CalculiX Benchmark Study: Two-Dimensional Blisk

Süleyman Muti

May 19, 2020

Abstract

The analytical and finite element analysis results of a two-dimensional rotating blisk problem are compared. The free and open-source finite element analysis software CalculiX is used. The blisk is modelled using axisymmetric elements for the disk portion and plane strain elements for the blade portion.

1 Rotating Blisk Problem

Consider the following two-dimensional rotating blisk (disk with integral blades) problem shown in Figure 1 based on Reference [1].

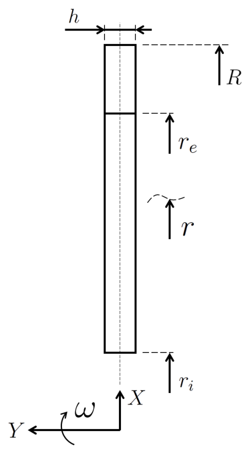

Figure 1: Parametric rotating blisk.

There are N number of blades in 360°. Blades are rectangular prisms and span from r_e to R, i.e., the blade height is $R - r_e$. The blisk is made of a linear elastic metal with an elasticity modulus of E, Poisson's ratio of ν, and density of γ. A blade has a width of w.

Following ratio between the total volume of the blades and the volume of the corresponding solid ring between radii r_e and R apply.

$$k{=}\frac{V_{blades}}{V_{ring}}, \quad 0{<}k{<}1. \tag{1}$$

Using the following given parameters, calculate the radial stresses, hoop stresses, and radial displacements along the blisk symmetry axis between the bore and hub radii.

Geometry:
$r_i = 100\ mm$,
$r_e = 400\ mm$,
$R = 660\ mm$,
$h = 10\ mm$,
$N = 24$,
$k = 0.5$.

Material properties:
$E = 2.1 \times 10^5\ MPa$,
$\nu = 0.3$,
$\gamma = 7.8 \times 10^{-9}\ tonne/mm^3$.

Loading:
$\omega = 1800\ rpm$.

The problem geometry with given parameters is illustrated in Figure 2.

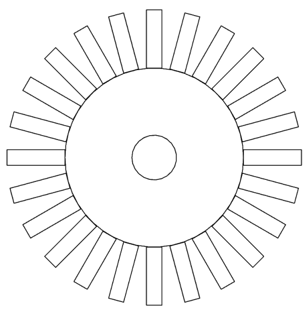

Figure 2: Rotating blisk with given parametric values.

2 Solution: The Theory of Elasticity

The rotating blisk problem can be considered as the superposition of two elastic problems.

- Non-rotating annular disk with radial stress applied at the outer radius,
- Rotating annular disk of same size.

The rotating blisk cannot carry hoop stress between the hub radius r_e and tip radius R. The centrifugal force due to the rotating mass of the blades is reflected as a boundary condition to the non-rotating annular disk problem.

Solution to non-rotating annular disk loaded at the outer radius:
The solution is given by the following equations [1].

$$\sigma_r = \sigma_{re}\frac{1}{1-\beta^2}\left(1-\frac{\beta^2}{\rho^2}\right) \tag{2}$$

$$\sigma_t = \sigma_{re}\frac{1}{1-\beta^2}\left(1+\frac{\beta^2}{\rho^2}\right) \tag{3}$$

$$u_r = \sigma_{re}\frac{r_e}{E}\frac{\rho}{1-\beta^2}\left[(1-\nu)+(1+\nu)\frac{\beta^2}{\rho^2}\right] \tag{4}$$

Solution to rotating annular disk problem:
The solution is given by the following equations [1].

$$\sigma_r = \frac{(3+\nu)}{8}\sigma_0\left(1+\beta^2-\frac{\beta^2}{\rho^2}-\rho^2\right) \tag{5}$$

$$\sigma_t = \frac{(3+\nu)}{8}\sigma_0\left(1+\beta^2+\frac{\beta^2}{\rho^2}-\frac{(1+3\nu)}{(3+\nu)}\rho^2\right) \tag{6}$$

$$u_r = \frac{r_e}{E}\rho\frac{(3+\nu)}{8}\sigma_0\left[\left(1+\beta^2\right)(1-\nu)+(1+\nu)\frac{\beta^2}{\rho^2}-\rho^2\left(\frac{1-\nu^2}{3+\nu}\right)\right] \tag{7}$$

where
$\rho = r/r_e$,
$\beta = r_i/r_e$,
$\sigma_{re} = \sigma_r|_{r_e}$,
$\sigma_0 = \gamma\omega^2{r_e}^2$.

