Wave Energy Calculations

C.Boocock

1 Overview

This document will give a detailed explanation of the calculations made in *wave_energy.pro*. *wave_energy.pro* uses outputs from *Wigglewave* to calculate the wave energy flux across magnetic surfaces at different heights. Before using *wave_energy.pro*, however, the outputs must be converted to .sav format using *makesavs.pro*.

The wave energy flux Π is calculated across magnetic surfaces defined by ϕ = constant. Each surface is uniquely identified by its height of intersection with the z-axis, hence we can define the energy flux as a function of height z. We use the following formula to calculate $\Pi(z)$,

$$\Pi(z) \;=\; -\frac{1}{\mu_0}\int_{\Sigma_b} B\langle vb\rangle \; d\Sigma \;=\; \pi\frac{HB_0}{\mu_0}\int_0^{\psi_b} v_p b_p \; d\psi$$

2 Inputs

The user must specify the parameters at the top of the code. The parameters used must match those used to generate the Wigglewave outputs being analysed. The parameter to be specified are:

- H the magnetic scale height
- B_0 the characteristic magnetic field strength
- ρ_0 the characteristic density
- α the ratio between magnetic and density scale heights $\alpha = H/H_\rho$
- r_{size} and z_{size} the number of grid cells in radial and vertical directions
- r_{dim} dimension of domain in the radial direction
- z_{dim} dimension of domain in the vertical direction
- ζ the density contrast between the centre of the density tube and the background
- u_0 the Alfvén wave amplitude
- r_0 the width of the density tube at the lower boundary
- *save_dir* the location of the Wigglewave outputs

After reading these parameters the code generates arrays for the coordinates r and z. Then the *Wigglewave* outputs for ρ, ϕ, ψ, v_{env} and b_{env} are read from .sav files in the specified location. The arrays are then cropped according to nr and nz removing any boundary damping regions from the outputs. The wave energy flux can now be calculated.

3 Calculations

We begin by calculating the wave energy flux density,

$$\epsilon = \frac{\pi B_0 H}{\mu_0} v_p b_p$$

A contour of ϵ is then plotted.

To find the wave energy flux Π we need to integrate ϵ with respect to ψ across magnetic surfaces defined by $\phi = const.$. We define each magnetic surface by the height at which it intersect the $z-$axis.

We begin by defining the lowest magnetic surface that can fit inside the domain. We label the height at which this surface intersects the $z-$axis as z_0. Using the definition for ϕ we have,

$$\begin{aligned}
\phi(r_0, 0) &= \phi(0, z_0) \\
-HJ_0\left(\frac{r_0}{H}\right) &= -He^{-z_0/H} \\
z_0 &= -H\ln\left(J_0\left(\frac{r_0}{H}\right)\right)
\end{aligned}$$

With that defined we loop over each height index k for which $z > z_0$ and calculate the wave energy flux for each corresponding surface. These surfaces do not coincide with the grid points, we therefore need to use interpolation to find the correct values for ϵ and ψ along the magnetic surfaces.

We begin by finding the height z at which the surface intersects the $z-$axis, this is $z_{en} = kz/z_{max}$. Then we find the value of ϕ for the surface, $\phi_1 = -He^{-z_{en}/H}$.

Now for each radial index i between zero and the tube boundary ψ_b we find the two closest points above and below the magnetic surface with height indices p_1 and p_2 such that $\phi(i, p_1) < \phi_1 < \phi(i, p_2)$. To find ϵ and ψ for the point at this radius on the magnetic surface we perform simple linear interpolation,

$$
\begin{aligned}
\psi_{int} &= \psi(i,p_1) + (\psi(i,p_2) - \psi(i,p_1))\frac{\phi_1 - \phi(i,p_1)}{\phi(i,p_2) - \phi(i,p_1)} \\
\epsilon_{int} &= \epsilon(i,p_1) + (\epsilon(i,p_2) - \epsilon(i,p_1))\frac{\phi_1 - \phi(i,p_1)}{\phi(i,p_2) - \phi(i,p_1)}
\end{aligned}
$$

During this process the loop continuously checks whether the calculated value $\psi_{int} < \psi_b$ and also save the position of each point across the magnetic surface. Each magnetic surface line will be drawn onto the contour of ϵ.

For each magnetic surface the total wave energy flux Π is calculated by integrating ϵ_{int} with respect to ψ_{int}. The wave energy flux Π as well as the intersection height z_{en} for each surface are saved in arrays.

The total wave energy flux $\Pi(z)$ is then plotted against the intersection heights of the magnetic surfaces z_{en}.

4 Outputs

The code now saves 1D arrays for z_{en}, z_{en} in units of the scale height H, $\Pi(z)$ and $\Pi(z)$ normalized by the lowest magnetic surface (i.e. $\Pi(z)/\Pi(z_0)$)