

## Group members

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## Project topic

Implementing parallel sorting algorithms (mergeSort, sampleSort, enumerationSort, oddEvenTranspositionSort) using MPI and CUDA, and subsequently analyzing and comparing the respective performances.

## Brief Project Description

We have implemented the following algorithms in different paradigms:

1. MergeSort
  - MPI
  - CUDA
2. SampleSort
  - MPI
  - CUDA
3. EnumerationSort
  - MPI
  - CUDA
4. OddEvenTranspositionSort
  - MPI
  - CUDA

# Pseudocode

## OddEven Sort in MPI:

Variables in main:

p : number of processes  
global\_n : number of elements in global array  
local\_n : number of elements in processes' local array  
local\_A : each processes' local array

Main Procedure:

1. Perform MPI initialization functions
2. Allocate local\_A
3. Populate local\_A
4. Call Sort on each process
5. MPI Finalize

Interpreting the sorted array:

The global array can be formed by concatenating each local array in ascending order by process rank. In MPI, this can be done with MPI\_Gather()

Variables in Sort:

my\_rank : the current processes' rank  
comm : MPI\_COMM\_WORLD  
phase : counter to track current phase, ranges from 0 to p  
temp\_B : temporary local storage for MPI\_Sendrecv  
temp\_C : temporary local storage for Merge\_high or Merge\_low  
local\_n : Same as in the Main Procedure  
local\_A : Same as in the Main Procedure  
p : Same as in the Main Procedure  
even\_partner : the process that will be communicated with during the even phase  
odd\_partner : the process that will be communicated with during the odd phase

Sort Procedure:

1. Allocate local storage temp\_B
2. Allocate local storage temp\_C
3. Determine even\_phase\_partner
4. Determine odd\_phase\_partner
5. Call quickSort on local\_A
6. For each phase:
  7. Call Odd-Even-Iterate
  8. Deallocate temp\_B
  9. Deallocate temp\_C

Variables in Odd-Even-Iterate:

my\_rank : the current processes' rank  
comm : MPI\_COMM\_WORLD  
status : MPI\_Status  
phase : counter to track current phase, ranges from 0 to p  
temp\_B : temporary local storage for MPI\_Sendrecv  
temp\_C : temporary local storage for Merge\_high or Merge\_low  
local\_n : Same as in the Main Procedure  
local\_A : Same as in the Main Procedure  
p : Same as in the Main Procedure  
even\_partner : the process that will be communicated with during the even phase  
odd\_partner : the process that will be communicated with during the odd phase

Odd-Even-Iterate Procedure:

1. if phase is even:
2. Call MPI\_Sendrecv to exchange values with even\_partner
3. if my\_rank is odd:
4. Call Merge\_high
5. else:
6. Call Merge\_low
7. else:
8. Call MPI\_Sendrecv to exchange values with odd\_partner
9. if my\_rank is odd:
10. call Merge\_low
11. else:
12. call Merge\_high

Merge\_high Variables:

local\_A : local array for a process  
temp\_B : contains array sent from partner process  
temp\_C : to be filled with the larger half of values from local\_A and temp\_B  
A\_ptr : iterator for local\_A  
B\_ptr : iterator for temp\_B  
C\_ptr : iterator for temp\_C

Merge\_high Procedure:

1. Initialize A\_ptr, B\_ptr, C\_ptr to point at the end of their respective arrays
2. While C\_ptr >= 0:
3. if local\_A[A\_ptr] >= temp\_B[B\_ptr]:
4. temp\_C[C\_ptr] = local\_A[A\_ptr]
5. decrement C\_ptr, A\_ptr
6. else:
7. temp\_C[C\_ptr] = temp\_B[B\_ptr]
8. decrement C\_ptr, B\_ptr

Merge\_low Variables:

- local\_A : local array for a process
- temp\_B : contains array sent from partner process
- temp\_C : to be filled with the larger half of values from local\_A and temp\_B
- A\_ptr : iterator for local\_A
- B\_ptr : iterator for temp\_B
- C\_ptr : iterator for temp\_C

Merge\_low Procedure:

1. Initialize A\_ptr, B\_ptr, C\_ptr to 0
2. While C\_ptr < local\_n:
  - 3. if local\_A[A\_ptr] <= temp\_B[B\_ptr]:
  - 4. temp\_C[C\_ptr] = local\_A[A\_ptr]
  - 5. increment C\_ptr, A\_ptr
  - 6. else:
  - 7. temp\_C[C\_ptr] = temp\_B[B\_ptr]
  - 8. increment C\_ptr, B\_ptr

## OddEven Sort in CUDA

Variables:

- n : number of elements to sorted
- t : number of threads
- A : array to be sorted

Idea: avoid coordination between threads by launching kernels for each phase (odd or even) and subsequently synchronizing CUDA threads after each phase.

Main Procedure:

1. Copy A to GPU memory
2. for each phase in {0, 1, ..., n-1}:
  - 3. if phase is even:
    - 4. launch EvenPhase kernel
    - 5. else:
    - 6. launch OddPhase kernel
    - 7. synchronize device
  - 8. Copy A to device memory

EvenPhase Kernel:

1. Id = threadId + blockDim \* blockIdx
2. index1 = 2 \* Id
3. index2 = index1 + 1
4. compare and swap array elements at index1, index2

OddPhase Kernel:

1.  $Id = \text{threadId} + \text{blockDim} * \text{blockId}$
2.  $\text{index1} = 2 * Id + 1$
3.  $\text{index2} = \text{index1} + 1$
4. if thread is not the last thread
5. compare and swap array elements at index1, index2

### EnumerationSort MPI

MPI Implementation Pseudocode

Define variables needed:

- Numprocs - Number of processes
- Myid - Process ID
- reg\_x, reg\_y, reg\_z - Register variables for data exchange
- compare - Counter for comparisons
- z\_count - Count of elements in reg\_z
- N - Size of data
- T - Number of threads
- a, b, c, d - Data arrays
- size - Size of data arrays

Main Procedure:

1. User input
2. Allocate the necessary memory for each array being used
3. Generate random numbers to save to a file
4. If myid is 0
  - Distribute the numbers among the processors
  - Send numbers to reg\_x
  - Send reg\_y to next process
5. For all other processes:
  - Receive reg\_x from process 0
  - Iterate through each processor
6. Swap reg\_z values:
  - Set values
  - Broadcast the values using MPI
  - Send and receive reg\_x and reg\_z
7. Print the result for process 0
8. If a process is not 0:
  - Send and receive reg\_z and z\_count
9. Print time

EnumerationSort CUDA

## CUDA Implementation Pseudocode

Define variables needed

- N - Array size
- B - Blocks
- T - Number of threads per block
- a, b, c, d - Arrays for data
- dev\_a, dev\_b, dev\_c - Device memory for data
- size - Size of data arrays
- start, stop - CUDA events for timing

Main Procedure:

1. Import libraries
2. Initialize CUDA
3. User Input
4. Use user input to initialize CUDA grid and block dimensions
5. Allocate the necessary memory for each array being used
6. Generate numbers and populate array
7. Record start time
8. Use a CUDA kernel to parallel sort
9. Record end time
10. Perform sequential sorting to compare GPU results
11. Print the time for GPU and CPU execution.
12. Compare GPU and CPU

## MergeSort MPI:

MPI Implementation Pseudocode

Declare a variable to track if the sorting check passed.

Define the CUDA kernel 'gpu\_merge\_sort' to merge subarrays:

Determine the thread's portion of the array to work on.

Merge the two sorted subarrays into one sorted array.

Define the 'merge\_sort' function to sort the array using GPU:

Allocate memory on the GPU.

Copy the array from the host to the GPU.

Perform merge sort in a loop, doubling the width each time:

Launch the 'gpu\_merge\_sort' kernel with appropriate blocks and threads.

Swap the pointers to work on the latest sorted data.

Copy the sorted array back to the host.

Free the GPU memory.

Define the 'main' function:

Initialize Caliper for profiling.

Check for the correct command line argument for THREADS\_PER\_BLOCK.  
Initialize an array with random integers.  
Print the array before sorting.  
Sort the array using 'merge\_sort'.  
Print the sorted array.  
Check if the array is sorted and print the result.  
Clean up and finalize Caliper.  
Return from the program.

### **MergeSort CUDA:**

CUDA Implementation Pseudocode  
Declare a variable to track if the sorting check passed.

Define the CUDA kernel 'gpu\_merge\_sort' to merge subarrays:  
Determine the thread's portion of the array to work on.  
Merge the two sorted subarrays into one sorted array.

Define the 'merge\_sort' function to sort the array using GPU:  
Allocate memory on the GPU.  
Copy the array from the host to the GPU.  
Perform merge sort in a loop, doubling the width each time:  
Launch the 'gpu\_merge\_sort' kernel with appropriate blocks and threads.  
Swap the pointers to work on the latest sorted data.  
Copy the sorted array back to the host.  
Free the GPU memory.

Define the 'main' function:  
Initialize Caliper for profiling.  
Check for correct command line argument for THREADS\_PER\_BLOCK.  
Initialize an array with random integers.  
Print the array before sorting.  
Sort the array using 'merge\_sort'.  
Print the sorted array.  
Check if the array is sorted and print the result.  
Clean up and finalize Caliper.  
Return from the program.

### **Sample Sort MPI:**

Variables in main:

Pivots: Array of random double to be sorted. Bucket\_Num: Array to track the number of values that are being sent to each processor Displacements: Storage for displacement data for scatter and gather functionality D\_list: final doubles list to store the sorted value

#### Main Function Logic:

1. MPI\_Init, Size, and Rank to set up MPI parallel functionality
2. Create all the random values in the main thread only
3. Set the displacements and sort each value into their respective buckets in the main thread only
4. Scatter, Sort, and Gather each individual bucket in their own thread

#### Compare Function Logic:

This is a customer comparator function that is designed to help the custom\_qsort function sort through doubles.

1. Cast and dereference the two input values
2. Subtract the two values
3. If the first is greater than the second return 1
4. If the second is greater than the first return -1
5. If both values are equal return 0

#### Custom Quicksort Function Logic:

This function simply is a wrapper for the stl qsort function. The only modification to the stl function is the use of the custom comparator mentioned above in the compare function logic.

#### Find Bucket Logic:

In order to complete the sample sort operation it is important to sort values into buckets for each processor.

#### Loop through every processor

Check if the current input value is lower than the upper limit of each processor  
Once a processor is found return the index number of the processor

### Sample Sort Cuda:

#### Main Function Logic:

1. Create vector of random floats
2. Call bucket sort function
3. Print out the sorted vector

#### Bucket Sort Logic:

1. Create arrays for data and bucket value counters
2. Call the bucket\_sort\_kernel function and synchronize
3. Use thrust sort to sort each individual bucket
4. Free all the previously created data structures for data and bucket value counters

#### Bucket Sort Kernel Logic:

1. Check if the current value being passed to the function is less than the max bucket values
2. Modify the bucket that we add the value to... If the current bucket is within the bound of the number of buckets we can add it if not we must go back a bucket

- Add the value to the appropriate bucket

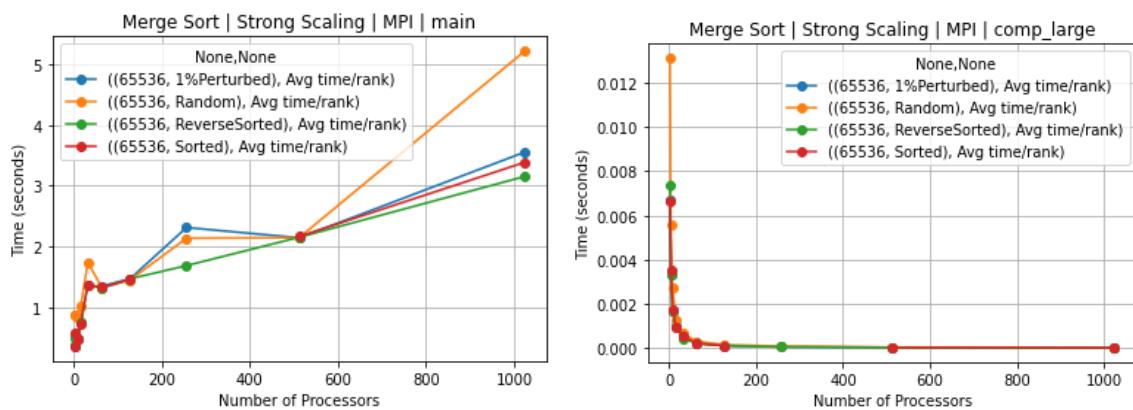
## Evaluation Plan

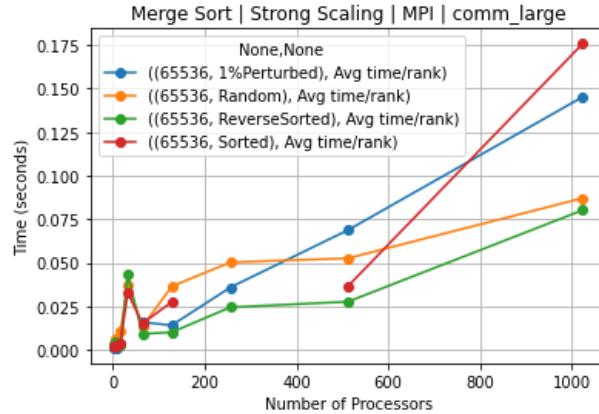
- For each algorithm, we plan to measure the runtime of various components with our programs with respect to input size.
- In testing, we plan to vary the array input sizes with the following counts  $\{2^{16}, 2^{20}, 2^{24}\}$ .
- Input properties of float arrays to be tested: { randomized, reversed, sorted }.
- We plan to evaluate how our algorithms behave with respect to Strong Scaling (same problem size  $\{2^{24}\}$ , increase number of processors/nodes)
- MPI Strong Scaling: increase number of cores {2, 4, 8, 16, 32, 64}
- CUDA Strong Scaling: number of threads per block {64, 128, 512, 1024}
- Weak scaling (increase problem size, increase number of processors)
- MPI Weak Scaling:
- Increase number of cores {2, 4, 8, 16, 32, 64}
- Increase input array size  $\{2^4, 2^8, 2^{12}, 2^{16}, 2^{20}, 2^{24}\}$
- CUDA Weak Scaling:
- Increase number of threads per block {64, 128, 512, 1024}
- Increase input array size  $\{2^{12}, 2^{16}, 2^{20}, 2^{24}\}$

## Performance Evaluation

### Merge Sort MPI:

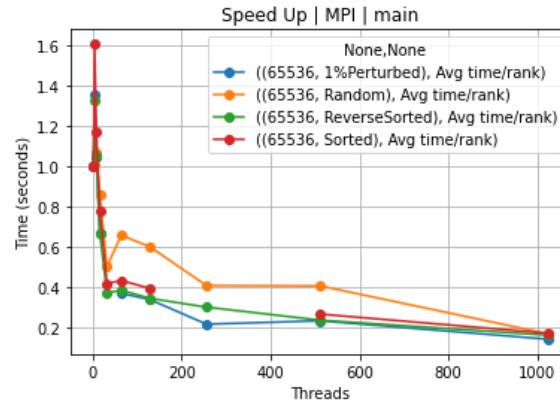
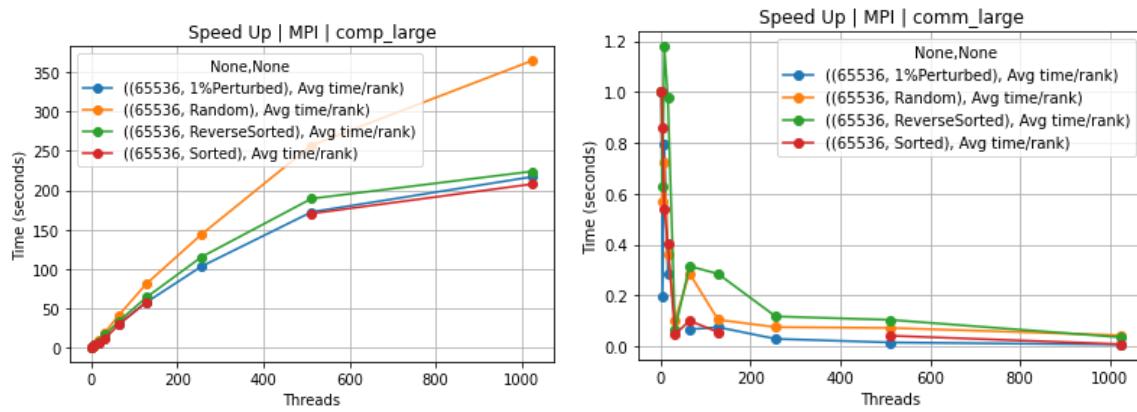
**Strong Scaling 65536**