The radial stress at hub radius r_e:

The radial stress at hub radius r_e is given by Equation 8.

$$\sigma_{re} = \frac{F_c}{A_{r_e}} \tag{8}$$

where F_c is the centrifugal force due to the rotating mass of the blades, and A_{r_e} is the area of the cylindrical surface at the hub radius r_e.

The centrifugal force due to the rotating mass of the blades is given by Equation 9.

$$F_c = \frac{k\gamma\pi h(R^2 - {r_e}^2)\omega^2(R + r_e)}{2} \tag{9}$$

The area of the cylindrical surface at the hub radius r_e is given by Equation 10.

$$A_{r_e} = 2\pi h r_e \tag{10}$$

The radial stress at hub radius r_e is obtained by substituting Eqs. 9 and 10 into Eq. 8.

$$\begin{aligned} \sigma_{re} &= \frac{k\gamma\pi h\omega^2(R^2 - {r_e}^2)(R + r_e)}{4\pi h r_e} \\ &= \frac{k\gamma\omega^2(R^2 - {r_e}^2)(R + r_e)}{4r_e} \end{aligned} \tag{11}$$

The radial stress value at hub radius r_e is obtained by substituting the given parameter values into Equation 11.

$$\sigma_{re} = 25.3\, MPa \tag{12}$$

The reference stress σ_0:

$$\begin{aligned} \sigma_0 &= \gamma\omega^2{r_e}^2 \\ &= 44.34\, MPa \end{aligned} \tag{13}$$

A MATLAB code is written to calculate and plot the radial stresses, hoop stresses, and radial displacements along the blisk symmetry axis from the bore radius r_i to te hub radius r_e. The radial and hoop stresses are shown in Figure 3, and the radial displacements are shown in Figure 4.

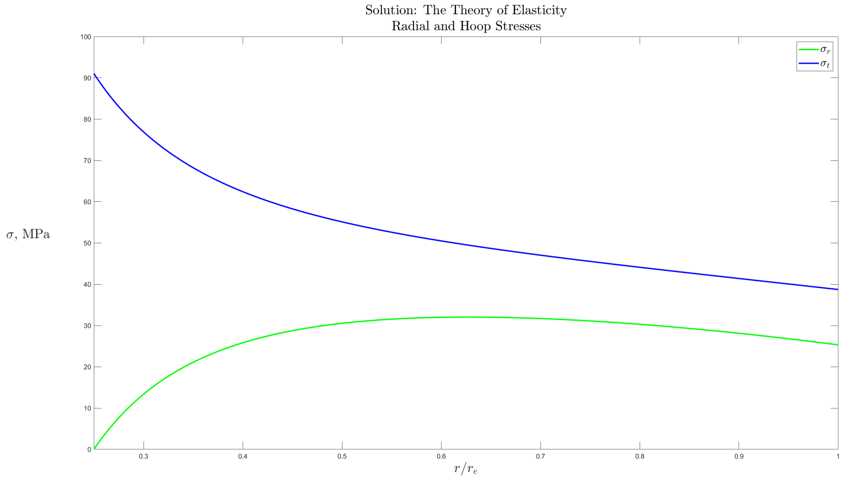

Figure 3: Theory of Elasticity solution for radial and hoop stresses.

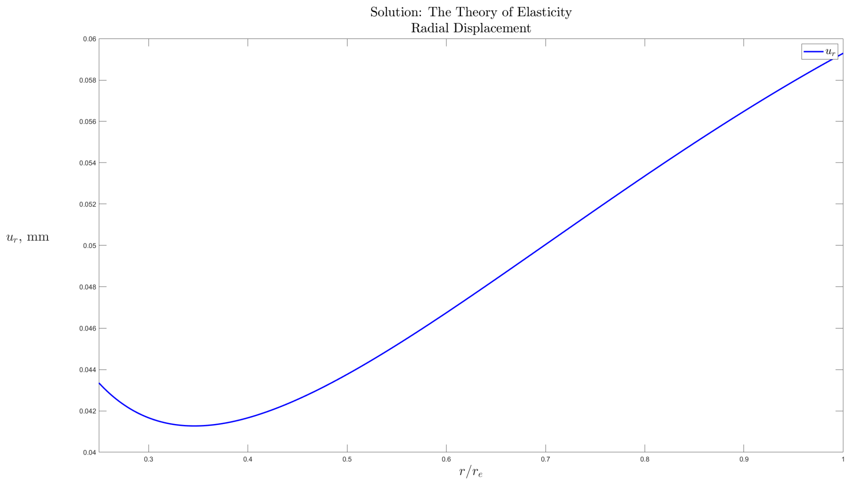

Figure 4: Theory of Elasticity solution for radial displacements.

