$mg \downarrow \quad \uparrow Kv^2$

$$\sum \vec{F} = m\vec{a}$$

$$mg - Kv^2 = ma$$

$$mg - Kv^2 = mv'$$

$$\frac{dv}{dt} = g - \frac{k}{m} v^2$$

$$\int \frac{dv}{g - \frac{k}{m} v^2} = \int dt$$

$$\int \frac{dv}{\frac{k}{m}\left(\frac{mg}{k} - v^2\right)} = \int dt$$

$$\int \frac{dv}{\frac{mg}{k} - v^2} = \frac{k}{m} \int dt$$

$$\int \frac{dv}{\left(\sqrt{\frac{mg}{k}} - v\right)\left(\sqrt{\frac{mg}{k}} + v\right)} = \frac{k}{m} \int dt$$

$$\frac{1}{\left(\sqrt{\frac{mg}{k}}+v\right)\left(\sqrt{\frac{mg}{k}}-v\right)} = \frac{A}{\sqrt{\frac{mg}{k}}+v} + \frac{B}{\sqrt{\frac{mg}{k}}-v}$$

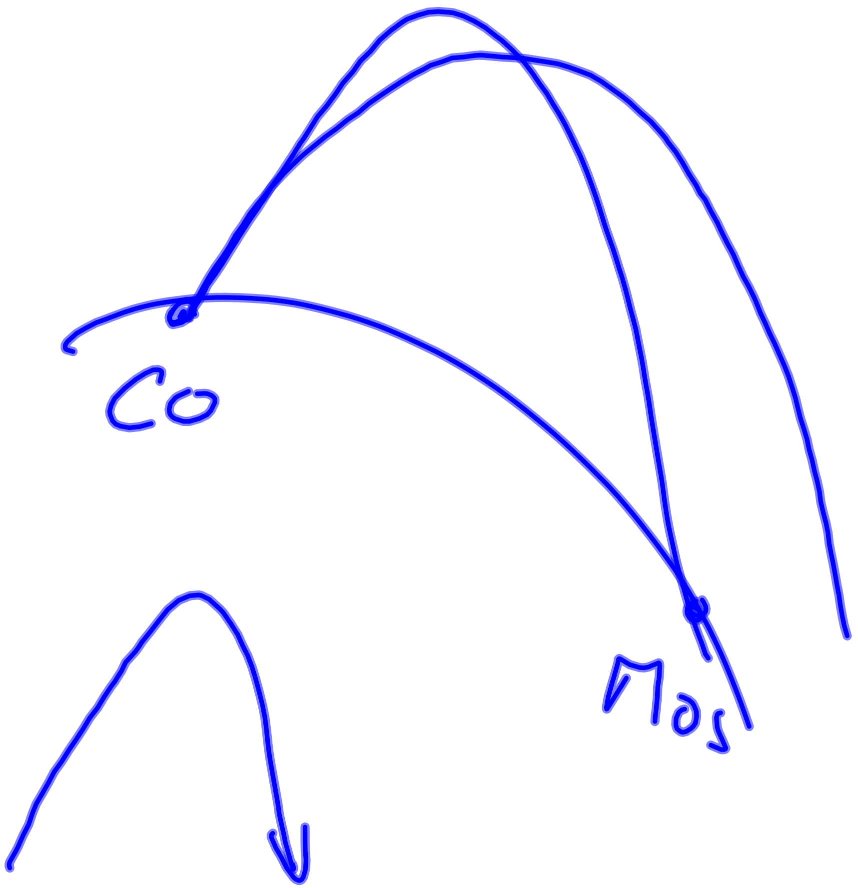
$$0v + 1 = A\sqrt{\frac{mg}{k}} - Av + B\sqrt{\frac{mg}{k}} + Bv$$

$$0 = -A + B$$

$$1 = A\sqrt{\frac{mg}{k}} + B\sqrt{\frac{mg}{k}}$$

$$1 = B\sqrt{\frac{mg}{k}} + B\sqrt{\frac{mg}{k}} \qquad 1 = 2B\sqrt{\frac{mg}{k}}$$

$$B = \frac{1}{2}\sqrt{\frac{k}{mg}}$$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$$\frac{1}{2}\sqrt{\frac{k}{mg}}\int\frac{dv}{\sqrt{\frac{mg}{k}}+v}+\int\frac{dv}{\sqrt{\frac{mg}{k}}-v}=\frac{k}{m}\int dt$$

$$\frac{1}{2}\sqrt{\frac{k}{mg}}\left(\ln\left|\sqrt{\frac{mg}{k}}+v\right|-\ln\left|\sqrt{\frac{mg}{k}}-v\right|\right)=\frac{kt}{m}+C$$

$$\frac{1}{2}\sqrt{\frac{k}{mg}}\ln\left|\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right|=\frac{kt}{m}+C$$

$$e^{\ln\left|\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right|}=e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\left(t+D\right)}$$

