

(107) (1) $y'' = -y, y(0) = 0, y'(0) = 1$

$$y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + \dots$$

$$y' = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \dots$$

$$y'' = 2C + 6Dx + 12Ex^2 + 20Fx^3 + 30Gx^4 + \dots$$

$$\begin{array}{lllll} 2C = A & 6D = -B & 12E = -C & 20F = -D & 30G = -E \\ 2C = 0 & 6D = -1 & 12E = 0 & 20F = -\frac{1}{6} & 30G = 0 \\ C = 0 & D = -\frac{1}{6} & E = 0 & F = \frac{1}{120} & G = 0 \end{array}$$

$$y = \frac{x^1}{1} - \frac{x^3}{6} + \frac{x^5}{120}$$

$$y = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+1}}{(2m+1)!} = \sin(x)$$

$$y'' = -y$$

$$(10) \quad y'' = -y, \quad y(0) = 1, \quad y'(0) = 0$$

$$y = \overset{1}{A} + Bx + Cx^2 + \overset{0}{D}x^3 + Ex^4 + Fx^5 + Gx^6 + \dots$$

$$y' = \overset{0}{B} + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \dots$$

$$y'' = 2C + 6Dx + 12Ex^2 + 20Fx^3 + 30Gx^4 + \dots$$

$$\begin{array}{lllll} 2C = -A & 6D = -B & 12E = -C & 20F = -D & 30G = -E \\ 2C = -1 & 6D = 0 & 12E = \frac{1}{2} & 20F = 0 & 30G = -\frac{1}{24} \\ C = -\frac{1}{2} & D = 0 & E = \frac{1}{24} & F = 0 & G = -\frac{1}{720} \end{array}$$

$$y = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$$

$$y = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \cos(x)$$