Nitrate Leaching Model Design Proposal

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August 2, 2021

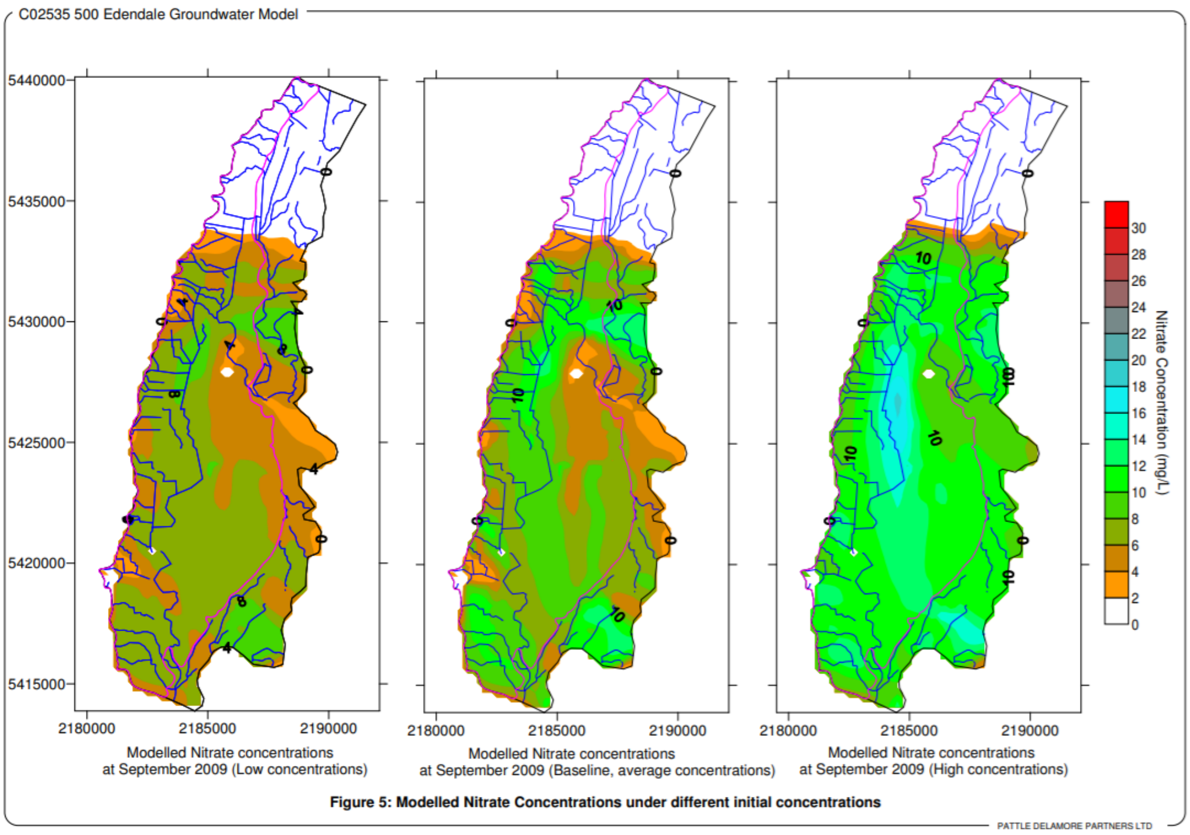

Figure 1: Nitrate Concentrations in Edendale under different initial conditions

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1 Problem Summary

1.1 Problem Description

In order to address groundwater quality concerns for the review of their resource consent by Environment Southland at a demonstration farm in Edendale, Dairy NZ is proposing to use MAR (managed aquifer recharge) to improve aquifer quality. There are two main points of interest. First, to find out whether the installation of an extensive active carbon sink (wood chip layer) at the farm in 2010 has been effective in reducing nitrate concentration levels in groundwater (as Dairy NZ claim). And second, to design the proposed MAR program which will be used to improve aquifer quality. If both options prove ineffective, other options, such as reducing the scale of dairy cattle farming, may have to be considered.

