Circuit Analysis I ELE1121

Course Given by

Emmanuel Wokulira Miyingo

Dept of ELE & Computer Engineering

Email: miyingo@yahoo.com

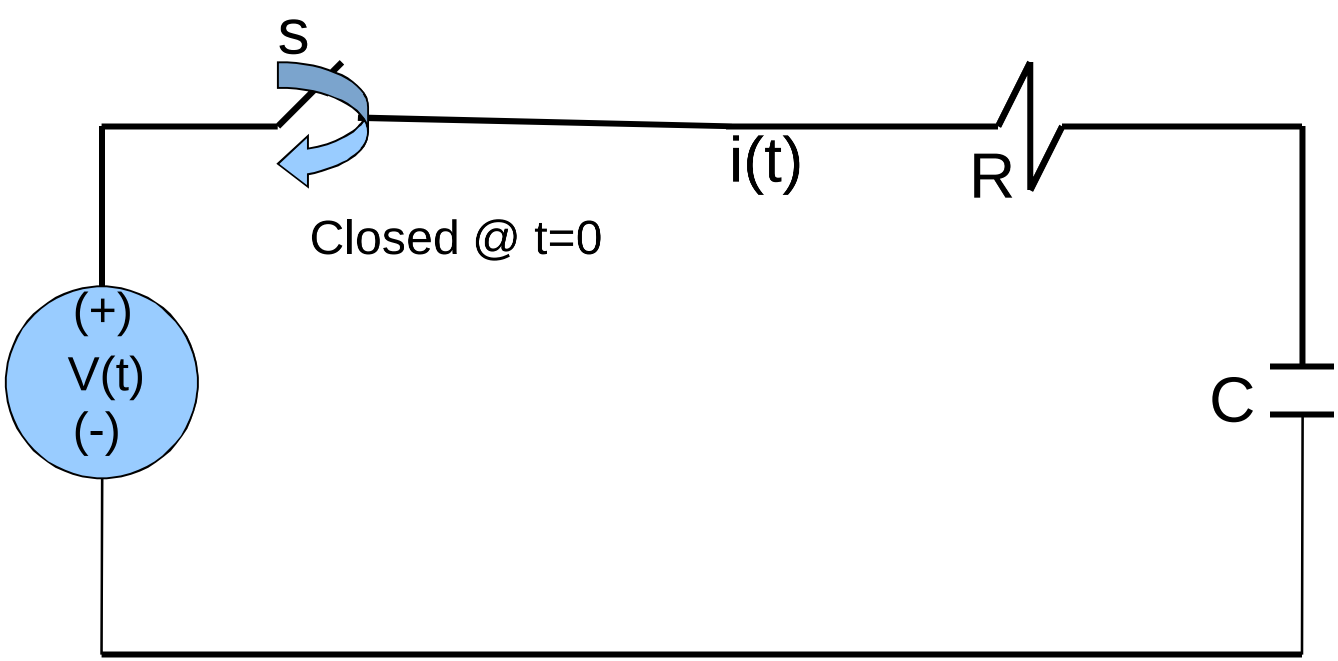
Mob: 0702423774/0782423774

Transient Signals

- Consider the circuit (next slide):
- A switch, which closes at a time $t = 0$ is included in a series circuit of R & C.
- In practice, this is done; otherwise the practical analysis (using an oscilloscope) is not possible.

Transient Signals: R-C Circuit

- A capacitive circuit is modified to include a switch:



Transient Signals: R-C Circuit

- From the knowledge of capacitor behaviour; it will act as a short @ t = 0, & open circuit as

  $t \rightarrow \infty$

$$clearly;$$

$$i_0 = \frac{V_0}{R} = current\ when\ t = 0$$

$$i_\infty = 0 = current\ when\ t \rightarrow \infty$$

Transient Signals: R-C Circuit Cont’d

- It is in our interest to find

$$values \ of \ i$$

$$0 \le t \le \infty;$$

$$this \ is \ called \ transient \ current \ / \ response$$

$$of \ the \ circuit.$$

$$When \ the \ switch \ (s) \ is \ closed;$$

$$i(t) = \frac{v(t)}{R + Z_C}$$

Transient Signals: R-C Circuit Cont'd

- We have 2 ways of determining i(t):

1. Using Laplace Transforms.
2. Using Ordinary Differential Equations. DE Table

- In the interest of time we shall use Laplace

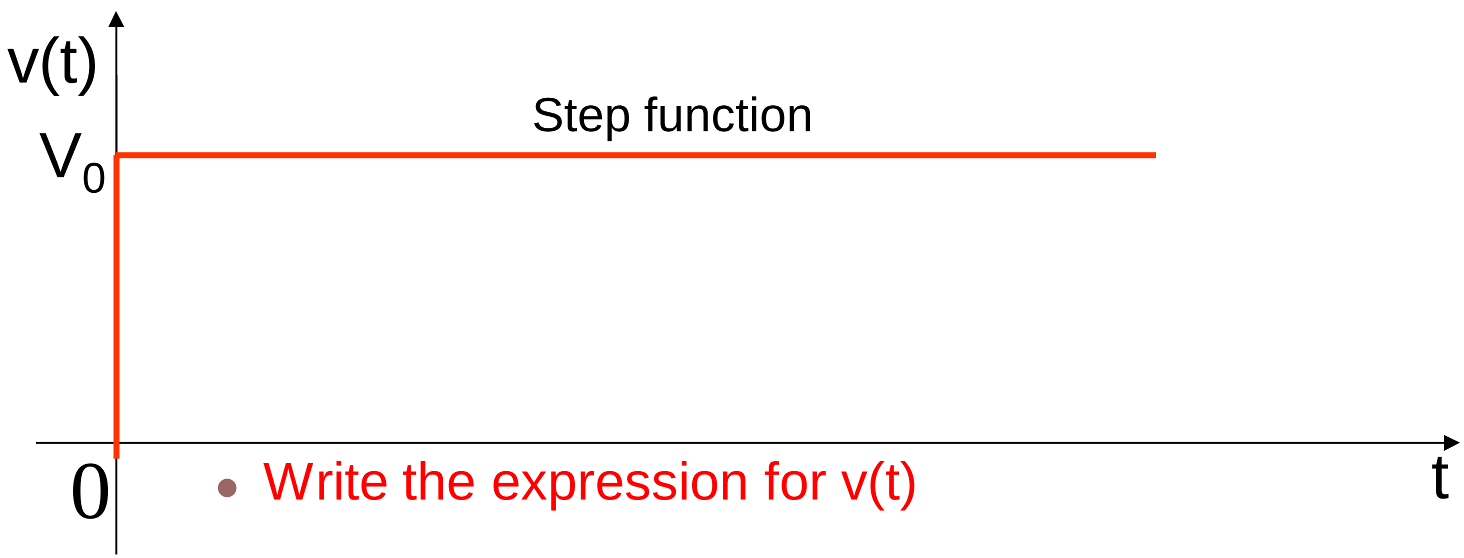
$$Let$$

$$s = j\omega = Laplace\ operator$$

$$\therefore Z = R + \frac{1}{j\omega C} = R + \frac{1}{sC}$$

Transient Signals: R-C Circuit Cont'd

- The operation of suddenly closing the switch results in a voltage waveform below