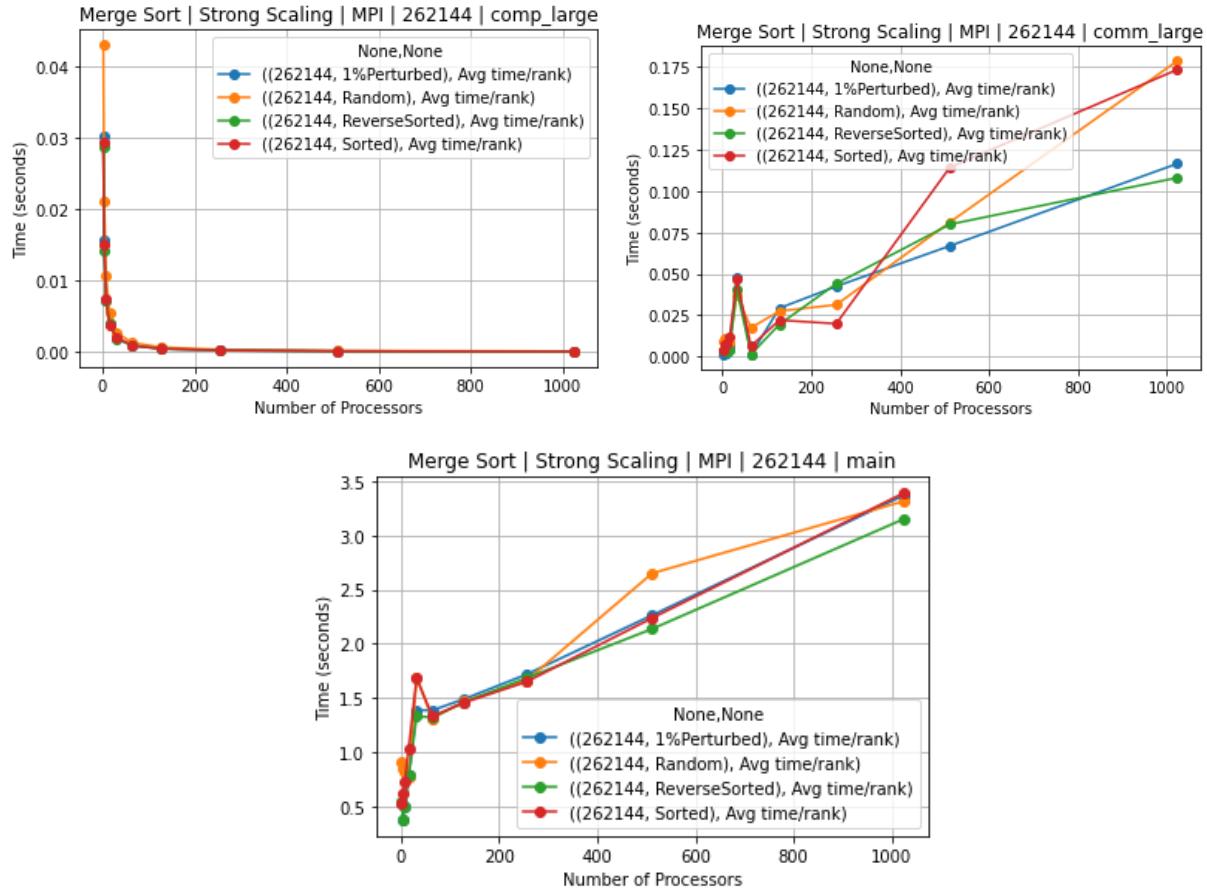
You can see at 65536 comp\_large has quite a big drop off at the beginning and starts to straighten out at a higher number of processes. But while with comm\_large, the time it takes seems to only increase over time most likely making it the overhead for main.

### Strong Scaling 65536 Speed Up



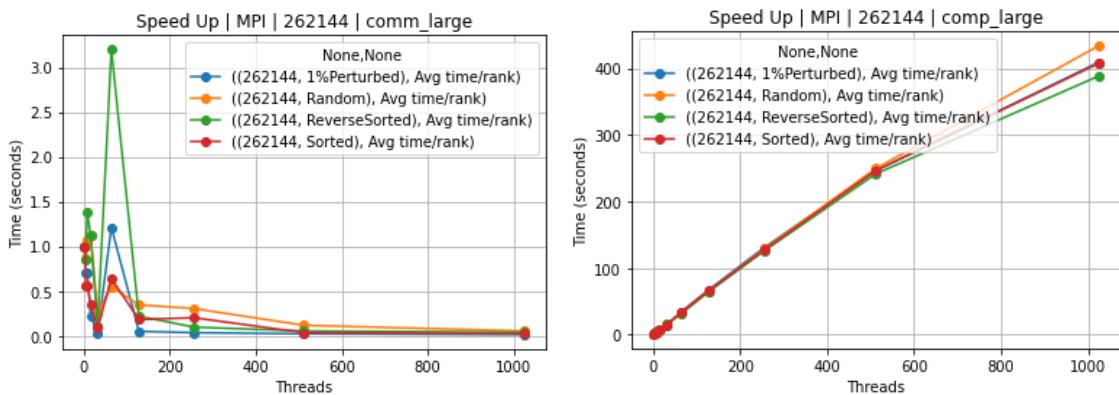
The speed up here shows that the comp\_large experiences quite a bit of speed up while the comm\_large does not which is fairly straight forward.

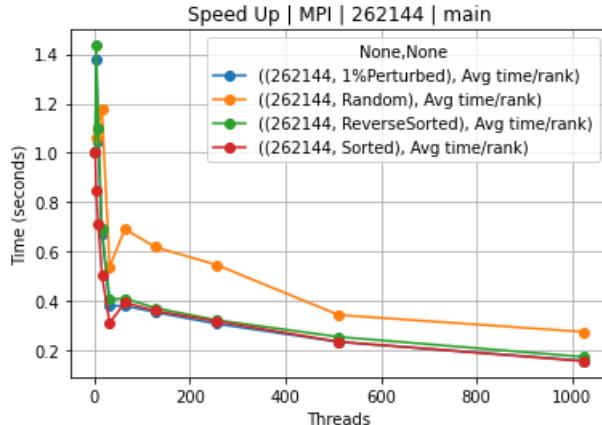
## Strong Scaling 262144



This continues to follow the trend from 65536 in that comp\_large has a drop off over time, and comm\_large continues to take a majority of the time.

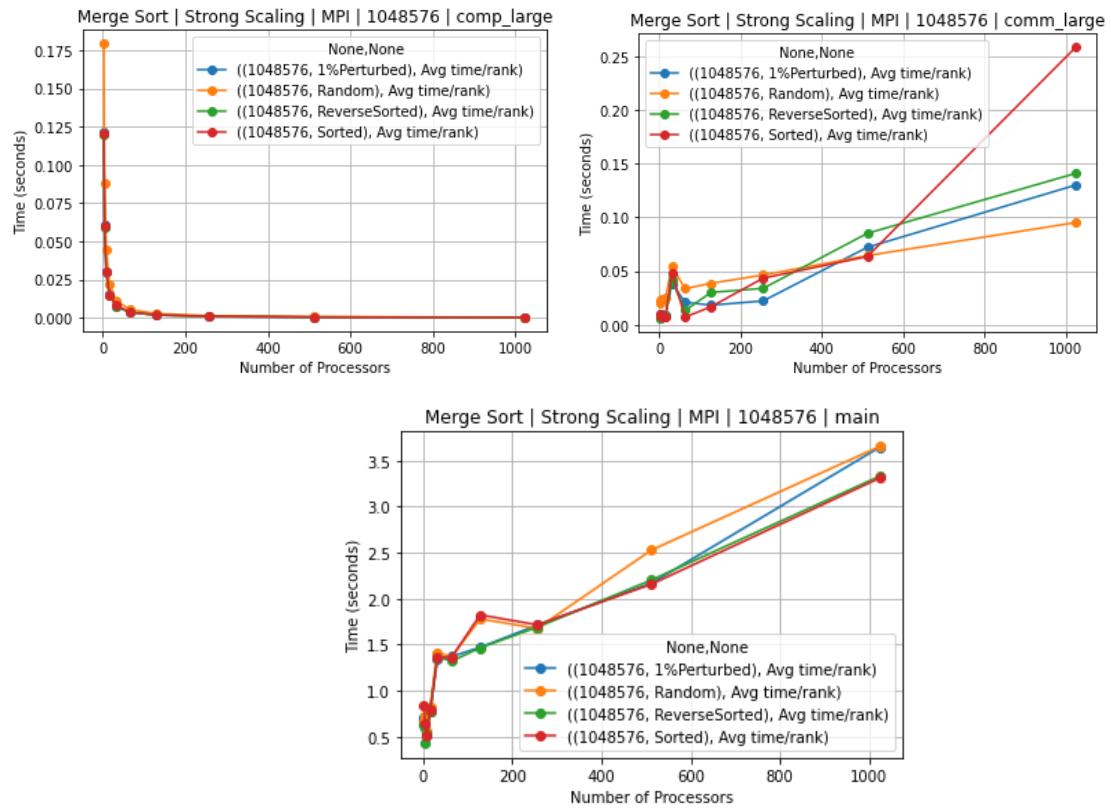
## Strong Scaling 262144 Speed Up





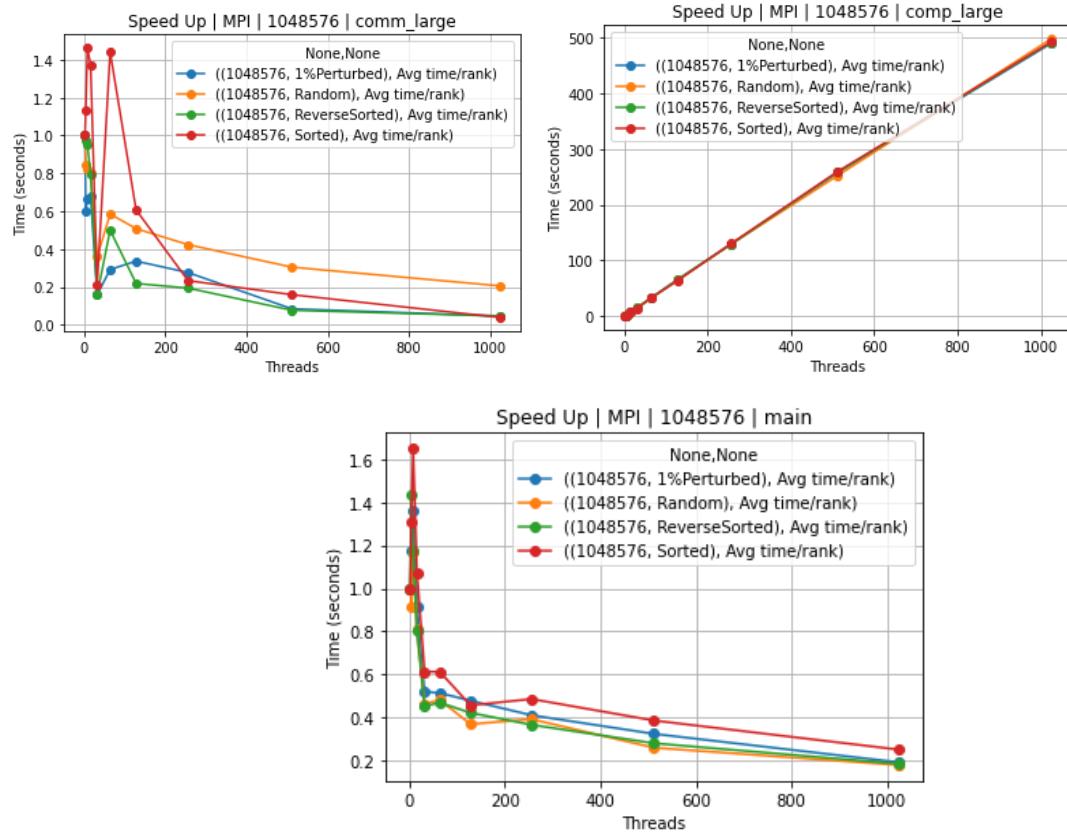
The speed up here is also similar to 65536 so there isn't much to say which may be a trend with this MergeSort.

### Strong Scaling 1048576



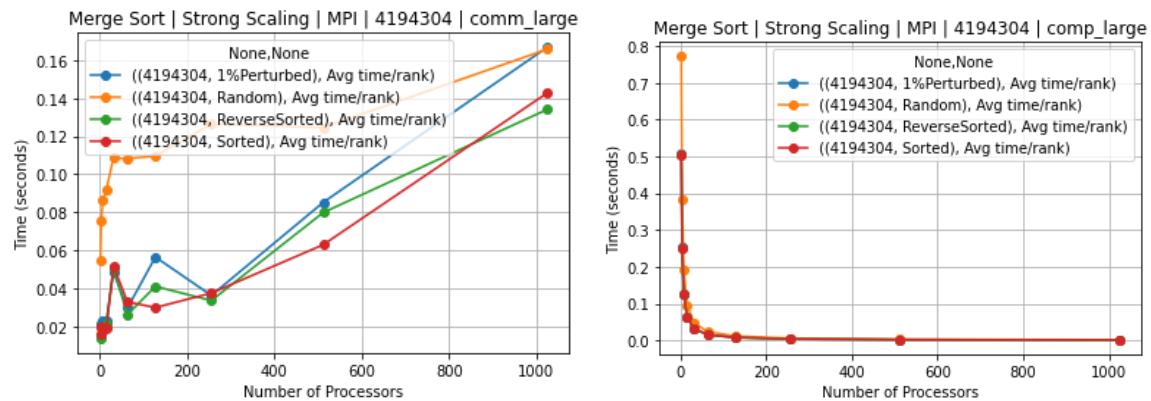
Once again following the same trends as before.

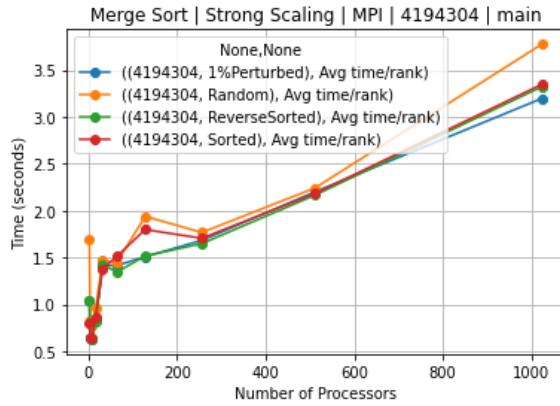
## Strong Scaling 1048576 Speed Up



The speed up also seems very similar to the previous input sizes with more jumps at the beginning in the comm\_large.

