

# Vibration of a Rotating Cantilever Beam

Süleyman Muti

May 19, 2020

## Abstract

This study demonstrates the natural frequency analysis of a cantilever beam subjected to centrifugal loading. This is analogous to the vibrations of compressor and turbine blades.

The free and open-source finite element analysis software CalculiX is used [1].

## 1 Description

The beam is built into a rigid disk of  $r$  *mm* radius, rotating about its axis at a speed of  $\Omega$  *rpm*. It is  $\ell$  *mm* long,  $w$  *mm* wide,  $t$  *mm* thick, and makes an angle of  $\alpha$  *degrees* with the rotation axis, as shown in Figure 1.

The plate is made of steel, with a Young's modulus  $2.17 \times 10^5$  *MPa* and a density of  $7.85 \times 10^{-9}$  *tonne/mm<sup>3</sup>*.

## 2 Pre-processing

Dimensions are given below:

$$\begin{aligned} r &= 150 \text{ mm}, \\ \ell &= 328 \text{ mm}, \\ w &= 28 \text{ mm}, \\ t &= 3 \text{ mm}, \\ \alpha &= 0 \text{ degrees.} \end{aligned}$$

The general purpose quadratic brick element with reduced integration (C3D20R) is used. See Figure 2 for the finite element model of the rotating cantilever beam. The the beam is cantilevered at  $r = 150$  *mm*. A rotational speed of  $\Omega = 4500$  *rpm* about the axis of the rigid disk is applied to the whole model.

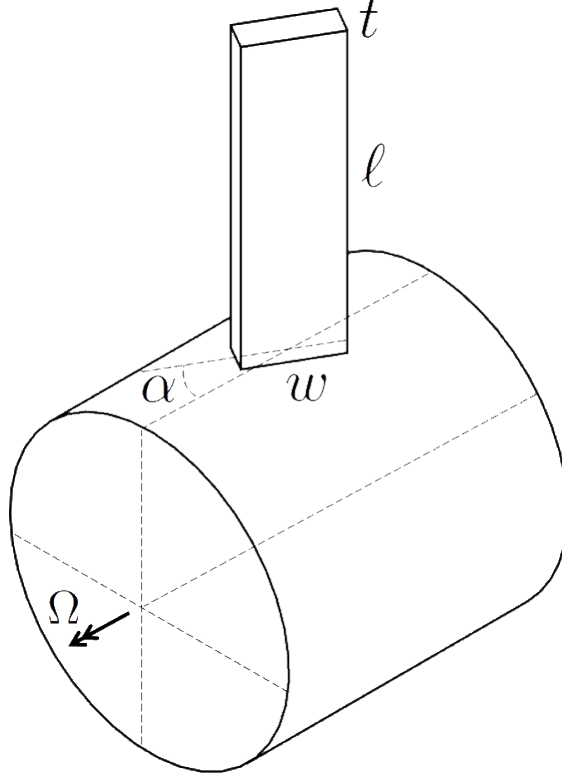


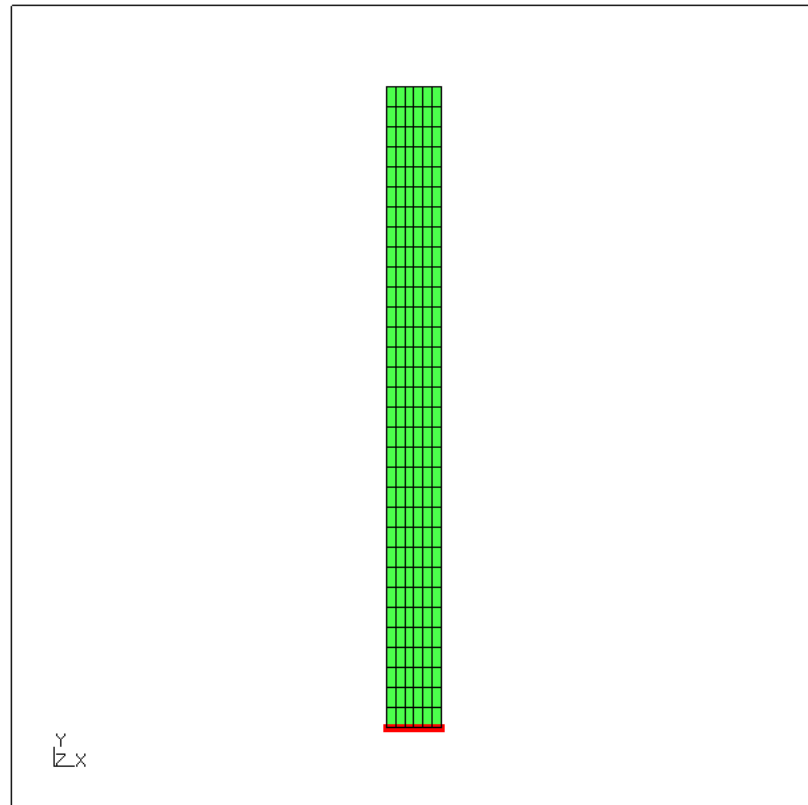
Figure 1: Rotating cantilever beam.