- Write the expression for v(t)

Transient Signals: R-C Circuit Cont'd

- Thus v(t) is a Step Function of magnitude V_p
- The Laplace transform of the step function voltage above and the Laplace transform of the current are given by: Recall: Transformation Table

$$V(s) = \frac{V_0}{s}$$

$$\therefore I(s) = \frac{V_0 / s}{R + 1 / sC} = \frac{V_0 C}{sCR + 1}$$

Transient Signals: R-C Circuit Cont'd

$$\therefore I(s) = \frac{V_0}{R} \cdot \frac{1}{s + \frac{1}{CR}}$$

Transform Table

$$\therefore Inverse\ Laplace\ Transform\ of\ \ I(s)$$

$$is\ i(t)\ given\ by:$$

$$i(t) = \frac{V_0}{R} \cdot [e^{-\frac{t}{CR}}]u(t)$$

Inverse Laplace Transform (Standard Signal)

- Recall: the Laplace Transform ↔ Inverse Laplace Transform Table:

$$if\ \ I(s) = \frac{1}{s+a}$$

$$i(t)\ is\ \ given\ by:$$

$$i(t) = e^{-at}u(t)$$

Inverse Laplace Transform (Standard Signal) Cont’d

- the Inverse Laplace Transform of:

$$similarly\ I(s) = \frac{1}{s+a} + \frac{1}{s+b}$$

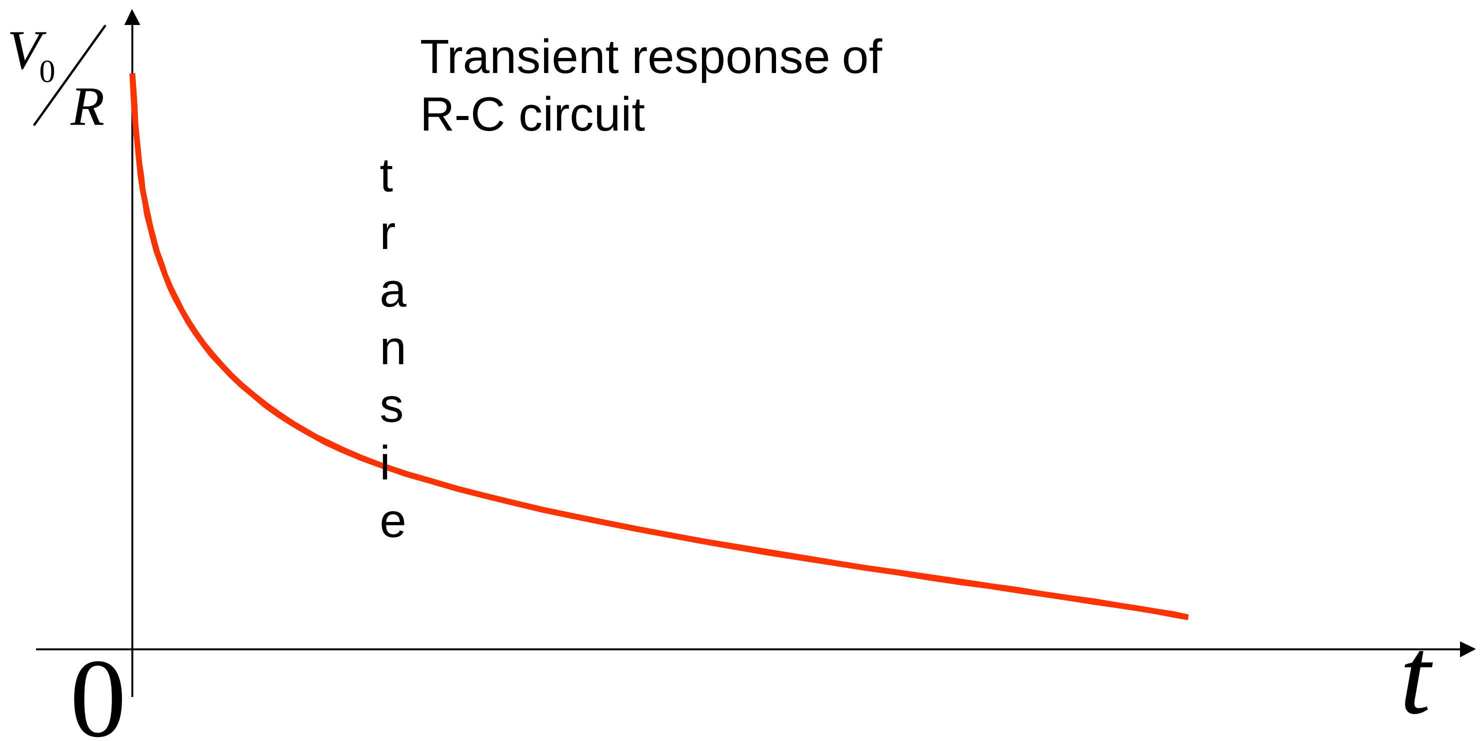
$$i(t)\ is\ given\ by:$$

$$i(t) = [e^{-at} + e^{-bt}]u(t)$$

Transient Signals: R-C Circuit

- We can easily check for the initial & final values of $i(t)$ by substituting for values of t.
- We can also sketch i vs. t.
- The results confirm our previous knowledge of a charging capacitor.

