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1 %EGM3601 stoplight project
2
3 clc, clear all, close all;
4
5 printf("EGM3601 Solid Mechanics stoplight Design Project\n")
6 printf("Name: Caleb Gibson\nNID: ca727627")
7 disp(date())
8 printf("Due date: Friday April 22, 2022\n")
9
10 %{
11 %declare and define constants
12 F_g = 164.59; %Newton [N]
13 rho_w = 76518; %Newton per cubic meter [N/m^3] material weight density
14 ro = 0.177; %meters [mm]
15 %R_y = 3*F_g + w*L; %Newton
16 E = 200e9; %Newtons per m^2 [N/m^2]
17 L = 14; %meters [m]
18 v = 0.1; %meters [m] deflection
19 w = @(ri) rho_w*pi*(ro^2 - ri^2);
20 I = @(ri) (pi/2)*(ro^4 - ri^4);
21
22 %declare and define functions
23 v_tip = @(ri) (1/(E*I(ri)))*((-w(ri)*(14^4))/24+((14^3)*(-3*F_g+w(ri)*L))/6 ...
24 +((14^2)*(89*F_g+98*w(ri)))/2) - 0.1;
25
26 %solve for given params
27 ri_guess1 = 0.17;
28 ri_guess2 = 0.18;
29 tol = 1e-5;
30 max_iters = 20;
31 ri_actual = secant(v_tip,ri_guess1,ri_guess2,max_iters,tol);
32 printf("The inner radius for the deflection to be 0.1m is %.6f.\n",ri_actual)
33
34 %}
35
36 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
37
38 %AFTER ATTENDING DR. YAVAS'S REVIEW SESSION: It has been discovered that the max
39 % deflection for a point load at ANY POINT along the length "L" is easily given
40 %by D_MAX = ((P*a^2)/(6*E*I)) * (3*L - a), where "a" is the length from the
41 %origin to the point of the application of the point load AND "P" is the POINT
42 %LOAD.
43 % The max deflection for a distributed load ALONG THE WHOLE LENGTH of a
44 %CANTILEVERED beam of length "L" and weight distribution "w" is given by:
45 %D_MAX = (w*(L^4))/(8*E*I). IN EACH CASE, "E" is the MODULUS OF ELASTICITY, "I"
46 %is the MOMENT OF INERTIA ABOUT THE Z OR Y AXIS OF THE CROSS-SECTIONAL AREA OF
47 %THE BEAM, and the MAX DEFLECTIONS CAN BE ADDED TOGETHER to get the TOTAL MAX
48 %DEFLECTION!!!
49 % THEREFORE, (**side note: "UDL" is UNIFORM DISTRIBUTED LOAD**)
50 L = 14; %m
51 E = 207.5e9; %N/m^2; according to
52 https://matmatch.com/materials/minfm52552-astm-a595-grade-a-carbon-steel [1], average
53 Modulus of Elasticity of Grade A Structural A595 carbon steel. This material was chosen
54 because it has been consistently seen as being used by different Departments of
55 Transportation (DOTs) as one of their materials of choice for the construction of traffic
56 light pole arms.
57 P = 16.7829*9.8; %N; according to
58 https://www.hillsboroughcounty.org/en/newsroom/2019/10/31/traffic-signals [2], average
59 3-light aluminum traffic signals weigh 37 lbs, which equates to 16.7829 "kilograms"
60 according to Google's built in unit converter (these are actually Newtons, as when we
61 measure the weight of something we are NOT measuring its MASS -- we are measuring its
62 WEIGHT, which is a FORCE, not a MASS, and therefore we must measure it in terms of
63 NEWTONS, NOT KILOGRAMS)
64 rho_w = 7850*9.8; %N/m^3; according to [1], average density of A525 steel in kilograms
65 per cubic meter, multiplied by 9.81 m/s^2 - the acceleration due to gravity - in order to
66 convert it to Newtons per cubic meter.
