$G = \langle V, E \rangle$
$V = \{ v_1, \ldots, v_n \}$
$E = \{ \langle v_{i1}, w_{i1} \rangle, \ldots, \langle v_{im}, w_{im} \rangle \}$

Given set, length k, of colors
$C = \{ c_1, \ldots, c_k \}$

Assign $c \in C$ to each $v \in V$ s.t.
every edge $\langle v, w \rangle \in E$, $color(v) \neq color(w)$

<u>HW</u> Encode K-Coloring into propositional Formula, F.

a) Every vertex v has a color c.

$$\boxed{\bigwedge_{v \in V} \left(\bigvee_{c \in C} p_v^c \right)}$$? This is saying every vertex must have a coloring that is one of C

<u>Thoughts:</u> $(p_{v_1}^{c_1} \vee p_{v_1}^{c_2} \vee p_{v_1}^{c_3}) \wedge (p_{v_2}^{c_1} \vee p_{v_2}^{c_2} \vee p_{v_1}^{c_3})$

" "

v_1 has c_1 or c_2 or c_3

but both $v_1 + v_2$ must have a color

$c_1 \neq c_2$

b) Every vertex has @ most one color

$$\bigwedge_{v \in V} \left(\bigvee_{c_1, c_2 \in C} \text{where } c_1 \neq c_2, \neg (p_v^{c_1} \wedge p_v^{c_2}) \right) \longrightarrow$$

cont... Every vertex has @ most one color

$$\bigwedge_{v \in V} \left(\bigvee_{c_1, c_2 \in C} \text{ where } c_1 \neq c_2 \, \neg \left(p_v^{c_1} \wedge p_v^{c_2} \right) \right)$$

$$\boxed{\bigwedge_{v \in V} \left(\bigvee_{c_1, c_2 \in C} \text{ where } c_1 \neq c_2 \left(\neg p_v^{c_1} \vee \neg p_v^{c_2} \right) \right)}$$

c) $$\bigwedge_{\langle v, w \rangle \in E} \left(\bigwedge_{c \in C} \neg \left(p_v^c \wedge p_w^c \right) \right)$$

$$\boxed{\bigwedge_{\langle v, w \rangle \in E} \left(\bigwedge_{c \in C} \left(\neg p_v^c \vee \neg p_w^c \right) \right)}$$

So we just combine all these?

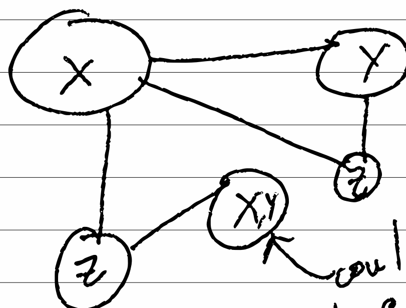
$$\left(\bigwedge_{v \in V} \left(\bigvee_{c \in C} p_v^c \right) \right) \wedge \left(\bigwedge_{v \in V} \left(\bigvee_{\substack{c_1 c_2 \in C \\ c_1 \neq c_2}} \left(\neg p_v^{c_1} \vee \neg p_v^{c_2} \right) \right) \right) \wedge$$

$$\bigwedge_{\langle v, w \rangle \in E} \left(\bigwedge_{c \in C} \left(\neg p_v^c \vee \neg p_w^c \right) \right)$$

d) So constraint 1 is necessary(?) as all other constraints rely on each vertex having a color.

Do 2 & 3 overlap?

colors: (x, y, z)



could put 2 here & 3 wouldn't catch it?
But this graph is still colored, so it's ok?

If we are simply trying to find any coloring, I constraint 2 can be dropped, as you can just select either color.

e) ???