Transient Signals: R-C Circuit Cont'd



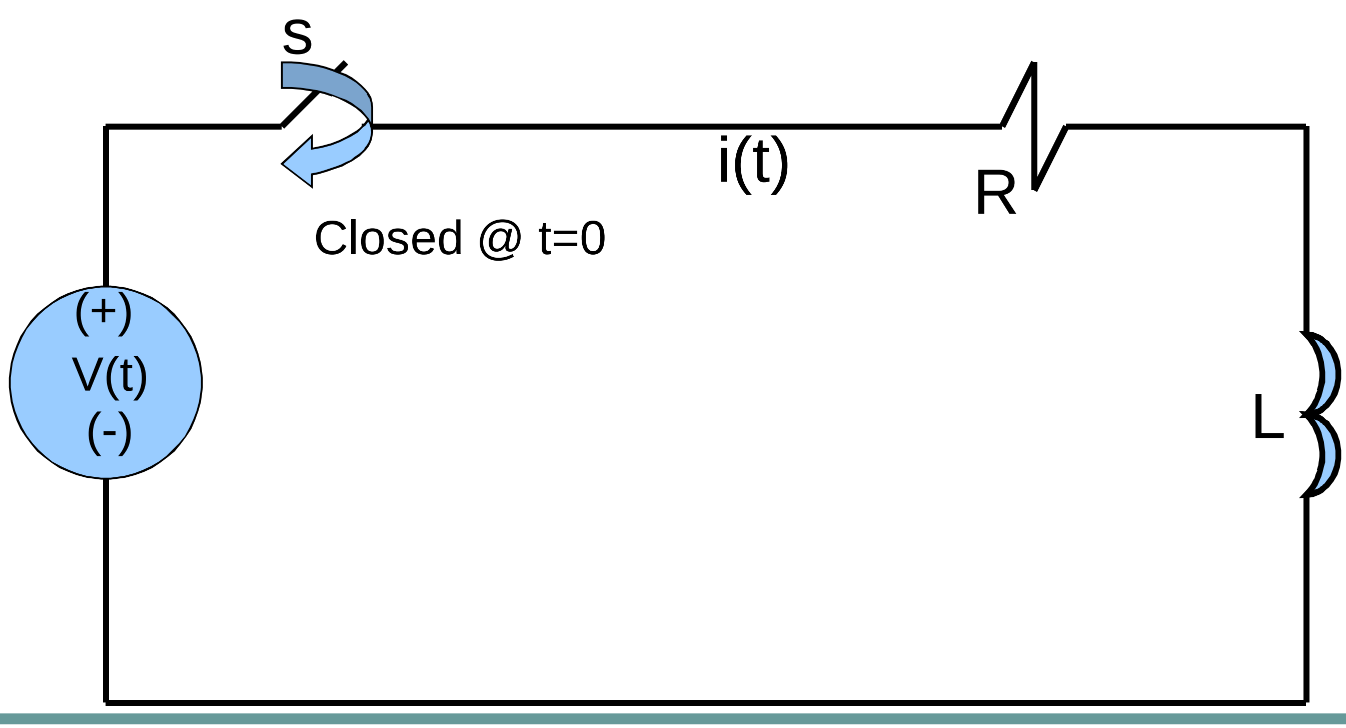
Transient Signals: R-L Circuit

- Similarly, let us consider the transient behaviour of an R-L circuit next slide:
- The same conditions apply:

1. Switch s is closed suddenly.
2. The signal received by R & L is a step function shown above.

Transient Signals: R-L Circuit Cont'd

- An inductive circuit is modified to include a switch:



Transient Signals: R-L Circuit Cont'd

- From the knowledge of inductor behaviour; it will act as open @ t = 0, & short circuit as

  $t \rightarrow \infty$

$$clearly;$$

$$i_0 = 0 = current\ when\ t = 0$$

$$i_\infty = \frac{V_0}{R} = current\ when\ t \rightarrow \infty$$

Transient Signals: R-L Circuit Cont'd

- The Laplace Transforms are as follows:
- The Laplace transform of the step function voltage above and the Laplace transform of the current are given by:

$$V(s) = \frac{V_0}{s}$$

$$Z = R + j\omega L;$$

$$Z(s) = R + sL$$

$$\therefore I(s) = \frac{V_0 / s}{R + sL} = \frac{V_0}{s[R + sL]}$$

Transient Signals: R-L Circuit Cont'd

- Using Partial Fractions
- $I(s) = \frac{V_0}{s(R+sL)} \equiv V_0\left[\frac{A}{s} + \frac{B}{(R+sL)}\right]$
- Determining A and B
- $A = \frac{1}{R}\ and\ B = -\frac{L}{R}$
- $\therefore I(s) = \frac{V_0}{R}\left[\frac{1}{s} - \frac{L}{(R+sL}\right] = \frac{V_0}{R}\left[\frac{1}{s} - \frac{1}{(s+{}^{R}/{}_{L}}\right]$

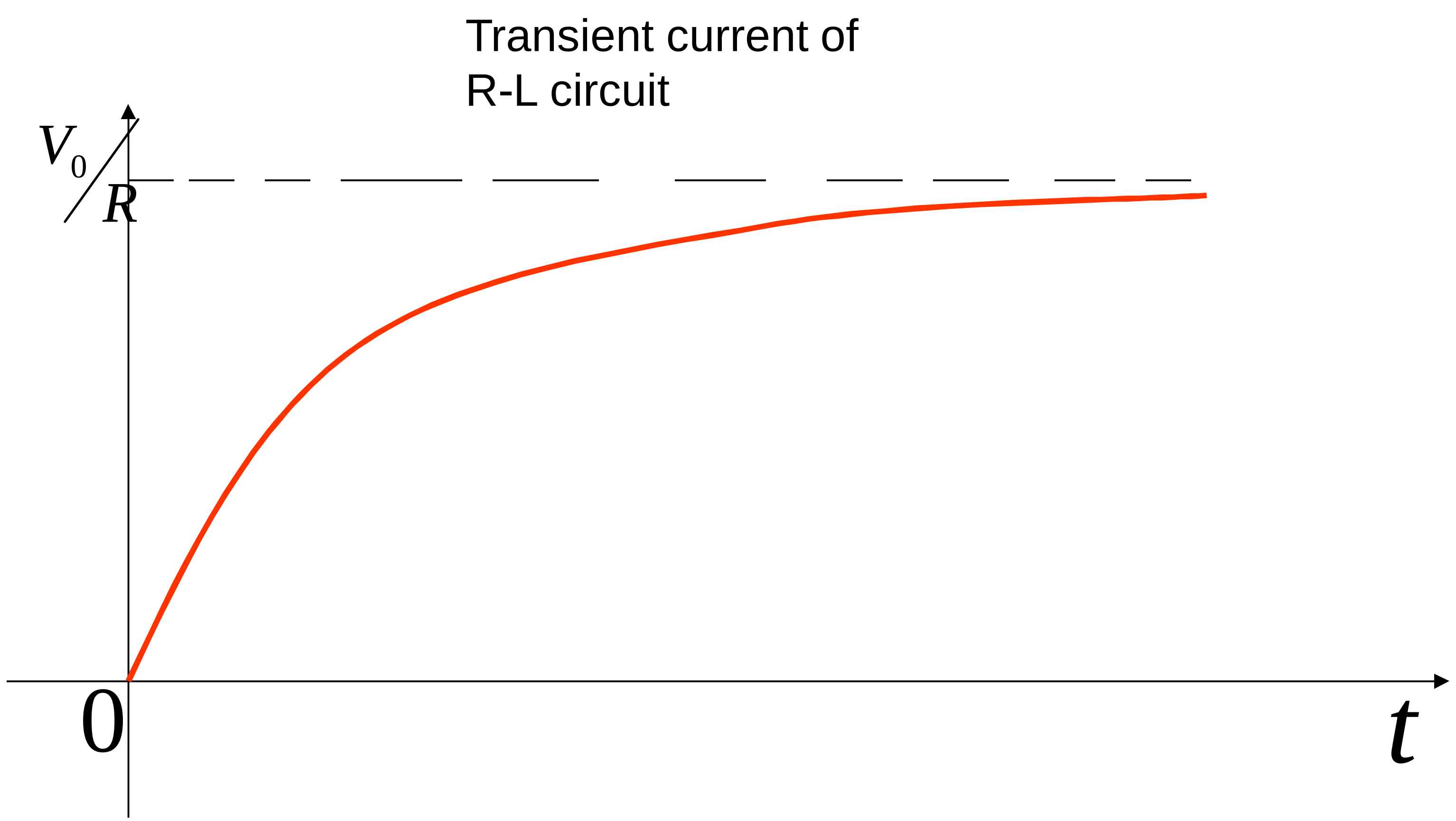
Transient Signals: R-L Circuit Cont'd

- The transient current is obtained from the Inverse Laplace Transform of I(s) to give i(t).

