Ordinary First Order Differential Equations
Uniformly Accelerated Motion: Free Fall without Friction

(1) Consider the Baseball problem and pretend that a major cataclysm has befallen the Earth such that there is no atmosphere! Well, the game must go on anyway.

You're at bat, you hit the ball and calculate its trajectory before it hits the ground to see if you hit a home run. You can accomplish this easily if you decompose the trajectory into horizontal and vertical components.

Assume that the horizontal component complies with the constraints of Uniform Motion and that the vertical component complies with the constraints of Uniformly Accelerated Motion.

At t = 0 sec, you hit the ball when it is 3 ft off the ground. The velocity of the ball at the time of impact is $\overline{v}(0) = 207\frac{ft}{s}\angle 42°$ ***and*** $g = -32\frac{ft}{s^2}$.

(1a) Write $\overline{a}(0)$, $\overline{v}(0)$ and $\overline{r}(0)$ as vectors in $\Re^2$ in Cartesian form.

(1b) Write $\overline{a}(t)$, $\overline{v}(t)$ and $\overline{r}(t)$ as vectors in $\Re^2$ in Cartesian form.

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(1c) Find the maximum height of the baseball and when it reaches this zenith.

(1d) Find the range of the baseball and when it hits the ground.

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(1e) Eliminate the parameter t to find the trajectory as y=f(x).

(1f) Graph the trajectory using parametric mode showing that it is parabolic. Label the (x,y) coordinates when you hit the ball, when the ball reaches the zenith and when the ball hits the ground.

(1g) If this is Fenway Park, did the ball go over the outfield wall?

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(2) Consider the Baseball problem and pretend that a major cataclysm has befallen the Earth such that you have to play on the Moon!

At t = 0 sec, you hit the ball when it is 3 ft off the ground. The velocity of the ball at the time of impact is $\overline{v}(0) = 207\,\frac{ft}{s}\,\angle 42°$ ***and*** $g = -5.2\,\frac{ft}{s^2}$.

(2a) Write $\overline{a}(0)$, $\overline{v}(0)$ and $\overline{r}(0)$ as vectors in $\Re^2$ in Cartesian form.

(2b) Write $\overline{a}(t)$, $\overline{v}(t)$ and $\overline{r}(t)$ as vectors in $\Re^2$ in Cartesian form.

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(2c) Find the maximum height of the baseball and when it reaches this zenith.

(2d) Find the range of the baseball and when it hits the ground.

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(2e) Eliminate the parameter t to find that $\frac{dy}{dx}=f(x)$.

(2f) Graph the trajectory using diffEqu mode showing that it is parabolic. Label the (x,y) coordinates when you hit the ball, when the ball reaches the zenith and when the ball hits the ground.

(2g) If we moved Fenway Park to the Moon, did the ball go over the outfield wall?

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(3) Consider the Baseball problem and pretend that a major cataclysm has befallen the Earth such that you don't know where you're going to play next!

At $t = 0$ sec, you hit the ball when it is h ft off the ground. The velocity of the ball at the time of impact is $\overline{v}\,(0) = v_o\angle\theta$ and

$|\,\overline{a}\,(0)\,| = -g$.

(3a) Write $\overline{a}\,(0)$, $\overline{v}\,(0)$ and $\overline{r}\,(0)$ as vectors in $\Re^2$ in Cartesian form.

(3b) Write $\overline{a}\,(t)$, $\overline{v}\,(t)$ and $\overline{r}\,(t)$ as vectors in $\Re^2$ in Cartesian form.

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(3c) Find the maximum height H of the baseball as a function of θ.

(3d) Use the First Derivative Test to find the value of θ that maximizes the Height, $H(\theta)$.

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(3d) Find the range R of the baseball as a function of θ.

(3e) Use the Second Derivative Test to find the value of θ that maximizes the range, $R(\theta)$.

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(3f) Note that $R(\theta) = R(90° - \theta)$. What does this mean. Use your function $R(\theta)$ to confirm this.

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Teacher lectures:

Define Uniform Motion (a=0) vs. Uniformly Accelerated Motion (a>0 but constant)

Discuss Newton's Second Law of Motion ($\Sigma F = ma = m\frac{dv}{dt}$ and $\Sigma F = \frac{dp}{dt}$)

Discuss Newton's Law of Gravitation ($F = \frac{GmM}{r^2}$)

Combine the 2 Laws to calculate g on different planets and show that g is not constant.

Solve Variable Separable DiffEqus
84AB1
85AB2
90AB1
91AB1
92AB6