Summary:

Maximum radial stress is 32 MPa.
Maximum hoop stress is 91 MPa.
Radial displacement at the inner radius is 0.043353 mm.
Radial displacement at the outer radius is 0.059294 mm.

3 Solution: Finite Element Analysis with CalculiX

A parametric finite element model for CalculiX [2] is created to compare finite element results with the theory of elasticity results.

In CalculiX, the disk portion is modelled using axisymmetric elements (qu8cr, CAX8R), and the blade portion is modelled using plane stress elements (qu8sr, CPS8R) with thickness.

The thickness of the plane stress elements needs to be determined such that the centrifugal force due to the rotating mass of the blades is correctly included in the two-dimensional finite element analysis.

The blades are assumed as rectangular prisms as show in Figure 5.

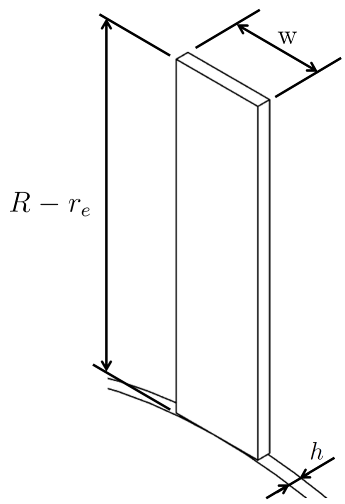

Figure 5: Parametric blade dimensions.

Equation 1 defines a relationship between the volume of the corresponding full ring prior to the machining of the blades, visualized in Figure 6, and the total volume of the blades visualized in Figure 7.

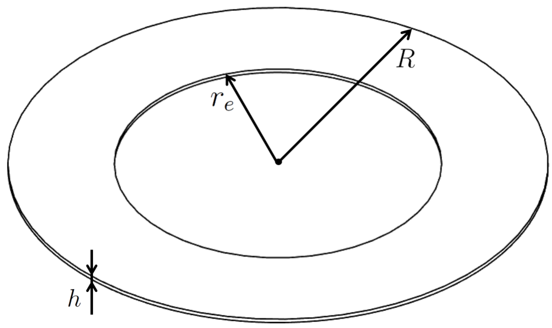

Figure 6: The volume of the corresponding full ring prior to the machining of the blades.

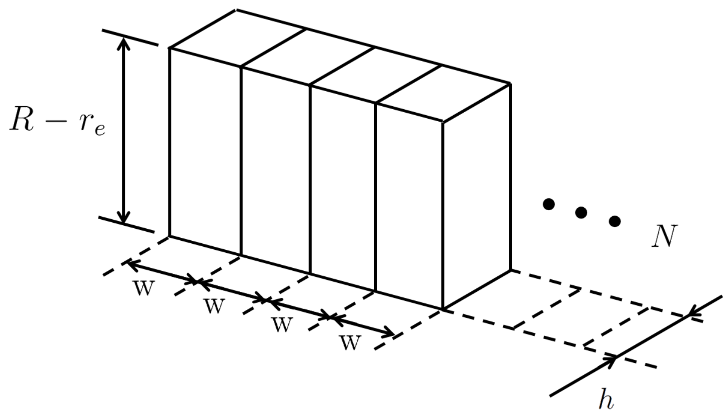

Figure 7: The total volume of blades.

Equation 1 is rewritten as

$$V_{blades} = kV_{ring}. \tag{14}$$

The volume of the corresponding full ring prior to the machining of the blades is

$$\begin{aligned} V_{ring} &= \pi R^2 h - \pi r_e^2 h \\ &= \pi h (R^2 - r_e^2). \end{aligned} \tag{15}$$

The total volume of the blades is

$$V_{ring} = N\mathrm{w}h(R - r_e). \tag{16}$$

The thickness of the plane stress elements in the finite element analysis is defined as

$$t = N\mathrm{w}. \tag{17}$$

Substituting Eq. 17 into Eq. 16, we have

$$V_{ring} = th(R - r_e). \tag{18}$$

Substituting Eqs. 18 and 15 into Eq. 14, we have

$$\begin{aligned} th(R - r_e) &= k\pi h(R^2 - r_e^2) \\ t\cancel{h}\cancel{(R - r_e)} &= k\pi \cancel{h}\cancel{(R - r_e)}(R + r_e), \end{aligned} \tag{19}$$

and

$$t = k\pi(R + r_e). \tag{20}$$

The radial stress, hoop stress, and radial displacement results from CalculiX are shown in Figures, 8, 9, and 10, respectively.