$$i(t) = \frac{V_0}{R}(1 - e^{-(R/L)t})u(t)$$

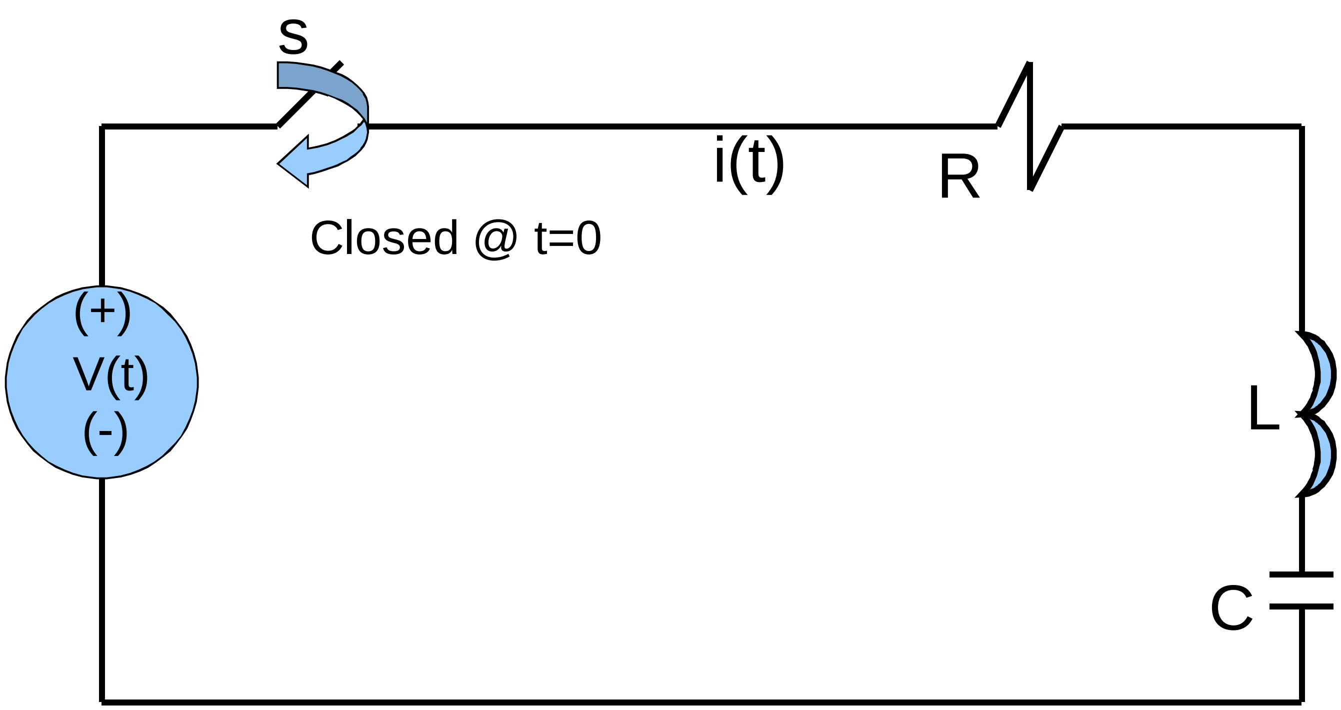
- We can again check the initial & final values of i(t) & also plot i(t) v's t .

Transient Signals: R-L Circuit Cont'd



Transient Signals: R-L-C Circuit

- A circuit is designed to include R, L, & C



Transient Signals: R-L-C Circuit Cont'd

$$Z(s) = R + sL + \frac{1}{sC}$$

$$Z(j\omega) = R + j\omega L - j\frac{1}{\omega C}$$

$$\therefore There\ is\ a\ critical\ \ freq = \omega_{cr}$$

$$= \omega_{cr} = \frac{1}{\sqrt{LC}}$$

$$\therefore Z(\omega_{cr}) = R$$

Transient Signals: R-L-C Circuit Cont'd

$$We\ can\ show\ that:$$

$$\frac{V_0}{s} = I(s)[R + sL + \frac{1}{Cs}]$$

$$\therefore I(s) = \frac{V_0 C}{LCs^2 + RCs + 1}$$

$$\equiv \frac{V_0 C}{bs^2 + as + 1}; where\ a = RC, b = LC$$

Transient Signals: R-L-C Circuit Cont'd

- We can solve the quad equation below for two values of s:

$$bs^2 + as + 1 = 0$$

$$s = \frac{-a \pm \sqrt{(a^2 - 4b)}}{2b}$$

$$\therefore if$$

$$(a^2 - 4b) > 0; real\ roots\ of\ s$$

$$(a^2 - 4b) = 0; repeated\ roots\ of\ s$$

$$(a^2 - 4b) < 0; complex\ roots\ of\ s$$

Transient Signals: R-L-C Circuit Cont'd

$$Alternatively, we\ may\ solve\ the\ quad\ eqn$$

$$s^2 LC + RCs + 1 = 0$$

$$\therefore s_1, s_2 = \frac{-RC \pm \sqrt{(R^2C^2 - 4LC)}}{2LC}$$

$$\therefore Depending\ on\ values\ of\ R, L \,\&\, C;$$

$$s_1, s_2\ may\ be\ real\ or\ complex$$

Transient Signals: R-L-C Circuit Cont'd

- We may obtain the Inverse Laplace Transform of I(s) to obtain i(t) for the two cases where roots of s are real or complex.
- Let us write the expression of I(s) as:

$$\frac{V_0 C}{1 + as + bs^2} = \frac{V_0 C}{(s - s_1)(s - s_2)}$$

Transient Signals: R-L-C Circuit Cont'd

$$\therefore \frac{I(s)}{V_0 C} = \frac{1}{(s-s_1)(s-s_2)}$$

$$By\ partial\ fractions;$$

$$\frac{1}{(s-s_1)(s-s_2)} \equiv \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