1.2 Stakeholders

- Environment Southland – Primarily concerned with nitrate leeching.
- Dairy farmers (Dairy NZ) – Concerned with the financial well being of the farmers and dairy industry.
- Ngāi Tahu – have financial interest. However, due to the harvest of mahinga kai from the Mataura River, also have concerns with the nitrate leaching.

2 Contribution of Modelling Study

Our modelling study aims to capture the relationship between the pressure in the aquifer and concentration of nitrate in groundwater over time. By doing so, we expect to quantify the effectiveness that the active carbon sink has had on reducing nitrate levels in groundwater. Additionally, we aim to see how different MAR programs will reduce nitrate concentrations in groundwater over time and how long it will take to see these reductions. This will help us to determine the best MAR program to be proposed for the resource consent.

3 Data

3.1 Data Visualisation

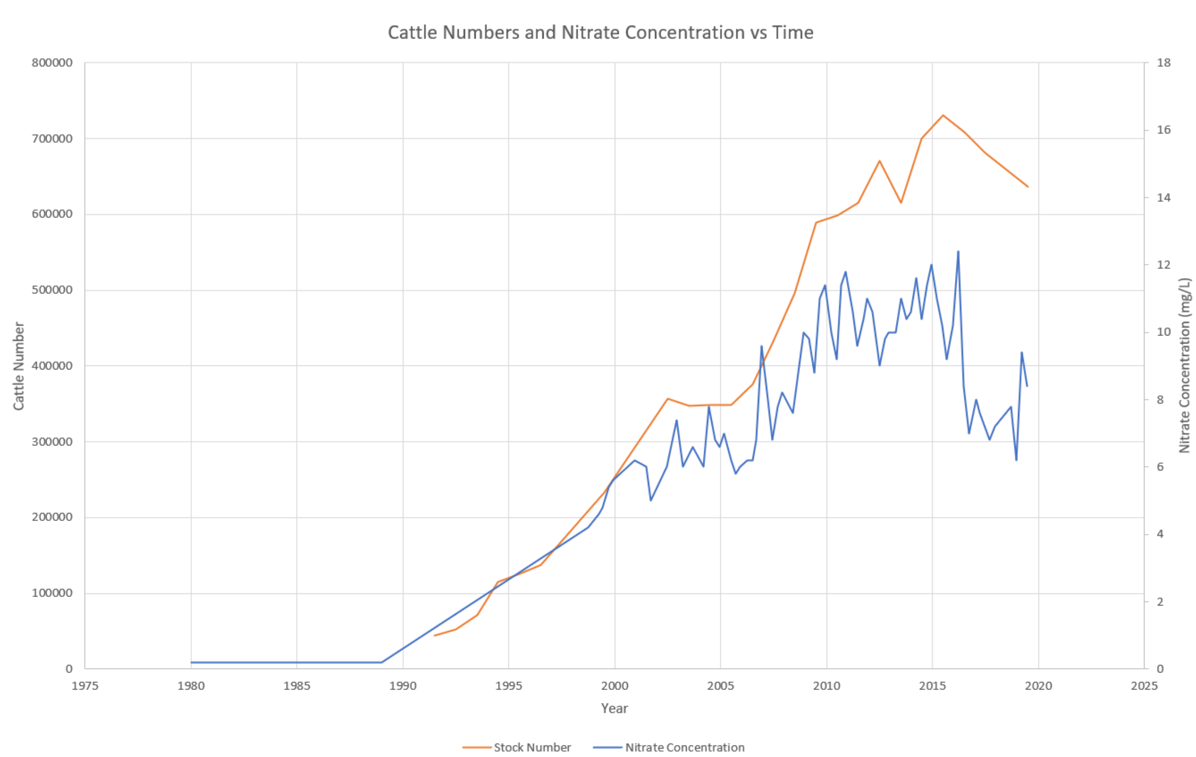

Figure 2: Time series graph of the given data

3.2 Data Description

We observe that there is a roughly steady increase in the cattle numbers in Southland and following along this increase are the nitrate concentration measurements (mg/L). There does seem to be some variation in the concentration measurements, however much of this can be attributed to natural effects such as variations in annual rainfall which affects the overall mass of nitrate that leaches into the system. Prior to 1990 there is no available data for the number of cattle, and the concentration of nitrate is taken as a constant level of 0.2 mg/L. As the number of cattle increases approximately linearly from 40,000 to 360,000, the nitrate concentration also increases approximately linearly to about 6, beyond which the nitrate concentration exhibits various peaks and drops. The nitrate concentration experiences a peak of about 12.5 mg/L in 2015, where the annual cattle numbers are experiencing a decrease. This indicates the time delay between changes in the nitrate source (cattle waste and fertilizers) at the surface and the nitrate concentration inside the aquifer itself.

Pressure effects are assumed to be immediate in the aquifer system, with the driving force for nitrate transport across the boundary being a surface over-pressure of 0.05MPa. The pressure drop across the local aquifer is roughly 0.1MPa which we observe as the pressure gradient across the aquifer.

3.3 Parameters

Parameter	Symbol	Value	Reference
Edendale aquifer area	A	Unknown	-
Permeability	k	Unknown	-
Porosity	ϕ	Unknown	-
Density	ρ	Unknown	-
Average depth of aquifer	h	Unknown	-
Dynamic viscosity	μ	Unknown	-
Average time Lag	t_{lag}	Unknown	-
Initial mass of nitrate	M	Unknown	-
Critical overpressure	P_c	0.05MPa	Project Document
Pressure gradient	P_o	0.1MPa	Project Document
Maximum transmissivity	T_{max}	9300m^2/day	Thomas N. (2012). Edendale Groundwater Model
Minimum transmissivity	T_{min}	4m^2/day	Thomas N. (2012). Edendale Groundwater Model

Figure 3: Parameters considered in the model, alongside respective values (if known)

