

$$\vec{F} = -k \vec{s}$$

$$m \vec{a} = -k \vec{s}$$

$$m s'' = -k s$$

$$s'' = -\frac{k}{m} s$$

$$s'' + \frac{k}{m} s = 0$$

$$S'' + S = 0$$

$$S'' = -S$$

$$S = \cos t$$

$$S' = -\sin t$$

$$S'' = -\cos t$$

$$S = \sin t$$

$$S' = \cos t$$

$$S'' = -\sin t$$

$$S'' + S = 0$$

$$S(t) = A \cos t + B \sin t$$

$$S'' + \omega^2 S = 0$$

$$S = \sin(\omega t) \quad S'' = -\omega^2 S$$

$$S' = \omega \cos(\omega t)$$

$$S'' = -\omega^2 \sin(\omega t)$$

$$S = \cos(\omega t)$$

$$S' = -\omega \sin(\omega t)$$

$$S'' = -\omega^2 \cos(\omega t)$$

$$s'' + \omega^2 s = 0$$

$$s(t) = A \sin(\omega t) + B \cos(\omega t)$$

$$\sum \vec{F} = m \vec{a}$$

$$-a s' - k s = m s''$$

$$m s'' + a s' + k s = 0$$

$$s'' + \frac{a}{m} s' + \frac{k}{m} s = 0$$

$$\begin{array}{l|l}
 s = e^{rt} & s'' + as' + bs = 0 \\
 s' = r e^{rt} & r^2 e^{rt} + a r e^{rt} + b e^{rt} = 0 \\
 s'' = r^2 e^{rt} & e^{rt} (r^2 + ar + b) = 0 \\
 & r^2 + ar + b = 0 \\
 & r = \frac{-a \pm \sqrt{a^2 - 4(1)(b)}}{2(1)} = \frac{-a \pm \sqrt{a^2 - 4b}}{2} \\
 & s(t) = \begin{cases} e^{\frac{-a + \sqrt{a^2 - 4b}}{2} t} \\ e^{\frac{-a - \sqrt{a^2 - 4b}}{2} t} \end{cases}
 \end{array}$$

$$\begin{aligned}
 S &= e^{rt} \\
 S' &= r e^{rt} \\
 S'' &= r^2 e^{rt}
 \end{aligned}$$

$$S'' + 3S' + 2S = 0$$

$$r^2 e^{rt} + 3r e^{rt} + 2e^{rt} = 0$$

$$e^{rt}(r^2 + 3r + 2) = 0$$

$$(r+1)(r+2) = 0$$

$$r = -1 \text{ or } r = -2$$

$$s = 2e^{-t} - \frac{1}{2}e^{-2t}$$

$$s'' + 3s' + 2s = 0$$

$$s = Ae^{-t} + Be^{-2t}$$

$$s(0) = -\frac{1}{2}$$

$$s' = -Ae^{-t} - 2Be^{-2t}$$

$$s'(0) = 3$$

$$-\frac{1}{2} = A + B$$

$$-\frac{1}{2} = A + B$$

$$3 = -A - 2B$$

$$-\frac{1}{2} = A - \frac{1}{2}$$

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$$\begin{array}{rcl} \frac{5}{2} & = & -B \\ -\frac{5}{2} & = & B \end{array}$$

$$2 = A$$



$$\begin{array}{l}
 s = e^{rt} \\
 s' = r e^{rt} \\
 s'' = r^2 e^{rt}
 \end{array}
 \left. \vphantom{\begin{array}{l} s = e^{rt} \\ s' = r e^{rt} \\ s'' = r^2 e^{rt} \end{array}} \right)
 \begin{array}{l}
 s'' + 2s' + 2s = 0 \\
 r^2 e^{rt} + 2r e^{rt} + 2e^{rt} = 0 \\
 e^{rt}(r^2 + 2r + 2) = 0 \\
 r = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{-4}}{2} \\
 = \frac{-2 \pm 2i}{2} \\
 = -1 \pm i
 \end{array}$$

$$s'' + 2s' + 2 = 0$$

$$s(t) = Ae^{(-1+i)t} + Be^{(-1-i)t}$$

$$s'(t) = A(-1+i)e^{(-1+i)t} + B(-1-i)e^{(-1-i)t} \quad \left( \begin{array}{l} s(0) = 2 \\ s'(0) = 0 \end{array} \right.$$

$$2 = A + B$$

$$0 = A(-1+i) + B(-1-i)$$