$v(0)=0$

$y'(0)=0$

$$\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}=E\,e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}$$

$$\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}=e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}$$

$$\sqrt{\frac{mg}{k}}+v=\sqrt{\frac{mg}{k}}\,e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}-v\,e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}$$

$$v+v\,e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}=\sqrt{\frac{mg}{k}}\,e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}-\sqrt{\frac{mg}{k}}$$

$$v\left(1+e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}\right)=\sqrt{\frac{mg}{k}}\left(e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}-1\right)$$

$$v=\sqrt{\frac{mg}{k}}\left(\frac{e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}-1}{1+e^{\frac{2k}{m}\sqrt{\frac{mg}{k}}\,t}}\right)$$

$$v=\sqrt{\frac{mg}{k}}\tanh\left(\frac{k}{m}\sqrt{\frac{mg}{k}}\;t\right).$$

$$f(x) = \sinh(x) = \frac{e^{x} - e^{-x}}{2}, \quad f'(x) = \cosh x$$

$$g(x) = \cosh(x) = \frac{e^{x} + e^{-x}}{2} \qquad g'(x) = \sinh x$$

$$h(x) = \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$h'(x) = \frac{(e^{x} + e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^2}$$

$$= \left(\frac{e^{x} + e^{-x}}{e^{x} + e^{-x}}\right)^2 - \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)^2 = 1 - h^2(x)$$

$$\int \tanh(x)\,dx = \int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$$

$$= \ln\left|e^x + e^{-x}\right| + A$$

$$\left(\begin{array}{l} u = e^x + e^{-x} \\ du = (e^x - e^{-x})\,dx \end{array}\right.$$

$$= \ln\left|2\cdot\cosh(x)\right| + A$$

$$= \ln|2| + \ln\left|\cosh(x)\right| + A$$

$$= \ln\left|\cosh(x)\right| + B$$

$$\frac{dy}{dt} = \sqrt{\frac{mg}{k}} \tanh\left(\frac{k}{m}\sqrt{\frac{mg}{k}}\, t\right)$$

$$\int dy = \int \sqrt{\frac{mg}{k}} \tanh\left(\frac{k}{m}\sqrt{\frac{mg}{k}}\, t\right) dt$$

$$y = \frac{m}{k} \ln\left|\cosh\left(\frac{k}{m}\sqrt{\frac{mg}{k}}\, t\right)\right| + C$$

$$\left(\begin{aligned} u &= \frac{k}{m}\sqrt{\frac{mg}{k}}\, t \\ du &= \frac{k}{m}\sqrt{\frac{mg}{k}}\, dt \end{aligned} \right.$$

$$y = \frac{m}{k} \ln\left|\cosh\left(\frac{k}{m}\sqrt{\frac{mg}{k}}\, t\right)\right|$$

$S' = S$ $\quad S(0) = 1$

$S = \overset{=1}{A} + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6$

$S' = \underset{=1}{B} + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \cdots$

$2C = 1$ $\quad 3D = 1$ $\quad 4E = \frac{1}{2}$ $\quad 5F = \frac{1}{24}$

$C = \frac{1}{2}$ $\quad D = \frac{1}{6}$ $\quad E = \frac{1}{24}$

$F = \frac{1}{120}$

$S = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$

$$S'' = S \qquad S(0) = 1$$

$$S'(0) = 1$$

$$S = \overset{1}{A} + \overset{1}{B}x + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + \cdots$$

$$S' = \overset{1}{B} + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \cdots$$

$$S'' = 2C + 6Dx + 12Ex^2 + 20Fx^3 + 30Gx^4$$

$$C = \frac{1}{2} \qquad D = \frac{1}{6} \qquad E = \frac{1}{24}$$