4 Conceptual Model

4.1 Sketch of Conceptual Model

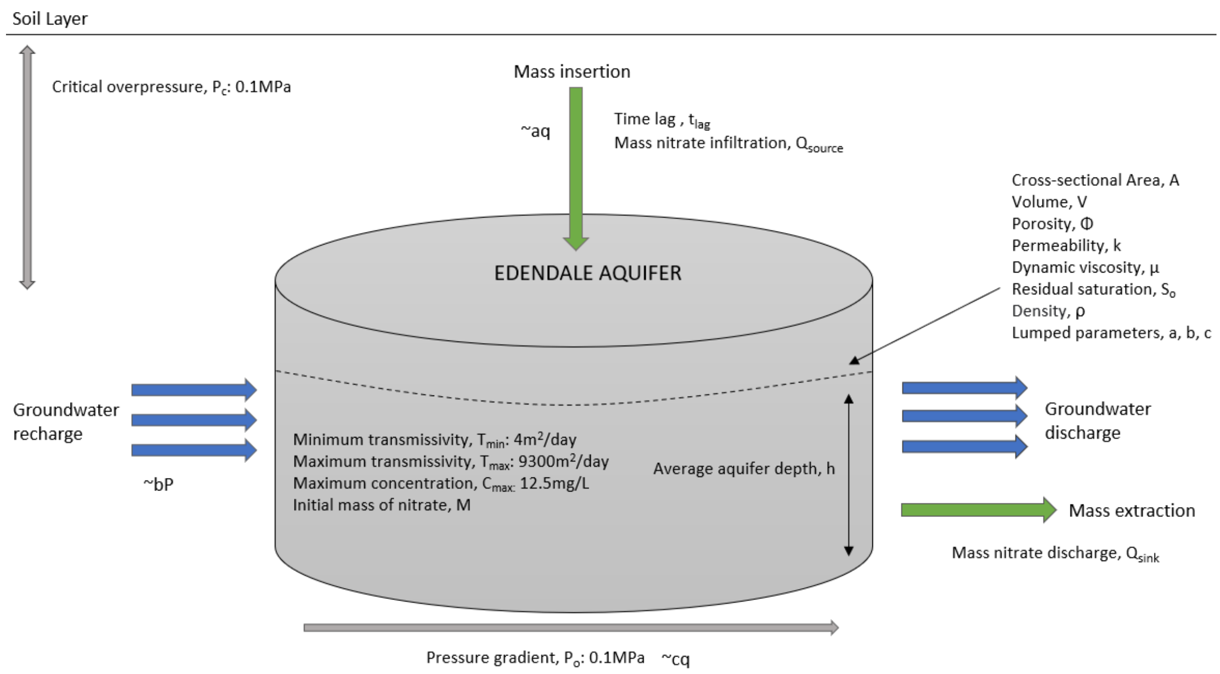

Figure 4: Conceptual model of the Edendale Aquifer

4.2 Justification of Conceptual Model

The physical theories underlying this problem are conservation of mass (Darcy's Law) and conservation of solute (Fick's Law). We will be using an LPM to analyse the concentration of nitrate within the aquifer groundwater.

The physical processes shown by the model above are as follows:

- The leaching of nitrate into the groundwater is modelled as the mass nitrate infiltration, while mass nitrate discharge quantifies the rate at which nitrate leaves the aquifer through groundwater discharge areas.
- The groundwater recharge models the rate at which freshwater enters the aquifer (through freshwater injection, precipitation, percolation or irrigation). Meanwhile, groundwater discharge models the rate at which groundwater (which is mixed in with other solutes) leaves the aquifer into streams and tributaries (or through evapotranspiration).
- The rate of movement of groundwater from areas of recharge to areas of discharge within the aquifer is driven by the pressure gradient. This pressure gradient can be modelled to observe the effects of MAR on aquifer rehabilitation.
- The effect of slow drainage is also modelled as a time lag, meaning that there will be an observable time between changes made and significant reduction in nitrate concentrations.

The domain of the model is the Edendale aquifer boundary, which does not include any surrounding land. As such, the model will include the physical processes that relate to mass leaving and entering the aquifer's boundaries. However, we will assume there is negligible spatial variation of nitrate concentrations within the Edendale aquifer meaning the effects of localised temperature, elevation and point/anthropogenic sources will be considered negligible and will therefore not be considered.

