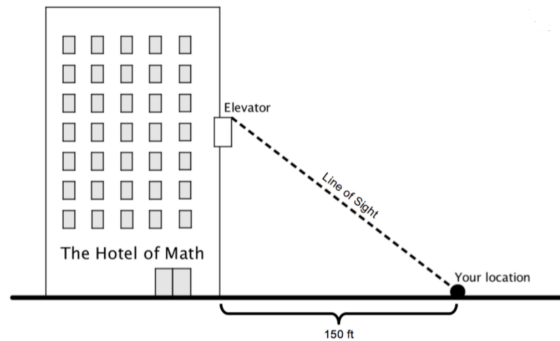


## 0.1 Related Rates

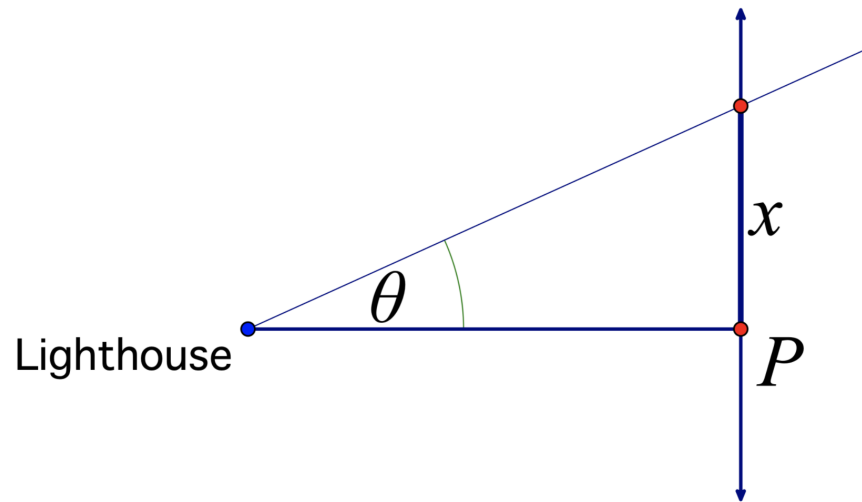
1. While sitting at an outdoor restaurant in a large city you notice that a hotel has an elevator on the outside of the building. You are 150 feet away from the hotel.



What is the relationship between the rate of change of the height,  $h$ , of the elevator and the rate of change of the angle,  $\theta$ , between the ground and your line of sight?

- (a)  $\frac{d\theta}{dt} = (h^2 + 150^2) \frac{dh}{dt}$
- (b)  $\tan \frac{d\theta}{dt} = \frac{1}{150} \frac{dy}{dt}$
- (c)  $\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{150} \frac{dy}{dt}$
- (d)  $\frac{d\theta}{dt} = \sqrt{150^2 - 2h} \frac{dh}{dt}$
- (e)  $\frac{d\theta}{dt} = 75h \frac{dh}{dt}$

2. A lighthouse is located on a small island 3 kilometers away from the nearest point  $P$  on a straight shoreline. Let  $x$  represent the distance between  $P$  and the light beam's intersection with the shoreline. Also let  $\theta$  represent the measure of the angle created by the beam of light and the line connecting the lighthouse and  $P$ . Which formula defines the relationship between the rate of change of the angle's measure and the rate at which the beam of light is moving along the shoreline?



- (a)  $\tan\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$
- (b)  $\sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{1}{3} \cdot \frac{dx}{dt}$
- (c)  $\sec^2\left(\frac{d\theta}{dt}\right) = \frac{1}{3} \cdot \frac{dx}{dt}$
- (d)  $\frac{d\theta}{dt} = \frac{1}{3 \sec^2(\theta)}$
- (e)  $\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{dx}{dt}\right)^2}$

3. A cylindrical container of fixed radius  $r$  is being filled with water. Which of the following equations expresses the relationship between the rate of change of the volume  $V$  of the water in the container (with respect to time) and the rate of change of the height  $h$  of the water in the container (with respect to time)?

(a)  $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$   
(b)  $V = \pi r^2 h$   
(c)  $\frac{dV}{dt} = 2\pi r h + \pi r^2 \cdot \frac{dh}{dt}$   
(d)  $V = 2\pi r h$   
(e)  $\frac{dh}{dt} = \pi r^2 h$

4. A spherical ice ball of radius  $r$  is melting in a liquid. It melts in a uniform fashion so that it remains a sphere while melting. Which of the following equations expresses the relationship between the rate of change of the volume  $V$  of the ice (with respect to time) and the rate of change of its radius  $r$  (with respect to time)?

(a)  $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$   
(b)  $\frac{dV}{dt} = 4\pi r^2$   
(c)  $V = \frac{4}{3}\pi r^3$   
(d)  $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$   
(e)  $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt}$

5. An ice cube that is initially three inches wide is placed on a table and starts to melt. (Assume that the ice cube melts in a uniform fashion so that at every instant it remains a cube.) Let  $V$  denote the volume of the cube, measured in cubic inches, let  $t$  denote the number of minutes elapsed since the cube began to melt, and let  $x$  denote the width of the cube, measured in inches. If we know that the values of  $x$  are related to the values of  $t$  according to the formula  $x = 3e^{-t}$ , then which of the following formulas correctly gives the instantaneous rate of change in  $V$  with respect to  $t$ ?

(a)  $V'(t) = -81e^{-3t}$   
(b)  $V'(t) = \frac{27e^{-3t} - 27}{t}$   
(c)  $V'(t) = 3e^{-t}$   
(d)  $V'(t) = -3e^{-t}$   
(e)  $V'(t) = 9e^{-3t}$

6. A stone thrown into a pond produces a circular ripple. If the radius of the ripple increases at a rate of 1.5 ft/sec, how fast is the area growing when the radius is 8 feet? State your answer as an exact value (i.e., in terms of  $\pi$ ).
7. The hour hand of a clock tower is 5 meters long. The minute hand is 7 meters long. How fast, in meters/hr, is the distance between the tips of the hands changing at 3:00 AM?
8. Suppose gravel is being poured into a conical pile at a rate of  $5 \text{ m}^3/\text{s}$ , and suppose that the radius  $r$  of this cone is always half its height  $h$ . How fast is the height of the pile increasing when the height is 10 m?

(Note that the formula for the volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ ).

