

Design and Implementation of an LDPC-based FEC encoder/decoder suitable for Storage devices

15. November 2018

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- 1 Motivation
- 2 LDPC Codes
- 3 Encoding
- 4 Decoding
- 5 Performance
- 6 Results
- 7 Conclusion and Outlook

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- Build a Hardware encoder and decoder suitable for storage devices
- Create a system for testing the encoder and decoder


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Low Density Parity Check (LDPC) Codes

- Multiple parity check equations
- Each bit is contained in multiple equations
- With these equations we can correct errors

Low Density Parity Check (LDPC) Codes

- Multiple parity check equations
- Each bit is contained in multiple equations
- With these equations we can correct errors
- We construct the check matrix with the check equations

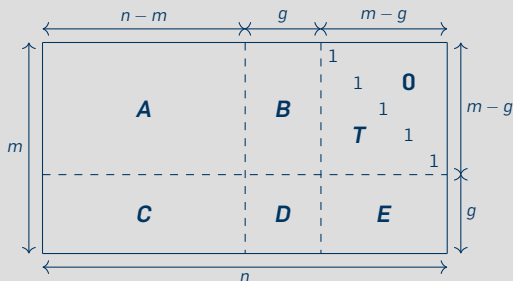
$$x_1 \oplus x_3 \oplus x_4 \oplus x_7 = 0$$

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- High complexity of LDPC codes
- Reduce complexity by adding structure to the PCM
- Split PCM into submatrices
- Only allow shifted version of a submatrix

$$\mathbf{B} = \begin{bmatrix} -1 & 0 & 2 & -1 \\ 0 & 1 & -1 & -1 \\ -1 & 1 & 0 & 2 \end{bmatrix}$$

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- Is usually done with generator matrix
- The generator matrix is dense due to the inversion
- With long codes the dense matrix multiplication is large
- Use transforms on the PCM to convert it into a more desirable form



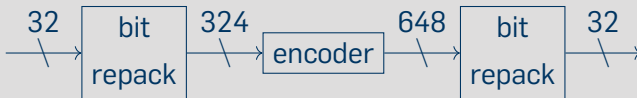
- Reach minimum gap g by doing only row and column permutations
- Only need an inverted matrix of size $g \times g$

- Only large sparse matrix multiplication and back substitution
- One small dense matrix multiplication

Operation	Type
$\mathbf{A}s^T$	sparse multiplication
$\mathbf{T}^{-1}\mathbf{A}s^T$	sparse back substitution
$-\mathbf{E}\mathbf{T}^{-1}\mathbf{A}s^T$	sparse multiplication
$\mathbf{C}s^T$	sparse multiplication
$(-\mathbf{E}\mathbf{T}^{-1}\mathbf{A}s^T) + (\mathbf{C}s^T)$	vector addition
$\phi^{-1}(-\mathbf{E}\mathbf{T}^{-1}\mathbf{A}s^T + \mathbf{C}s^T)$	dense $g \times g$ multiplication

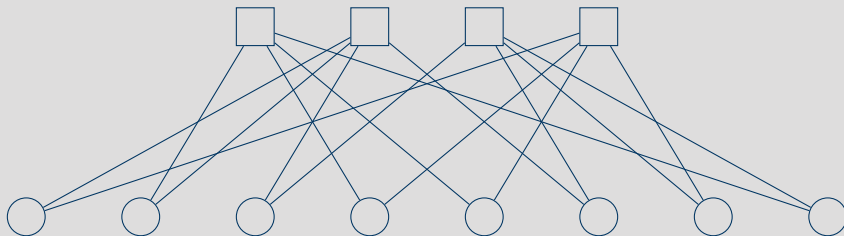
- Implemented as combinatorial logic
- Connects to the other modules with an axi stream bus
- Repacking is needed as bit counts dont evenly divide
- Encoder is generated from the QC PCM with Python scripts

For wifi ldpc with rate 0.5 it looks like this



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- I implemented a message passing decoder
- Messages are passed along the edges on the tanner graph
- Computations are done on the nodes



$$r_{mn} = \left(\prod_{n' \in M(m) \setminus n} \text{sign}(q_{n'm}) \right) \min_{n' \in M(m) \setminus n} (|q_{n'm}|)$$