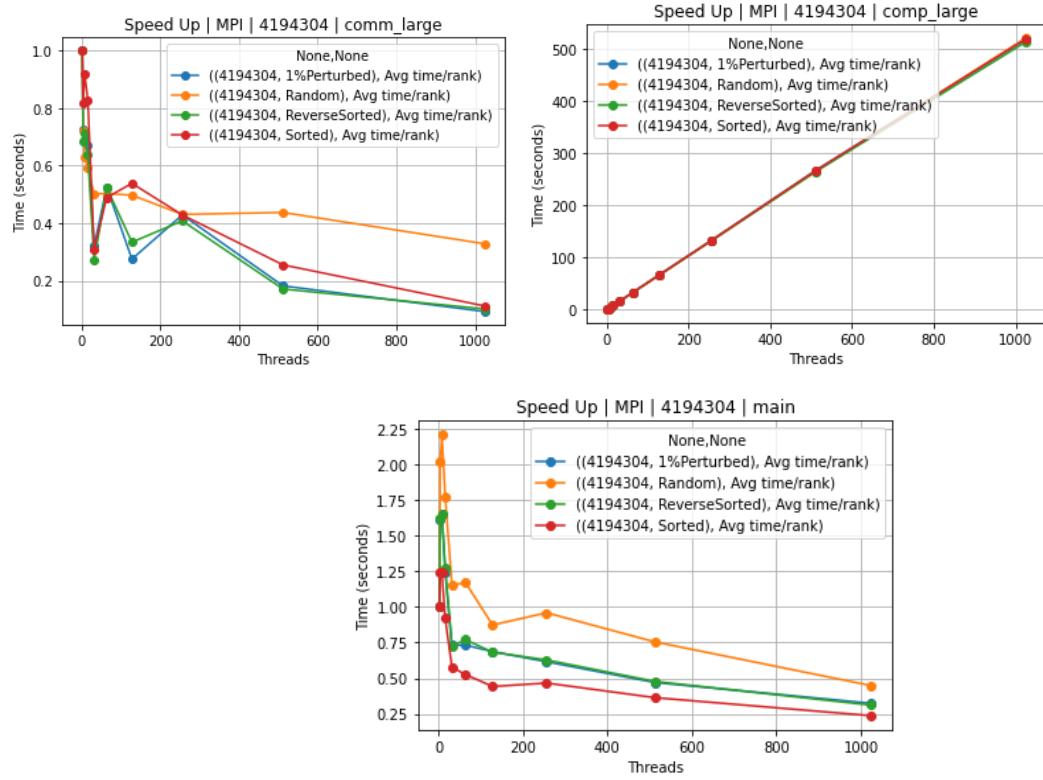
## Strong Scaling 4194304





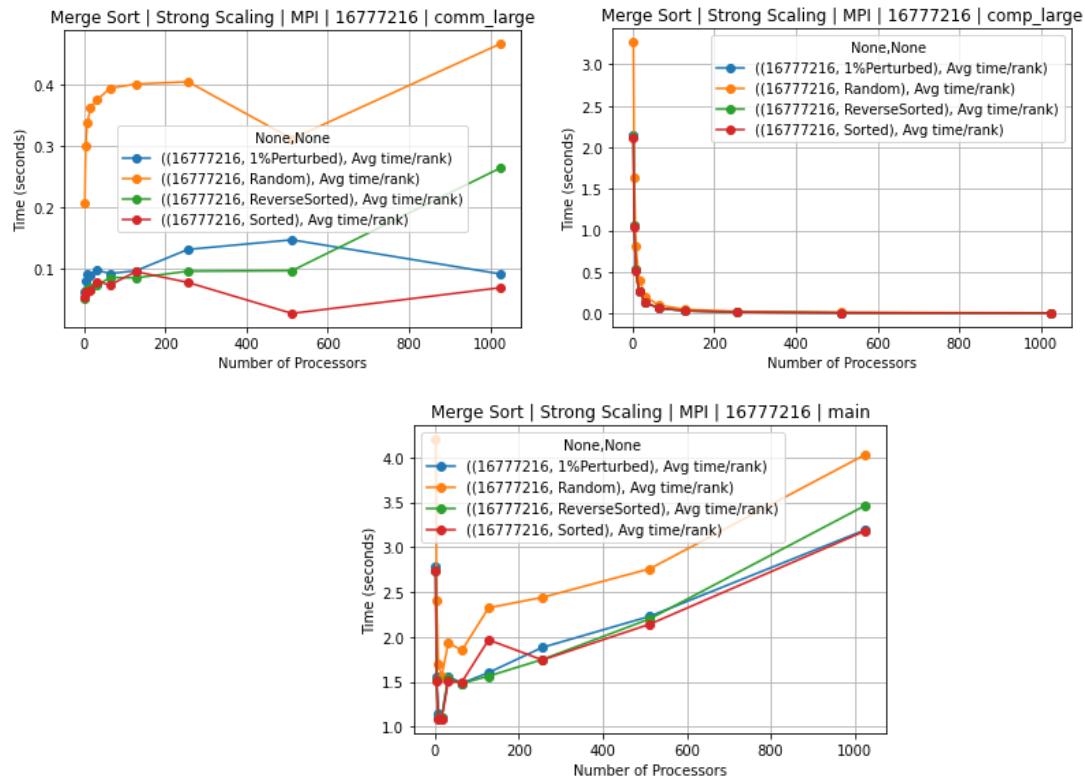
Still very similar to the previous input sizes which is very interesting since it seems to continue following this trend.

### Strong Scaling 4194304 Speed Up



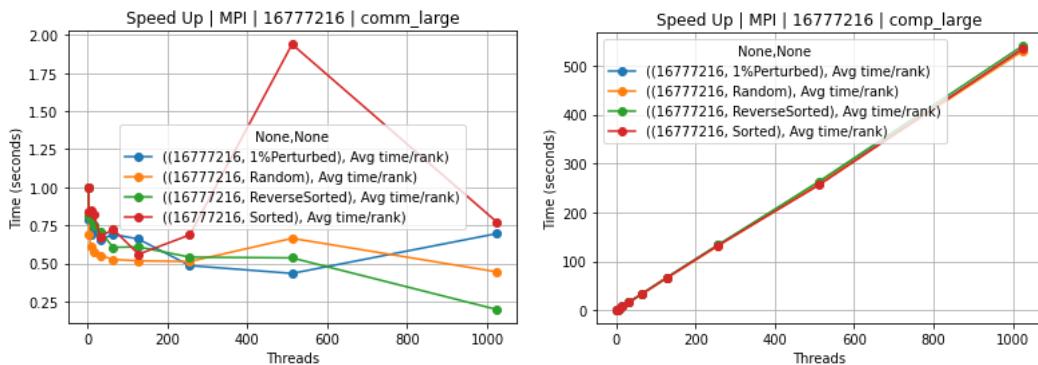
Again, very similar to the other speed ups with comp\_large, speeding up substantially, but comm\_large decreasing as the processors increase.

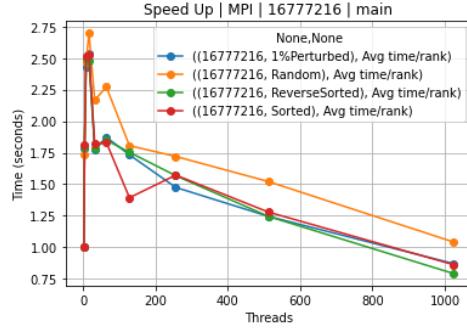
## Strong Scaling 16777216



Here while it looks similar, the `comm_large`, specifically the `Random` input type, has an interesting curve to it that isn't seen previously.

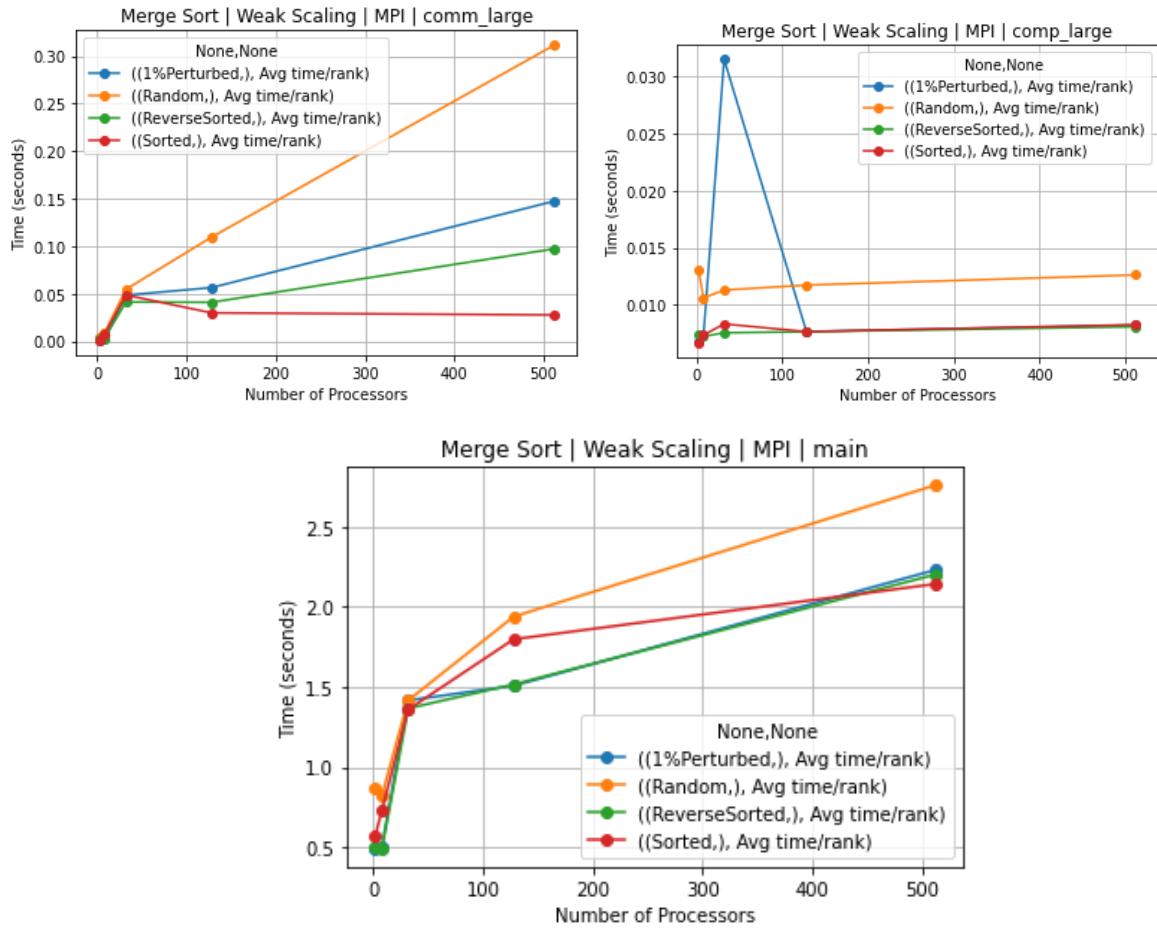
## Strong Scaling 16777216 Speed Up





And here it is a little different as well in that there is a moment of speed up for the sorted input type in the comm\_large that hasn't been seen before.

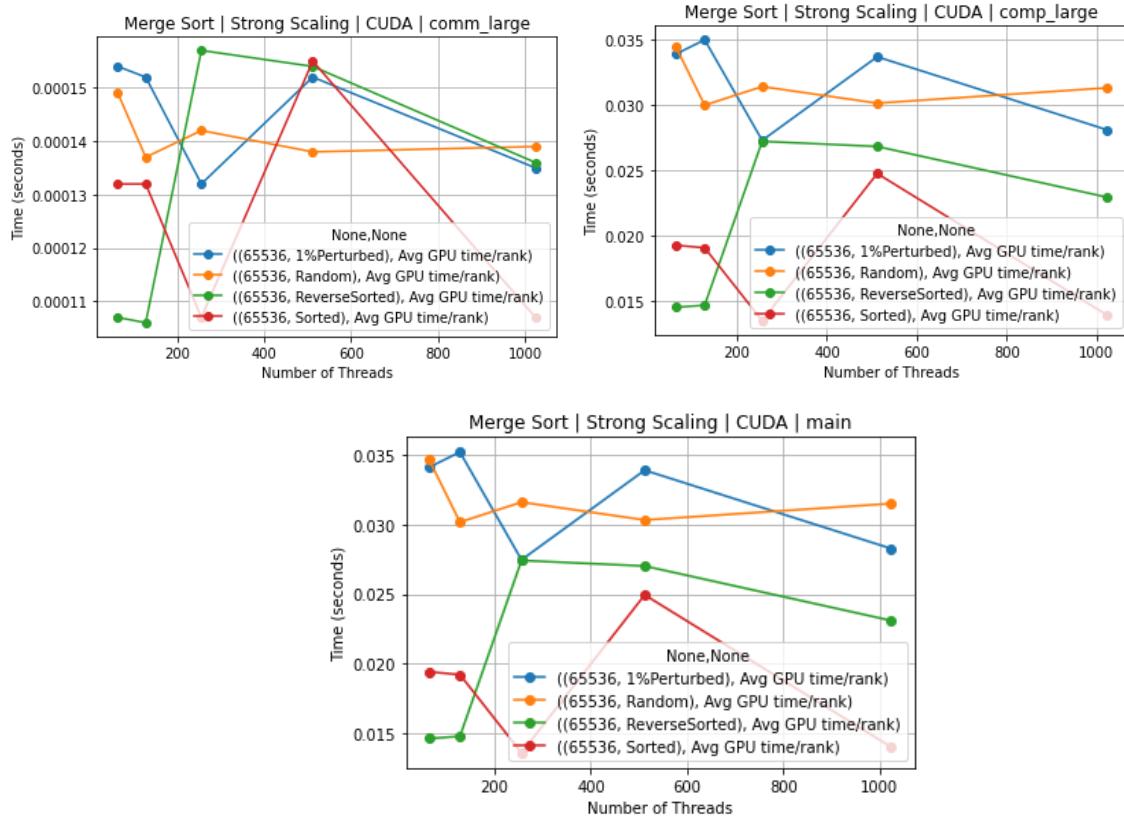
## Weak Scaling



In the Weak Scaling it can be seen that the comp\_large graph tapers off into a somewhat straight line as the processors increase but the comm\_large continues to take more and more time especially for the Random input type.

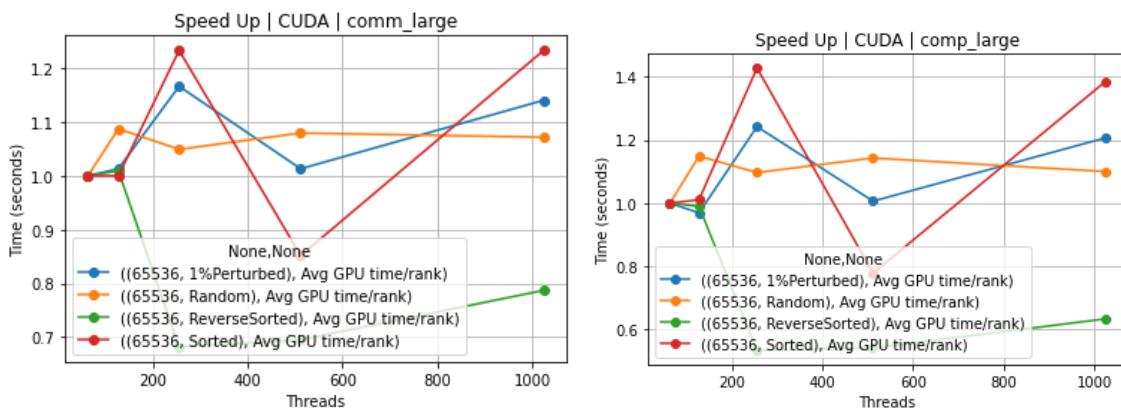
## Merge Sort CUDA:

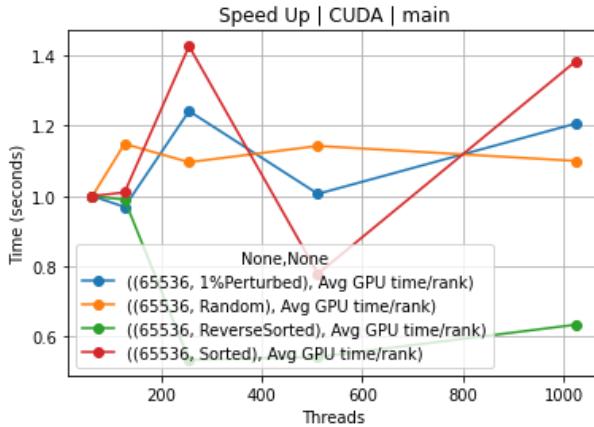
### Strong Scaling 65536



Here, the lines are all over the place with ups and downs in both comp\_large and comm\_large with the Random and 1%Perturbed input types seemingly taking the most time which makes sense.