67 %FROM PAGE 6 OF THE NYDOT TRAFFIC DESIGN MANUAL [3], IT IS KNOWN THAT THE DIA-
68 %-METER OF THE BASE OF THE MAST ARM IS 1.75 M FOR A STOPLIGHT OF THIS LENGTH (14
69 % m =~ 45.9318 feet), SO THE TOTAL DIAMETER OF THE POLE ARM can be taken as
70 %being HALF this base diameter, which is 1.75/2 m = *0.875 METERS.*

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59 %[1]https://matmatch.com/materials/minfm52552-astm-a595-grade-a-carbon-steel
60 %[2] https://www.hillsboroughcounty.org/en/newsroom/2019/10/31/traffic-signals
61 %[3] https://www.dot.ny.gov/portal/pls/portal/mexis_app.pa_eiEb_admin_app.show_pdf?id=13512
62
63 d_O = 0.875; %m
64 rO = d_O/2; %m
65
66
67
68 %SINCE we are calculating for the geometry such that the MAX DEFLECTION <= 100
69 %MM, ALSO KNOWN AS 0.1 M, CAN MAKE THE WHOLE DEFLECTION FUNCTION A FUNCTION OF
70 %INNER RADIUS "ri" AND REMOVE THE EI FROM THE DENOMINATOR TO SIMPLIFY THE
71 %EQUATION. (**side note: "rO" = outer radius)
72 % D_MAX_TOTAL = 0.1 = D_MAX_UDL + D_MAX_P1 + D_MAX_P2 + D_MAX_P3, THEREFORE,
73 % 0 = D_MAX_UDL + D_MAX_P1 + D_MAX_P2 + D_MAX_P3 - 0.1, THEREFORE
74 %ri_calculatn = 0 = 3wL^4 + 4PL1^2(3L-L1) + 4PL2^2(3L-L2) + 4PL3^2(3L-L3) -
75 % 0.1EI , WHERE "I" = (PI/4)*(rO^4 - ri^4) AND "w" = rho_wpi(rO^2-ri^2)
76 ***SPECIAL NOTE TO SELF: here, "I" refers to the second PLANAR moment of area,
77 %NOT the second polar moment of area (also known as the PLANAR MOMENT OF
78 %INERTIA, NOT the polar moment of inertia). This can be confusing, since dif-
79 %ferent professions and sub-specializations use "moment of inertia" and "second
80 % moment of area" to refer to different moments. However, THE MOMENT OF INERTIA
81 %FOR CALCULATING DEFLECTION IS THE *PLANAR* MOMENT OF INERTIA, NOT the polar
82 %moment of inertia. [2]
83
84 %[2]
85 https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html#:~:text=Area%20Moment%20o
86 f%20Inertia%20or,bending%20and%20stress%20in%20beams.
87
88 %I_y = @(ri) (pi/4)*(rO^4-ri^4);
89
90 w = @(ri) rho_w*pi*(rO^2-ri^2);
91 ri_calculatn = @(ri) ((3*w(ri)*L^4) + ((4*P*(5^2))*(3*L-5)) + ((4*P*L*(9^2))*(3*L-9)) +
92 ((4*P*L*(13^2))*(3*L-13))) - (24*0.1*E*(pi/4)*(rO^4 - ri^4));
93
94 %ri_calculatn = @(ri) (w(ri)*L^4)/(8*E*I_y(ri)) + ((4*P*(5^2))*(3*L-5))/(6*E*I_y(ri)) +
95 ((4*P*L*(9^2))*(3*L-9))/(6*E*I_y(ri)) + ((4*P*L*(13^2))*(3*L-13))/(6*E*I_y(ri)) - 0.1;
96
97 %(**special note: L1=5m, L2=9m, L3=13m**) **"24" is LCD of equatn.
98 %NOW, HAVING THE EQUATION WE NEED, WE CAN USE OUR "secant" SOLVER FUNCTION TO
99 %SOLVE FOR THE ROOTS OF THE "D_MAX_TOTAL" EQUATION SUCH THAT THE MAX DEFLECTION
100 %IS <=100MM ALSO KNOWN AS 0.1M.
101
102 tol = 1e-3;
103 max_iters = 1000;
104 ri_guess=0.4; %m
105 ri_guess2=0.3; %m
106 ri_actual = secant(ri_calculatn,ri_guess,ri_guess2,max_iters,tol);
107
108 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
109
110 %NOW, we can CALCULATE *STEPS 4 AND 5 AND 6* of the project. Using what we know
111 %now, we can calculate the force acting along on the beam due to a wind gust
112 %with a velocity of 50 m/s. The total DRAG FORCE "D" due to the wind gust is the dynamic
113 pressure
114 % dynamic pressure due to the air being forced against it, given by
115 %p = CD*(0.5*rho_air*v_air^2), where "CD" is the drag coefficient of the shape
116 %(in our case, a cylinder), "rho_air" is air's density at standard temperature
117 %and pressure (STP) which is 1.225 kg/m^3, using the International Standard
118 %Atmosphere (ISA) values of temperature = 15 degrees Celsius and pressure
119 %= 1 atm = 101325 Pa at sea level (according to macinstruments.com [^1]), all
120 %that TIMES the PLATFORM AKA PROJECTED AREA of the SURFACE IT IS ACTING UPON
121 %(in our case, the HORIZONTAL of the stoplight arm, MINUS THE END [so
122 %OUTER DIAMETER TIMES TOTAL LENGTH, or "D_OUTER" times "L"]).
