Unit III Exam

Reviewer

Mathematics 53

Institute of Mathematics (UP Diliman)

Using the Mean Value Theorem, show that for the function $f(x) = \dfrac{x^2 + 4x}{x - 7}$, there exists a c in the interval $(2, 6)$ for which $f'(c) = -\dfrac{72}{5}$.

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Is $f(x)$ differentiable in $(2,6)$?

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$$\lim_{x \to \pm\infty} \frac{x^3 - 4}{x^2 - 2}$$

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$$\lim_{x \to \pm\infty} \frac{x^3 - 4}{x^2 - 2} = \lim_{x \to \pm\infty} \frac{x^3 - 4}{x^2 - 2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

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V.A. $x = \sqrt{2}$ and $x = -\sqrt{2}$

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O.A.

$$\begin{array}{r|l} & \quad\quad\quad\;\; x \\ \hline x^2 - 2 & \; x^3 \quad\quad\quad - 4 \\ & -x^3 + 2x \\ \hline & \quad\quad\; 2x \end{array}$$

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x \\
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\cline{2-2}
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$$\lim_{x \to \infty} \frac{2x - 4}{x^2 - 2}$$

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O.A. $y = x$

$$
\begin{array}{r|l}
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C.N. $x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$

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$$f'(x) = \cos x - \sin x = 0 \qquad \Longrightarrow \qquad \cos x = \sin x$$

$$f\left(-\frac{7\pi}{4}\right) = f\left(\frac{\pi}{4}\right) = \sqrt{2} \qquad f\left(\frac{-3\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = -\sqrt{2} \qquad f(\pm 2\pi) = 1$$

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C.N. $x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$

$$f'(x) = \cos x - \sin x = 0 \quad \Longrightarrow \quad \cos x = \sin x$$

$$f\left(-\frac{7\pi}{4}\right) = f\left(\frac{\pi}{4}\right) = \sqrt{2} \qquad f\left(\frac{-3\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = -\sqrt{2} \qquad f(\pm 2\pi) = 1$$

f has abs. maxima at $\left(-\frac{7\pi}{4}, \sqrt{2}\right)$ and $\left(\frac{\pi}{4}, \sqrt{2}\right)$, and abs. minima at $\left(-\frac{3\pi}{4}, -\sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$.

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domain

x-intercepts

y-intercepts

horizontal asymptote

vertical asymptote

oblique asymptote

Sketch the graph of $f(x) = \dfrac{(x-2)(x-1)}{(x+1)(x-4)}$.

domain $\mathbb{R} - \{-1, 4\}$

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oblique asymptote

Sketch the graph of $f(x) = \dfrac{(x-2)(x-1)}{(x+1)(x-4)}$.

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x-intercepts $x = 1, 2$

y-intercepts

horizontal asymptote

vertical asymptote

oblique asymptote

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vertical asymptote

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x-intercepts $x = 1, 2$

y-intercepts $y = -\frac{1}{2}$

horizontal asymptote $y = 1$ $\lim_{x \to \pm\infty} f(x) = 1$

vertical asymptote

$$\lim_{x \to -1^-} f(x) = \infty \qquad \lim_{x \to -1^+} f(x) = -\infty$$

oblique asymptote

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x-intercepts $x = 1, 2$

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oblique asymptote

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y-intercepts $y = -\frac{1}{2}$

horizontal asymptote $y = 1$ $\qquad \lim_{x \to \pm\infty} f(x) = 1$

vertical asymptote $x = -1$ and $x = 4$

$$\lim_{x \to -1^-} f(x) = \infty \qquad \lim_{x \to -1^+} f(x) = -\infty$$

$$\lim_{x \to 4^-} f(x) = -\infty \qquad \lim_{x \to 4^+} f(x) = \infty$$

oblique asymptote

Sketch the graph of $f(x) = \dfrac{(x-2)(x-1)}{(x+1)(x-4)}$.

domain $\mathbb{R} - \{-1, 4\}$

x-intercepts $x = 1, 2$

y-intercepts $y = -\frac{1}{2}$

horizontal asymptote $y = 1$ $\qquad \lim_{x \to \pm\infty} f(x) = 1$

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$$f'(x) = -\frac{6(2x-3)}{(x+1)^2(x-4)^2} \qquad f''(x) = \frac{12(13-9x+3x^2)}{(-4+x)^3(1+x)^3}$$

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CNs $x = \dfrac{3}{2}$

PPOI NONE

Sketch the graph of $f(x) = \dfrac{(x-2)(x-1)}{(x+1)(x-4)}$.

