Exponential Decay and Exponential Approach
nonUniformly Accelerated Motion: Free Fall with Friction

Reconsider the Baseball problem and forget about that major cataclysm. You must have had a nightmare! Maybe it was that burrito you had at Taco Bell?

Given Initial Conditions:

$$\overline{a}\ (0) = \begin{pmatrix} 0 \\ -32 \end{pmatrix} \frac{ft}{s^2}$$

$$\overline{v}\ (0) = \begin{pmatrix} 207\cos(42°) \\ 207\sin(42°) \end{pmatrix} \frac{ft}{s}$$

$$\overline{r}\ (0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix} ft$$

Given Differential Model:

$$\overline{a}\ (t) = \begin{pmatrix} 0 - \frac{v_x(t)}{4} \\ -32 - \frac{v_y(t)}{4} \end{pmatrix} \frac{ft}{s^2}$$

These initial conditions and differential equations are supposed to model the first homerun ever hit at Dodger Stadium. You will test this model to see if it predicts Willie Stargell's homerun in 1969!

Remember that:

$$\overline{a}\ (t) = \begin{pmatrix} a_x(t) \\ a_y(t) \end{pmatrix} = \begin{pmatrix} x''\ (t) \\ y''\ (t) \end{pmatrix} = \begin{pmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{pmatrix} \frac{ft}{s^2}$$

You will be calculating:

$$\overline{v}\ (t) = \begin{pmatrix} v_x(t) \\ v_y(t) \end{pmatrix} = \begin{pmatrix} x'\ (t) \\ y'\ (t) \end{pmatrix} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} \frac{ft}{s}$$

You will be calculating and analyzing:

$$\overline{r}\ (t) = \begin{pmatrix} r_x(t) \\ r_y(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}\ ft$$

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(1) Use deSolve() to find the General Solution x '(t). Apply the given Initial Conditions to find the Particular Solution x '(t).

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(2) Use deSolve() to find the General Solution x (t). Apply the given Initial Conditions to find the Particular Solution x (t).

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(3) Use deSolve() to find the General Solution y '(t). Apply the given Initial Conditions to find the Particular Solution y '(t).

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(4) Use deSolve() to find the General Solution y (t). Apply the given Initial Conditions to find the Particular Solution y (t).

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(5) Use Solve() and FIX9 to estimate $t_r>0$ such that $y(t_r)=0$. This is the time it takes for the ball to hit the ground. Use your estimate for t_r to estimate $x(t_r)$. This is the horizontal range of flight. Willie's homerun had a horizontal range of 506.5ft. This model should predict this value much more accurately than the UAM model of C&P103.

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(6) The outfield wall at Dodger Stadium is 8ft high and is 395ft from the batter. Use Solve() and FIX9 to estimate t_w such that $x(t_w) = 395$. This is the time it takes for the ball to go over the wall. Use your estimate for t_w to estimate $y(t_w)$. Did Willie Stargell get a homerun based on this model?

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(7) What is the maximum height, or vertical range, for this trajectory? Use Solve() and FIX9 to estimate t_m such that $y'(t_m) = 0$. Use your estimate for t_m to estimate $x(t_m)$ and $y(t_m)$, the point of maximum height.

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(8) Use Parametric Mode to graph the ballistic trajectory:

$$\overline{r}\ (t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}.$$

You should have a complete graph of the ball's path, from when its hit by the bat to when it hits the ground using the following window:

tmin = 0, tmax = 7, xstep = .1
xmin = 0, xmax = 600, xscl = 100
ymin = 0, ymax = 200, yscl = 100

Label the points when t=0, $t=t_r$, $t=t_w$ and $t=t_m$. Is this path parabolic?

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(9) Estimate $\overline{v}\,(t_r)$ and $|\,\overline{v}\,(t_r)\,|$. What angle does the ball's trajectory make with the ground at the point of impact? Interpret these results.

(10) Estimate $\int_{0}^{t_r} |\overline{v(t)}|\,dt$ and interpret this result.

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Teacher's notes:

Introduce the TI89 function deSolve() to investigate the general solutions to differential equations modeling:

Exponential Growth and Decay
Newton's Law of Heating and Cooling (Exponential Approach)
Logistic Growth and Decay

Do word problems from exercises in section 11.5 of Hughes-Hallett's Calculus: Single Variable 4^{th} ed. © 2005 from Wiley.