

Algorithm to Pseudocode

This is very very basic pseudocode in an attempt to understand these algorithms.

Basic Program Structure

```
// Load the clauses from the source problem file
loadClauses();

// Evaluate and simplify clauses before main computation
initialEval();

// Loop through rest of clauses and evaluate the satisfiability
processClauses();
```

Algo 1 - SAT by backtracking

Give non-empty clauses $C_1 \wedge \dots \wedge C_m$ on $n > 0$ boolean variables $x_1 \dots x_n$, represented as above, this algorithm finds a solution if and only if the clauses are satisfiable. It records its current progress in an array $m_1 \dots m_n$ of "moves", whose significance is explained below.

1. [Initialise.] Set $a \leftarrow m$ and $d \leftarrow 1$. (Here a represents the number of active clauses, and d represents the depth-plus-one in an implicit search tree.)
2. [Choose.] Set $L \leftarrow 2d$. If $C(L) \geq C(L + 1)$, set $L \leftarrow L + 1$. Then set $md \leftarrow (L \& 1) + 4[C(L \oplus 1) = 0]$. Terminate successfully if $C(L) = a$.
3. [Remove $\neg L$.] Delete $\neg L$ from all active clauses; but go to 5 if that would make a clause empty. (We want to ignore $\neg L$, because we are making L true.)
4. [Deactivate L 's clauses.] Suppress all clauses that contain L . (Those clauses are now satisfied.). Then set $a \leftarrow a - C(L)$, $d \leftarrow d + 1$, and return to step 2.
5. [Try again.] If $md < 2$, set $md \leftarrow 3 - md$, $L \leftarrow 2d + (md \& 1)$, and go back to step 3.
6. [Backtrack.] Terminate unsuccessfully if $d = 1$ (the clauses are unsatisfiable). Otherwise set $d \leftarrow d - 1$ and $L \leftarrow 2d + (md \& 1)$.
7. [Reactivate L 's clauses.] Set $a \leftarrow a + C(L)$, and unsuppress all clauses that contain L . (Those clauses are now unsatisfied, because L is no longer true.)
8. [Unremove $\neg L$.] Reinstall $\neg L$ in all the active clauses that contain it. Then go back to step 5.

```
// This will contain the pseudocode for the above algorithm.
```

Algo 2 - SAT by watching

This algorithm works best with smaller problems.

Given non-empty clauses $C_1 \wedge \dots \wedge C_m$ on $n > 0$ boolean variables $x_1 \dots x_n$, represented as above, this algorithm finds a solution if and only if the clauses are satisfiable. It records its current progress in an array $m_1 \dots m_n$ of "moves", whose significance is explained below.

1. [Initialize.] Set $d \leftarrow 1$
2. [Rejoice or choose.] If $d > n$, terminate successfully. Otherwise set $m_d \leftarrow [W_{2d} = 0 \wedge \neg W_{2d+1} \neq 0]$ and $l \leftarrow 2d + m_d$
3. [Remove $\neg \bar{l}$ if possible.] For all j such that \bar{l} is watched in C_j , watch another literal of C_j . But go to B5 if that can't be done.
4. [Advance.] Set $W_{\bar{l}} \leftarrow 0$, $d \leftarrow d + 1$, and return to B2.
5. [Try again.] If $m_d < 2$, set $m_d \leftarrow 3 - m_d$, $l \leftarrow 2d + (m_d \& 1)$, and go to B3.
6. [Backtrack.] Terminate unsuccessfully if $d = 1$ (the clauses are unsatisfiable). Otherwise set $d \leftarrow d - 1$ and go back to B5.

Algo 3 - SAT by cyclic DPLL

"explained above" means the previous section from the book. Which in this instance is page 32.

Given non-empty clauses $C_1 \wedge \dots \wedge C_m$ on $n > 0$ Boolean variables $x_1 \dots x_n$, represented with lazy data structures and an active ring as explained above, this algorithm finds a solution if and only if the clauses are satisfiable. It records its current progress in an array $h_1 \dots h_n$ of indices and an array $m_0 \dots m_n$ of "moves," whose significance is explained below.

1. [Initialize.] Set $m_0 \leftarrow d \leftarrow h \leftarrow t \leftarrow 0$, and do the following for $k = n, n - 1, \dots, 1$: Set $x_k \leftarrow -1$ (denoting an unset value); if $W_{2k} \neq 0$ or $W_{2k+1} \neq 0$, set $\text{NEXT}(k) \leftarrow h$, $h \leftarrow k$, and if $t = 0$ also set $t \leftarrow k$. Finally, if $t \neq 0$, complete the active ring by setting $\text{NEXT}(t) \leftarrow h$.
2. [Success?] Terminate if $t = 0$ (all clauses are satisfied). Otherwise set $k \leftarrow t$.
3. [Look for unit clauses.] Set $h \leftarrow \text{NEXT}(k)$ and use the subroutine in exercise 129 to compute $f \leftarrow [2h \text{ is a unit}] + 2[2h + 1 \text{ is a unit}]$. If $f = 3$, go to D7. If $f = 1$ or 2 , set $m_{d+1} \leftarrow f + 3$, $t \leftarrow k$, and go to D5. Otherwise, if $h \neq t$, set $k \leftarrow h$ and repeat this stage.
4. [Two-way branch.] Set $h \leftarrow \text{NEXT}(t)$ and $m_{d+1} \leftarrow [W_{2h} = 0 \wedge \neg W_{2h+1} \neq 0]$