$$\therefore A = \frac{1}{s_1 - s_2}, B = \frac{1}{s_2 - s_1}; A = -B$$

Transient Signals: R-L-C Circuit Cont'd

$$\therefore \textit{By Inverse Laplace Transform;}$$

$$i(t) = V_0 CA(e^{s_1 t} - e^{s_2 t})u(t)$$

$$\textit{Clearly,}$$

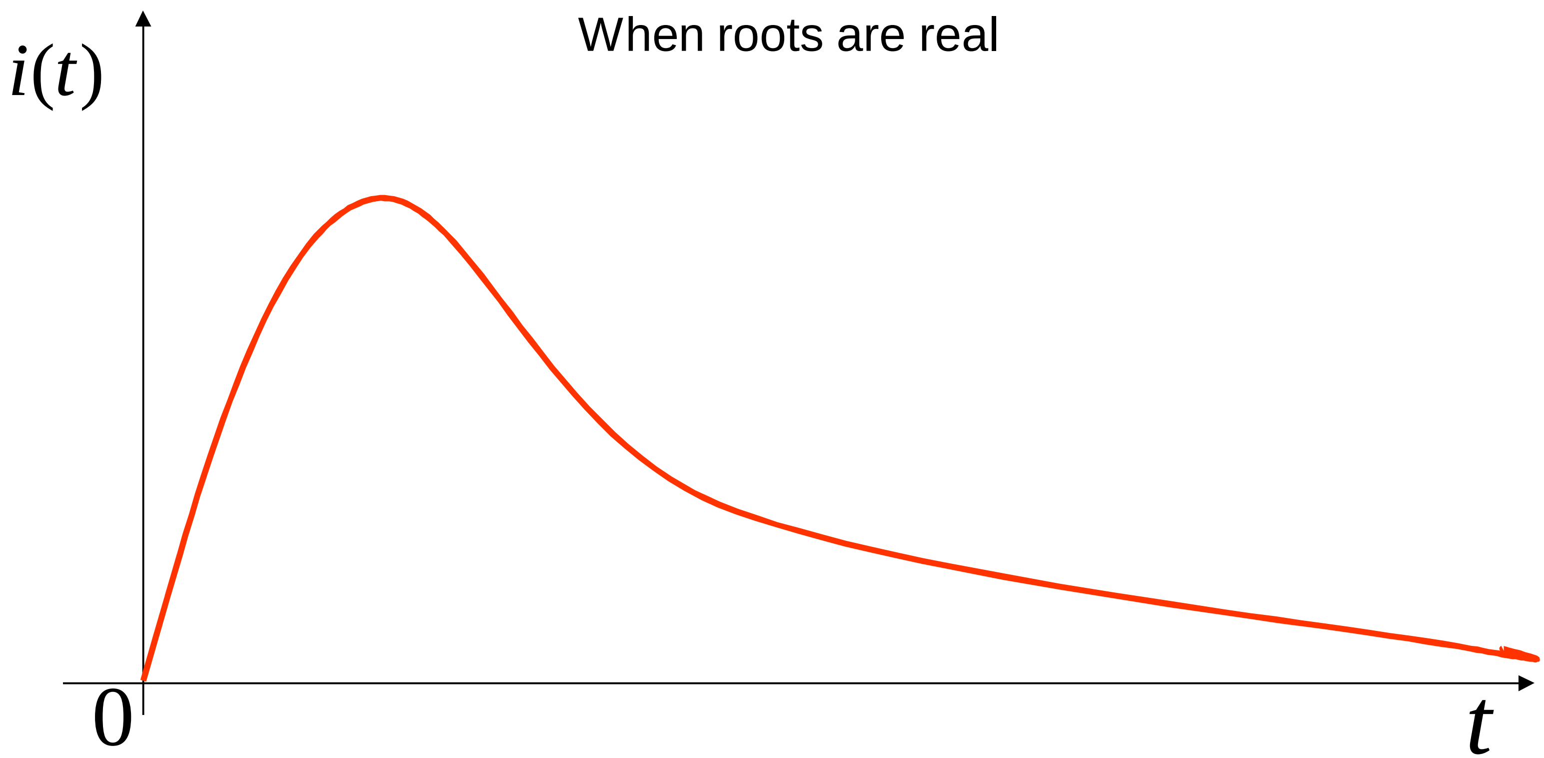
$$i(0) = 0$$

$$i(\infty) = 0; \textit{if } s_1, s_2 \textit{ have negative real parts;}$$

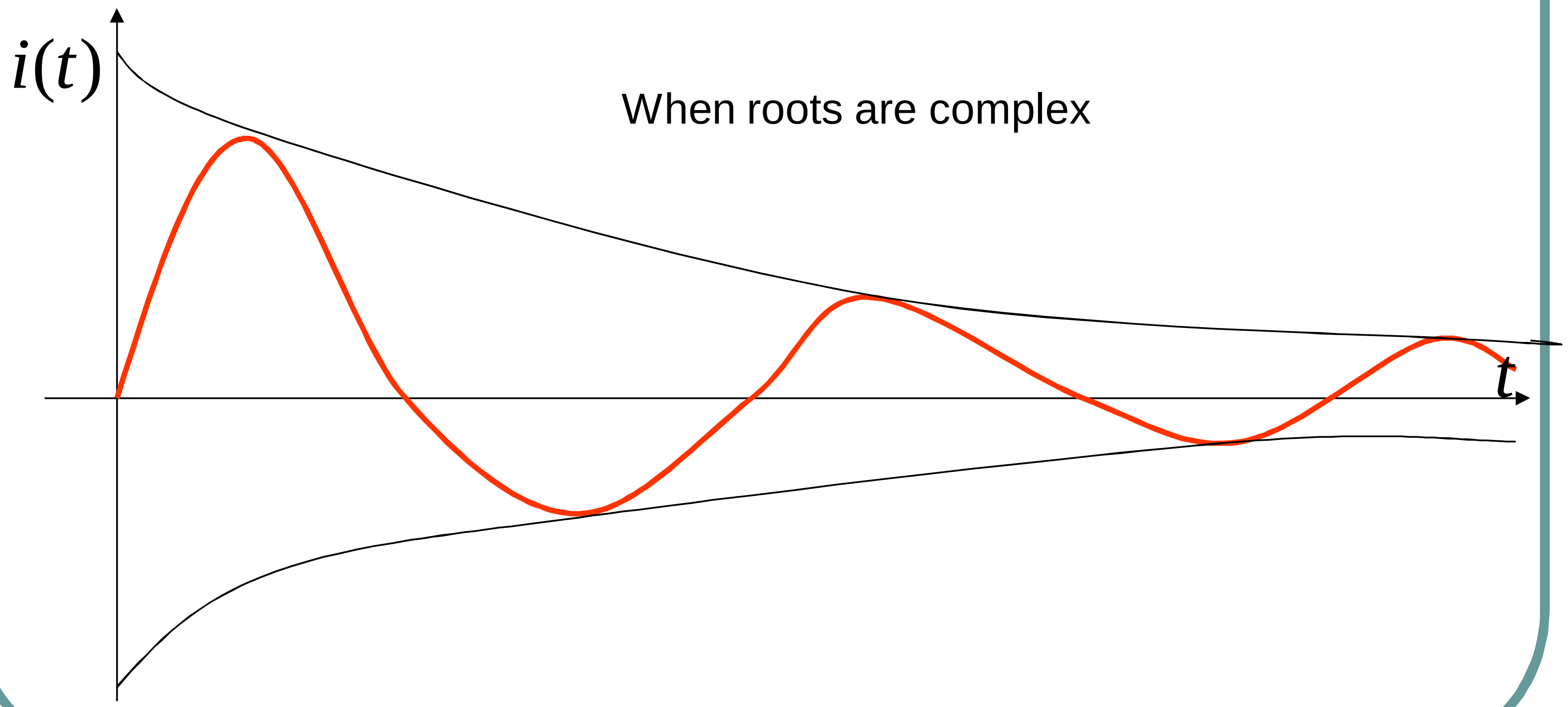
$$\textit{we can show that it is true}$$

Transient Signals: R-L-C Circuit Cont'd

- We can sketch i(t) vs. t. Its general shape is:



