2. $\neg(\neg r \rightarrow \neg(p \wedge q))$
$\Leftrightarrow$ $(a1) \wedge (a1 \leftrightarrow \neg a2) \wedge (a2 \leftrightarrow (a3 \rightarrow a4)) \wedge \wedge (a4 \leftrightarrow \neg a5) \wedge (a5 \leftrightarrow p \wedge q)$
$\Leftrightarrow$ $(a1) \wedge (\neg a1 \vee \neg a2) \wedge (a2 \vee a1) \wedge (\neg a2 \vee \neg a3 \vee a4) \wedge (\neg(\neg a3 \vee a4) \vee a2) \wedge (\neg a4 \vee \neg a5) \wedge (a4 \vee a5) \wedge (\neg a5 \vee (p \wedge q)) \wedge (a5 \vee \neg(p \wedge q))$
$\Leftrightarrow$ $(a1) \wedge (\neg a1 \vee \neg a2) \wedge (a2 \vee a1) \wedge (\neg a2 \vee \neg a3 \vee a4) \wedge (a3 \vee a2) \wedge (\neg a4 \vee a2) \wedge (\neg a4 \vee \neg a5) \wedge (a4 \vee a5) \wedge (\neg a5 \vee p) \wedge (\neg a5 \vee q) \wedge (a5 \vee \neg p \vee \neg q)$

3. Proof by induction: Assume that $pos(I, \varphi) \subseteq pos(I', \varphi)$
<u>Basis step</u>: literall. If $I \models l \rightarrow l \in pos(I, \varphi) \rightarrow l \in pos(I', \varphi) \rightarrow I' \models l$
<u>Inductive hypothesis</u>: For some k, assume that if $I \models F_k$ (formula at level k) then $I' \models F_k$
<u>Inductive step</u>: At level k+1 there are two possibilities for formula F_{k+1}:

- $F_{k+1} = F_{k1} \wedge F_{k2}$. If $I \models F_{k+1} \rightarrow I \models F_{k1} \wedge I \models F_{k2} \rightarrow I' \models F_{k1} \wedge I' \models F_{k2}$ (inductive hypothesis) $\rightarrow I' \models F_{k+1}$
- $F_{k+1} = F_{k1} \vee F_{k2}$. If $I \models F_{k+1} \rightarrow I \models F_{k1} \vee I \models F_{k2} \rightarrow I' \models F_{k1} \vee I' \models F_{k2}$ (inductive hypothesis) $\rightarrow I' \models F_{k+1}$

By induction, we've shown that if $I \models \varphi$, then $I' \models \varphi$.