CSC418 - ASSIGNMENT 2 CALVIN CHENG

$\boxed{1}$ $w = -\frac{1}{\sqrt{11}}\langle 1,1,3\rangle$

$\vec{e} = \langle 1,2,2\rangle$

$\vec{g} = \langle 1,1,3\rangle$

$\vec{t} = \langle 0,1,0\rangle$

$u = \frac{t \times w}{\|t \times w\|}$

$t \times w = \begin{vmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & -\frac{3}{\sqrt{11}} \end{vmatrix}$

$= \frac{\langle -\frac{3}{\sqrt{11}}, 0, \frac{1}{\sqrt{11}}\rangle}{\sqrt{\frac{10}{11}}}$

$= \frac{3}{\sqrt{11}} - 0 + \frac{1}{\sqrt{11}}$

$u = \frac{1}{\sqrt{10}}\langle -3,0,1\rangle$

$v = w \times u$

$= \frac{1}{\sqrt{110}}\langle -1,10,-3\rangle$

$M_{wc} = \begin{bmatrix} [\vec{u}\ \vec{v}\ \vec{s}]^T & -[\vec{u}\ \vec{v}\ \vec{s}]^T \cdot \vec{e} \\ 0\ 0\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{110}} & \frac{-10}{\sqrt{110}} & \frac{3}{\sqrt{110}} \\ \frac{1}{\sqrt{11}} & \frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{\sqrt{10}} \\ -\frac{13}{\sqrt{110}} \\ \frac{9}{\sqrt{11}} \end{bmatrix}$

$= \begin{bmatrix} -\frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ -\frac{1}{\sqrt{110}} & \frac{10}{\sqrt{110}} & -\frac{3}{\sqrt{110}} & -\frac{13}{\sqrt{110}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & -\frac{3}{\sqrt{11}} & \frac{9}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. $p'_x = \frac{-f}{p_z} p_x$, $p'_y = \frac{-f}{p_z} p_y$,

$q'_x = \frac{-f}{q_z} q_x$, $q'_y = \frac{-f}{q_z} q_y$,

$m'_x = \frac{-f}{m_z} m_x$; $m'_y = \frac{-f}{m_z} m_y$

plugging in for $m = 0.5(p+q)$ ...

~~$m = 0.5(p+q)$~~ $m'_x = \frac{-f}{p_z + q_z} p_x + q_x$

(1) $\begin{bmatrix} m'_x \\ m'_y \end{bmatrix} = \frac{-f}{p_z + q_z} \begin{bmatrix} p_x + q_x \\ p_y + q_y \end{bmatrix} \Leftarrow$ $m'_y = \frac{-f}{p_z + q_z} p_y + q_y$

if we assume $m' = 0.5(p' + q')$ then:

$$\begin{bmatrix} m'_x \\ m'_y \end{bmatrix} = 0.5 \begin{bmatrix} p'_x + q'_x \\ p'_y + q'_y \end{bmatrix}$$

using computed values above:

$$\begin{bmatrix} m'_x \\ m'_y \end{bmatrix} = 0.5 \begin{bmatrix} \frac{-f \cdot p_x}{p_z} + \frac{-f \cdot q_x}{q_z} \\ \frac{-f p_y}{p_z} + \frac{-f \cdot q_y}{q_z} \end{bmatrix} = 0.5 \begin{bmatrix} \frac{-f p_x q_z - f q_x p_z}{q_z p_z} \\ \frac{-f p_y q_z - f q_y p_z}{p_z q_z} \end{bmatrix}$$

(2) $\begin{bmatrix} m'_x \\ m'_y \end{bmatrix} = \frac{-0.5f}{q_z p_z} \begin{bmatrix} p_x q_z + q_x p_z \\ p_y q_z + q_y p_z \end{bmatrix}$

we can see $(1) \neq (2)$, $\therefore m' \neq 0.5(p'+q')$, and that means the midpoint will ~~page~~ not map to the same point after the perspective projection.

~~The~~ For orthographic projection:

$p' = \alpha p$ and $q' = \alpha q$, so

$m' = 0.5(p'+q') = 0.5(\alpha p + \alpha q)$

$= 0.5\alpha(p+q)$

which is equivalent to

$m = 0.5(p+q)$ at $\alpha = 1$

$\boxed{\text{Yes for ortho}}$

3. Applying perspective projection to general line $l(u)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} P_x + u d_x \\ P_y + u d_y \\ P_z + u d_z \\ 1 \end{bmatrix} = \begin{bmatrix} P_x + u d_x \\ P_y + u d_y \\ P_z + u d_z \\ \frac{P_z + u d_z}{f} \end{bmatrix}$$