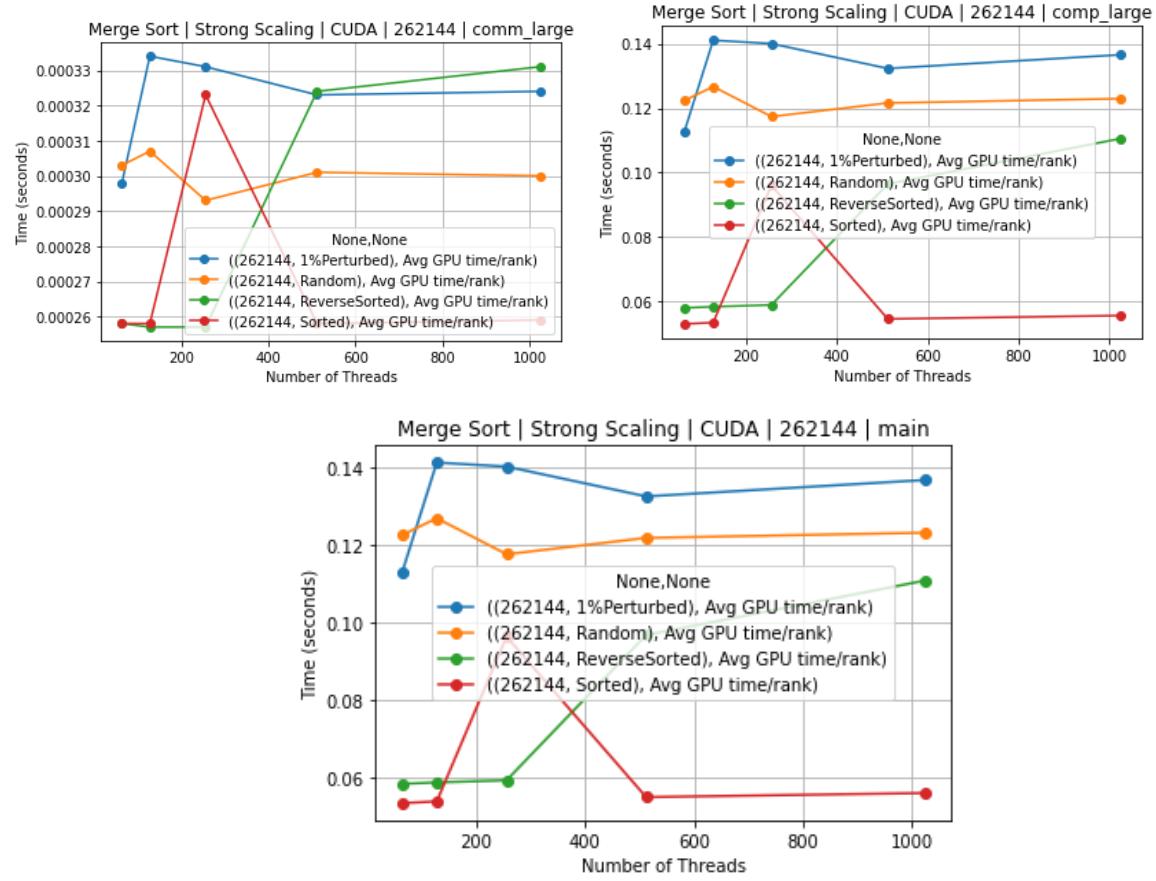
### Strong Scaling 65536 Speed Up





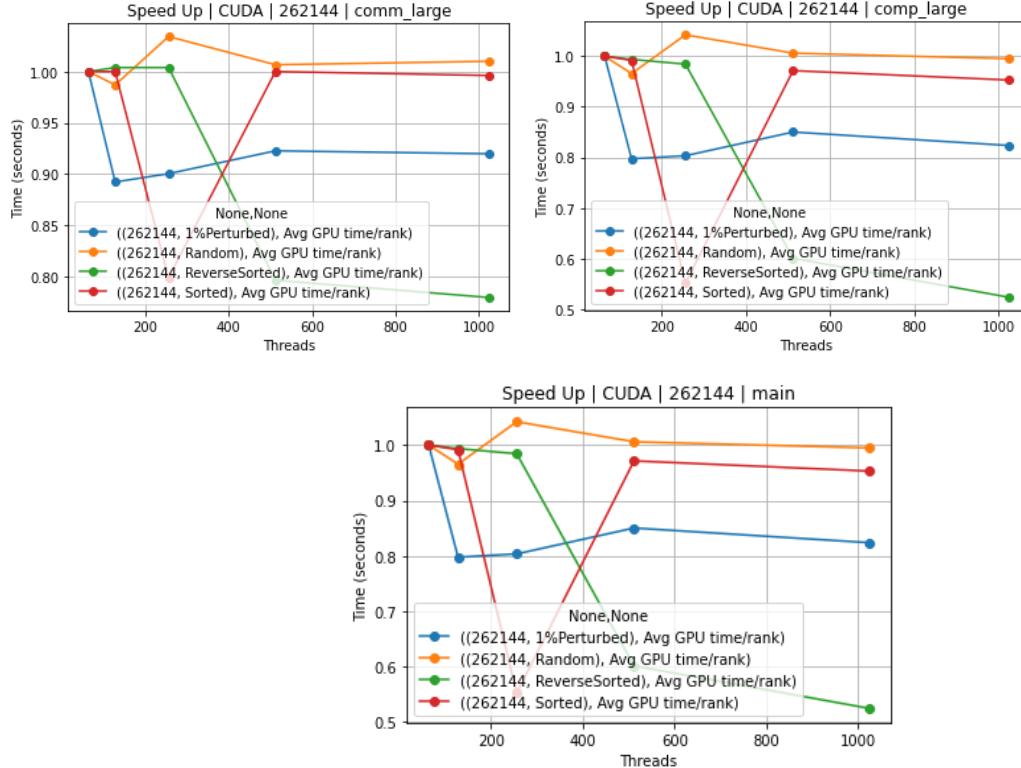
The speed up is also similar in that it goes up and down quite a bit with speeds up from one processor to the next while also speeding down a bit as well.

### Strong Scaling 262144



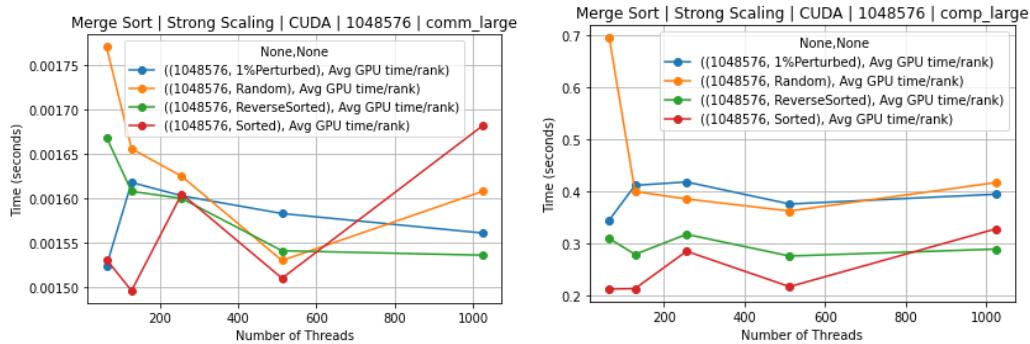
This looks more calm compared to 65536 in that the lines have less chaotic trends but overall seem to straighten out in the end.

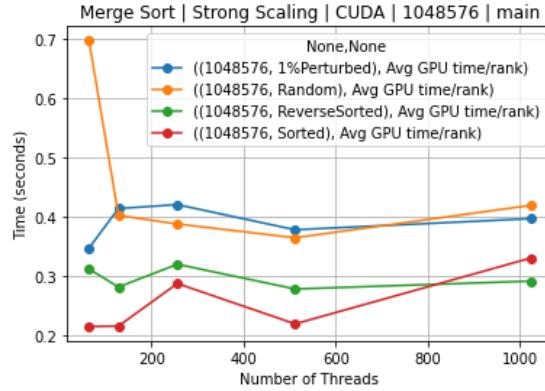
### Strong Scaling 262144 Speed Up



The speed up doesn't seem to show anything substantial with little to no speed up at all.

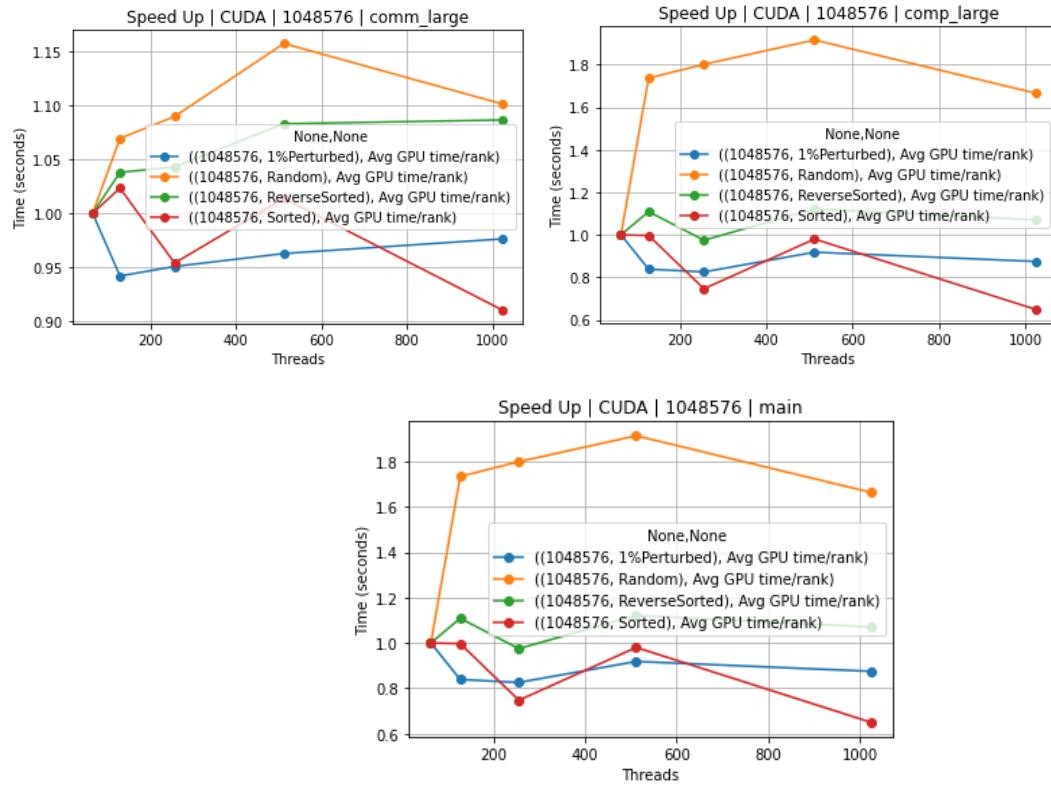
### Strong Scaling 1048576





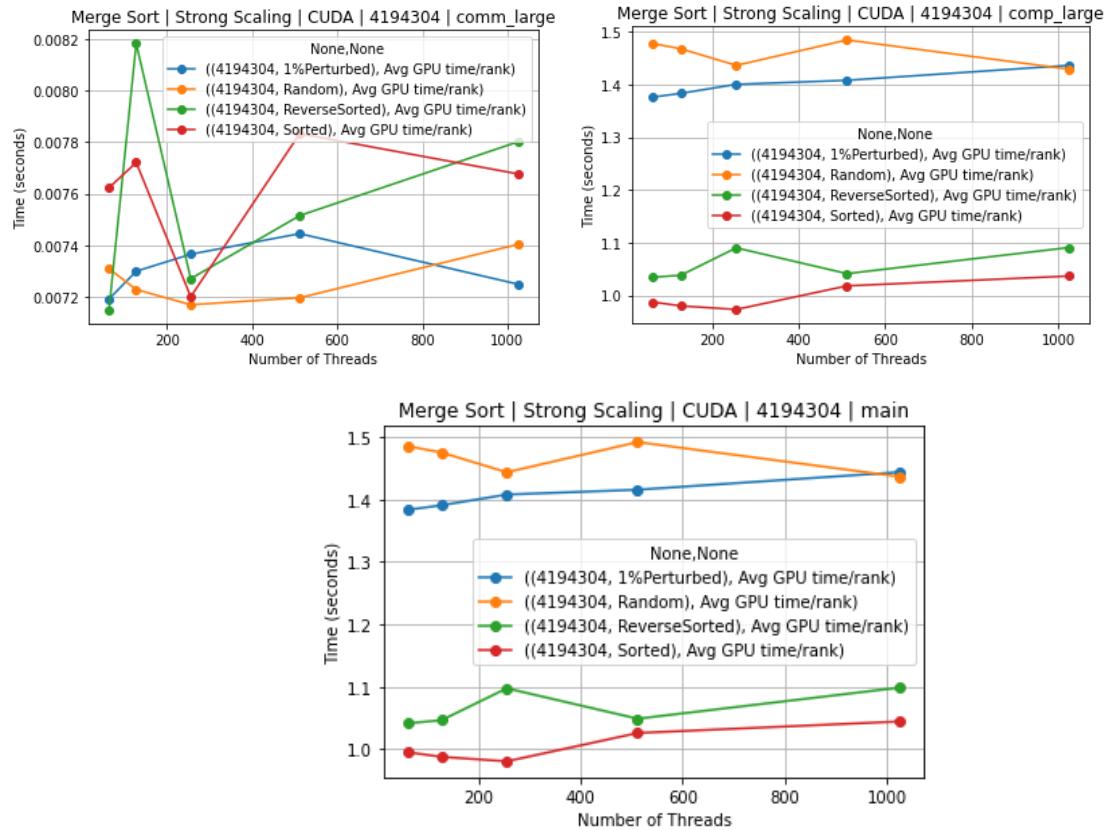
Here we actually start to see the graphs move down a bit more than before and seem to have a pattern decreasing for the most part.

### Strong Scaling 1048576 Speed Up



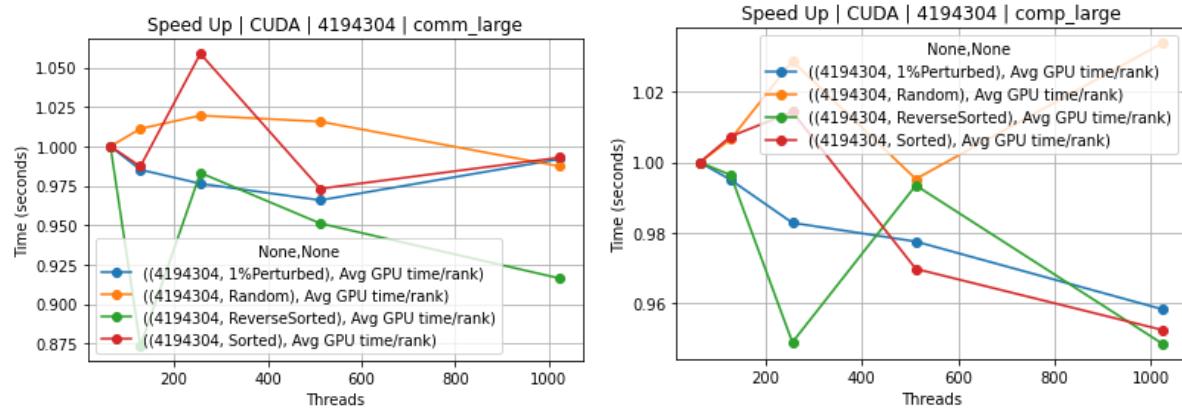
The speed up also reflects that in the comp\_large almost hitting 2x speed up for the Random input type for comp\_large, and a very little speed up for Random in comm\_large.

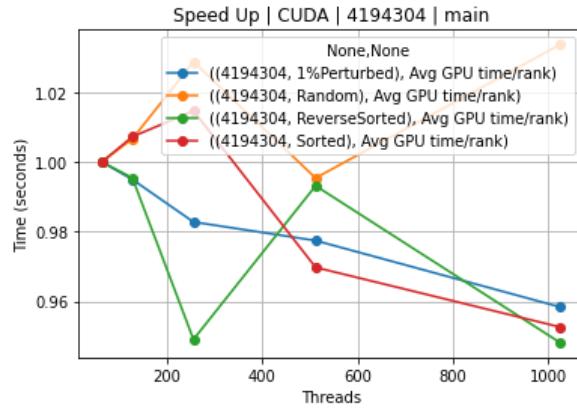
## Strong Scaling 4194304



Here we seem to go back to the previous input sizes with some interesting lines in comm\_large and fairly linear lines in comp\_large.

