Relations

Rashidah Kasauli & Nasser Kimbugwe

Department of Networks
Block A, Level 3,Room 301
COCIS

Overview

Introduction

- We could want to describe relations between objects, e.g.,
  - This record is stored under that key
  - This input channel is connected to that output channel
  - This action takes priority over that one
- These can be described using mathematical objects called relations
- This topic coversexplains and describes the following:
  - How to define and extract info from relations.
  - Classification of relations.
  - Inversion or composed to form new objects, and explain what these objects represent

Binary relations

- These express links between pairs of objects
- A relation is a set of ordered pairs - a subset of a Cartesian product.
- Definition: Let A and B be sets. A binary relation from A to B is a subset of AXB.
- In other words, for a binary relation R we have $R \subseteq (AXB)$.
- We use the notation aRb to denote that $(a, b) \in R$ and $a\underline{R}b$ to denote that $(a, b) \notin R$.

Example

- Given set $A = \{a, b\}$ and set $B = \{0, 1\}$, a relation aRb where $a \in A$ and $b \in B$ is a subset of

$$\{(a, 0), (a, 1), (b, 0), (b, 1)\}$$

- The power set of AXB lists the possible subsets from

Equality

- In arithmetic we learn that $1 + 1$ equals 2;
- In the Christian religion, the 25th of December equals Christmas Day.
- Such statements indicate that the two expressions concerned have the same value, or that they denote the same object.
- In a formal description, we write $e = f$ when e is identical to f , in the sense that we cannot distinguish between them.

Equality....

- Example: In an identity parade, a witness may state that *'The man on the right is the man who stole my idea'*,

  making the following identification:

  The man on the right = *the man who stole my idea*

  That is, the man on the right is identical to the man who stole the idea.

Equality...

- Used to represent that two values (such as numbers) and not predicates are identical

- *Law of reflection*: Everything is identical to itself e.g $t = t$
- Doesn't hold for all logics and thus an *axiom*

- *Leibniz's law*, or the substitution of equals: if $s = t$, then whatever is true of s is also true of t.

Equality...

- Example: If we know that Christmas Day = 25th December, and that 25th December falls on a Sunday this year then we may apply the substitution of equals rule and conclude that Christmas Day falls on a Sunday this year.
- If two expressions e and f are not identical, then we write $e \neq f$. This is simply an abbreviation for $\neg(e = f)$.
- Equality is symmetric: for any expressions s and t , if $s = t$, then $t = s$.
- Equality is transitive: for any expressions s, t and u, if $s = t$ and $t = u$ then $s = u$.

Uniqueness

- Example: Let x loves y mean that x is in love with y, and let domain be the set of all people. We may symbolise the proposition

  'only Romeo loves Juliet' using a conjunction:

  $$Loves(Romeo, Juliet) \wedge Loves(x, Juliet) \rightarrow x = Romeo$$

  That is, any person who loves Juliet must be Romeo.
- This technique can be use to formalise expressions containing 'at most' and 'no more than'

Definite description

- We often use a descriptive phrase to denote an object, rather than a name.

  For example, 'the driver of the white car' or 'the cat in the hat'.

  In both of these examples, it is the word 'the' that is important
- 'The' is used to indicate *existence* and *uniqueness*.
- Each of the following phrases indicates that there is a unique object with a certain property:
  - The man who shot John Lennon
  - The woman who discovered radium
  - The oldest university in Uganda
- A special notation for this definite description of objects: the μ-notation. We write $(\mu x P(x))$ or $(\mu x : a | P)$ to denote the unique object x from a such that P.

Definite description

- Each of the following phrases indicates that there is a unique object with a certain property:
  - The man who shot John Lennon
    $\mu x\, Shot(x, John)$ and $x \in \{People\}$
  - The man who discovered the source of the nile
    $\mu y\, discovered(y, SourceNile)$ and $y \in \{People\}$
  - The oldest university in Uganda
    $\mu m\, oldest(m, Uganda)$ and $m \in \{Universities\}$
- So how are these statements formalised?
  - John Speke discovered the source of the nile.
    $John\ Speke = \mu y\, discovered(y, SourceNile)$

The End