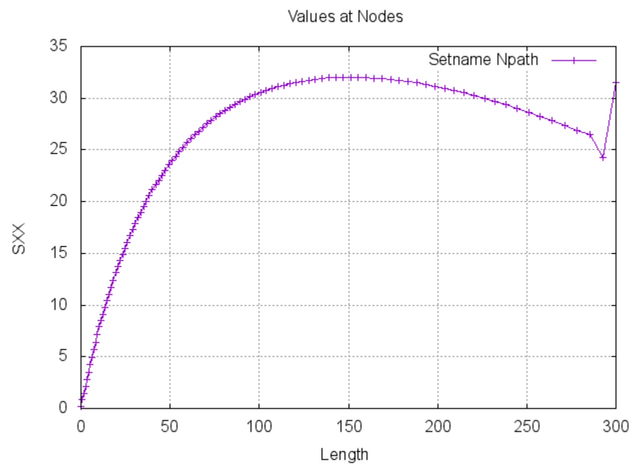

Figure 8: CalculiX radial stress plot.

Summary:

Maximum radial stress is 32 MPa.
Maximum hoop stress is 91 MPa.
Radial displacement at the inner radius is 0.043348 mm.
Radial displacement at the outer radius is 0.059241 mm.

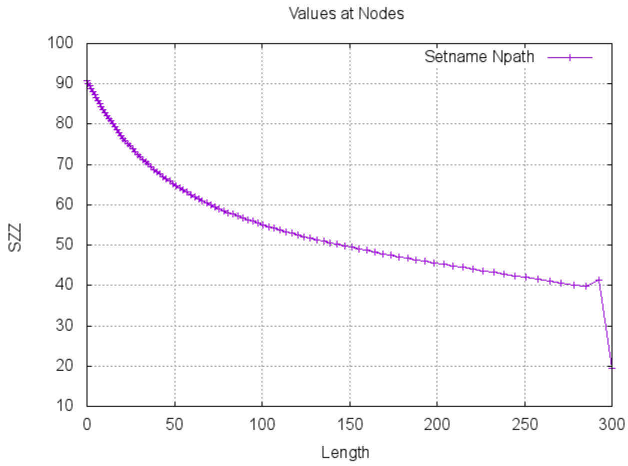

Figure 9: CalculiX hoop stress plot.

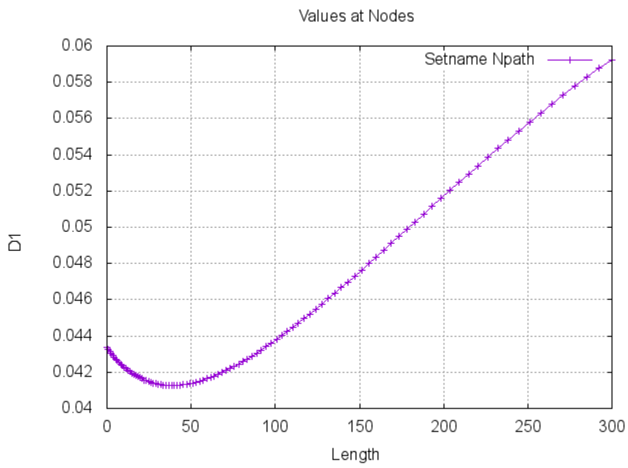

Figure 10: CalculiX radial displacement plot.

4 Comparison

Dimensionless stresses and displacement are defined below (see Eqs. 21, 22, and 23).

$$\overline{\sigma}_r = \frac{8}{(3+\nu)\rho\omega^2 r_e^2}\sigma_r \tag{21}$$

$$\overline{\sigma}_t = \frac{8}{(3+\nu)\rho\omega^2 r_e^2}\sigma_t \tag{22}$$

$$\overline{u}_r = \frac{8E}{(3+\nu)(1-\nu)\rho\omega^2 r_e^3}u_r \tag{23}$$

A MATLAB code is written to perform the following tasks:

- Run a Python code that performs the following tasks:
  - Run the pre-processing file that contains parametric model information. Create the geometry and mesh, and export the finite element pre-processing data in CalculiX format.
  - Run the finite element input file in CalculiX.
  - Run the post-processing file that generates result plots and saves them.
- Load CalculiX results into MATLAB workspace,
- Define the problem geometry with given parametric values for analytical solution.
- Calculate the analytical results for radial stresses, hoop stresses, and radial displacements and plot them.
- Derive the dimensionless stresses, and displacements from the analytical and CalculiX results, and compare them in a single plot, from r_i to r_e.

