(108) (1) $y'' = -y, \; y(0) = 5, \; y'(0) = 0$

$y = A\cos t + B\sin t$ $\quad 5 = A\cos(0) + B\sin(0)$

$y' = -A\sin t + B\cos t$ $\quad 0 = A\sin(0) + B\cos(0)$

$y'' = -A\cos t - B\sin t$

$y(t) = 5\cos t$

(2) $y'' + \omega^2 y = 0$, $y(0) = 1$, $y'(0) = -6$

$y = A\cos(\omega t) + B\sin(\omega t)$

$y' = -\omega A \sin(\omega t) + \omega B \cos(\omega t)$

$y'' = -\omega^2 A \cos(\omega t) - \omega^2 B \sin(\omega t) = -\omega^2 y$

$y'' + 4y = 0$, $y(0) = 1$, $y'(0) = -6$

$y = A\cos(2t) + B\sin(2t)$ $\quad 1 = A\cos(0) + B\sin(0)$

$y' = -2A\sin(2t) + 2B\cos(2t)$ $\quad -6 = -2A\sin(0) + 2B\cos(0)$

$y = \cos(2t) - 3\sin(2t)$

(4) $x'' + \frac{g}{\ell}x = 0$

$$x(t) = A\cos\left(\sqrt{\frac{g}{\ell}}\,t\right) + B\sin\left(\sqrt{\frac{g}{\ell}}\,t\right)$$

$$x'(t) = -\sqrt{\frac{g}{\ell}}A\sin\left(\sqrt{\frac{g}{\ell}}\,t\right) + \sqrt{\frac{g}{\ell}}B\cos\left(\sqrt{\frac{g}{\ell}}\,t\right)$$

$x(0) = 0,\ x'(0) = v_0$

$0 = A\cos 0 + B\sin 0$

$v_0 = -\sqrt{\frac{g}{\ell}}A\sin 0 + \sqrt{\frac{g}{\ell}}B\cos 0$

$x(t) = \sqrt{\frac{\ell}{g}}v_0\sin\left(\sqrt{\frac{g}{\ell}}\,t\right)$

IF ℓ increases

THEN { amp inc.
Freq dec.

$x(0) = x_0 \quad x'(0) = 0$

$x_0 = A\cos 0 + B\sin 0$

$0 = -\sqrt{\frac{g}{\ell}}\sin(0) + \sqrt{\frac{g}{\ell}}B\cos 0$

$x(t) = x_0\cos\left(\sqrt{\frac{g}{\ell}}\,t\right)$

IF x_0 increases
THEN amp inc

IF ℓ increases
THEN Freq dec

(6a) $36Q'' + \frac{Q}{9} = 0 \quad Q'' + \frac{Q}{324}$

$\rightarrow Q(t) = A\cos\left(\frac{t}{18}\right) + B\sin\left(\frac{t}{18}\right)$

$\Rightarrow Q'(t) = -\frac{A}{18}\sin\left(\frac{t}{18}\right) + \frac{B}{18}\cos\left(\frac{t}{18}\right)$

$Q(0) = 6$

$Q'(0) = 0$

$6 = A\cos 0 + B\sin(0)$

$0 = -\frac{A}{18}\sin(0) + \frac{B}{18}\cos(0)$

$Q(t) = 6\cos\left(\frac{t}{18}\right)$

(6b) $10Q'' + \frac{Q}{C} = 0 \quad Q'' + \frac{Q}{10C} = 0$

$$Q(t) = A\cos\frac{t}{\sqrt{10C}} + B\sin\left(\frac{t}{\sqrt{10C}}\right)$$

$$Q'(t) = -\frac{A}{\sqrt{10C}}\sin\left(\frac{t}{\sqrt{10C}}\right) + \frac{B}{\sqrt{10C}}\cos\left(\frac{t}{\sqrt{10C}}\right)$$

$$0 = A\cos(0) + B\sin(0)$$

$$4 = -\frac{A}{\sqrt{10C}}\sin(0) + \frac{B}{\sqrt{10C}}\cos(0)$$

$$Q(t) = 4\sqrt{10C}\sin\left(\frac{t}{\sqrt{10C}}\right)$$

$$2\sqrt{2} = 4\sqrt{10C}$$

$$\frac{\sqrt{2}}{2} = \sqrt{10C}$$

$$\frac{1}{2} = 10C$$

$$\frac{1}{20} = C$$