(109) 1.)

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$e^{i\theta} = 1 + (i\theta) + \frac{(i\theta)^2}{2} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots$$

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} + \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$

$$e^{i\theta} = \cos\theta + i\sin\theta = \operatorname{cis}(\theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$$e^{i\pi} = -1$$

$$e^{i\pi} + 1 = 0$$

(2) $s'' + 3s' + 2s = 0$

$s = e^{rt}$

$s' = re^{rt}$

$s'' = r^2 e^{rt}$

$r^2 e^{rt} + 3re^{rt} + 2e^{rt} = 0$

$r^2 + 3r + 2 = 0$

$(r+1)(r+2) = 0$

$r = -1$ or $r = -2$

$s(t) = Ae^{r_1 t} + Be^{r_2 t}$

$s(t) = Ae^{-t} + Be^{-2t}$

$$s = Ae^{-t} + Be^{-2t}$$

$$s' = -Ae^{-t} - 2Be^{-2t}$$

$$-\frac{1}{2} = Ae^{0} + Be^{0}$$

$$3 = -Ae^{0} - 2Be^{0}$$

$$* \quad s(t) = 2e^{-t} - \frac{5}{2}e^{-2t}$$

$$A + B = -\frac{1}{2}$$

$$A - \frac{5}{2} = -\frac{1}{2}$$

$$A = 2$$

$$A + B = -\frac{1}{2}$$

$$-A - 2B = \frac{6}{2}$$

$$-B = 5/2$$

$$B = -5/2$$

③ $s'' + 2s' + 2s = 0$

$$r^2 + 2r + 2 = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$s(t) = Ae^{(-1+i)t} + Be^{(-1-i)t}$$

$$s = Ae^{(-1+i)t} + Be^{(-1-i)t}$$

$$s' = (-1+i)Ae^{(-1+i)t} + (-1-i)Be^{(-1-i)t}$$

$$2 = Ae^{0} + Be^{0} \qquad A+B=2$$

$$0 = (-1+i)Ae^{0} + (-1-i)Be^{0} \qquad (-1+i)A + (-1-i)B = 0$$

$$s = (1-i)e^{(-1+i)t} + (1+i)e^{(-1-i)t}$$

$$(1-i)A + (1-i)B = 2-2i$$

$$(-1+i)A + (-1-i)B = 0$$

$$-2iB = 2-2i$$

$$B = \frac{2-2i}{-2i}$$

$$B = \frac{1-i}{0-i} \cdot \frac{0+i}{0+i}$$

$$B = 1+i$$

$$A+B = 2$$

$$A + 1+i = 2$$

$$A = 1-i$$

$$s=(1-i)e^{-t+it}+(1+i)e^{-t-it}$$

$$s=(1-i)e^{-t}e^{it}+(1+i)e^{-t}e^{-it}$$

$$s=e^{-t}\left[(1-i)(\cos t+i\sin t)+(1+i)(\cos t-i\sin t)\right]$$

$$s=e^{-t}\Big[(\cos t+i\sin t-i\cos t+\sin t)$$

$$+$$

$$(\cos t-i\sin t+i\cos t+\sin t)\Big]$$

$$s=e^{-t}(2\cos t+2\sin t)$$

$$r=-1\pm i$$

$$s=Ce^{\alpha t}\cos\beta t+De^{\alpha t}\sin\beta t \qquad r=\alpha\pm\beta i$$

(4) $$Q'' + 2Q' + \frac{Q}{4} = 0$$

$B^2 - 4AC > 0$

OVER DAMPED

$$r^2 + 2r + \frac{1}{4} = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 1}}{2(1)} = \frac{-2 \pm \sqrt{3}}{2}$$

$$Q(t) = Ae^{\left(\frac{-2+\sqrt{3}}{2}\right)t} + Be^{\left(\frac{-2-\sqrt{3}}{2}\right)t}$$

$$Q'' + Q' + \frac{Q}{4} = 0$$

$$r^2 + r + \frac{1}{4} = 0$$

$B^2 - 4AC = 0$

CRITICALLY DAMPED

$$r = \frac{-1 \pm \sqrt{1-1}}{2(1)} = -\frac{1}{2}$$

$$Q(t) = Ae^{-t/2} + Bte^{-t/2}$$

$$8Q'' + 2Q' + \frac{Q}{4} = 0$$

$B^2 - 4AC < 0$

UNDER DAMPED

$$8r^2 + 2r + \frac{1}{4} = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 8}}{2(8)} = \frac{-2 \pm 2i}{16} = \frac{-1}{8} \pm \frac{i}{8}$$

$$Q(t) = e^{-t/8}\left(A\cos\frac{t}{8} + B\sin\frac{t}{8}\right)$$