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EE126 HW #5

1\. a) Bernoulli R.V. $p = 1/4$

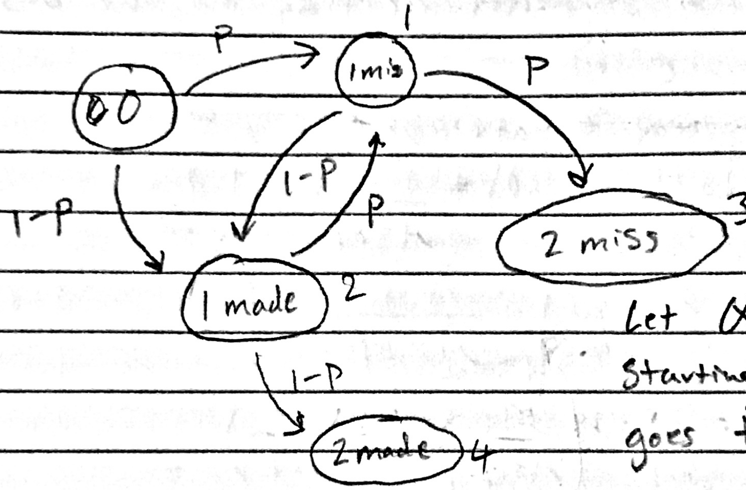
$E(\#\text{ shots missed}) = np = 3$ so 12 taken so $\boxed{9 \text{ made}}$

b)

$p(\text{event described}) = \binom{7}{1}(1-p)(p)^6(1-p)^2$ (since independent

$7\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^6 = \boxed{\frac{189}{262144}}$

c) P [



Let $\alpha(i)$ = Probability that Starting in state i markov goes to 2 misses before 2 makes

Pr [misses 2 before making 2] = ~~1/~~

$\alpha(0) = p(\alpha(1)) + (1-p)(\alpha(2))$ $(\alpha(3), \alpha(4) = 0)$

$\alpha(1) = p(1) + (1-p)\,\alpha(2)$

$\alpha(2) = p(\alpha(1))$

$\alpha(1) = p + (1-p)(p)\,\alpha(1)$

$\alpha(0) = p(\alpha(1)) + (1-p)(p)\,\alpha(1)$

$1/1\frac{3}{16} \cdot \frac{16}{13}$

$\alpha(1) = p + (1-p)(p)\left[p + (1-p)(p)\left[p + \ldots\right]\right]$

$= \frac{1}{4} + p\sum_{i=1}^{\infty}(1-p)^i(p)^i$

3/16

$= \frac{1}{4} + \frac{1}{4}\left[\frac{(3/4)(1/4)}{1 - 3/16}\right] = \frac{1}{4} + \left(\frac{3}{13}\right)\left(\frac{1}{4}\right) = \frac{16}{52} = \frac{4}{13}$

$\alpha(0) = \frac{1}{4}\left(\frac{16}{52}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{16}{52}\right) = \boxed{7/52}$

$\frac{}{K!}$

2. a) bus $\sim P(\lambda)$

Students $\sim$ Poisson(μ)

Because Poisson is memoryless distribution is just

$P_{N_x}(k) = P(k,x) = e^{-\mu x}\frac{(\mu x)^k}{k!}$

b) Let T = time when next bus arrives

K = # students getting on the bus

$f_T(t) = \lambda e^{-\lambda t}$

Merged poisson = rate $\lambda+\mu$

$P(\text{student} \mid \text{arrival}) = \frac{\mu}{\lambda+\mu}$

$P(\text{Bus} \mid \text{arrival}) = \frac{\lambda}{\lambda+\mu}$

$P(\text{student} = k) = \left(\frac{\mu}{\lambda+\mu}\right)^k\left(\frac{\lambda}{\lambda+\mu}\right)$

c) Let U be time of first arrival before 11 AM and V = time of first arrival after 11 AM.

L = interarrival interval containing 11 AM

$L = (t_{11AM} - U) + (V - t_{11AM})$

These 2 intervals model inde. exp. r.v w/ param λ

$f_L(t) = \frac{\lambda^2 t e^{-\lambda t}}{2} = \frac{\lambda^2 t}{2}e^{-\lambda t}$

$P(\text{Student arrived before last}) = \left(\frac{\mu}{\lambda+\mu}\right)^k\left(\frac{\lambda}{\lambda+\mu}\right)$

$P(\text{Student arrived before next}) = \left(\frac{\mu}{\lambda+\mu}\right)^k\left(\frac{\lambda}{\lambda+\mu}\right)$

$\boxed{P(\text{Student} = k) = \binom{k}{2}\left(\frac{\mu}{\lambda+\mu}\right)^k\left(\frac{\lambda}{\lambda+\mu}\right)}$

4. Police drive ~ Poisson rate λ

a) Let $T_1, T_2, T_3 \ldots$ = interarrival times for police

$$E[N] = \sum_{i=0}^{\infty} P(T_1 < t)^i = \sum_{i=0}^{\infty} (1 - e^{-\lambda t})^i = \frac{(1-e^{-\lambda t})}{1-(1-e^{-\lambda t})} =$$

$$\frac{1-e^{-\lambda t}}{e^{-\lambda t}} = \boxed{\frac{1}{e^{-\lambda t}} - 1}$$

b)