123
124 %[^3] https://macinstruments.com/blog/what-is-the-density-of-air-at-stp/
125
126 %*Note to self: the reason we only use "outer diameter" instead of "pi*diamtr/2"
127 % is because the "projected area" is not the area if we were to unwrap the

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123 %surface the dynamic fluid is blowing against, but rather simply our view of it
124 %from the singular direction (in this case) that the dynamic fluid is flowing
125 %from, which in this case would be the same no matter which direction it came at
126 % the cylinder at, as long as it is still normal to points on the wall - not the
127 % bases - of the cylinder. Henceforth, OUTER DIAMETER WILL BE REFERRED TO AS
128 % "d_o".
129 %"CD" ~= 0.5 for a cylinder.
130 %THEREFORE, "D" = 0.5*(0.5*1.225*50^2)*(d_o*L) = 0.25*1.225*2500*(0.875*14). The
131 %DISTRIBUTED FORCE DUE TO THE WIND, "w_DRAG", is this drag force "D" per unit
132 %length, in essence "w_DRAG" = "D" divided by the length "L", in another word,
133 %"w_DRAG" = D/L = (0.5*(0.5*1.225*50^2)*(d_o*L))/L, so = 0.5*(0.5*1.225*50^2)*d_o,
134 % and since d_o = 2r_o = 2*0.127 = 0.254 [m], "w_DRAG" = 0.25*1.225*2500*0.254
135
136 w_DRAG = 0.25*1.225*2500*d_o; % (kg/m^3) (m^2/s^2) (m)=[kg/s^2], since [N]=[kgm/s^2], then
    [kg/s^2]=[ (kgm/s^2) /m]=[ (N) /m]
137 printf("=====\n")
138 printf("NOW APPLYING A WIND GUST OF 50 M/S PARALLEL TO THE Y-AXIS, NORMAL \n\
139 TO THE Z AND X AXES ACTING ALONG THE ENTIRETY OF THE BEAM:\n")
140 printf("The total drag force acting on the beam is %.4f N.\n", w_DRAG*L)
141 printf("The drag force per unit length acting on the beam is %.4f N/m.\n", w_DRAG)
142 printf("The dynamic pressure due to drag is %.4f Pa.\n", (w_DRAG/d_o))
143
144 %the total moment acting in the "y" direction, normal to both the mast arm as
145 %well as the vertical upright pole it is attached to (acts as the "z" axis), is
146 %equivalent to the magnitude force times the magnitude (in other words the
147 %length) of the moment which extends out to the point upon which it acts, which,
148 % since it is a distributed force, can be modeled as having its total force -
149 %which is "D" - as acting on the centroid of its (w_DRAG's) application to the
150 %beam. THEREFORE, half the length of its (the UDL's) application distance, which
151 % since it acts across the whole length of the beam is L over 2 {L/2}.
152 %=> D(L/2). MOMENT ALONG THE *X* AXIS is given by the weight of the beam acting
153 % on its centroid (the distance to which is the moment arm), plus each point
154 % load times its respective moment arm.
155
156 M_z = -(w_DRAG*L)*L/2; % <- this is actually M in y, but the PowerPoint has it notated as
    M in z, so to be consistent I have notated it as M in z as well; but according to our
    reference system, it is actually M in y
157 M_y = -(w(ri_actual)*L/2 + P*27); %since M_y = w*L/2 + P*5 + P*9 + P*13, the "P" can just
    be factored out of the latter 3/4ths of the equation, to reduce math operations (MOPS),
    thereby reducing overall computational complexity (albeit only minimally)
158
159 %it is worth noting that although traditionally, the sign of every moment in the M_x
    equation would be positive, since each one causes the arm to rotate counterclockwise, here
    we take clockwise to be our positive convention, since we denote to the left of "x" as
    positive (in another word, LEFT OF ORIGIN is our POSITIVE direction along the x axis by
    our convention, so we must use clockwise along x as our positive moment convention along
    x; our upwards and outwards [z and y] conventions are the same as traditional however, so
    moment need not be converted to negative counterclockwise positive clockwise for these 2
    cardinal directions)
160 %now to find the combined stress-state at the critical cross-section of the
161 %pole, i.e. where the moment is maximized, which is at the base of the beam
162 %(just before the reaction forces act where it attaches to the rigid vertical light pole).
    Therefore, calculate the
163 %bending moment at different points (which are essentially infinitesimally small
164 %areas on the cross-section) of the cross-section. In order to calculate the max
165 % stresses experienced at these points due to the bending moments (which are dif
166 %-ferent along the y and z axes, since the bending moment in z is only due to the wind
    force blowing against the mast arm whereas the bending moment in x is due to the weight
    of the mast arm acting along its centroid of its application as well as the weights of
    each stoplight acting at a moment arm of the length from the base at which they act).
