APPENDIX C

SLOPES AND DEFLECTIONS OF BEAMS

Simply Supported Beam Slopes and Deflections			
Beam	Slope	Deflection	Elastic Curve
$ \begin{array}{c cccc} v & \mathbf{P} & \underline{L} \\ \hline 2 & \mathbf{V} & \underline{L} \\ \theta_{\text{max}} & v_{\text{max}} \end{array} $	$\theta_{\text{max}} = \frac{-PL^2}{16EI}$	$v_{\text{max}} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI} (3L^2 - 4x^2)$ $0 \le x \le L/2$
θ_1 θ_2 x	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\Big _{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL} (L^2 - b^2 - x^2)$ $0 \le x \le a$
v θ_1 M_0 x	$\theta_1 = \frac{-M_0 L}{6EI}$ $\theta_2 = \frac{M_0 L}{3EI}$	$v_{\text{max}} = \frac{-M_0 L^2}{9\sqrt{3} EI}$ at $x = 0.5774L$	$v = \frac{-M_0 x}{6EIL} \left(L^2 - x^2 \right)$
v L w v d	$\theta_{\rm max} = \frac{-wL^3}{24EI}$	$v_{\text{max}} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI} (x^3 - 2Lx^2 + L^3)$
v w θ_2 x L d	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v \bigg _{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{\text{max}} = -0.006563 \frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI} (16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI} (8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
v w_0 v	$\theta_1 = \frac{-7w_0 L^3}{360EI}$ $\theta_2 = \frac{w_0 L^3}{45EI}$	$v_{\text{max}} = -0.00652 \frac{w_0 L^4}{EI}$ $\text{at } x = 0.5193L$	$v = \frac{-w_0 x}{360EIL} (3x^4 - 10L^2 x^2 + 7L^4)$

Cantilevered Beam Slopes and Deflections Deflection Elastic Curve Beam Slope $\theta_{\text{max}} = \frac{-PL^2}{2EI} \qquad v_{\text{max}} = \frac{-PL^3}{3EI}$ $v = \frac{-Px^2}{6FL} (3L - x)$ $v = \frac{-Px^2}{12EI} (3L - 2x) \quad 0 \le x \le L/2$ $\theta_{\text{max}} = \frac{-PL^2}{8EI}$ $v_{\text{max}} = \frac{-5PL^3}{48EI}$ $v = \frac{-FA}{12EI} (3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI} (6x - L) \quad L/2 \le x \le L$ $\theta_{\text{max}} = \frac{-wL^3}{6EI}$ $v_{\text{max}} = \frac{-wL^4}{8EI}$ $v = \frac{-wx^2}{24EI} (x^2 - 4Lx + 6L^2)$ $\frac{\partial c}{\partial t} \theta_{\max}$ $\frac{\partial c}{\partial t} \theta_{\max}$ $\theta_{\max} = \frac{M_0 L}{EI}$ $v_{\max} = \frac{M_0 L^2}{2EI}$ $v = \frac{-wx^2}{24FL} \left(x^2 - 2Lx + \frac{3}{2}L^2 \right)$ $\theta_{\text{max}} = \frac{-wL^3}{48EI} \qquad v_{\text{max}} = \frac{-7wL^4}{384EI}$ $v = \frac{-wL^3}{384EI} (8x - L)$ $L/2 \le x \le L$ $\theta_{\text{max}} = \frac{-w_0 L^3}{24EI} \qquad v_{\text{max}} = \frac{-w_0 L^4}{30EI} \qquad v = \frac{-w_0 x^2}{120EIL} (10L^3 - 10L^2 x + 5Lx^2 - x^3)$