PHY 3650: Quantum Information Processing

Spring 2024

Homework Set #5 - Due on March 25, 2024

In this homework set you will make use of Qiskit to solve Problems 3, 4, and 5. When you submit your solutions for these problems, follow closely the instructions provided below.

1. **(10)** In class, when discussing teleportation, we used the Bell state $|B_{00}\rangle$ as the entangled resource shared by Alice and Bob. Find teleportation circuits appropriate for the other Bell states, namely, $|B_{01}\rangle$, $|B_{10}\rangle$, and $|B_{11}\rangle$.
2. **(10)** Write the following two-qubit states in a Bell basis:
   a) $|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle)$
   b) $|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$
3. **(40)** Using Qiskit, build a quantum circuit that implements a QFT (Quantum Fourier Transform) for a register containing 4 qubits. Your construction should include the following steps:
   - qubits numbered from 0 to 3;
   - an ordered list of gates (including gate type and which qubits the gate acts on);
   - a diagram showing all the gates in the circuit;
   - a brief explanation of the role played by all stages of the circuit;
   - the Qiskit code defining the circuit;
   - results of a few experiments to verify that the output probability distribution is consistent with what is expected from a QFT.
4. **(30)** Based on the results you obtained in Problem 3, build a circuit that implements the inverse QFT for a register containing 4 qubits. Your construction should include the following steps:
   - qubits numbered from 0 to 3;
   - an ordered list of gates (including gate type and which qubits the gate acts on);
   - a diagram containing all the gates in the circuit;
   - the Qiskit code defining the circuit;
   - results of a a few experiments to verify that the output probability distribution is consistent with what is expected from an inverse QFT.
5. **(10)** Show that the circuits obtained for Problems 3 and 4 (QFT and its inverse), when concatenated, produce the equivalent of an identity circuit. *Hint*: run experiments and show that, for any input state, an output state is equal to the input state.