$$L_n = y_n + \sum_{m \in N(n)} r_{m'n}$$

$$q_{nm} = y_n + \sum_{m' \in N(n) \setminus m} r_{m'n}$$

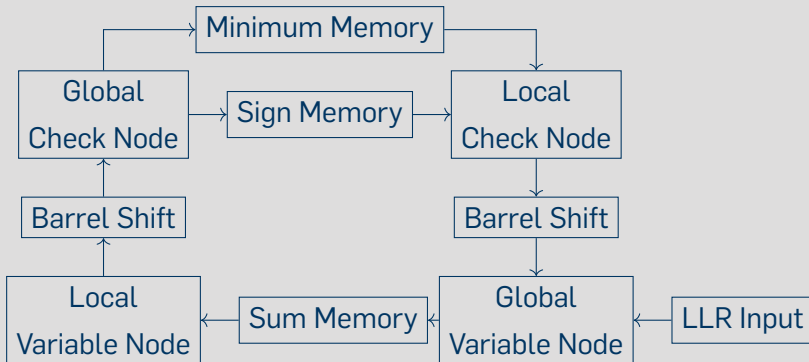
- We have to implement these equations
- Can exploit similarities

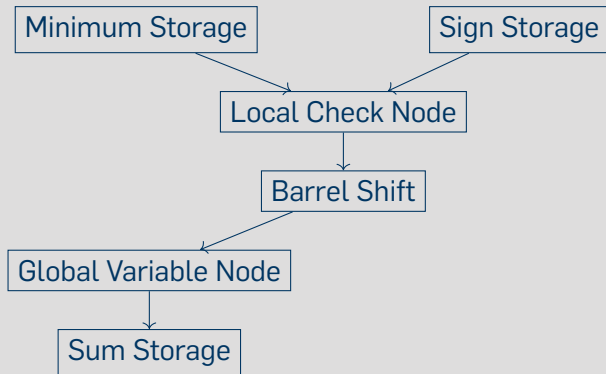
$$r_{mn} = \left(\prod_{n' \in M(m) \setminus n} \text{sign}(q_{n'm}) \right) \min_{n' \in M(m) \setminus n} (|q_{n'm}|)$$

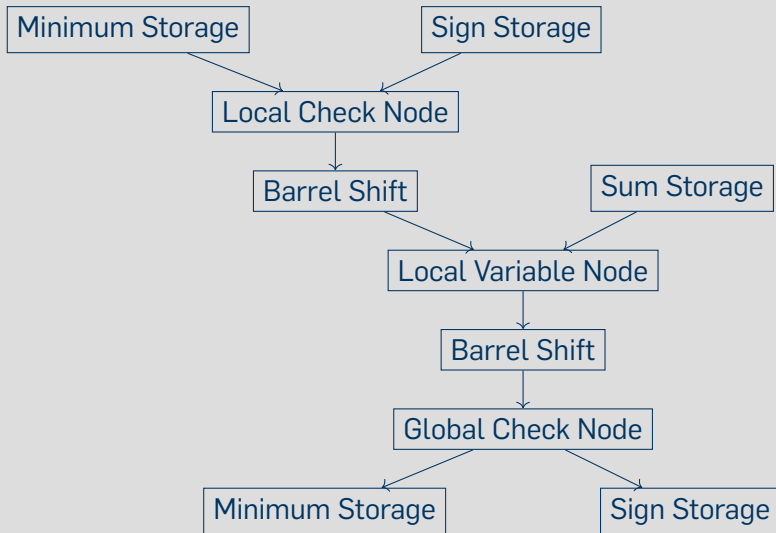
$$L_n = y_n + \sum_{m \in N(n)} r_{m'n}$$

