Vectors in $\Re^3$
Torque

(1) ***Let*** $\overline{u} = \overline{i} - 2\overline{j} + 5\overline{k}$ ***and*** $\overline{v} = -3\overline{i} + \overline{j} - \overline{k}$

(1a) Find $2\overline{u}$

(1b) Find $2\overline{v}$

(1c) Find $2\overline{u} + 2\overline{v}$

(1d) Find $2(\overline{u} + \overline{v})$

(1e) What have you just demonstrated?

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(2) Let A(0,1), B(3,0), C(4,4) and D(1,5)

(2a) Sketch $\overline{CD}$ and $\overline{CB}$

(2b) Find the components of $\overline{CD}$ and $\overline{CB}$

(2c) Find the cross product $\overline{CD}$ x $\overline{CB}$

(2d) Find m∠BCD using the cross product

(2e) Find the area of ΔABC using the cross product

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(3) ***Let $\overline{u}$ = <1",2",3">, $\overline{v}$ = <0",–2",5">, $\overline{w}$ = <1",1",0">***

(3a) Find $\overline{v} \times \overline{w}$

(3b) Find $\overline{u} \cdot (\overline{v} \times \overline{w})$

(3c) Find $| \overline{v} \times \overline{w} |$

(3d) What are the units of $| \overline{v} \times \overline{w} |$ and why?

(3e) What are the units of $\overline{u} \cdot (\overline{v} \times \overline{w})$ and why?

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(4) Confirm the following Lemma.

The area of a triangle with vertices (x_0, y_0), (x_1, y_1), (x_2, y_2) is given by:

$$A = \frac{1}{2}\sum_{i=0}^{2}(x_i\, y_{i+1} - y_i\, x_{i+1})$$

$$A = \frac{1}{2}\left[(x_0\, y_1 - y_0\, x_1) + (x_1\, y_2 - y_1\, x_2) + (x_2\, y_0 - y_2\, x_0)\right]$$

(4a) Construct ΔABC such that A(1, 2), B(4, 3) and C(0, 0)

(4b) Use the distance formula to find the length of each side of ΔABC.

(4c) Apply Heron's Formula to find the area of ΔABC.

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(4d) Now try the Lemma and see if you get the same area.

(4e) Find $\overline{CA} \times \overline{CB}$

(4f) How are the calculations in steps (4d) & (4e) related?

(4g) Does the order of the vector cross product make a difference?

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(5) Show that the following Theorem is based on triangulation and vector cross products!

Given the vertices of an n-sided polygon, (x_0, y_0), (x_1, y_1), (x_2, y_2), …, x_{n-1}, y_{n-1}), the area A is given by:

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i\, y_{i+1} - y_i\, x_{i+1})$$

Note: when i+1 = n, replace i+1 with 0.

This theorem is also known as the Surveyor's Formula. Surveyors use this formula to calculate the area of oddly shaped polygonal plots of land quickly and accurately.

(5a) Construct the pentagon ABCDE such that A(5,2), B(6, 4), C(4, 5), D(1, 4) and E(2, 2).

(5b) Apply the Surveyor's Formula to finding the area of the pentagon.

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(5c) Find the following vector cross products.

$\overline{OA} \times \overline{OB}$, $\overline{OB} \times \overline{OC}$, $\overline{OC} \times \overline{OD}$, $\overline{OD} \times \overline{OE}$, $\overline{OE} \times \overline{OA}$

(5d) Find the sum of all these vector cross products.

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(5e) What does this vector sum have to do with the Surveyor's Formula.

(5f) Some of the vector cross products contain negative components. Why is this significant?

(5g) Research the Shoelace Algorithm online. Recalculate the area of the pentagon using this algorithm. Is this different from the Surveyor's Formula?

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(6) ***Let $\overline{L} = \overline{r} \times \overline{p}$ represent angular momentum. Let $\overline{r}$ be the position vector for a particle's circular path measured from the axis of rotation. Let $\overline{p} = m\overline{v}$ be the linear momentum vector which is tangential to the trajectory. Let the torque $\overline{T} = \dfrac{d\overline{L}}{dt}$.***

$$[\overline{r}] = \text{meters},\ [m] = \text{kilograms},\ [t] = \text{seconds}$$

(6a) Show that the torque $\overline{T} = \overline{r} \times \overline{F}$ by applying the product rule of differentiation.

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(6b) Given that $\overline{r}$ = <sin(t), cos(t), 1> and m=2, find $\overline{p}$ (t).

(6c) Given that $\overline{r}$ = <sin(t), cos(t), 1> and m=2, find $\overline{L}$ (t).

(6d) Given that $\overline{r}$ = <sin(t), cos(t), 1> and m=2, find $\overline{T}$ (t).

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(6e) Given that $\overline{r} = \langle \sin(t), \cos(t), 1 \rangle$ and m=2,

find $\overline{p}(t)$, $\overline{L}(t)$ and $\overline{T}(t)$ when $t = \frac{\pi}{2}$.

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(6f) Given that $\overline{r} = <\sin(t), \cos(t), 1>$ and m=2,

find $p(\frac{\pi}{2})$, $L(\frac{\pi}{2})$ and $T(\frac{\pi}{2})$, the magnitudes of

$\overline{p}(t)$, $\overline{L}(t)$ and $\overline{T}(t)$ when $t = \frac{\pi}{2}$ (include units).

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Teacher lectures based on Larson's Calculus 3rd ed. ©1986
12.2 = Vectors in $\Re^3$
12.4 = Cross Product