$$\cong \begin{bmatrix} \frac{f(P_x + u d_x)}{P_z + u d_z} \\ \frac{f(P_y + u d_y)}{P_z + u d_z} \\ f \\ 1 \end{bmatrix}$$

since we know two lines run parallel to one another when they intersect at $u = \infty$, we shall find the point at ∞ in the previously compted image plane:

$$\begin{bmatrix} \lim_{u\to\infty} \frac{f(P_x + u d_x)}{P_z + u d_z} \\ \lim_{u\to\infty} \frac{f(P_y + u d_y)}{P_z + u d_z} \\ \lim_{u\to\infty} f \\ \lim_{u\to\infty} 1 \end{bmatrix} = \begin{bmatrix} \lim_{u\to\infty} \frac{f P_x}{P_z + u d_z} + \lim_{u\to\infty} \frac{f \cancel{u} d_x}{\frac{P_z}{u} + d_z} \\ \lim_{u\to\infty} \frac{f P_y}{P_z + u d_z} + \lim_{u\to\infty} \frac{f d_y}{\frac{P_z}{u} + d_z} \\ f \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f \frac{d_x}{d_z} \\ f \frac{d_y}{d_z} \\ f \\ 1 \end{bmatrix}$$

as you can see, the limits:

$$\lim_{u\to\infty} \frac{f P_x}{P_z + u d_z}, \quad \lim_{u\to\infty} \frac{f P_y}{P_z + u d_z} \quad \text{and} \quad \lim_{u\to\infty} \frac{P_z}{u},$$

all equal zero. ∴ The vanishing point of the parallel lines when $u = \infty$ is independent of the points p.

so, the line's vanishing point is:

$$\frac{f}{d_z}\begin{bmatrix} d_x \\ d_y \end{bmatrix},$$

in homogenous coordinates: $\begin{bmatrix} \frac{f d_x}{d_z} \\ \frac{f d_y}{d_z} \\ f \\ 1 \end{bmatrix}$

4. We will represent the canonical view transformation using a canonical space mapping for perspective projections:

$$M_p = \begin{bmatrix} \frac{2f}{L-R} & 0 & \frac{R+L}{L-R} & 0 \\ 0 & \frac{2f}{B-T} & \frac{B+T}{B-T} & 0 \\ 0 & 0 & \frac{f+F}{F-f} & \frac{2Ff}{F-f} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$L=-1,\ R=1,\ B=-1,\ T=1,\ f=1,\ F=1001$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1002}{1000} & \frac{2002}{1000} \\ 0 & 0 & 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

psuedo depth for point p is $1.002 + \frac{2.002}{z}$

psuedo-depths:

$P\begin{bmatrix}0\\0\\-1\end{bmatrix} = 1.002 - 2.002 = -1$

$P\begin{bmatrix}0\\0\\-10\end{bmatrix} = 1.002 - 0.2002 = 0.8018$

$P\begin{bmatrix}0\\0\\-100\end{bmatrix} = 0.98198$

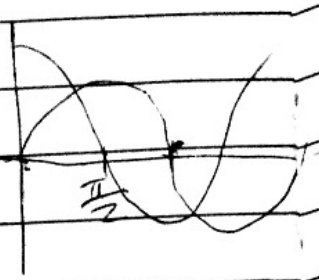
$P\begin{bmatrix}0\\0\\-1000\end{bmatrix} = 0.999998$

$\therefore$ no, the relationship between depth and pseudodepth is not linear. This is easily seen, as the equation for the pseudo-depth is not a linear function.

5. a) when $z=0$, then $\cos(\pi u) = 0$

$$\Downarrow$$

$$u = \frac{1}{2}$$



using $u = \frac{1}{2}$,

$$x(t) = a\cos(2\pi t)\sin\left(\frac{\pi}{2}\right)$$
$$= a\cos(2\pi t)$$
$$y(t) = b\sin(2\pi t)$$
$$z(t) = 0$$

~~N~~ to form another ellipse, set $y=0$, then $\sin(2\pi t) = 0$

$$\Downarrow$$

$$t = 0$$

usin $t=0$,

$$x(u) = a\cos(0)\sin(\pi u)$$
$$= a\sin(\pi u)$$
$$y(u) = \cancel{b\sin(0)} \rightarrow 0$$
$$z(u) = c\cos(u\pi)$$