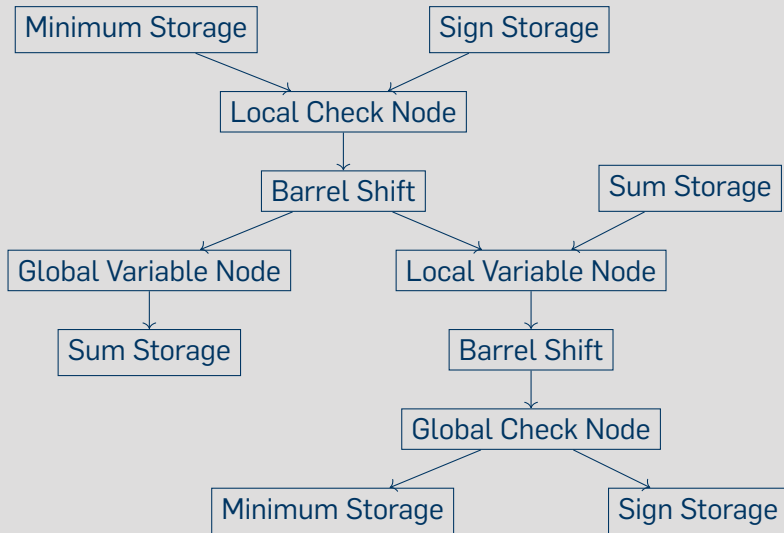
$$q_{nm} = L_n - r_{mn}$$

- two pass algorithm
- intermediate values stored in memory









Running minimum

0	0	0	1	0	0	0	0	1	0	0	0	∞
0	0	0	0	1	0	1	0	0	0	0	0	∞
0	0	0	0	0	1	0	1	0	0	0	0	∞
1	0	0	0	1	0	0	0	0	0	0	0	∞
0	1	0	0	0	1	0	0	0	0	0	0	∞
0	0	1	1	0	0	0	0	0	0	0	0	∞
0	0	0	0	1	0	1	0	0	0	0	1	∞
0	0	0	0	0	1	0	1	0	1	0	0	∞
0	0	0	1	0	0	0	0	1	0	1	0	∞

4	1	6	3	8	7	5	9	2	4	2	8
---	---	---	---	---	---	---	---	---	---	---	---

Shifted input

Running minimum

0	0	0	1	0	0	0	0	1	0	0	0	3	3
0	0	0	0	1	0	1	0	0	0	0	0	8	8
0	0	0	0	0	1	0	1	0	0	0	0	7	7
1	0	0	0	1	0	0	0	0	0	0	0	∞	
0	1	0	0	0	1	0	0	0	0	0	0	∞	
0	0	1	1	0	0	0	0	0	0	0	0	∞	
0	0	0	0	1	0	1	0	0	0	0	1	∞	
0	0	0	0	0	1	0	1	0	1	0	0	∞	
0	0	0	1	0	0	0	0	1	0	1	0	∞	

4	1	6	3	8	7	5	9	2	4	2	8	Shifted input	
---	---	---	---	---	---	---	---	---	---	---	---	---------------	--

Running minimum

0	0	0	1	0	0	0	0	1	0	0	0	2	2
0	0	0	0	1	0	1	0	0	0	0	0	5	5
0	0	0	0	0	1	0	1	0	0	0	0	7	9
1	0	0	0	1	0	0	0	0	0	0	0	∞	
0	1	0	0	0	1	0	0	0	0	0	0	∞	
0	0	1	1	0	0	0	0	0	0	0	0	∞	
0	0	0	0	1	0	1	0	0	0	0	1	∞	
0	0	0	0	0	1	0	1	0	1	0	0	∞	
0	0	0	1	0	0	0	0	1	0	1	0	∞	

4	1	6	3	8	7	5	9	2	4	2	8		
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Shifted input

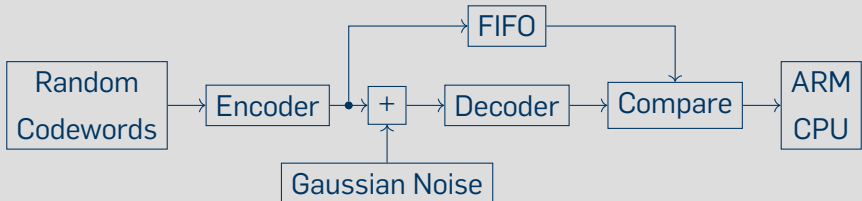
Running minimum

0	0	0	1	0	0	0	0	1	0	0	0	2
0	0	0	0	1	0	1	0	0	0	0	0	5
0	0	0	0	0	1	0	1	0	0	0	0	7
1	0	0	0	1	0	0	0	0	0	0	0	∞
0	1	0	0	0	1	0	0	0	0	0	0	∞
0	0	1	1	0	0	0	0	0	0	0	0	∞
0	0	0	0	1	0	1	0	0	0	0	1	∞
0	0	0	0	0	1	0	1	0	1	0	0	∞
0	0	0	1	0	0	0	0	1	0	1	0	∞

