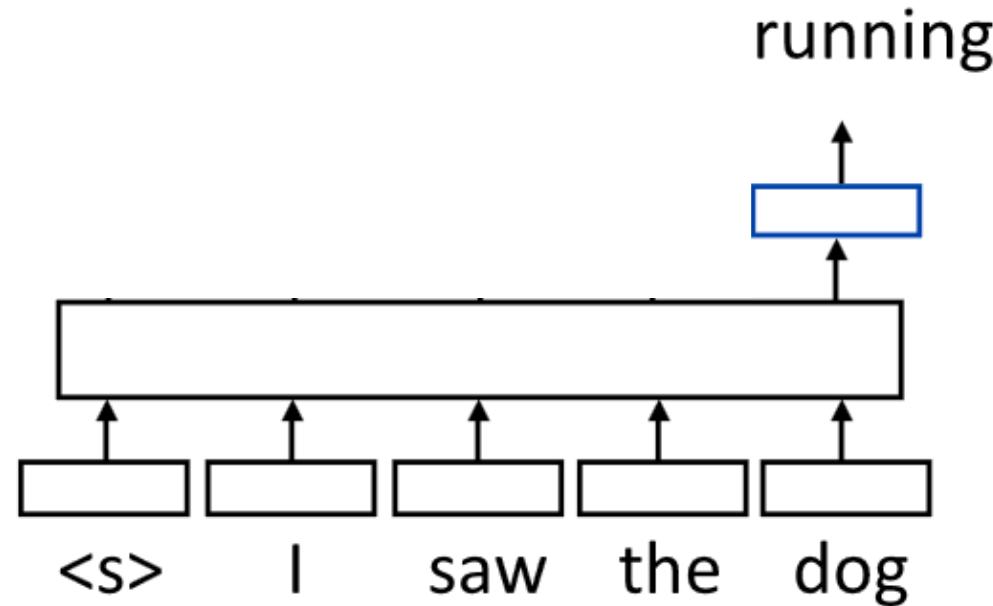


Next token prediction in Transformers





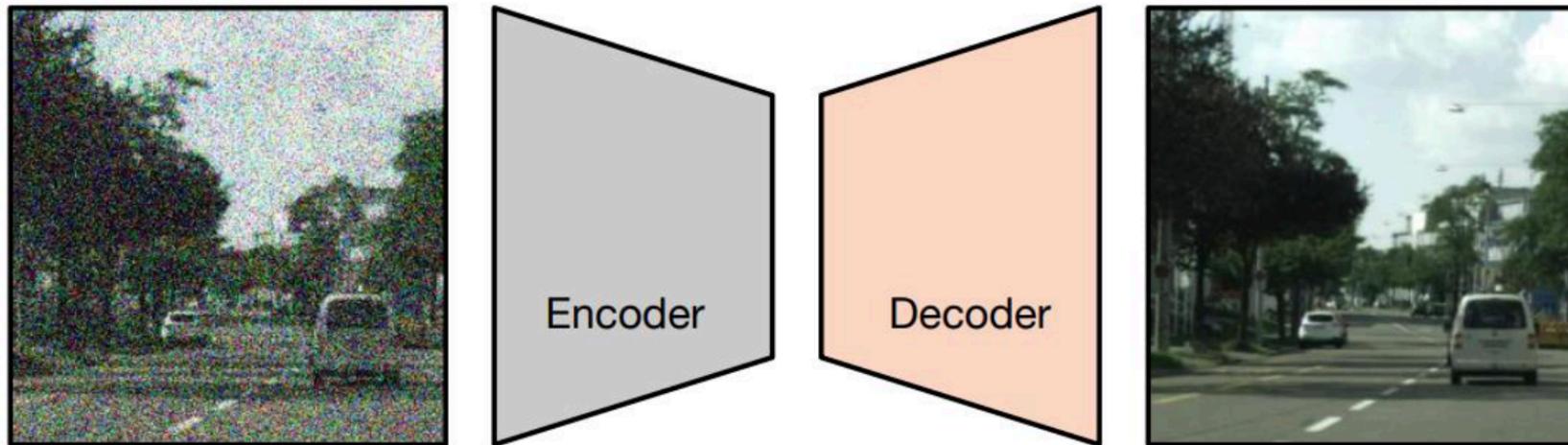
Next token prediction in Transformers





Denoising Objectives

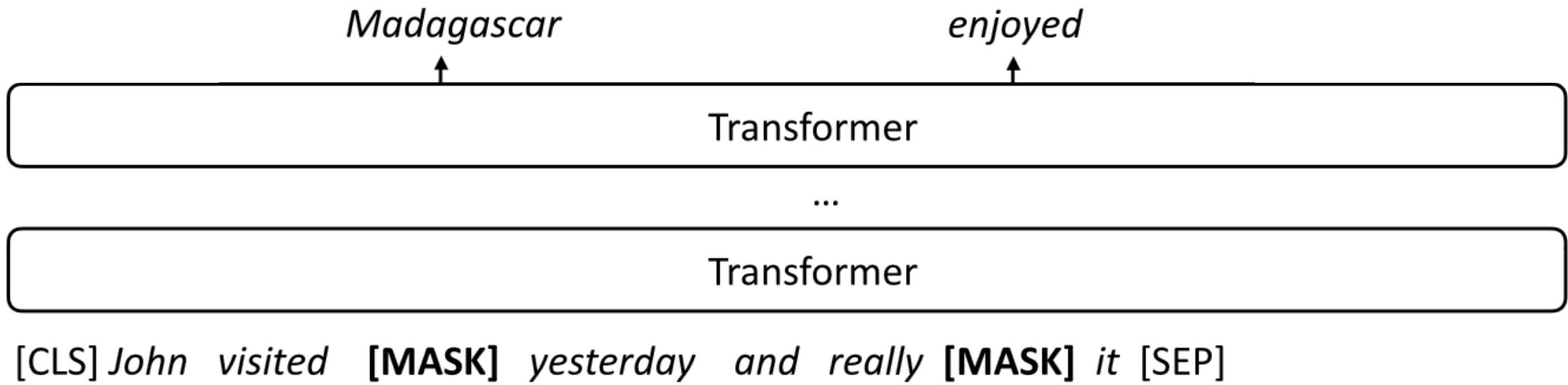
- Our goal: learn a distribution over text sequences
- Our assumption so far: this distribution is only backwards-looking (conditioned on prefix of the sequence)
- What if we remove this assumption?





Masking / Infilling Objectives