Transient Signals: R-L-C Circuit Cont'd

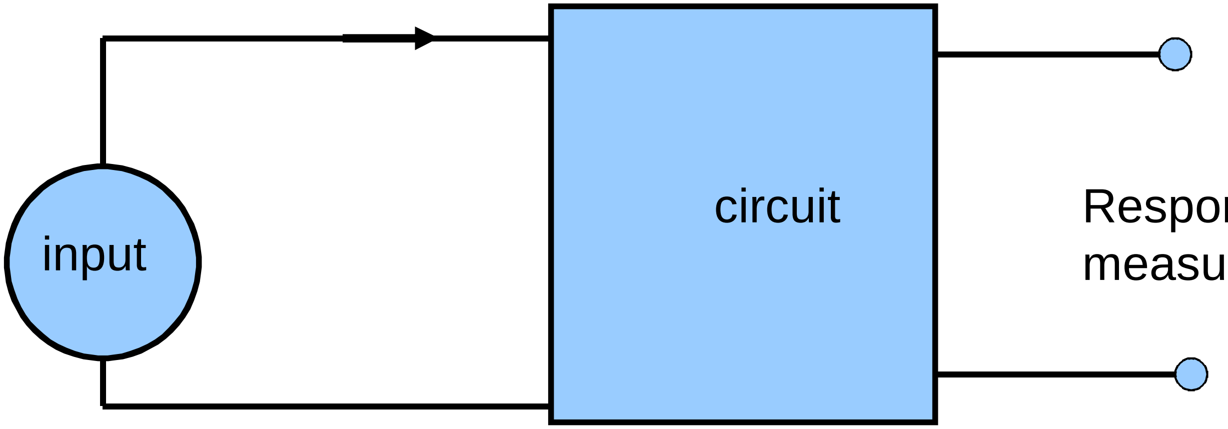


Transient Analysis of Circuits

- The above analysis of the transient response, analysed using the current i(t), used the step function as the input signal.
- It is possible to consider, in general, the transient response of any given circuit due to different input signals.
- If response to a given input is analysed, we can fairly accurately determine the circuit elements & their connections.

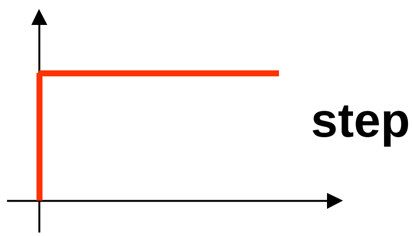
Transient Analysis of Circuits Cont'd

Circuit

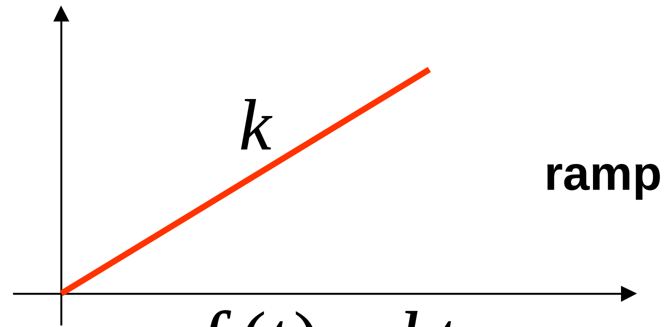


Standard Input Signals for Transient Analysis

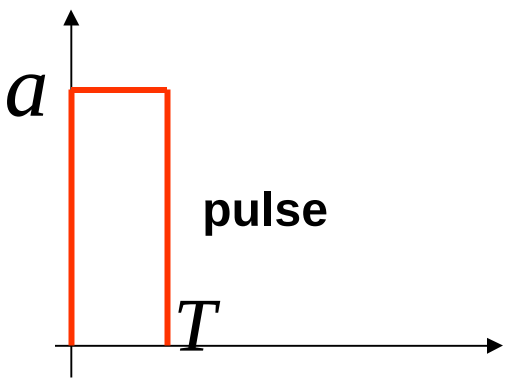
- Input Signals



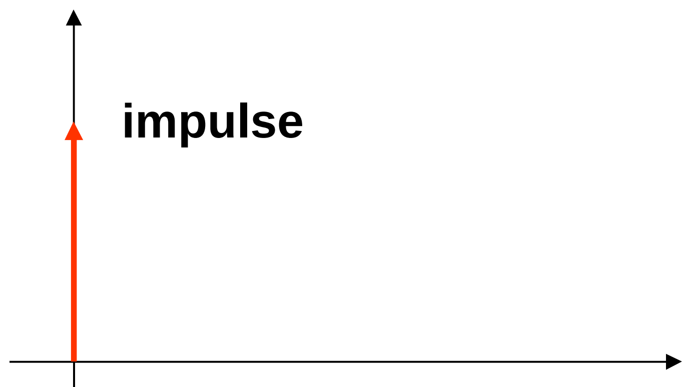
$$f(t) = u(t)$$



$$f(t) = kt$$



$$f(t) = a; 0 \le t \le T$$



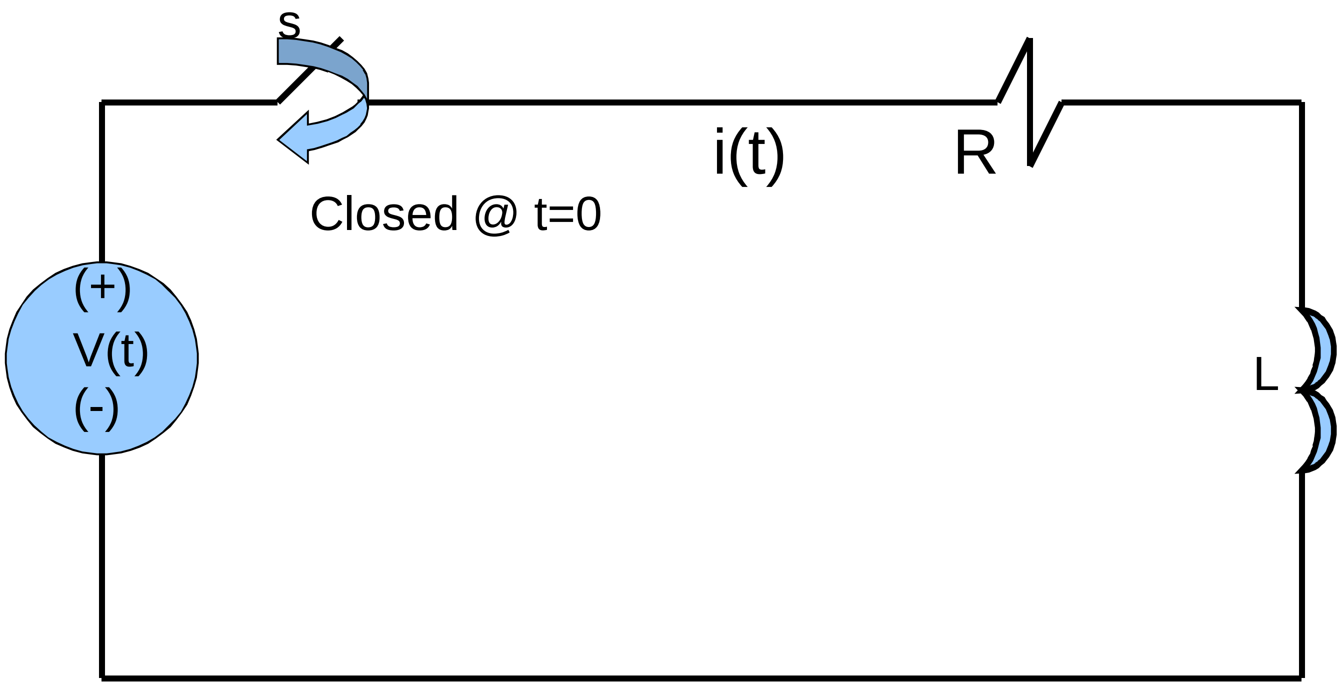
$$f(t) = \delta(t)$$

Input Signals

- It is possible to consider different inputs to the same circuit. Its response, as expected, will be different for each input.
- Consider the R-L circuit above and let the input signals be changed to
- (i) Impulse (ii) ramp (iii) pulse.