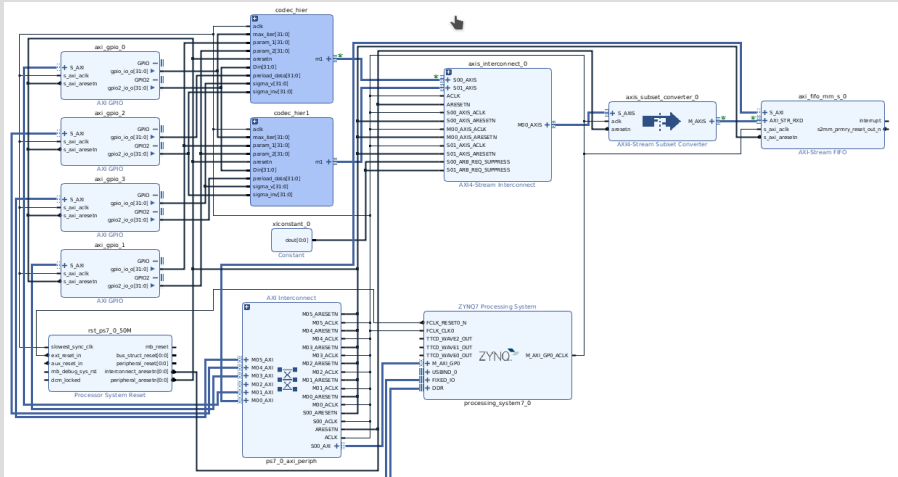
4	1	6	3	8	7	5	9	2	4	2	8	Shifted input
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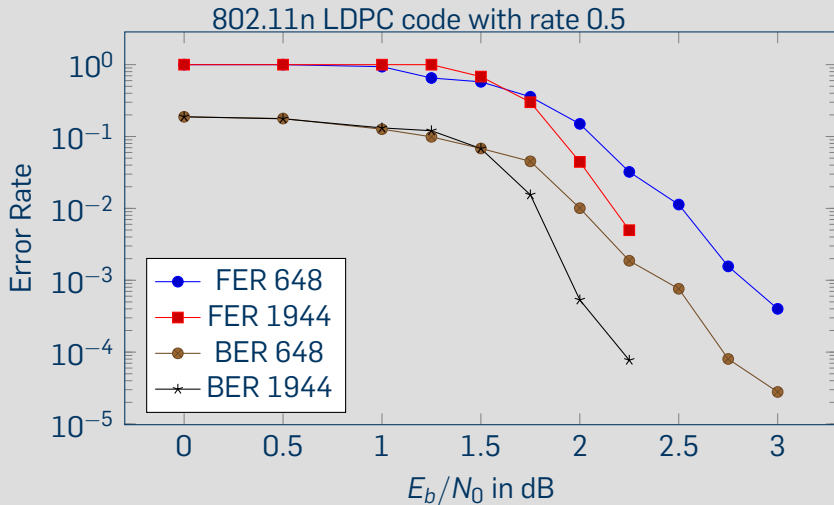
- Used a ZedBoard
- Testing requires an encoder, channel, decoder setup
- AWGN channel
- Whole performance test is implemented on Hardware
- ARM cpu calculates test parameters and reads the error counts