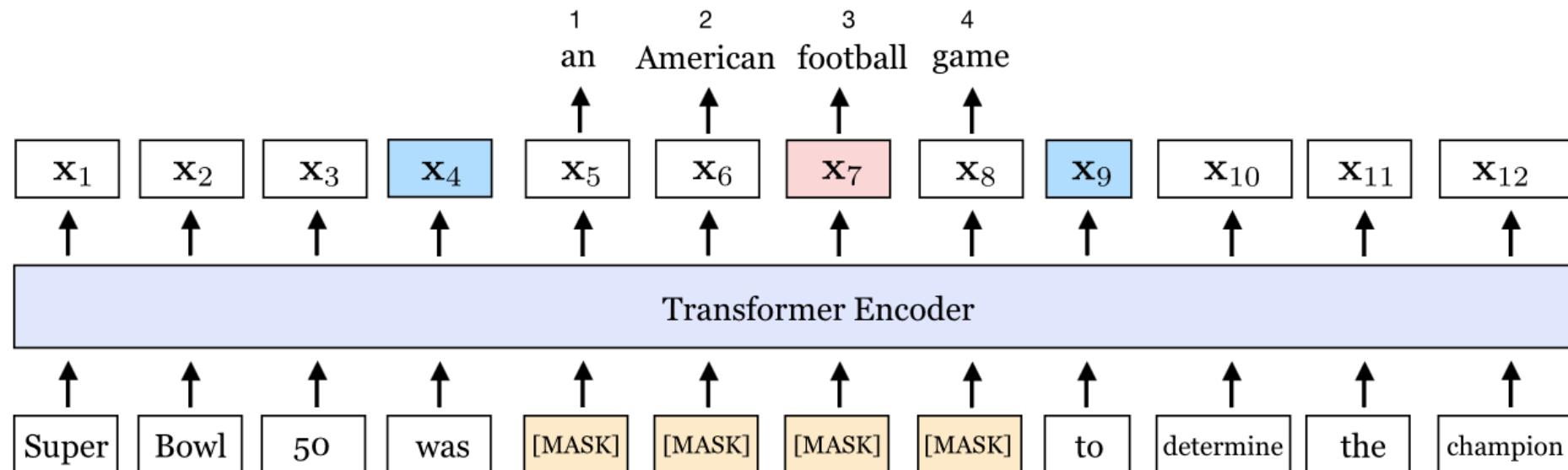
- Randomly mask out ~15% of tokens in the input, and try to predict them from past *and future* context





Masking / Infilling Objectives

- Randomly mask out ~15% of tokens in the input, and try to predict them from past *and future* context
- Or mask out spans of text





Auxiliary Objectives

A_C._E.
Token Masking

D E . A B C .
Sentence Permutation

C . D E . A B
Document Rotation

A . C . E .
Token Deletion

ABC . D E .
Text Infilling