Transient Signals: R-L Circuit

- Let the voltage supply/input be an impulse.



Response of R-L Circuit to an Impulse Input

- By Laplace:

$$I(s) = \frac{V(s)}{R + sL}$$

$$But$$

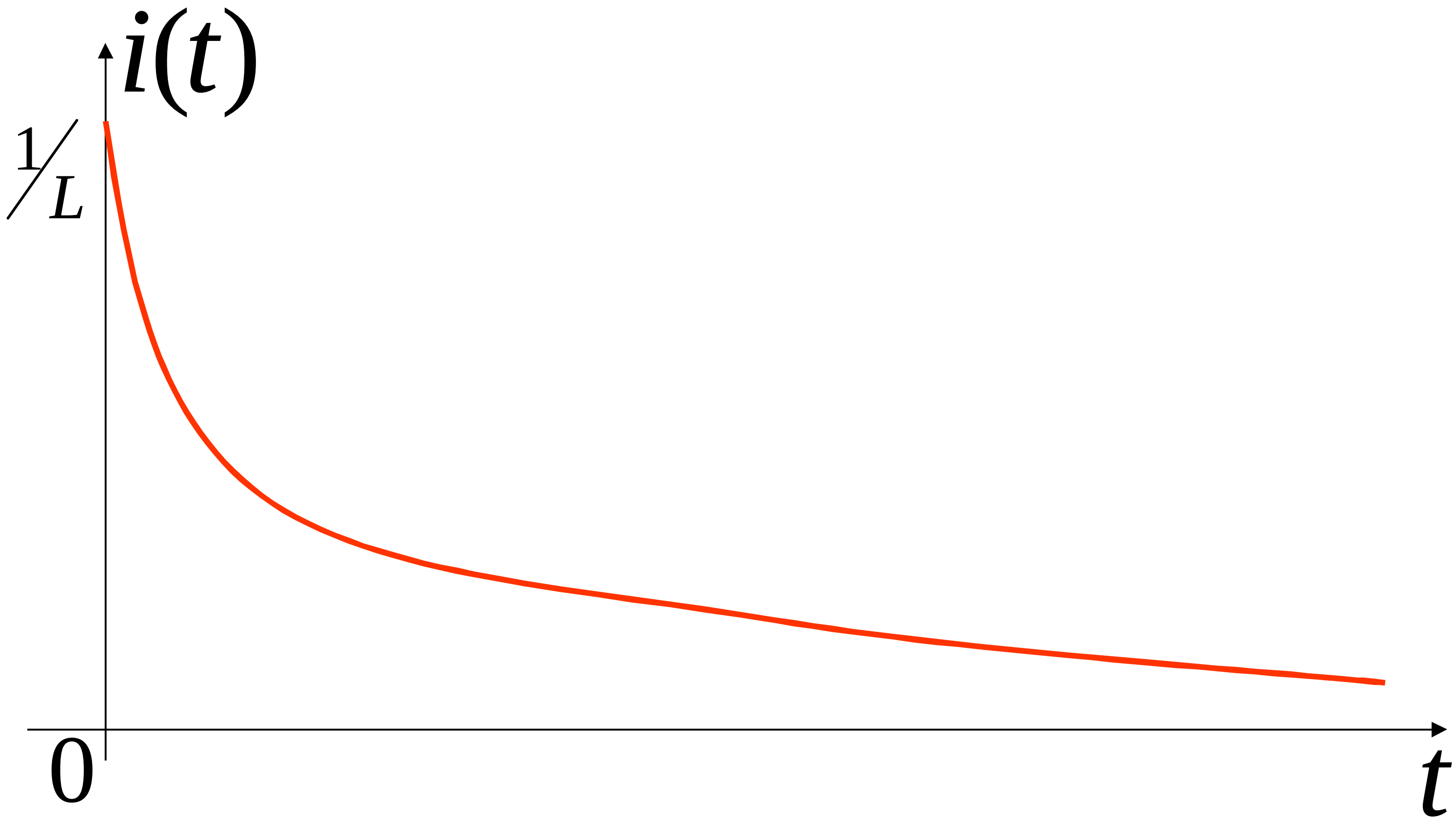
$$V(s) = Laplace\ \ Transform\ \ of\ \ an\ implse\ = 1$$

$$\therefore\ I(s) = \frac{1}{R + sL} = \frac{1/L}{s + R/L}$$

$$\therefore\ i(t) = \frac{1}{L}[e^{-Rt/L}]u(t)$$

Response of R-L Circuit to an Impulse Input Cont'd

- The transient response is:



Response of R-L Circuit to a Ramp Input

- Consider the input to be a ramp of gradient k.
- Needless to redraw the circuit:

$$\textit{Laplace Transform of a ramp is}:$$

$$\therefore I(s) = \frac{1/s^2}{R+sL} = \frac{1}{s^2(R+sL)}$$

Response of R-L Circuit to a Ramp Input Cont'd

$$\therefore \frac{1}{s^2(R+sL)} \equiv \frac{as+b}{s^2} + \frac{x}{R+sL}; partial \;\; fract.$$

$$1 \equiv (x+al)s^2 + (aR+bL) + bR$$

$$\therefore a = -L/_{R^2}, b = 1/_R, x = L^2/_{R^2}$$

$$\therefore \frac{1}{s^2(R+sL)} = \frac{1}{R^2}[\frac{R}{s^2} - \frac{L}{s} + \frac{L}{s+R/_L}]$$

