① $y' = x + 1 \qquad y(0) = 2$

$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + \cdots$

$y' = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \cdots$

$y' = 1 + x$

$B = 1$

$2C = 1$

$C = \frac{1}{2}$

$y = 2 + x + \frac{1}{2}x^2$

(3) $y' = y$ $\quad y(0) = 1$

$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + \dots$

$y' = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \dots$

$\| 1$

$2C = B \quad 3D = C \quad 4E = D$

$2C = 1 \quad 3D = \frac{1}{2} \quad 4E = \frac{1}{6}$

$C = \frac{1}{2} \quad D = \frac{1}{6} \quad E = \frac{1}{24}$

$y = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$

$y = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

⑤ $y' = \frac{1}{1+x}$ $\quad y(0) = 1$

$A = 1$

$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + 6x^6 \dots$

$y' = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \dots$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$

$B = 1 \quad 2C = -1 \quad 3D = 1 \quad 4E = -1 \quad 5F = 1 \quad 6G = -1$

$C = -\frac{1}{2} \quad D = \frac{1}{3} \quad E = -\frac{1}{4} \quad F = \frac{1}{5} \quad G = -\frac{1}{6}$

$y = 1 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \dots$

(7) $y' = x + y$ $\quad y(0) = 1$

$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + \cdots$

$A = 1$

$y' = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \cdots$

$\updownarrow$

$x + y = A + (B+1)x + Cx^2 + Dx^3 + Ex^4 + Fx^5$

$B = A \quad 2C = B+1 \quad 3D = C \quad 4E = D \quad 5F = E \quad 6G = F$

$B = 1 \quad 2C = 2 \quad 3D = 1 \quad 4E = \frac{1}{3} \quad 5F = \frac{1}{12} \quad 6G = \frac{1}{60}$

$C = 1 \quad D = \frac{1}{3} \quad E = \frac{1}{12} \quad F = \frac{1}{60} \quad G = \frac{1}{360}$

$y = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{360} + \cdots$