167 %Therefore, the bending stresses and shear stresses at the outermost points along the wall
    (of the mast arm) must be calculated, and each one respectively must be treated as most
    critical or not so by comparing its stresses with the others and then seeing which one is
    the mallest (since this is the most critical stress, since it will be able to resist only
    this amount of stress, which is smaller than all the other stresses). Then we have to
    calculate which stress is the greatest and whether or not it will break the material
    (because it exceeds the ultimate tensile strength of the material, which in our case is
    Structural A992 steel, which has an yield strength of 345 MPa and an ultimate strength of
    450 MPa.
168 %equation for max normal bending stress in y:bsy=-(M_z*c)/Iz; in z:bsz=-(M_y*c)/Iz

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169 %equation for max shear stress in y: tau_y = (V*Q_y)/(I_y*t); in z: tau_z = (V*Q_z)/(I_z*t)
170 %I_z = (pi/2)*(rO^4-ri^4) = I_y = I; Q_z = Q_y = A*distance to centroid normal to axis; the
    geometric center (centroid) for a half-circle is
    (pi*(1/3)*(rO^3-rO^2)/(pi*(rO^2-ri^2)/2) = (2/3)*((rO^3-ri^3)/(rO^2-ri^2)), therefore since
    Q_max = Q for the whole half-circle area, then the centroid will be:
171 %half_circ_centroid = (2/3)*((rO^3-ri_actual^3)/(rO^2-ri_actual^2));
172 %And therefore, Q will be this times the area of the half circle, (pi/2)*(rO^2-ri^2), => Q
    = (pi/3)*(rO^3-ri_actual^3)
173
174 Q = (2/3)*(rO^3-ri_actual^3);
175 %shear force in z direction is wL+3P; shear force in y direction is total drag force;
    thickness "t" is rO-ri; so t = rO-ri_actual:
176
177 t = 2*(rO-ri_actual);
178
179 %Therefore, we have
180
181 I = (pi/4)*(rO^4-ri_actual^4);
182 bsy = (M_z*rO)/I;
183 bsz = (M_y*rO)/I;
184 total_flexure = abs(bsy) + abs(bsz);
185 tau_y = ((w(ri_actual)*L+3*P)*Q)/(I*t);
186 tau_z = ((w_DRAG*L)*Q)/(I*t);
187 printf("=====\n")
188 printf("The maximum bending stress in the y direction of the cross-sectional \
189 area is %.8f Pa.\n", bsy)
190 printf("The maximum bending stress in the z direction of the cross-sectional \
191 area is %.8f Pa.\n", bsz)
192 printf("The magnitude of the total flexure of the cross-sectional area is %.8f \
193 Pa.\n", total_flexure)
194 printf("The maximum shear stress in the y direction of the cross-sectional area\
195 is %.8f Pa.\n", tau_y)
196 printf("The maximum shear stress in the z direction of the cross-sectional area\
197 is %.8f Pa.\n", tau_z)
198
199 stresses = [bsy; bsz; total_flexure; tau_y; tau_z];
200 max_stress = 0;
201
202 for i = 1:length(stresses)
203     if abs(stresses(i)) > abs(max_stress)
204         max_stress = stresses(i);
205     else
206         continue
207     endif
208 endfor
209 min_stress = max_stress;
210 for j = 1:length(stresses)
211     if abs(stresses(j)) < abs(min_stress)
212         min_stress = stresses(j);
213     else
214         continue
215     endif
216 endfor
217
218 printf("\nThe MOST CRITICAL STRESS is %.8f Pa.\n", max_stress)
219
220 %printf("The most critical of these stresses is %s.\n", min_stress)
221
222 %Now, it must be determined whether this most critical stress exceeds the ultimate tensile
    strength of the material selected, Structural A992 steel.
223
224 uts_A595_grade_a = 450e6; %N/m^2 aka Pa
225 ys_A595_grade_a = 380e6; %N/m^2 aka Pa
226 if abs(uts_A595_grade_a) > abs(max_stress)
227     printf("The material will NOT FAIL due to the stress.\n")
228     failure = 0;
229 else
230     printf("FAILURE!! The material will fail at this stress load.\n")
231     failure = 1;
232 endif

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233
234 if failure == 0
235     FS = abs(uts_A595_grade_a/max_stress); % Factor of Safety, equal to the 'Ultimate
        Tensile Strength' divided by the 'Critical stress'
236     printf("\nThe Factor of Safety (F.S.) of the material is %.6f.\n", FS)
237 else
238     printf("The factor of safety is less than 1, therefore either the pole needs to be
        re-designed or the selected material needs to be stronger.\n")
239 endif
240
```