Response of R-L Circuit to a Ramp Input Cont'd

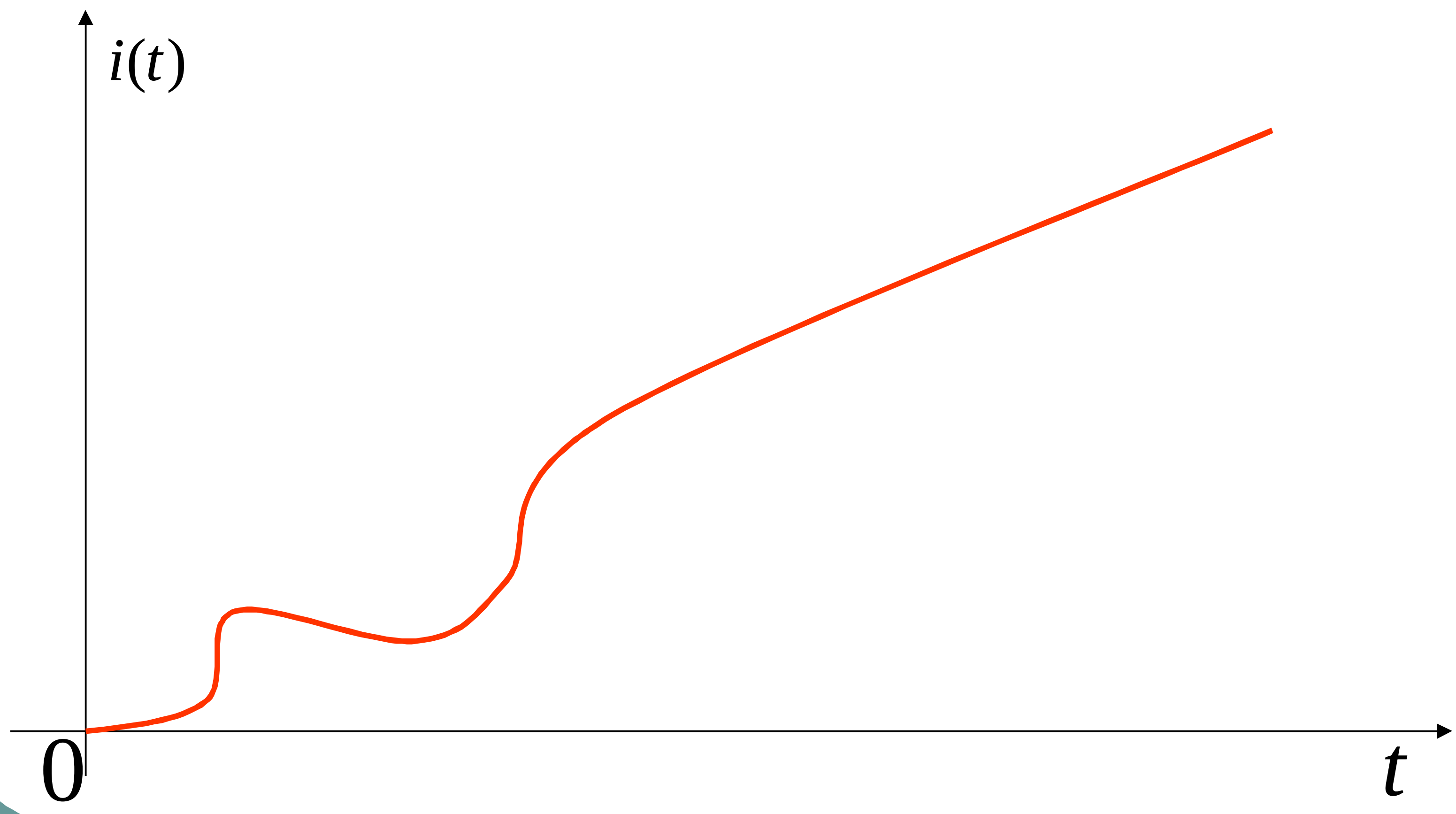
$$\therefore i(t) = \frac{1}{R^2}[Rt - L + Le^{-Rt/L}]u(t)$$

$$We\ can\ deduce\ that:$$

$$i(0) = 0,$$

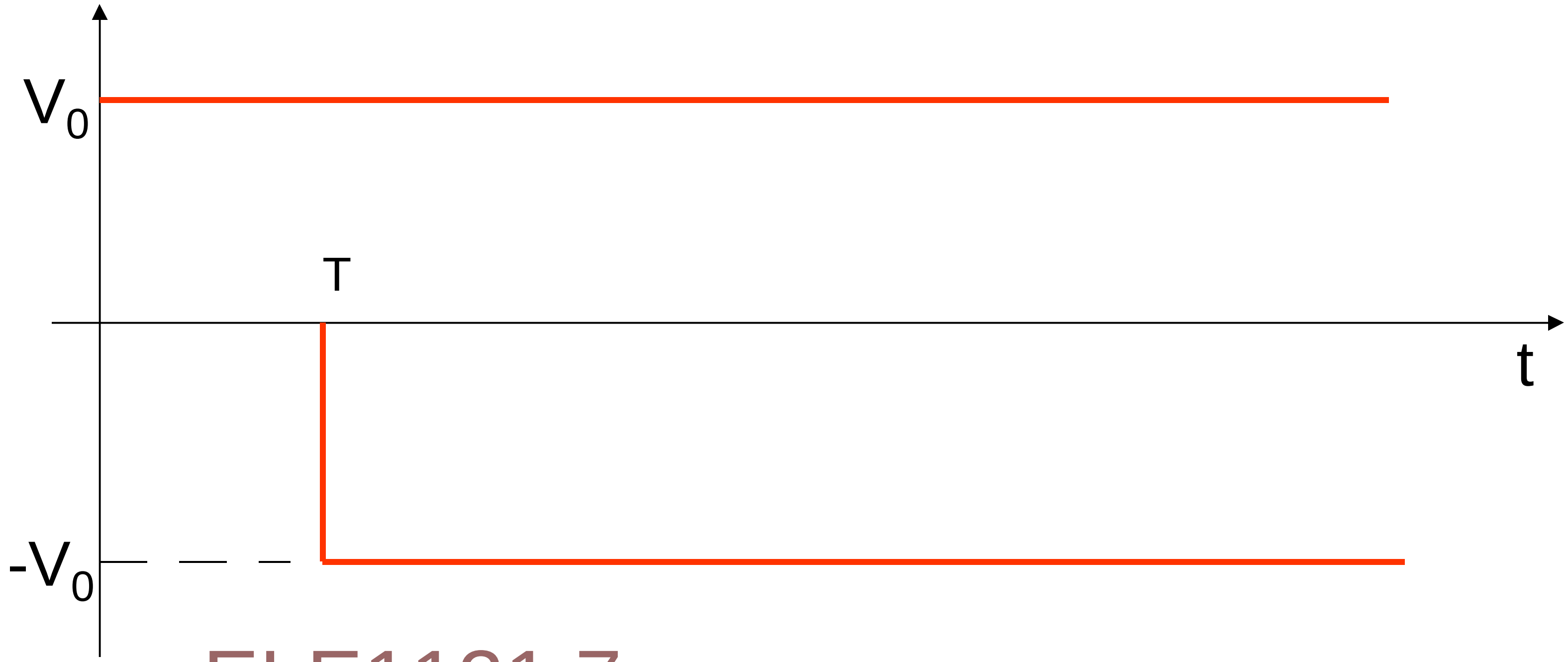
$$i(t)_{t \to \infty} = \frac{1}{R^2}[Rt] \equiv ramp$$

Response of R-L Circuit to a Ramp Input



Response of R-L Circuit to a Pulse Input

- The pulse input may be considered as two steps; where one is delayed by time T.



ELE1121 7