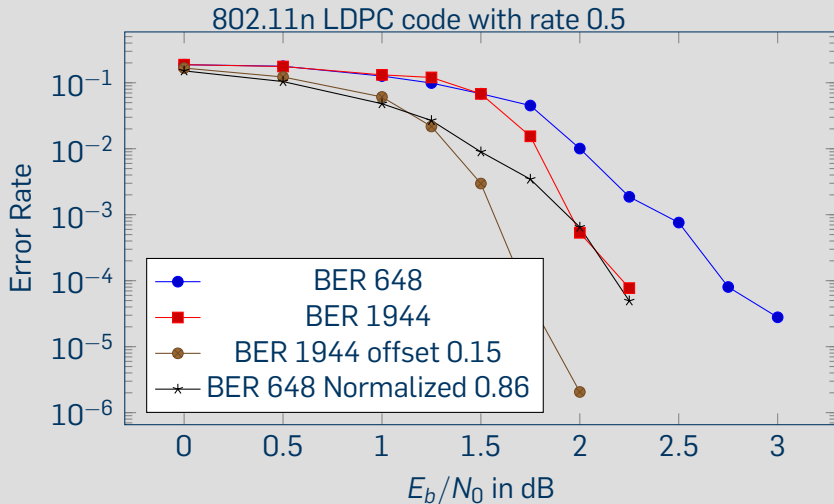




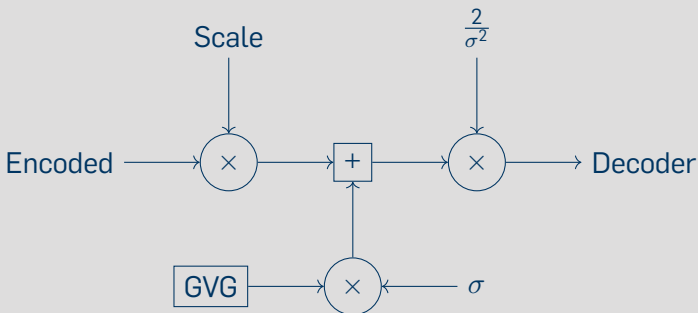


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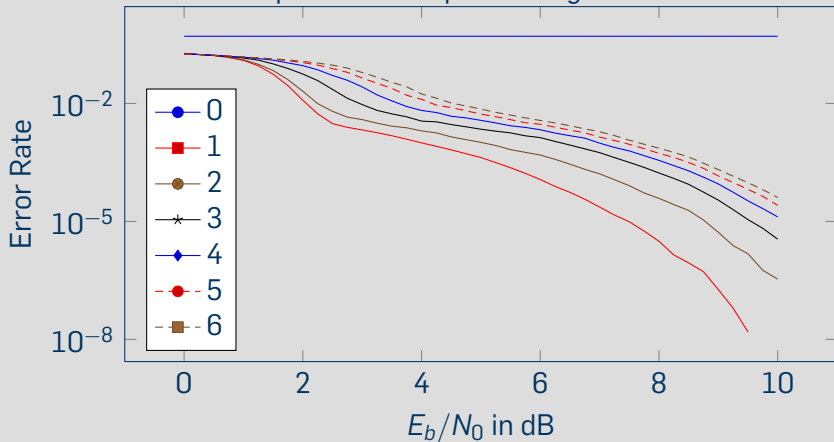




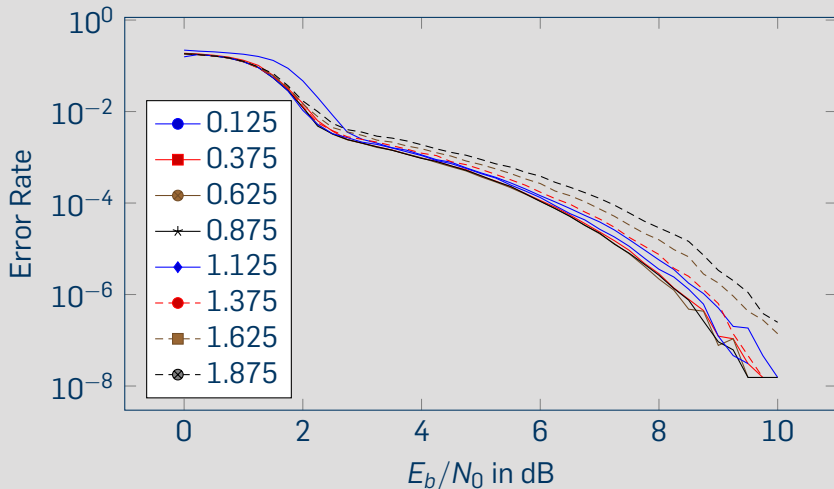
- Decoder expects LLR
- Convert channel output to LLR



Optimize the input scaling

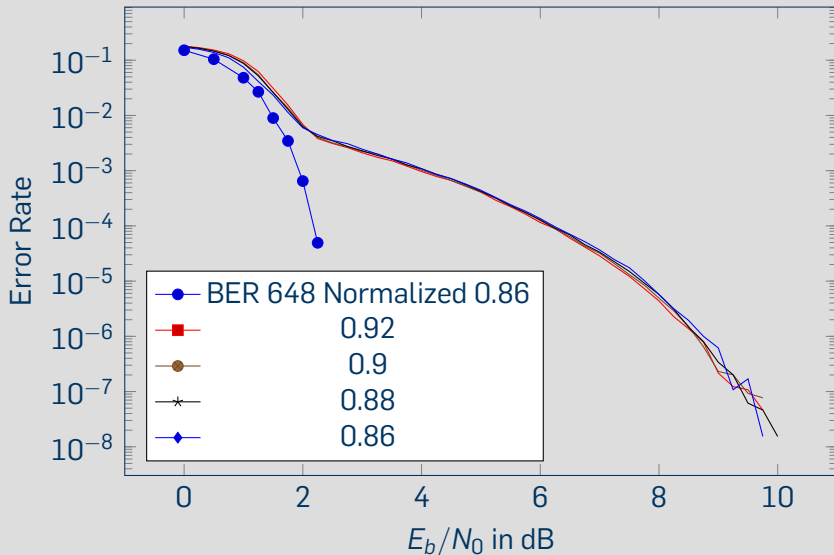


I chose a scaling of 0.75



Hardware Results

Normalized Min Sum



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Conclusion and Outlook

Conclusion

- LDPC codes can be implemented on hardware
- My Implementation has problematic error floor

Outlook

- Look at power consumption
- Optimize area utilization
- Utilize reconfiguration / change instructions

Questions?