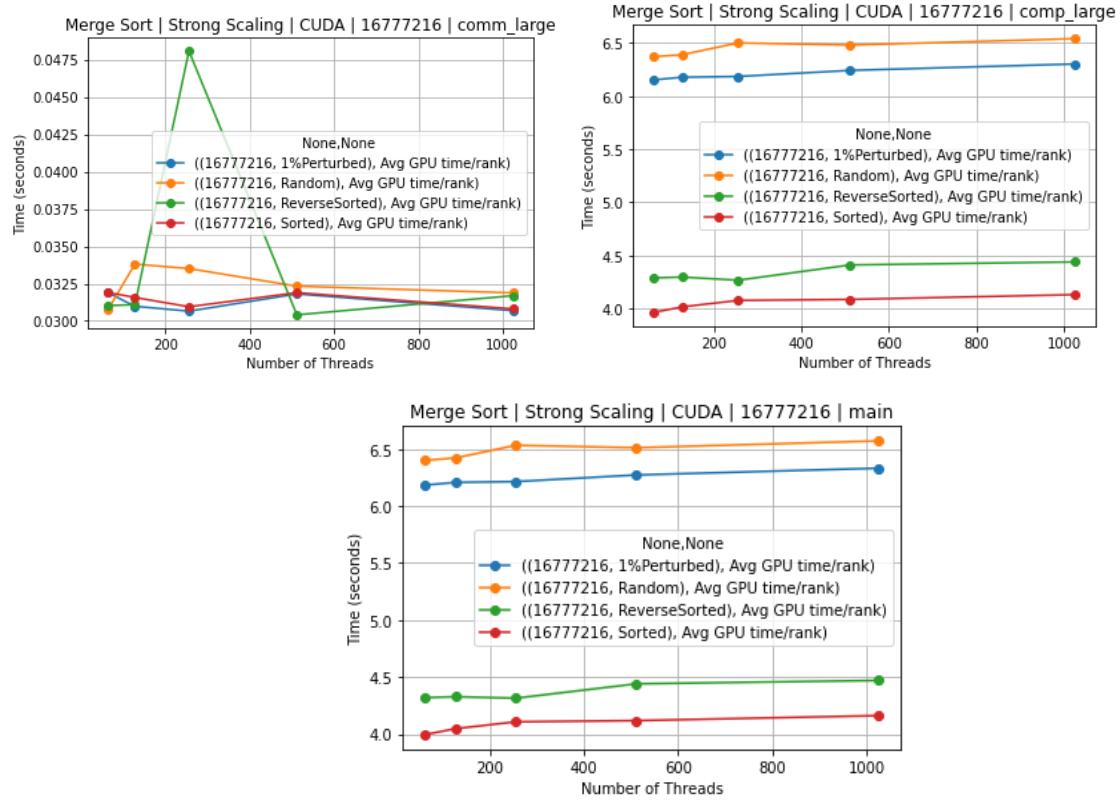
## Strong Scaling 4194304 Speed Up





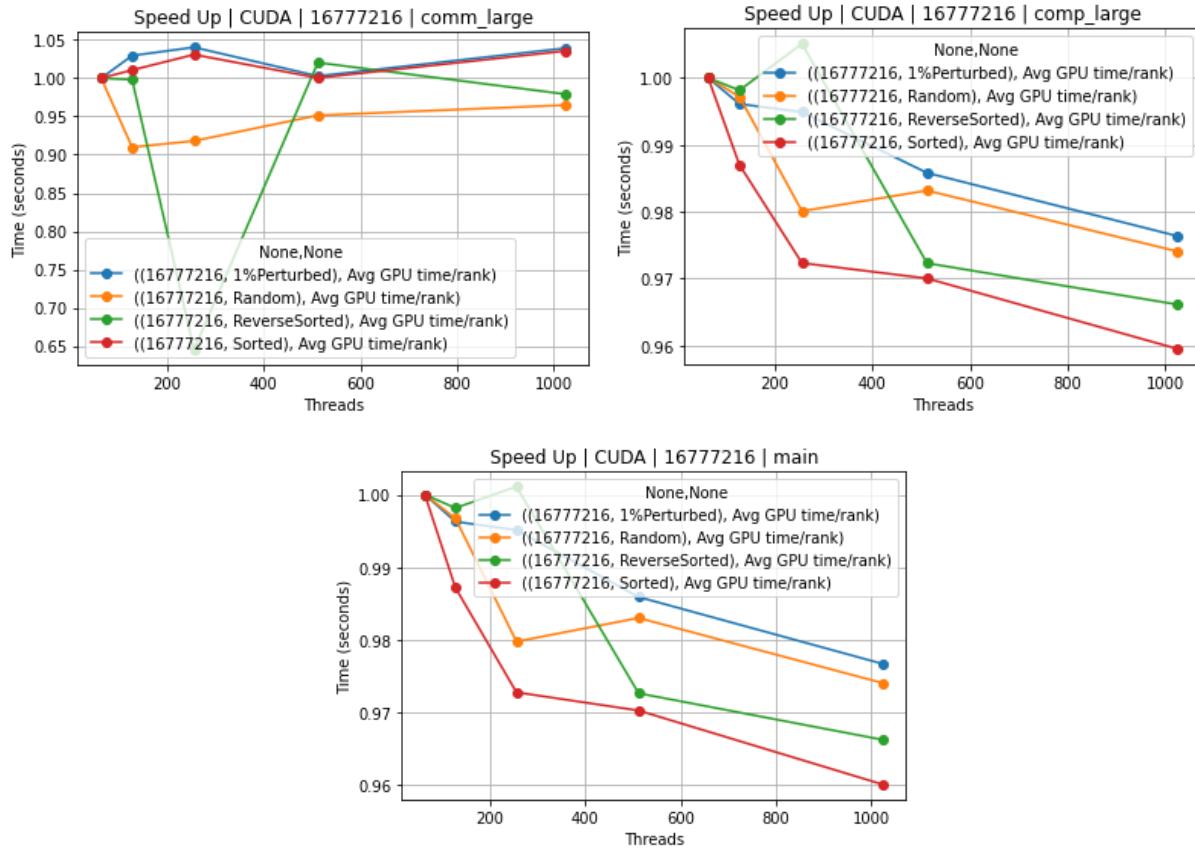
In the speed up, there seems to be little to none for either the comm\_large and comp\_large.

### Strong Scaling 16777216



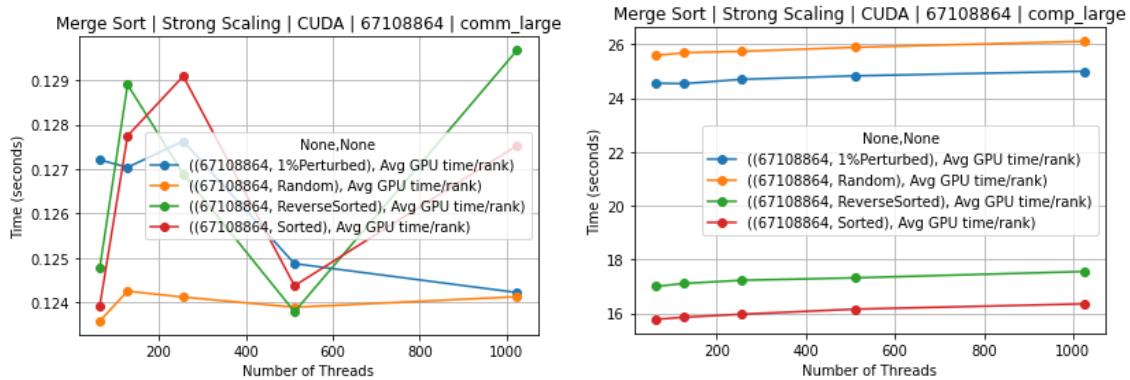
This follows the previous input size, but is quite linear for both comm\_large and comp\_large with a increase and decrease in comm\_large.

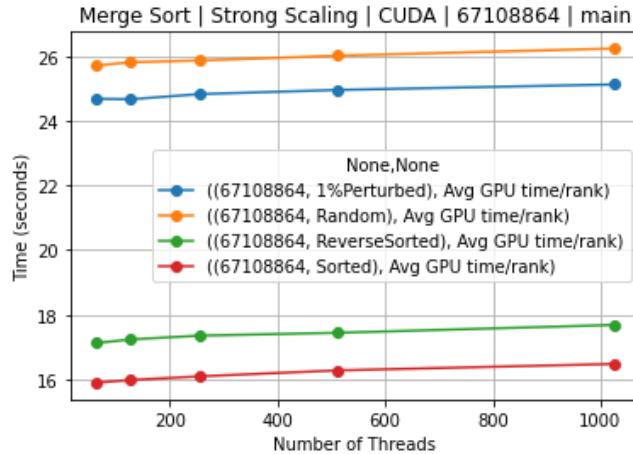
## Strong Scaling 16777216 Speed Up



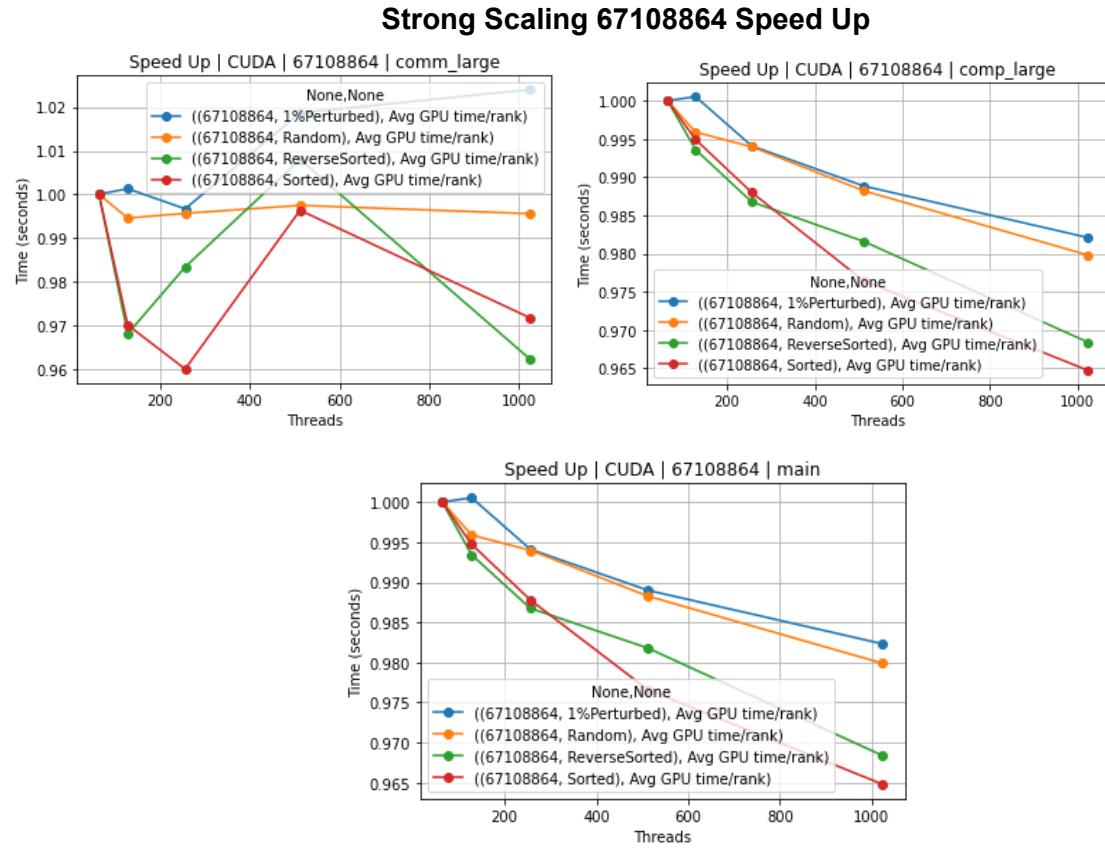
The speed up, does not show much results either, decreasing in performance as the processors increase.

## Strong Scaling 67108864



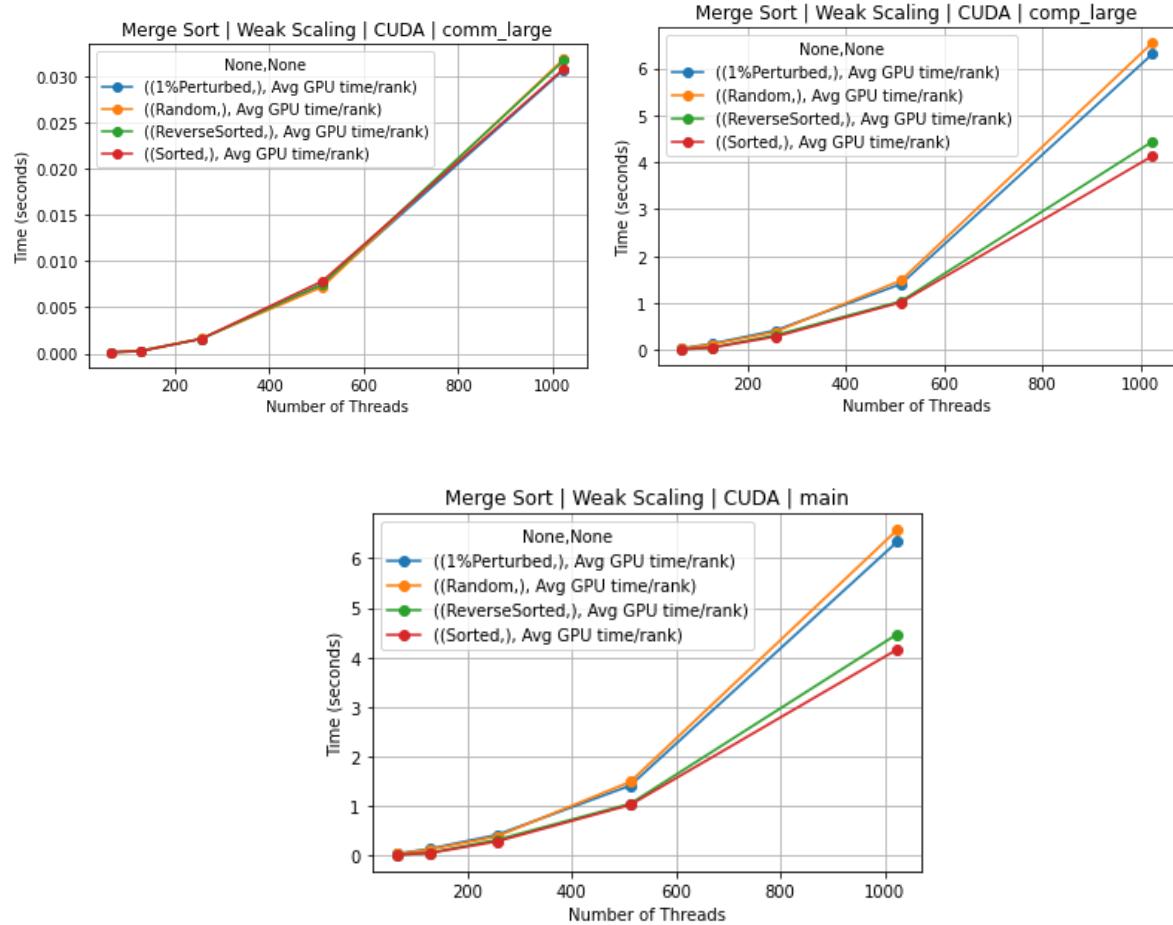


Finally this is slightly different, while the `comp_large` is still quite linear, the `comm_large` is a less stagnant but also the input types seem to follow a pattern aside from Random.



But similar to the other input sizes, the performance does seem to decrease as the processors increase.

