Second Order Differential Equations
Hooke's Law with Friction

(1) ***Proof of Euler's Identity***

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$
$$[\theta] = \text{radians}$$

(1a) Write the first 6 non-zero terms of the MacLaurin Expansion for e^x.
(1b) Write the first 6 non-zero terms of the MacLaurin Expansion for $e^{i\theta}$.
(1c) Rewrite your expansion for $e^{i\theta}$ in simplest terms.
(1c) Write the first 3 non-zero terms of the MacLaurin Expansion for $\cos(\theta)$.
(1d) Write the first 3 non-zero terms of the MacLaurin Expansion for $\sin(\theta)$.
(1e) Based on these expansions, explain why: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

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(2) Find the General Solution:

$$s'' + 3s' + 2s = 0$$

(2a) Write and solve the Characteristic Equation for r1 and r2.
(2b) Write the General Solution as: $s(t) = Ae^{r1t} + Be^{r2t}$.

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(2) *Find the Particular Solution:*

$$s'' + 3s' + 2s = 0$$

(2c) Given s(0) = -0.5 and s'(0) = 3, find A and B.

(2d) Rewrite your General Solution as a Particular Solution of the form:

$$s(t) = Ae^{r1t} + Be^{r2t}.$$

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(3) Find the General Solution:

$$s'' + 2s' + 2s = 0$$

(3a) Write and solve the Characteristic Equation for r1 and r2.
(3b) Write the General Solution as: $s(t) = Ae^{r1t} + Be^{r2t}$.

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(3) Find the Particular Solution:

$$s'' + 2s' + 2s = 0$$

(3c) Given s(0) = 2 and s'(0) = 0, find A and B.

(3d) Use Euler's Identity and your values for A and B to show that your General Solution reduces to a Particular Solution of the form:

$$s(t) = Ce^{\alpha t}\cos(\beta t) + De^{\alpha t}\sin(\beta t),$$
$$r = \alpha \pm \beta i.$$

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(4) The equation for the charge Q(t) on a capacitor in a circuit with inductance L, capacitance C and resistance R satisfies the differential equation:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0.$$

NB: [Q, L, R, C, t] = Coulomb, Henry, Ohm, Farad, and second

(4a) Given L=1, R=2 and C=4, use a characteristic equation to find the general solution for Q(t).

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(4b) Given L=1, R=1 and C=4, use a characteristic equation to find the general solution for Q(t).

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(4c) Given L=8, R=2 and C=4, use a characteristic equation to find the general solution for Q(t).

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(4d) How did reducing the resistance of the circuit affect the accumulated charge on the capacitor? Compare the results from parts (4a) and (4b).

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(4e) How did increasing the inductance of the circuit affect the accumulated charge on the capacitor? Compare the results from parts (4a) and (4c).

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Teacher's notes:

Develop differential model for springs with friction
Introduce the concept of a characteristic equation

Do word problems from exercises in section 11.11 of Hughes-Hallett's Calculus: Single Variable 4th ed. © 2005 from Wiley (p. 593, 594).