Complementary material for kids (and curious adults too)

cal-rmedina

The following sections give an geometrical interpretation of the square sum, for the series $1^2 + 2^2 + \cdots + n^2$, there is a formula for the sum of the first n terms:

$$\sum_{i=1}^{n} i^2 = \frac{n\,(n+1)\,(2n+1)}{6}.$$

Each term of the series can be seen as pyramid level (Fig. 1), it will help to give a geometrical interpretation to the sum, because $n\,(n+1)\,(2n+1)$ can be seen as a square prism of size $n \times (n+1) \times (2n+1)$.

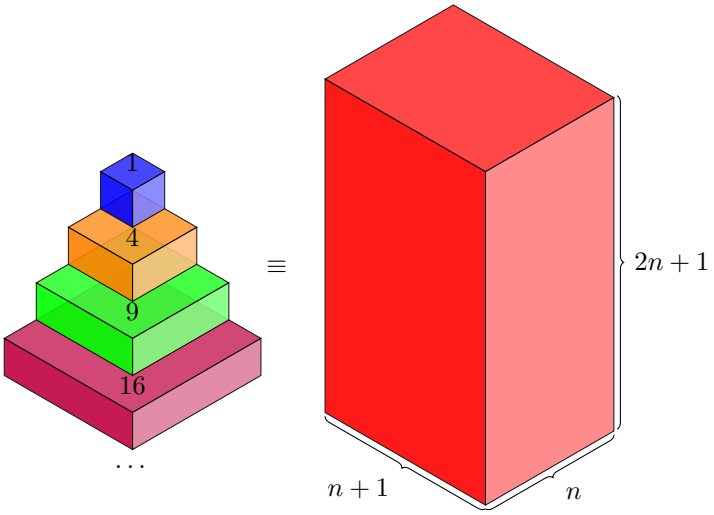
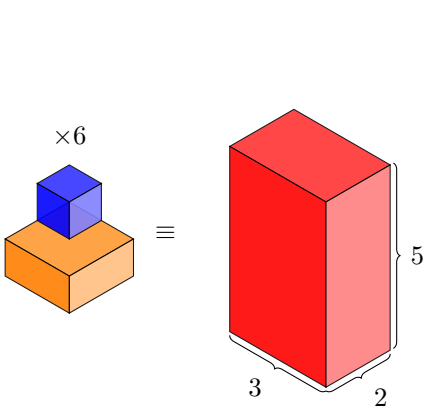


Figure 1: Each box represents a term in the series $1^2 + 2^2 + 3^2 + 4^2 \ldots$

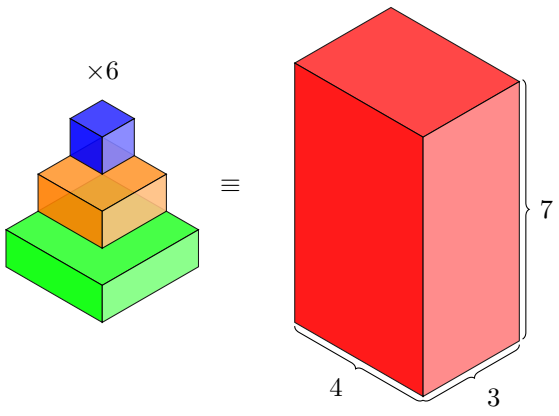
The pyramid **fits exactly 6 times** inside the square prism, cut-out material is provided to check the cases $n = 2, 3$, the trivial case for $n = 1$ won't be discussed, for a formal mathematical proof go to the link.

Two and three terms of the sum

Fig. 2 serves for the geometrical proof, check Assets, print and cut out the figures to construct the square prism.



(a) $1^2 + 2^2 = 5$



(b) $1^2 + 2^2 + 3^2 = 14$

Figure 2: 6 pyramids build a square prism; (2a) for $n = 2$ and (2b) for $n = 3$.

Assets