### 3 Results

The first vibration mode is obtained as  $71.81 \text{ Hz}$  from CalculiX. This result is can be compared with the results reported for Abaqus 6.6 in Reference [2]. Table 1 lists the first three modes obtained with CalculiX.

Table 1: Natural frequencies of the rotating cantilever beam.

Mode #	Frequency [Hz]
1	71.807009
2	243.222157
3	273.155573

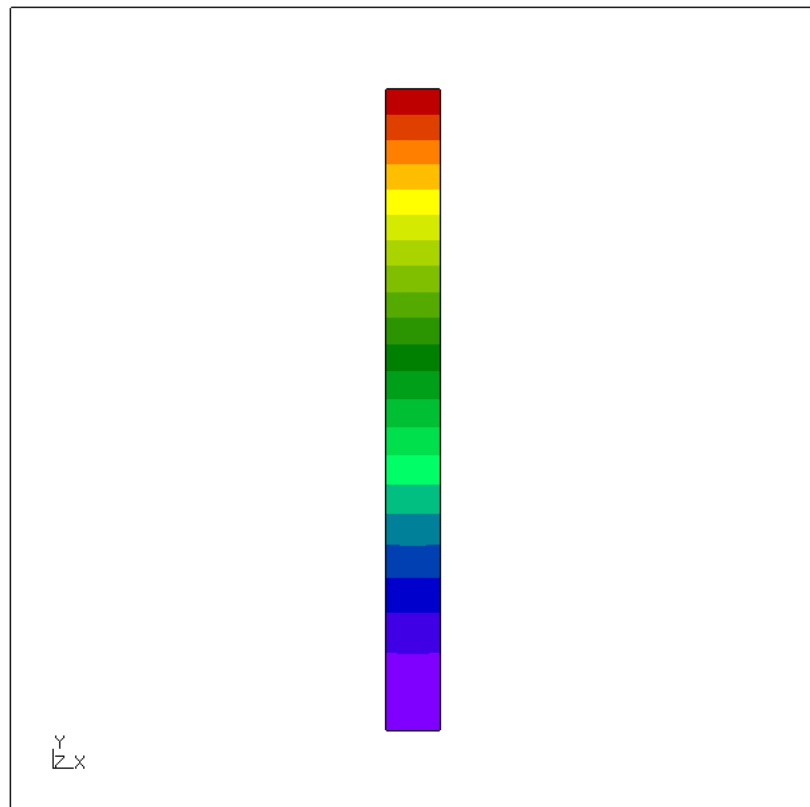
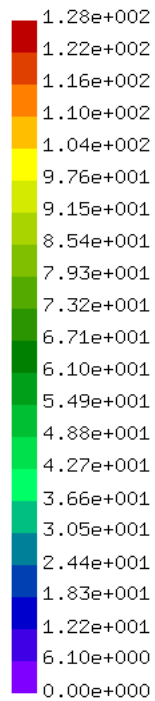


rotating\_cantilever\_beam\_pre.fbd  
FE model of the rotating cantilever beam (green). Boundary conditions (red).

Figure 2: Finite element model of the rotating cantilever beam.

2/1:DISP  
Time:71.807009  
Entity:D3

max: 1.28e+002  
min: 0.00e+000



rotating\_cantilever\_beam\_post.fbd  
Mode 1 displacements in z-axis.

Figure 3: Mode 1 displacements in z-axis.

## References

- [1] CalculiX, A Free Software Three-Dimensional Structural Finite Element Program. <http://www.calculix.de/>
- [2] Washington University in St. Louis, ABAQUS Benchmarks Manual, Vibration of a rotating cantilever plate. <https://classes.engineering.wustl.edu/2009/spring/mase5513/abaqus/docs/v6.6/books/bmk/default.htm?startat=ch01s04ach43.html>