

Convert to CNF using Tseitin's encoding  
 $\neg(\neg r \rightarrow \neg(p \wedge q))$

What is CNF?

**Conjunctive Normal Form**

Atom := Variable |  $\perp$  |  $\top$

Literal := Atom |  $\neg$ Atom

Formula := Clause  $\wedge$  Formula

Clause := Literal | Literal  $\vee$  Clause

Why is it important?

Bc SAT solvers use CNF as input language.

\* Equisatisfiable  $\equiv$  if formulas  $F$  &  $G$  are both satisfiable or both unsatisfiable

\* Tseitin's Transformation  $\equiv$  Converts  $F$  into equisatisfiable CNF formula that is linear to size of  $F$ .

\* Use auxiliary Variables \*

$x \rightarrow (y \wedge z)$        $a_1$

$a_1 \leftrightarrow (x \rightarrow a_2)$

$a_2 \leftrightarrow (y \wedge z)$

$$X \rightarrow (Y \wedge Z)$$

$a_1$

$$\left. \begin{array}{l} a_1 \rightarrow (X \rightarrow a_2) \\ (X \rightarrow a_2) \rightarrow a_1 \\ a_2 \leftrightarrow (Y \wedge Z) \end{array} \right\} \text{break down} \Leftrightarrow$$

$$\neg a_1 \vee (\neg X \vee a_2)$$

$$(X \rightarrow a_2) \rightarrow a_1$$

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$$\downarrow$$

$$\neg a_1 \vee \neg X \vee a_2$$

$$(X \wedge \neg a_2) \vee a_1$$

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$$\downarrow$$

$$\neg a_1 \vee \neg X \vee a_2$$

$$X \vee a_1$$

$$\neg a_2 \vee a_1$$

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~~Full~~ Conversion  $X \rightarrow (Y \wedge Z)$

$a_1$

$$\neg a_1 \vee \neg X \vee a_2$$

$$X \vee a_1$$

$$\neg a_2 \vee a_1$$

$$\neg a_2 \vee Y$$

$$\neg a_2 \vee Z$$

$$\neg Y \vee \neg Z \vee a_2$$

# HW #2

$$\neg(\neg r \rightarrow \neg(p \wedge q))$$

$$a \leftrightarrow \neg(\neg r \rightarrow \neg a \wedge q)$$

$$\boxed{a \wedge p \leftrightarrow (p \wedge q)}$$

$$\boxed{\begin{array}{l} \neg a \wedge p \vee p \\ \neg a \wedge p \vee q \\ \neg p \vee \neg q \vee a \wedge p \end{array}}$$

Easy conversion

$$a \leftrightarrow \neg(\neg r \rightarrow \neg a \wedge q)$$

$$\begin{array}{l} a \rightarrow \neg(\neg r \rightarrow \neg a \wedge q) \\ \neg(\neg r \rightarrow \neg a \wedge q) \rightarrow a \end{array}$$

$$\begin{array}{l} a \rightarrow \neg(r \vee \neg a \wedge q) \equiv a \rightarrow \neg(r \wedge a \wedge q) \\ \neg a \vee (\neg r \wedge a \wedge q) \end{array}$$

$$\boxed{\begin{array}{l} \neg a \vee \neg r \\ \neg a \vee a \wedge q \end{array}}$$

$$\neg(r \vee \neg a \wedge q) \rightarrow a$$

$$\boxed{r \vee \neg a \wedge q \vee a} \quad ? \text{ easy?}$$

Solution?

$a$

$$\neg a \vee \neg r$$

$$\neg a \vee a \wedge r$$

$$\neg a \wedge r \vee p$$

$$\neg a \wedge r \vee q$$

$$\neg p \vee \neg q \vee a \wedge r$$

$$r \vee \neg a \wedge r \vee a$$

$a$

$\neg a \vee \neg r$

$\neg a \vee a \wedge r$

$\neg a \wedge r \vee p$

$\neg a \wedge r \vee q$

$\neg p \vee \neg q \vee a \wedge r$