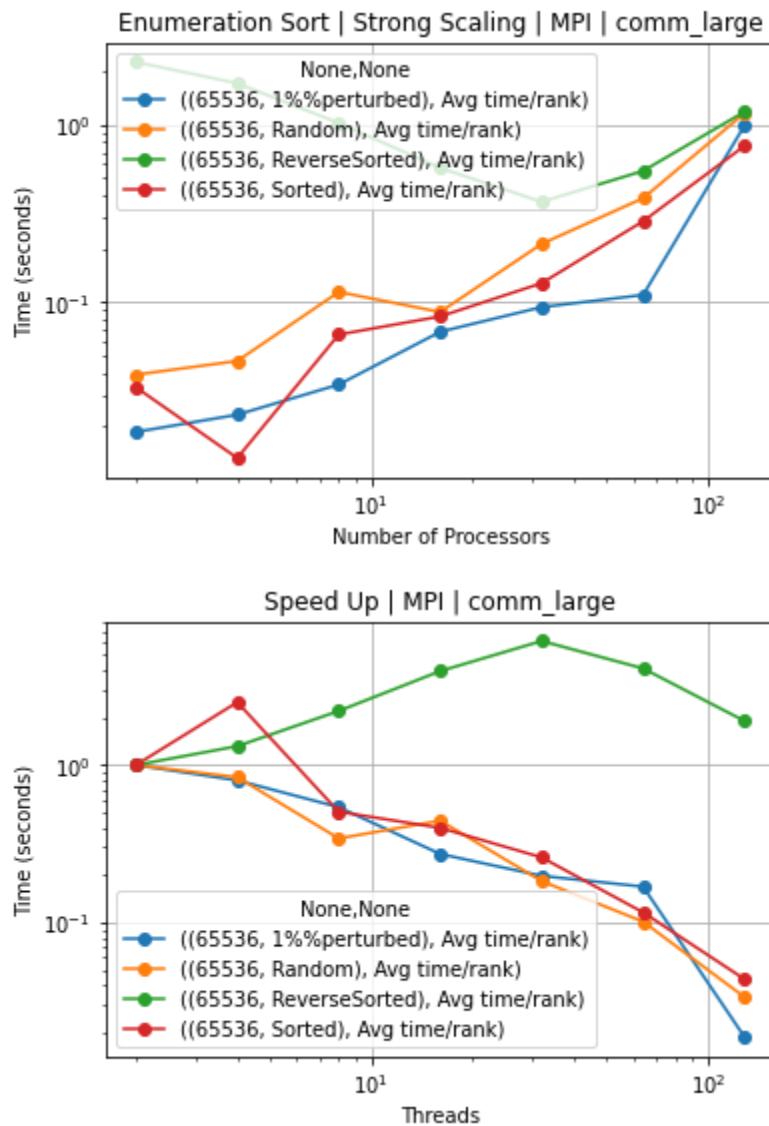
## Weak Scaling

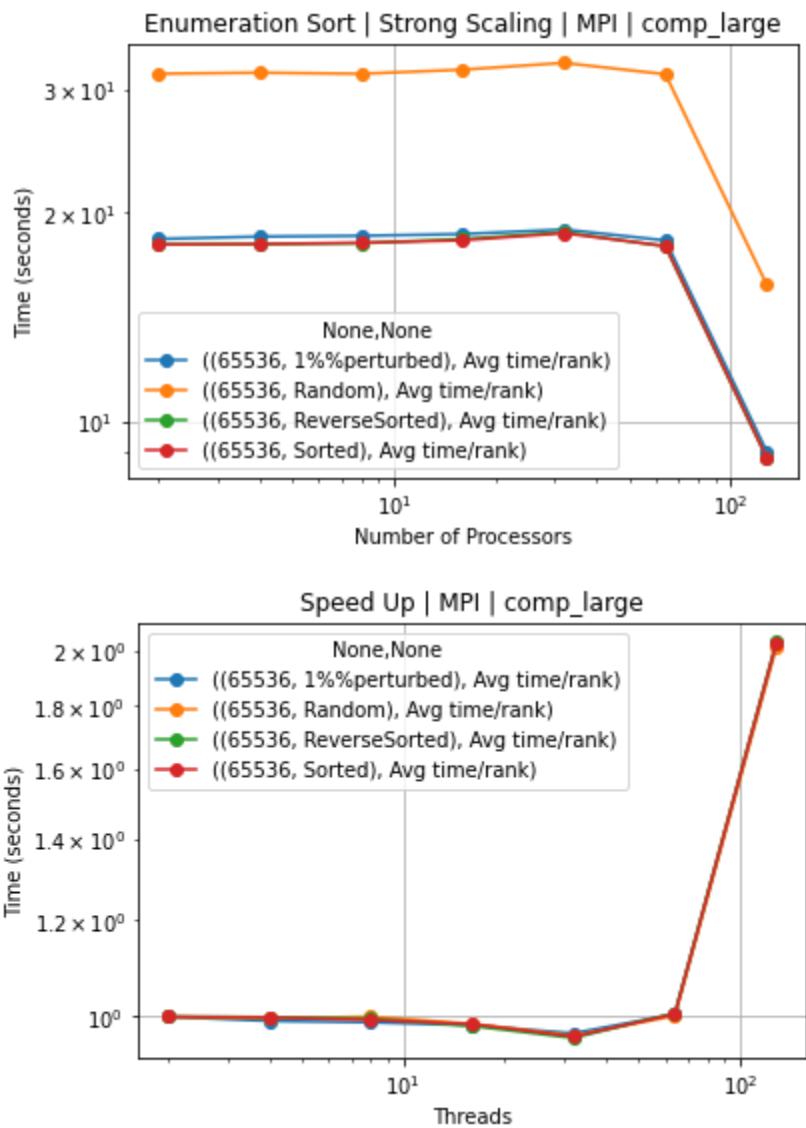


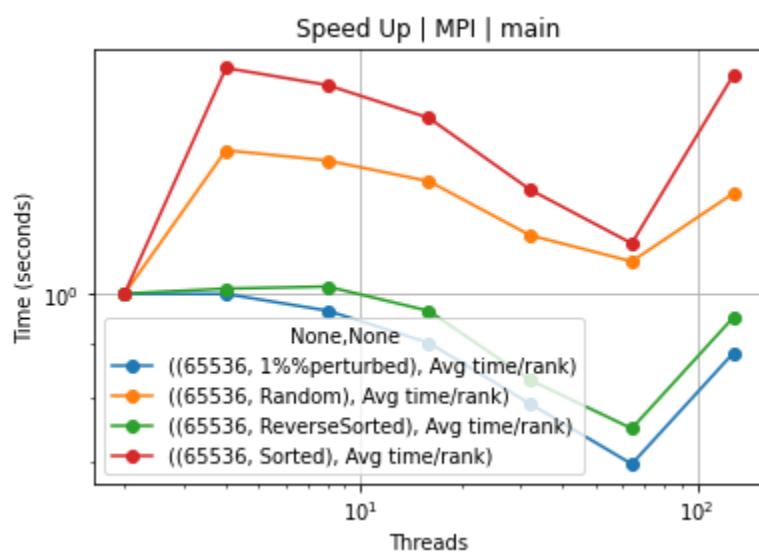
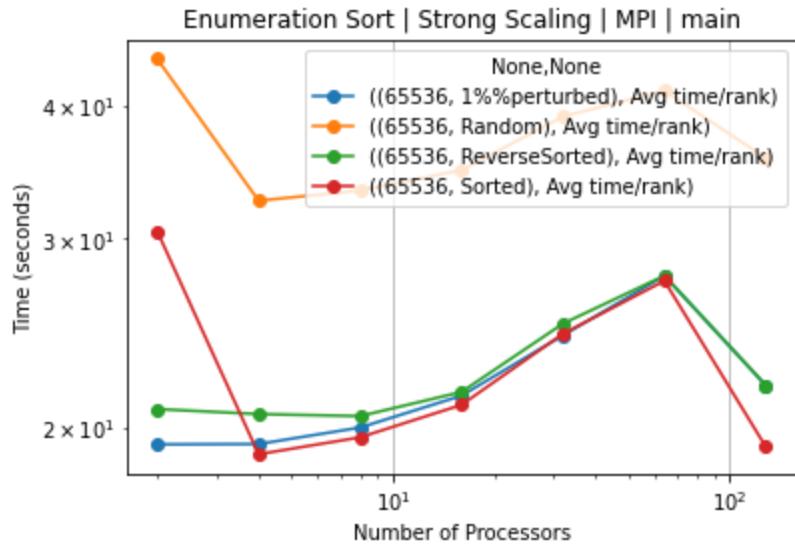
The Weak Scaling for CUDA follows a similar pattern for both comm\_large and comp\_large, taking more time as the number of processors increases, and doesn't seem to be flattening out like MPI.

## Enumeration Sort MPI:

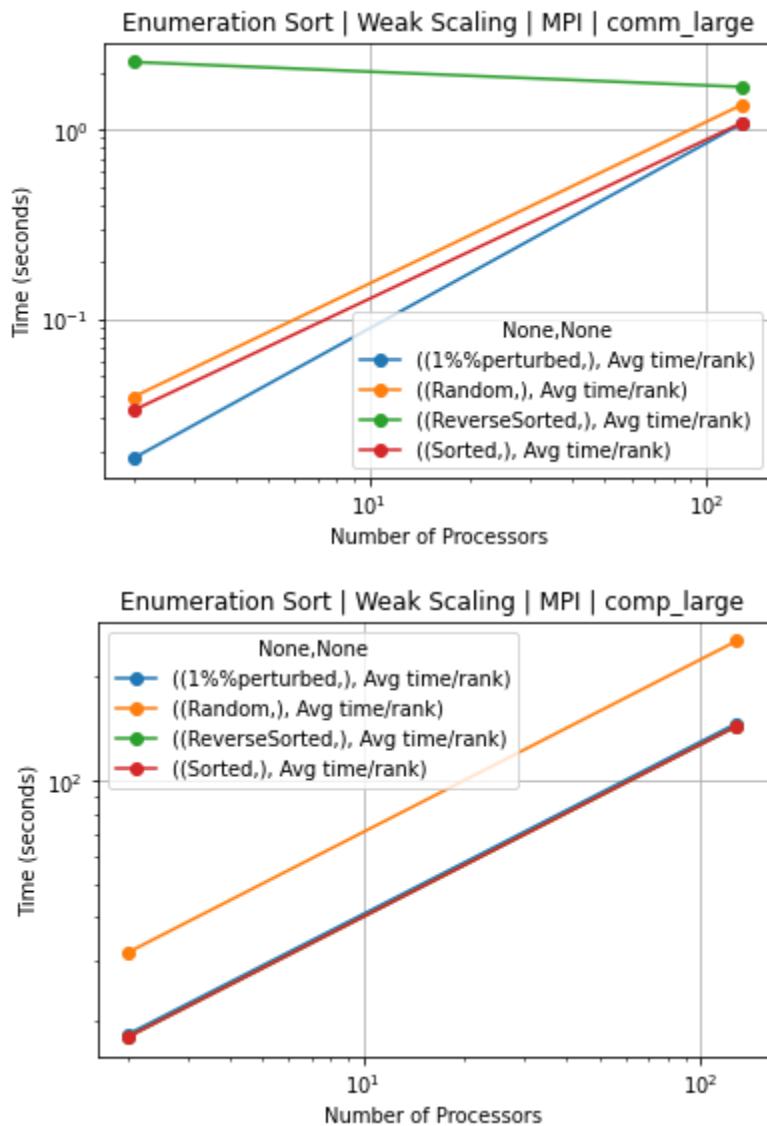
### Strong Scaling Graphs

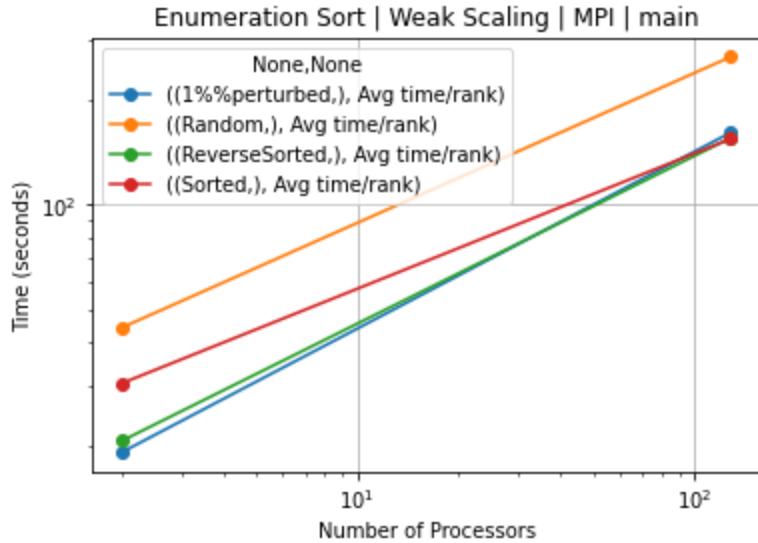






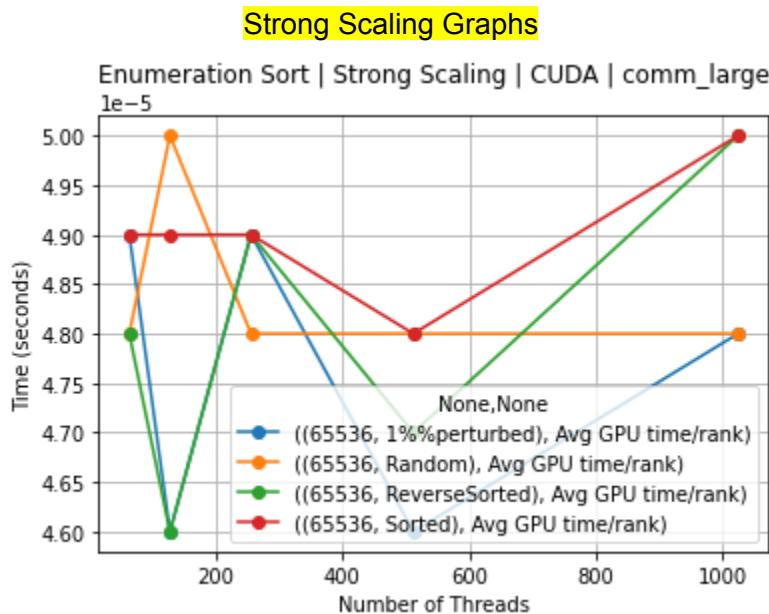
## Weak Scaling Graphs

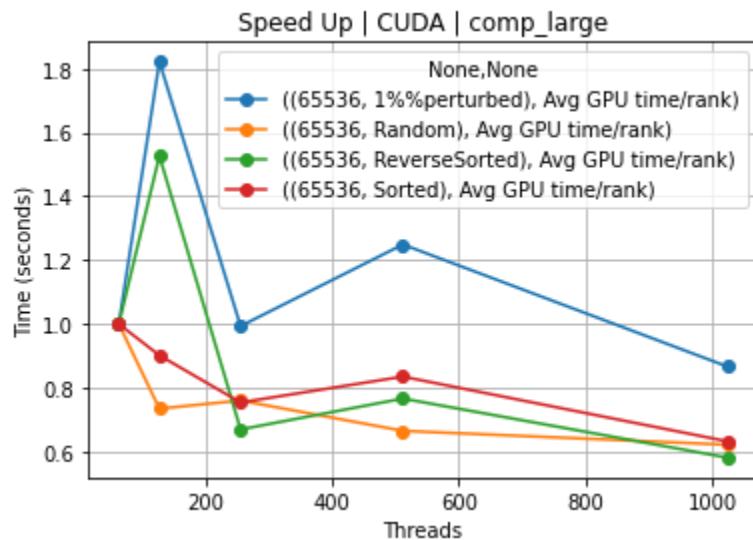
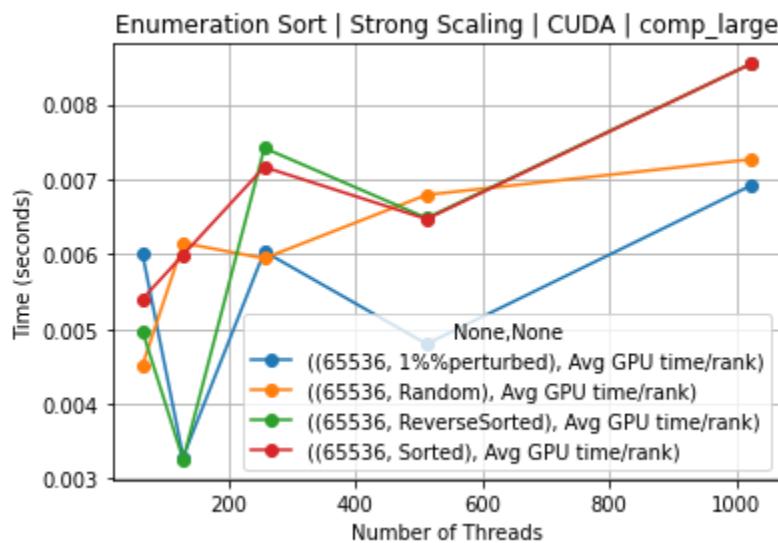
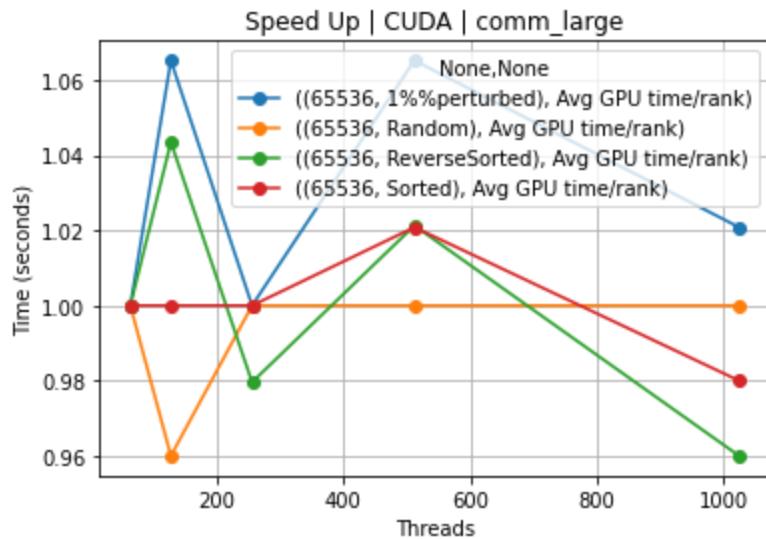