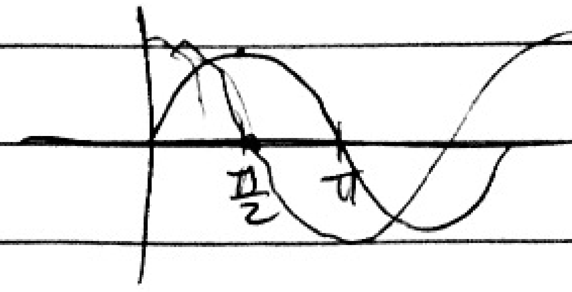
b) ~~Let S(u,t) be the surface of the ellipsoid~~

$$r_1 = \cancel{\frac{\partial S}{\partial t}} \langle x'(t), y'(t), z'(t) \rangle$$

$$= \langle -a\sin(2\pi t)2\pi, b\cos(2\pi t)2\pi, 0 \rangle$$

$$r_2 = \langle x'(u), y'(u), z'(u) \rangle$$
$$= \langle a\pi\cos(\pi u), 0, -c\pi\sin(u\pi) \rangle$$

$\vec{N} = T_1 \times T_2$



$$= \begin{vmatrix} & & \\ -2\pi a \sin(2\pi t) & 2\pi b \cos(2\pi t) & 0 \\ \pi a \cos(\pi u) & 0 & -\pi c \sin(\pi u) \end{vmatrix}$$

$$\vec{N} = \langle -2\pi^2 bc \cos(2\pi t)\sin(\pi u), -2\pi^2 ac \sin(2\pi t)\sin(\pi u), 2\pi^2 ab \cos(2\pi t)\cos(\pi u) \rangle$$

Finding the point of intersection...

$$y(t) = b\sin(2\pi t) = y(u) = 0 \Rightarrow t = 0$$
$$z(t) = 0 = z(u) = c\cos(u\pi) \Rightarrow u = \frac{1}{2}$$

$$\vec{N}\langle \overset{t=}{0}, \overset{u=}{\tfrac{1}{2}} \rangle = \langle -2\pi^2 bc(1)(1), -2\pi^2 ac(0)(1), 2\pi^2 ab(1)(0) \rangle$$
$$= \langle -2\pi^2 bc, 0, 0 \rangle$$

The normal of the surface at $t = 0$ and $u = \frac{1}{2}$ is:

$$\langle -2\pi^2 bc, 0, 0 \rangle$$

c) $x^2(t,u) = a^2\cos^2(2\pi t)\sin^2(u\pi)$
$y^2(t,u) = b^2\sin^2(2\pi t)\sin^2(u\pi)$
$z^2(t,u) = c^2\cos^2(u\pi)$

$$\frac{x^2(t,u)}{a^2} = \cos^2(2\pi t)\sin^2(u\pi)$$

$$\frac{y^2(t,u)}{b^2} = \sin^2(2\pi t)\sin^2(u\pi)$$

$$\frac{z^2(t,u)}{c^2} = \cos^2(u\pi)$$

$$\frac{x^2(t,u)}{a^2} + \frac{y^2(t,u)}{b^2} + \frac{z^2(t,u)}{c^2} = \cos^2(2\pi t)\sin^2(u\pi) + \sin^2(2\pi t)\sin^2(u\pi) + \cos^2(u\pi) =$$

$$= 1$$

$$\boxed{f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0}$$

d) $\vec{N} = \nabla f(x,y,z) = \left\langle \frac{2x}{a^2}, \frac{2y}{b^2}, \frac{2z}{c^2} \right\rangle$

when $t=0$, $u=\frac{1}{2}$; $x = a$

$y = 0$

$z = 0$

so, normal at $t=0$, $u=\frac{1}{2}$ is:

$\left\langle \frac{2}{a}, 0, 0 \right\rangle$. which is infact a scalar multiple of the normal computed in part B.

$$\left[\langle -2\pi^2 bc, 0, 0 \rangle \left(-\frac{1}{\pi^2 bca} \right) \right]$$