Interval	$f(x)$	$f'(x)$	$f''(x)$	Remarks
$(-\infty, -1)$		$+$	$+$	inc cu
$x = -1$	und	und	und	VA
$\left(-1, \frac{3}{2}\right)$		$+$	-	inc cd
$x = \frac{3}{2}$	$\frac{1}{25}$	0	-	rel. max.
$\left(\frac{3}{2}, 4\right)$		-	-	dec cd
$x = 4$	und	und	und	VA
$(4, \infty)$		-	$+$	dec cu

Note:

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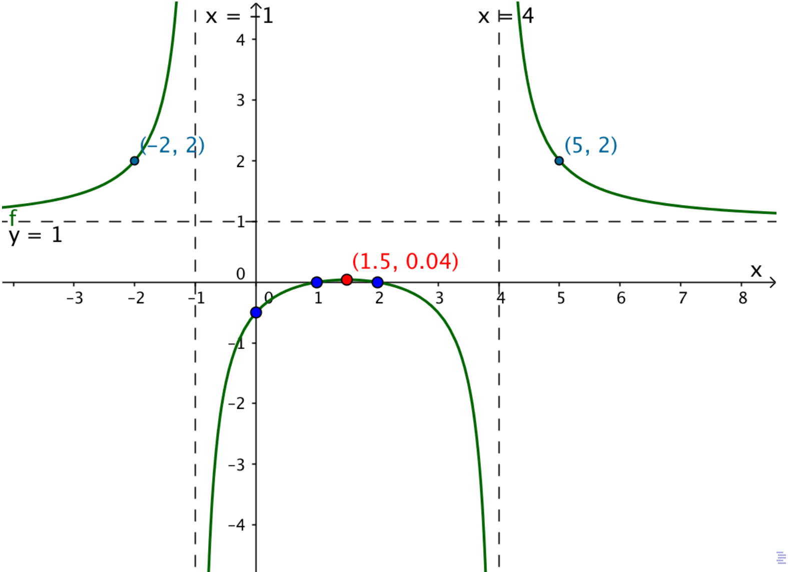

A cardboard poster containing 32 sq. in. of printed region is to have a margin of 2 in. at the top and bottom and 4/3 in. at each side. Determine the dimensions of the smallest piece of cardboard that can be used to make the poster.

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$A(h)$ has a relative minimum at $h = 4 + 4\sqrt{3}$ and it is the ONLY relative extremum in $(4, \infty)$.

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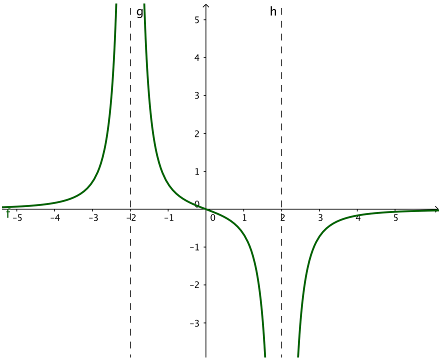
$A(h)$ has an absolute minimum at $h = 4 + 4\sqrt{3}$.

Therefore, the dimensions of smallest piece of cardboard that can be used to make a 32 sq. in. printed region is

$$h = 4 + 4\sqrt{3} \text{ inches and } l = \frac{8(1 + 4\sqrt{(3)})}{3} \text{ inches.}$$

Let f be a continuous function over $\mathbb{R}$. Describe the graph of f such that f' is drawn as follows:



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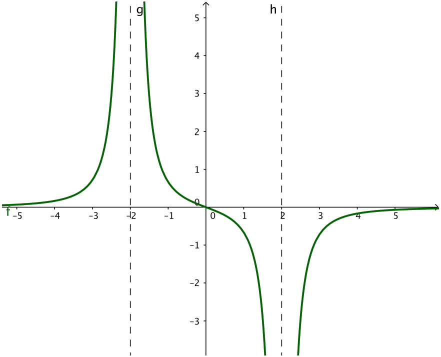

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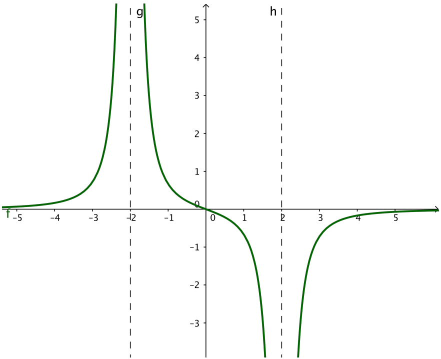

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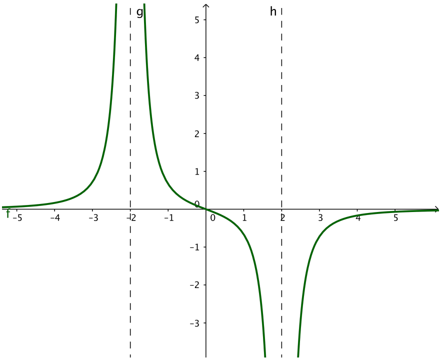

