Simultaneous Differential Equations
Competing Populations and Lachester's Law

(1) Let w(t) represent the population of worms (in millions) and r(t) the population of robins (in thousands) on an isolated island. A model for the interaction of these populations is given by the following system of simultaneous, first order, ordinary differential equations (NB: [t] = years).

$$\frac{dw}{dt} = w - wr \qquad \frac{dr}{dt} = -r + wr$$

(1a) Solve for w(t) assuming the robins have died out.

(1b) Describe the population growth for worms in the absence of robins: (circle one)

(1) Exponential Growth
(2) Exponential Decay
(3) Heating Curve
(4) Cooling Curve
(5) Logistic Growth
(6) Logistic Decay
(7) Stable Equilibrium
(8) Unstable Equilibrium

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(1c) Solve for r(t) assuming the worms have died out.

(1d) Describe the population growth for robins in the absence of worms: (circle one)

(1) Exponential Growth
(2) Exponential Decay
(3) Heating Curve (Exponential Approach)
(4) Cooling Curve (Exponential Approach)
(5) Logistic Growth
(6) Logistic Decay
(1) Stable Equilibrium
(2) Unstable Equilibrium

(1e) Find all Equilibrium points for the interacting populations.

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(1f) Using a Slope Field for $\frac{dr}{dw}$ in the window [0, 4]x[0, 4] to plot a trajectory for the initial populations of $2x10^6$ worms and $2x10^3$ robins. Use your trajectory to estimate the minimum and maximum populations for both species.

Wmin = _______________
Wmax= _______________
Rmin = _______________
Rmax = _______________

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(1g) Using a Slope Field for $\frac{dr}{dw}$ in the window [0, 4]x[0, 4] to plot a trajectory for the initial populations of $2x10^6$ worms and $3x10^3$ robins. Use your trajectory to estimate the minimum and maximum populations for both species.

Wmin = _______________
Wmax= _______________
Rmin = _______________
Rmax = _______________

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(1h) Describe the interaction of these two species: (circle one)

(1) Symbiotic
(2) Competitive
(3) Predator Prey
(4) Non-interactive

(1i) The people on this island do not usually interact with the robins or the worms. However, they do love robins! Does it make sense to introduce the additional 1000 robins at t = 0? Please explain why or why not in paragraph form.

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(2) Apply the Lanchester model (given below) to the Battle of Trafalgar (1805), when a fleet of 40 British ships expected to face a combined French and Spanish fleet of 46 ships. Suppose that there were x British ships and y opposing ships at time t. We assume that the ships are all identical so that the constants in the differential equations in Lanchester's model are equal:

$$\frac{dx}{dt} = -ay \qquad \frac{dy}{dt} = -ax$$

(2a) Write a differential equation involving $\frac{dy}{dx}$ and solve it using the initial sizes of the two fleets.

(2b) If the battle were fought until all the ships one fleet were put out of action, which side won the battle and how many ships were left according to your result in (a)?

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Admiral Nelson, who commanded the British fleet, did not in fact send his 40 ships against the 46 French and Spanish ships. Instead, he split the battle into two parts, sending 32 of his ships against 23 opposing ships and his other 8 ships against their other 23!

(2c) Using a Slope Field, plot a trajectory for the sub-battle with initial condition (32,23). Describe the outcome of this battle.

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(2d) Using a Slope Field, plot a trajectory for the sub-battle with initial condition (8,23). Describe the outcome of this battle.

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(2e) Using a Slope Field, plot a trajectory for the final battle with initial condition (remaining British, remaining French/Spanish). Describe the outcome of this battle. Is this the same result you predicted in part (b)?

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Teacher's notes:

Solving Simultaneous DiffEqus using Slope Fields

Do word problems from exercises in section 11.8 of Hughes-Hallett's Calculus: Single Variable 4th ed. © 2005 from Wiley (pp. 574–575).