PHY 3650: Quantum Information Processing

Spring 2024

Homework Set #4 - Due on March 4, 2024

1. **(10)** Construct a matrix representation for the two-qubit circuit shown below. *Hint*: Start by defining the two-qubit computational basis states and the action of the gates on these states.

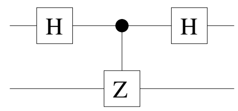

Figure 1.

2. Consider the three-qubit circuits in the figure shown below.

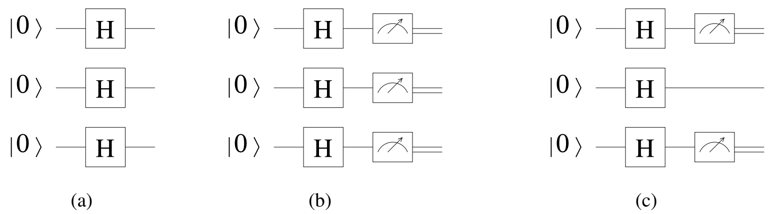

Figure 2.

   a) **(10)** Let $|000\rangle$ be the input state. Find the output state for the circuit in panel (a) of Fig. 2.

   b) **(10)** Find the probability that a measurement in the computational basis, as shown in panel (b) of Fig. 2, obtains the state $|010\rangle$. *Hint*: Define a projector operator for the state $|010\rangle$ and use it to compute the probability amplitude.

   c) **(10)** Find the probability that a measurement in the computational basis, as shown in panel (c) of Fig. 2, obtains the value 1 for the bottom qubit. *Hint*: Define a suitable project operator that only acts on the bottom qubit to produce the desired output value for that qubit.

3. Consider the two-bit function $f(x)$ defined by the truth table

x	$f(x)$
00	01
01	10
10	10
11	01

   a) **(10)** Construct a classical gate (reversible or irreversible), composed of Boolean gates, that evaluates this function.

   b) **(20)** Now construct a quantum gate that evaluates this function. *Hint*: You can build it using classical elementary gates but they need to be reversible. You will need two ancilla qubits; check the approach used in Deutsch's algorithm to turn a Boolean function into a unitary gate.

   c) **(10)** Obtain a unitary matrix $\hat{U}_f$ that implements the quantum gate. *Hint*: It is 16×16 matrix; it can be broken down into sixteen blocks, with each block being a 4×4 matrix. But only four of the sixteen blocks are non trivial.

   d) **(10)** Using this matrix, find the output state for the input state

$$|\Psi\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle).$$

   e) **(10)** Repeat the calculation in item d) for the input state $|\Psi\rangle = \hat{H}\otimes\hat{H}\,|00\rangle$, where $\hat{H}$ denotes a Hadamard gate (i.e., a Hadamard gate is applied to each qubit).