Math 275: Statistical Inference	**Winter 2014**
Homework #7	*Calder Coalson*

Chapter 6

28.

$$\hat{\theta}_1' = \frac{\hat{\theta}_1}{0.9} \qquad E[\hat{\theta}_1'] = \frac{E[\hat{\theta}_1]}{0.9} = \theta \qquad V[\hat{\theta}_1'] = \frac{V[\hat{\theta}_1]}{0.9^2} = 3.70$$

$$\hat{\theta}_2' = \frac{\hat{\theta}_2}{1.2} \qquad E[\hat{\theta}_2'] = \frac{E[\hat{\theta}_2]}{1.2} = \theta \qquad V[\hat{\theta}_2'] = \frac{V[\hat{\theta}_2]}{1.2^2} = 1.39$$

$V[\hat{\theta}_2'] < V[\hat{\theta}_1']$ so $\hat{\theta}_2'$ is more efficient.

29.

$$E[\hat{\theta}_1] = E[X_1] = \int_0^\infty x\frac{e^{-x/\theta}}{\theta}\ dx\ = \theta \qquad V[\hat{\theta}_1] = V[X_1] = \int_0^\infty x^2\frac{e^{-x/\theta}}{\theta}\ dx\ = 2\theta^2$$

$$E[\hat{\theta}_2] = E\left[\frac{X_1+X_2}{2}\right] = \frac{E[X_1]+E[X_2]}{2} = \theta \qquad V[\hat{\theta}_2] = V\left[\frac{X_1+X_2}{2}\right] = \frac{V[X_1]+V[X_2]}{4} = \theta^2$$

$$E[\hat{\theta}_3] = E\left[\frac{X_1+2X_2}{3}\right] = \frac{E[X_1]+2E[X_2]}{3} = \theta \qquad V[\hat{\theta}_3] = V\left[\frac{X_1+2X_2}{3}\right] = \frac{V[X_1]+4V[X_2]}{9} = \frac{10}{9}\theta^2$$

30. (a)

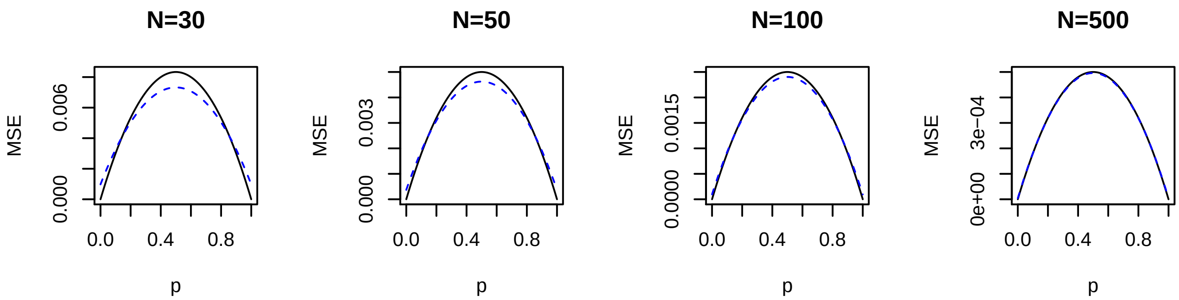
$$MSE[\hat{\theta}_1] = B[\hat{\theta}_1]^2 + V[\hat{\theta}_1] = 0^2 + 25 = 25$$

$$MSE[\hat{\theta}_2] = B[\hat{\theta}_2]^2 + V[\hat{\theta}_2] = 3^2 + 4 = 13$$

(b)

$$\begin{aligned} MSE[\hat{\theta}_2] &< MSE[\hat{\theta}_1] \\ b^2 + 4 &< 25 \\ b &< \sqrt{19} \end{aligned}$$

31.




```
f = function (n) {
  curve(x*(1-x)/n, from=0, to=1, main=sprintf("N=%d",n), xlab="p", ylab="MSE")
  curve(n * (1-x)*x/(n+2)^2 + (1-2*x)^2/(n+2)^2, add=TRUE, col="blue", lty=2)
}
f(30); f(50); f(100); f(200)
```

The MSE converges on the MSE of the original estimator as $N \to \infty$.

32. (a)

$$\begin{aligned}
B[\hat{\beta}_1] &= E[\hat{\beta}_1] - \beta \\
&= (n+1)E[X_{min}] - \beta \\
&= (n+1)\int_0^\beta x\frac{1}{\beta}\left(\frac{x}{\beta}\right)^{n-1} dx - \beta \\
&= (n+1)\frac{1}{\beta^n}\int_0^\beta x^n dx - \beta \\
&= (n+1)\frac{\beta}{n+1} - \beta \\
&= 0
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{V[\hat{\beta}_1]}{V[\hat{\beta}_2]} &= \frac{V[(n+1)X_{min}]}{V[((n+1)/n)X_{max}]} \\
&= n^2\frac{V[X_{min}]}{V[X_{max}]} \\
&= n^2\frac{V[X_{min}]}{V[X_{min}]} && \text{(by symmetry)} \\
&= n^2
\end{aligned}$$

33. (a)

$$E[X_i] = \int_0^{1/\theta} x 2\theta^2 x \, dx \ = \frac{2}{3\,\theta}$$

(b)

$$\begin{aligned} B[T] &= E[T] - \frac{1}{\theta} \\ &= \frac{E[X_i]}{9} + \frac{E[X_i]}{9} + \frac{E[X_i]}{3} - \frac{1}{\theta} \\ &= \frac{10}{27\,\theta} - \frac{1}{\theta} \\ &= -\frac{17}{27\,\theta} \end{aligned}$$

$$\begin{aligned} MSE[T] &= B[T]^2 + V[T] \\ &= \frac{17^2}{27^2\,\theta^2} + \int_0^{1/\theta} x^2 2\theta^2 x \, dx \\ &= \frac{17^2}{27^2\,\theta^2} + \frac{1}{2\,\theta^2} \\ &= \frac{1307}{1458\,\theta^2} \end{aligned}$$

(c) Let $T' = \frac{27}{10}T$.

$$\begin{aligned} B[T'] &= E[T'] - \frac{1}{\theta} \\ &= \frac{27}{10}E[T] - \frac{1}{\theta} \\ &= 0 \end{aligned}$$

Chapter 7

1. (a) The statement is about the *mean*, not individual cows.
   (b) True.
   (c) μ is either in the interval or it isn't. We don't know which it is, but that doesn't make it a random variable.
   (d) True.
   (e) That would be true of the actual mean, but all we have is a sample mean.

2.

$$\begin{aligned} \mu_{sample} \pm 2\sigma_{sample} &= \mu \pm 2\frac{\sigma}{\sqrt{n}} \\ &= 538 \pm 2\frac{116}{\sqrt{34}} \\ &= 538 \pm 39.8 \end{aligned}$$

3. (a) $210 \pm 8.2 \frac{mg}{dL}$

```
sigma = 50
sample.size = 100
sample.mean = 210
qnorm(0.05, sample.mean, sigma/sample.size^0.5)
qnorm(0.95, sample.mean, sigma/sample.size^0.5)
```

(b)

$$error = -Q\left(\frac{1-confidence}{2}\right)\frac{\sigma}{\sqrt{n}}$$

$$n = Q\left(\frac{1-confidence}{2}\right)^2\frac{\sigma^2}{error^2}$$

$$n = 96$$

```
f = function (conf, err, sigma) {
  qnorm((1-conf)/2)^2 * sigma^2 / err^2
}
f(0.95, 10, 50)
```

(c)

$$n = 166$$

```
f(0.99, 10, 50)
```

5. 4 times larger.