While some parameters of the model may not actually be constant, we will simplify the model by considering many of them to be constant. For example, it will be assumed that dissolved solutes and pressure will not impact on the viscosity of the groundwater. We will also assume that the aquifer is homogeneous i.e. porosity, permeability and density remain constant and we will assume that aquifer depth does not change over the cross-section of the Edendale aquifer. Finally, we will assume that nitrate leaching and discharge is constant i.e. $\frac{dq}{dt} = 0$.

The time domain of our model will have to be reasonably long (10+ years) due to the potentially large lag times before any changes in nitrate concentrations are observed.

Finally, we will further simplify the model by assuming that nitrates are not converted to other compounds (that may or may not also be pollutants) and that other chemical solutes do not pose any problems.

5 Formulation

Firstly, we will have our ODE for pressure within the system.

$$\frac{dP}{dt} = -a_p q - b_p(P - P_o) - c_p \frac{dq}{dt}$$

Since we have assumed $\frac{dq}{dt} = 0$, the ODE can be simplified to:

$$\frac{dP}{dt} = -a_p q - b_p(P - P_o)$$

$$a_p = \frac{g}{A\psi}$$

$$b_p = \frac{gKA_{rech}\rho}{AL\mu\psi}$$

In this differential equation the independent variable is time (years), t, while the dependent variable is pressure (MPa), P. The parameters that make up the coefficients, a_p and b_p can be located in the parameter table (figure 3) above.

Secondly, we have the ODE describing the nitrate concentration in the groundwater.

$$M\frac{dC}{dt} = Q_{source} + Q_{sink} + C\frac{b_p}{a_p}(P - P_o)$$

$$a_p = \frac{g}{A\psi}$$

$$b_p = \frac{gKA_{rech}\rho}{AL\mu\psi}$$

For this differential equation the independent variable is also time (years), t, however the dependent variable is concentration (mg/L), C. The coefficients, a_p and b_p are the same as described above, while Q_{source} and Q_{sink} are assumed to be constants. Furthermore, we will assume that the change in concentration due to injected fresh water is negligible, hence there is no term for this in our equation.