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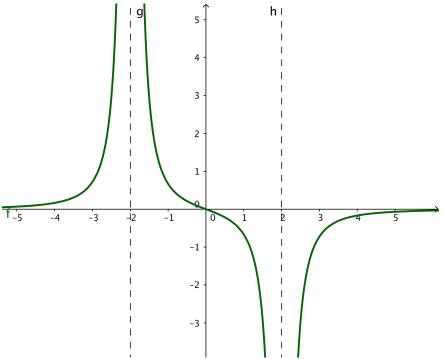

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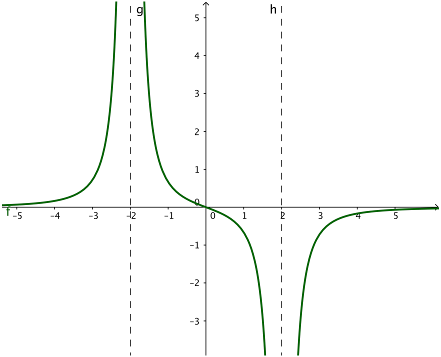

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Thus
f is increasing if $x \in (-\infty, -2) \cup (-2, 0)$
f is decreasing if $x \in (0, 2) \cup (2, \infty)$
f has a rel. max. at $x = 0$

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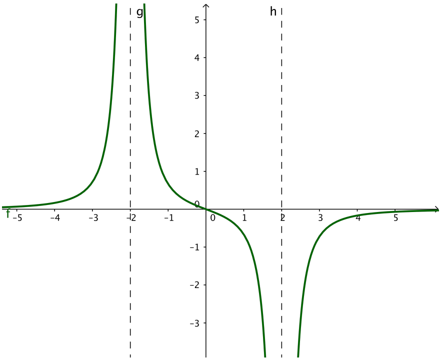

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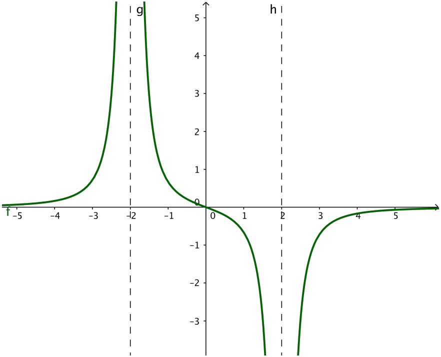

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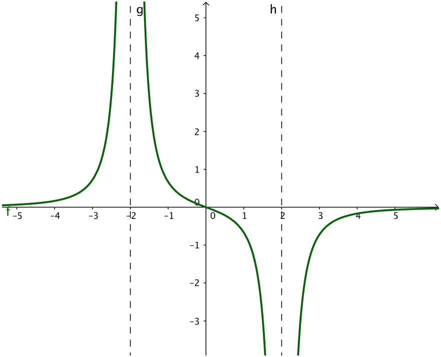

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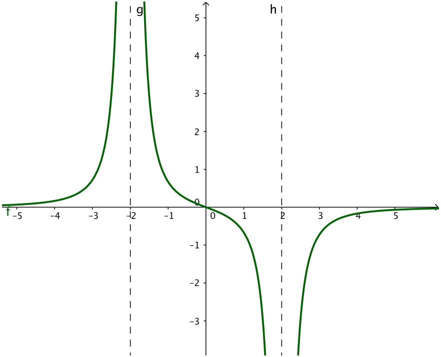

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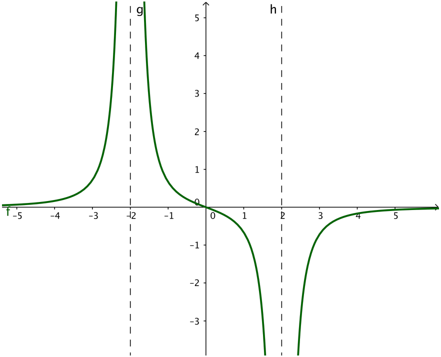

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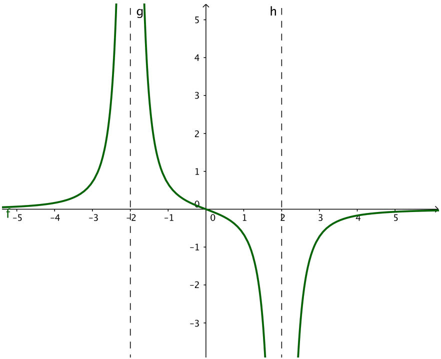

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Thus
f is conc. up if $x \in (-\infty, -2) \cup (2, \infty)$
f is conc. down if $x \in (-2, 0) \cup (0, 2)$
f has POIs at $x = -2, 2$

GOOD LUCK!!