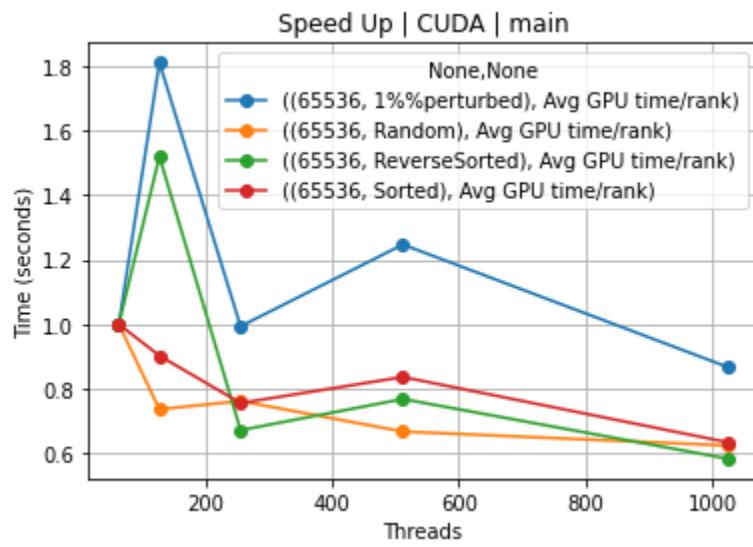
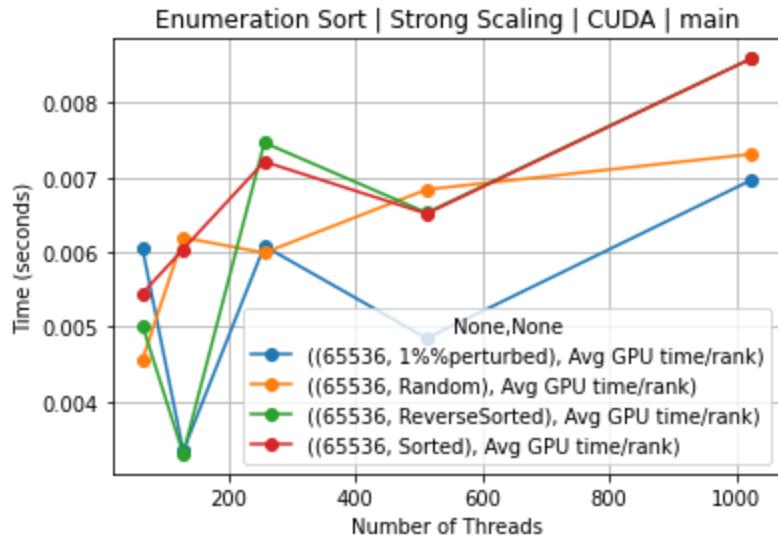


The current implementation of Enumeration Sort in MPI has many weak points. The execution time per processor stays constant with an increase in the number of processors. A reduction in runtime is also notable when increasing the number of nodes. The weak scaling graphs of MPI compare the input size with performance based on the processors. The strong scaling graphs compare the average time for different inputs. The comp graphs of strong and weak scaling show an increase in communication with correlation to the processors being increased. The main graphs show a decrease in performance as the input size is increased. In the comm\_large graphs, there is no clear trend being shown while all the other graphs show a clear trend. In terms of the input types, random seemed to perform the worse based on the graphs.

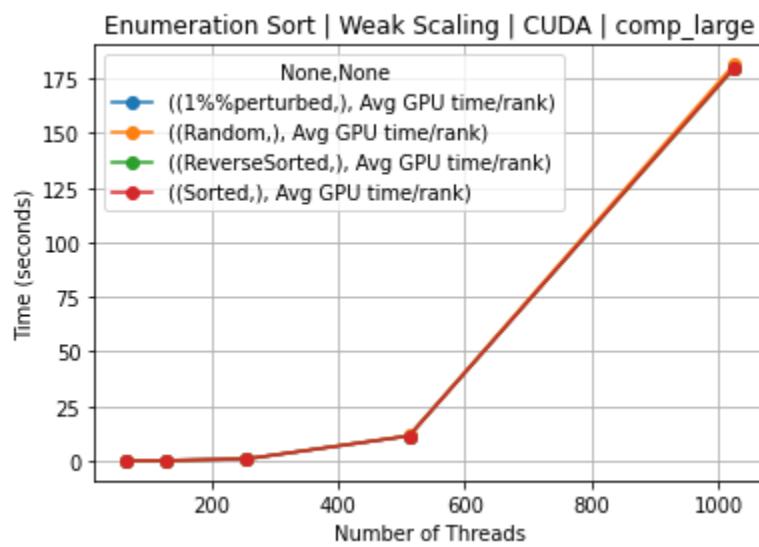
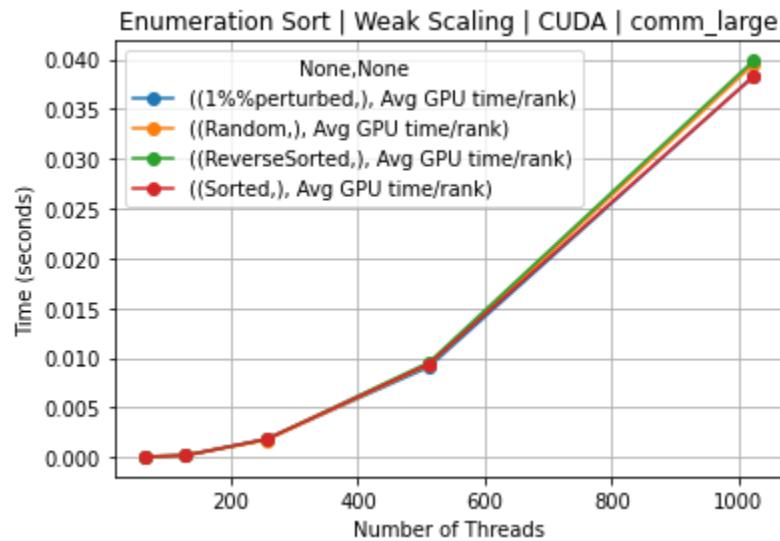
### Enumeration Sort CUDA:

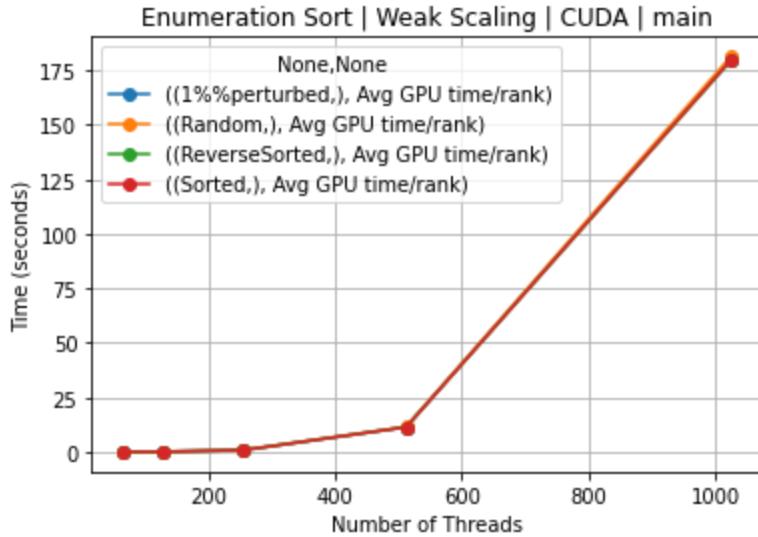






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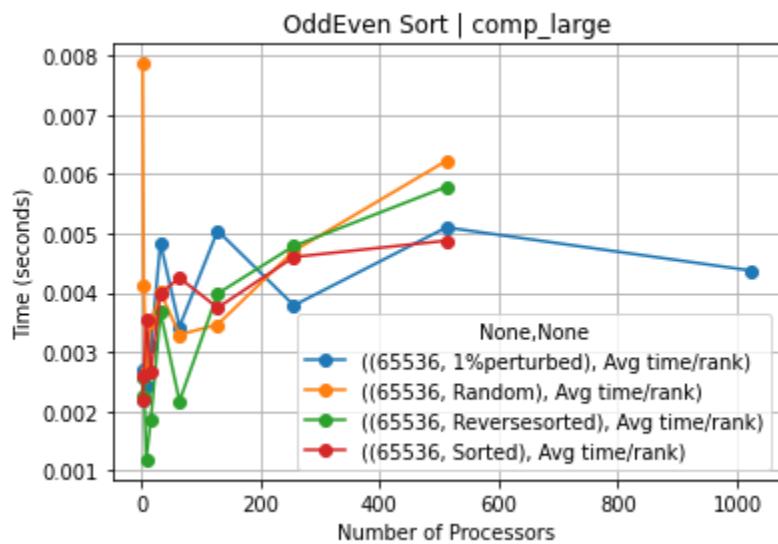
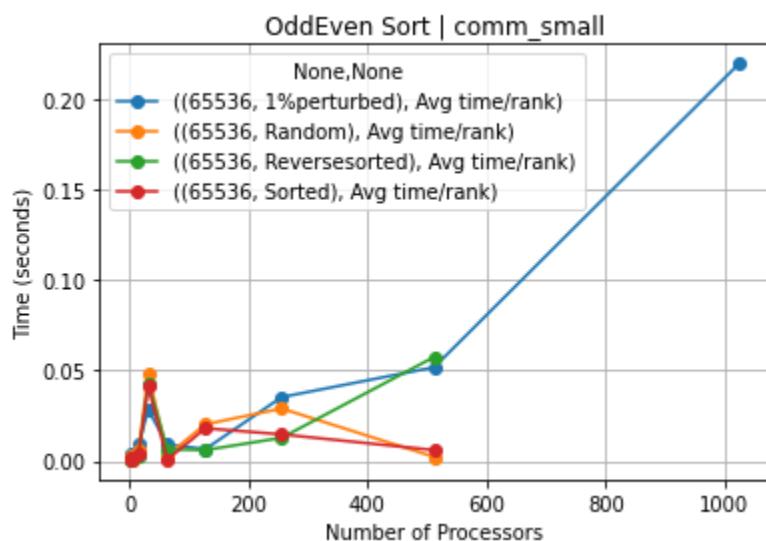
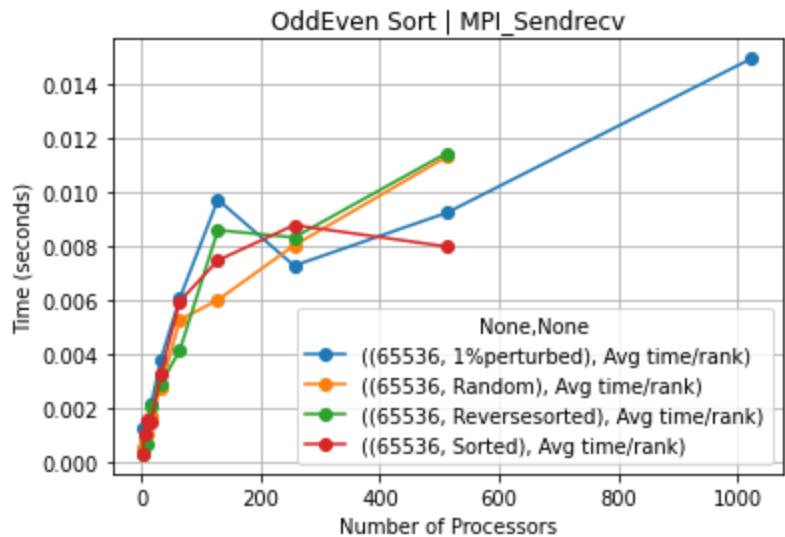


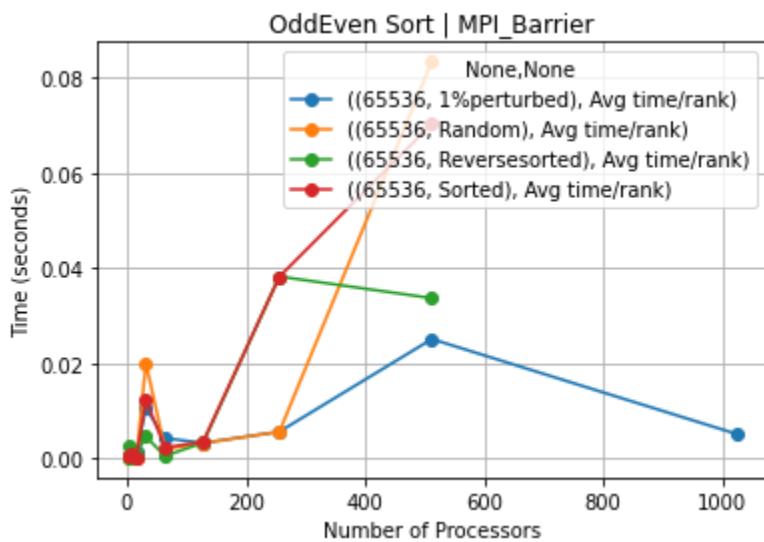
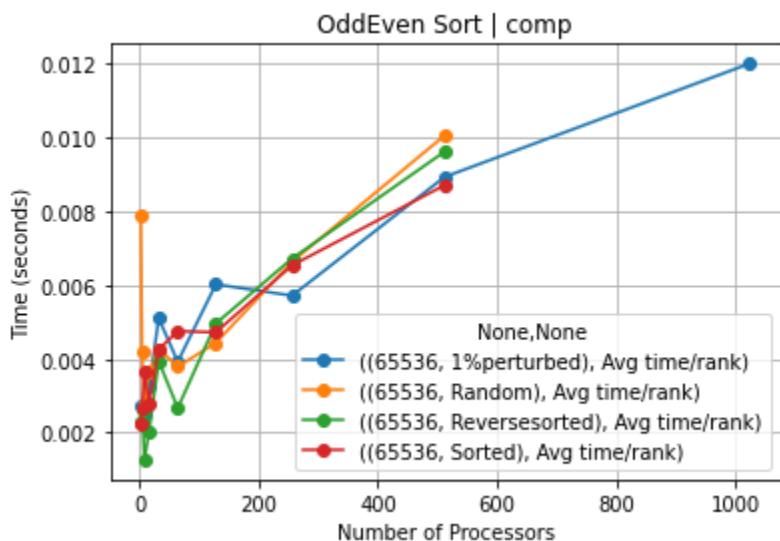
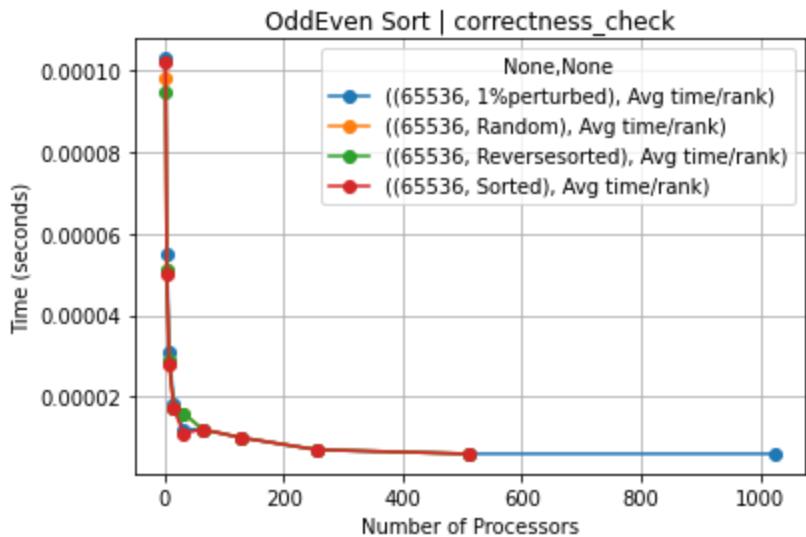