$$= \sin^2(u\pi)\left(\cos^2(2\pi t) + \sin^2(2\pi t)\right) + \cos^2(u\pi)$$

$$= \sin^2(u\pi) + \cos^2(u\pi)$$

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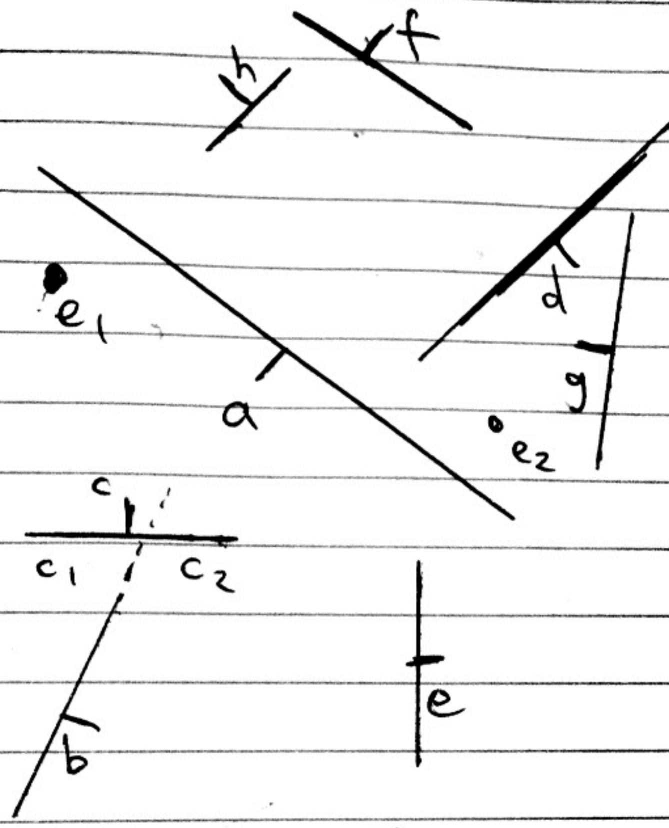
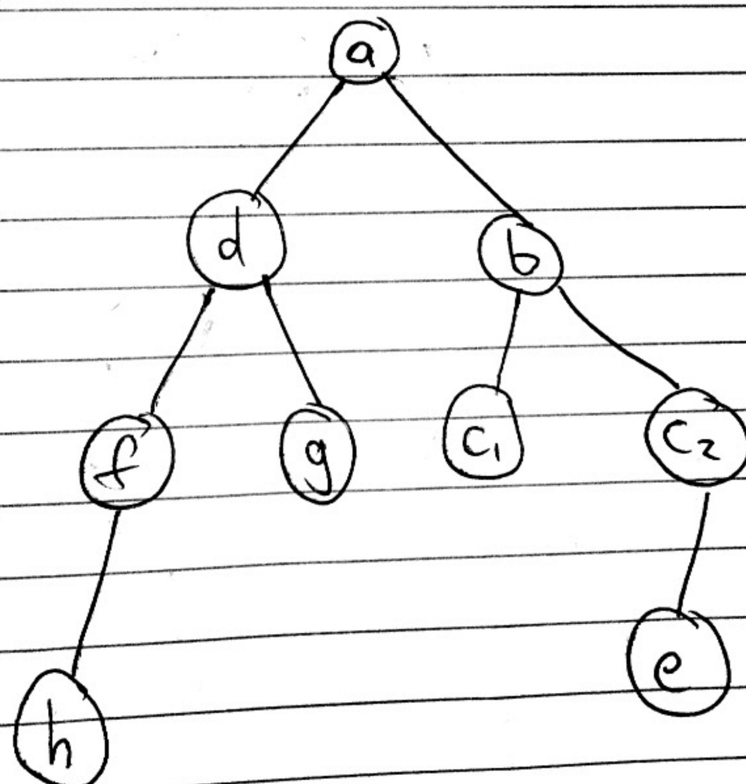
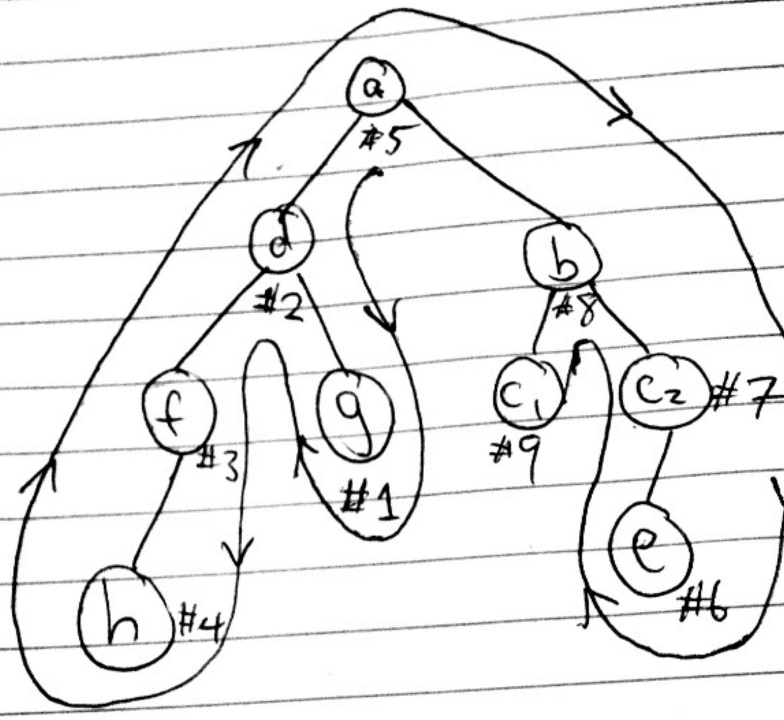


a)



(b) $\boxed{e_1}$



$\boxed{e_2}$