The comparison between the analytical solution and the finite element results obtained from CalculiX is presented in Figure 11. It can be observed that the finite element results from CalculiX are the same as the analytical solution except in the very close proximity to the plane stress elements.

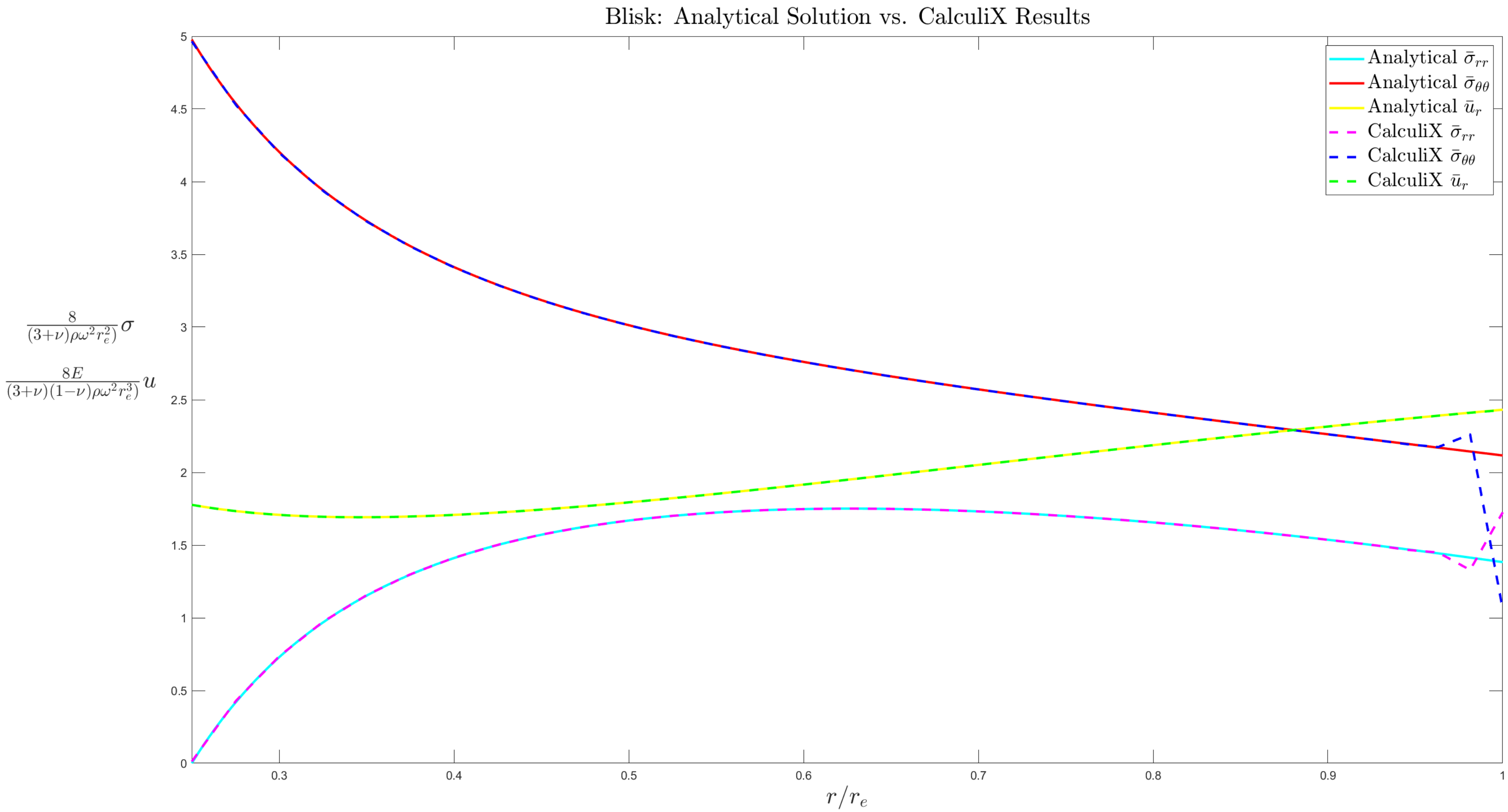

Figure 11: Two-dimensional rotating blisk: Analytical solution vs. CalculiX results.

References

[1] Rotors: Stress Analysis and Design, 2013. Page 39, Example 4.

[2] CalculiX, A Free Software Three-Dimensional Structural Finite Element Program. http://www.calculix.de/