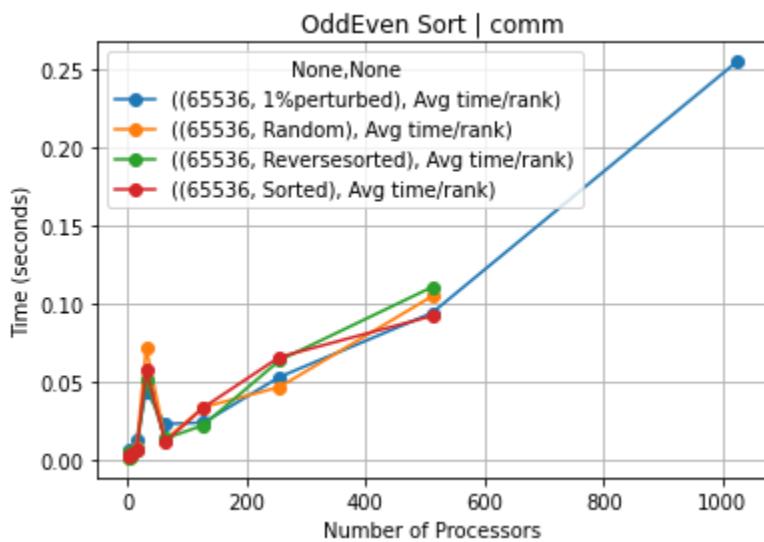
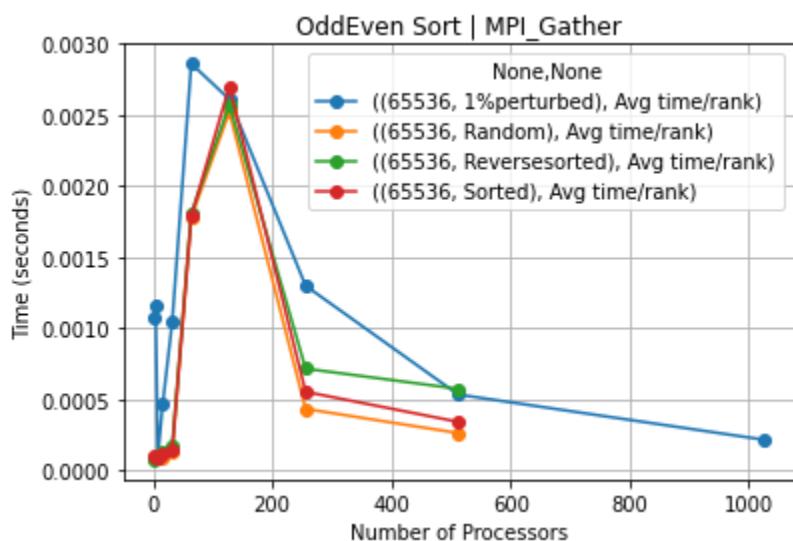
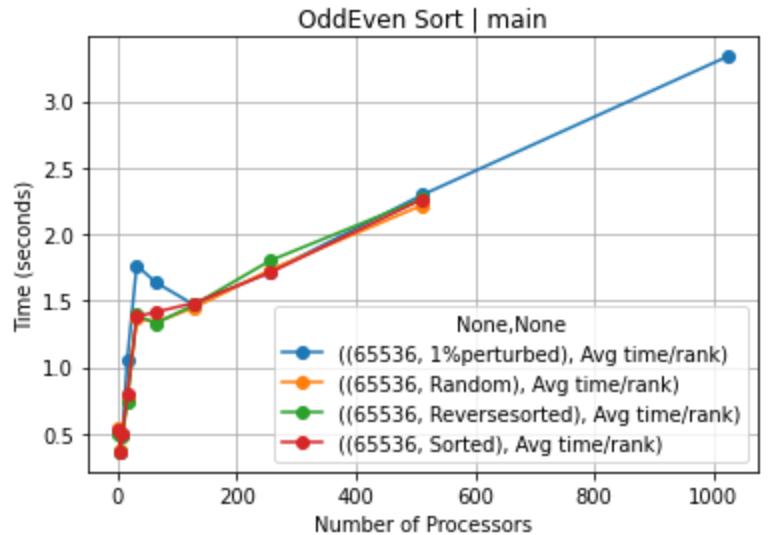
The current implementation of Enumeration Sort in CUDA shows that the increase in the number of threads does not lead to a decrease in time. This may be attributed to deficiencies of the enumeration sort algorithm. There is no speedup and weak and strong scaling are not shown for the current implementation. In addition, there are no evident performance improvements for the strong scaling graphs. This may be due to the implementation and how the arrays are accounted for. The strong scaling does not show a clear trend in CUDA which can be due to limitations for the algorithm. However, the weak scaling graphs performed relatively well in CUDA.

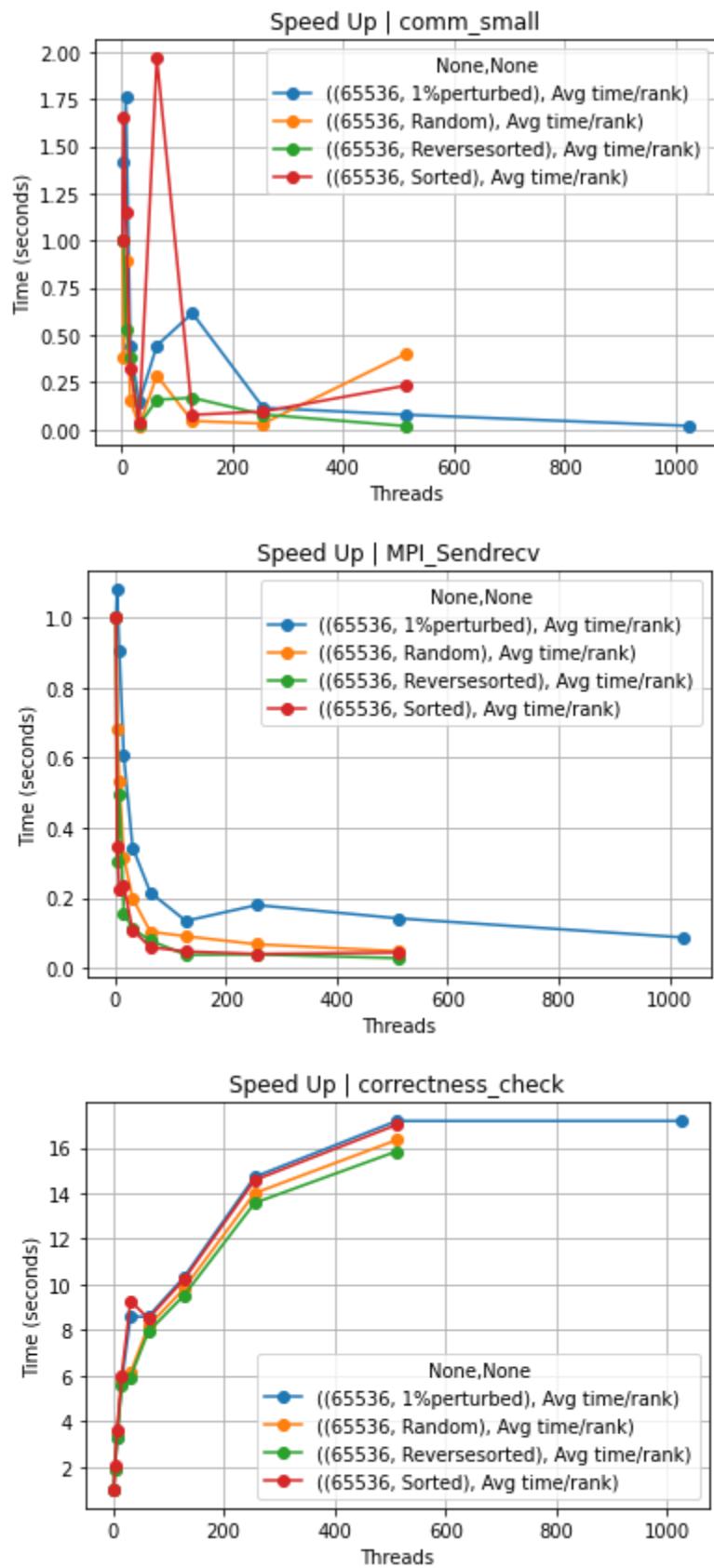
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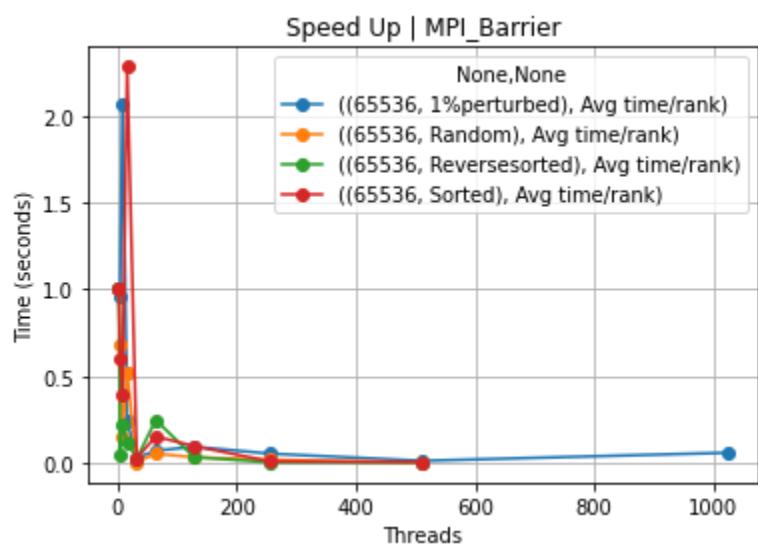
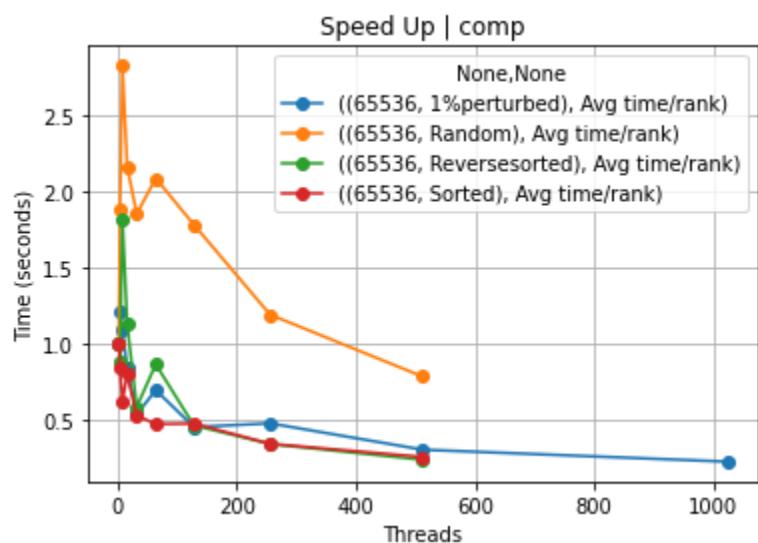
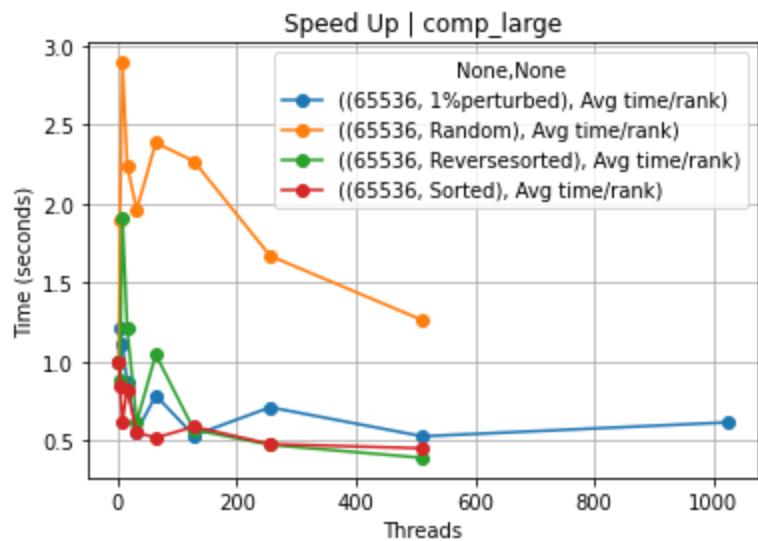
**Strong Scaling 65536**

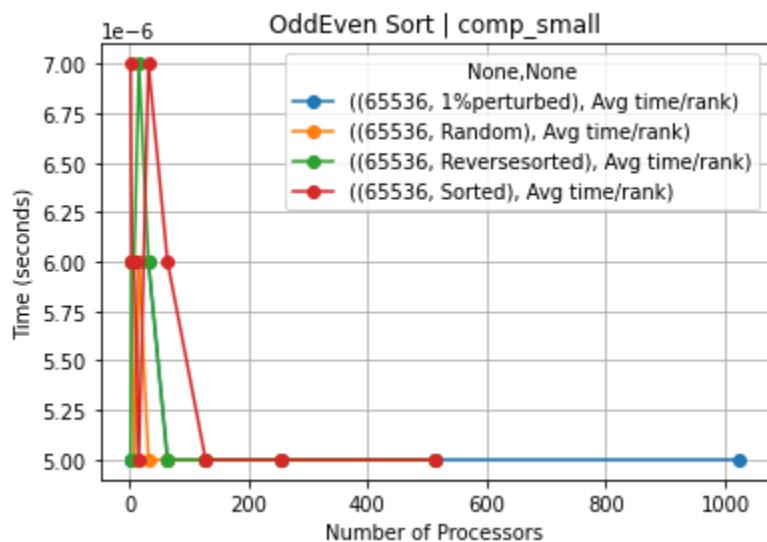
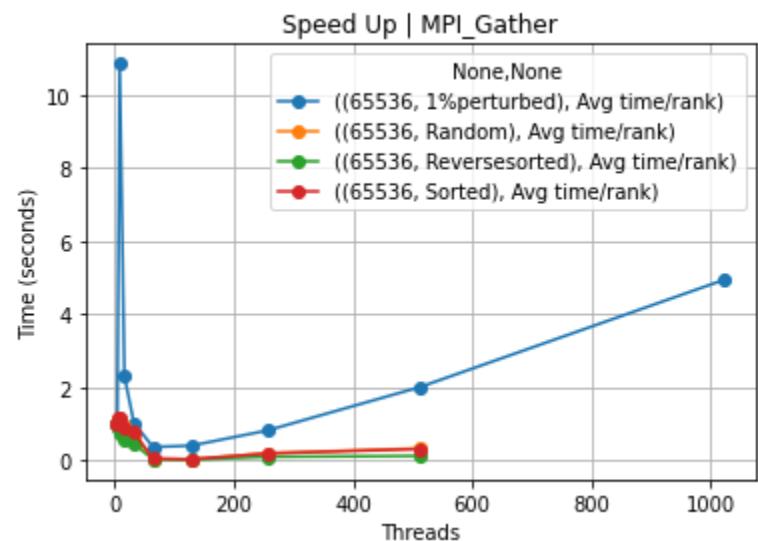
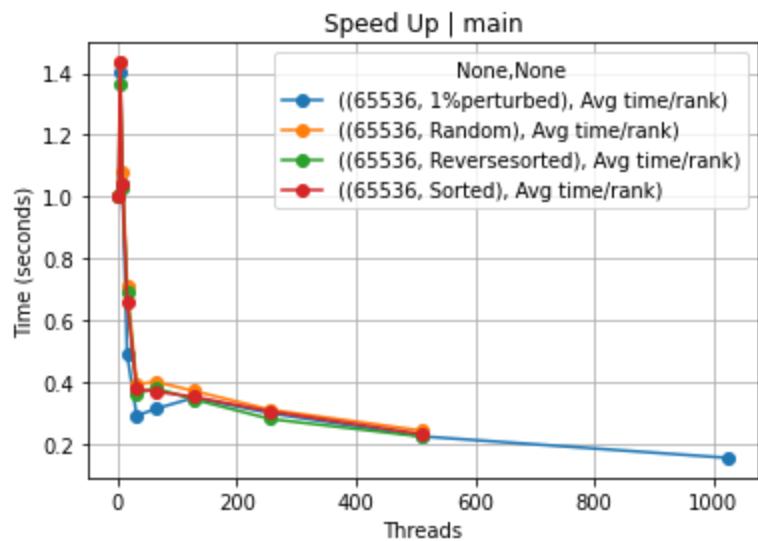


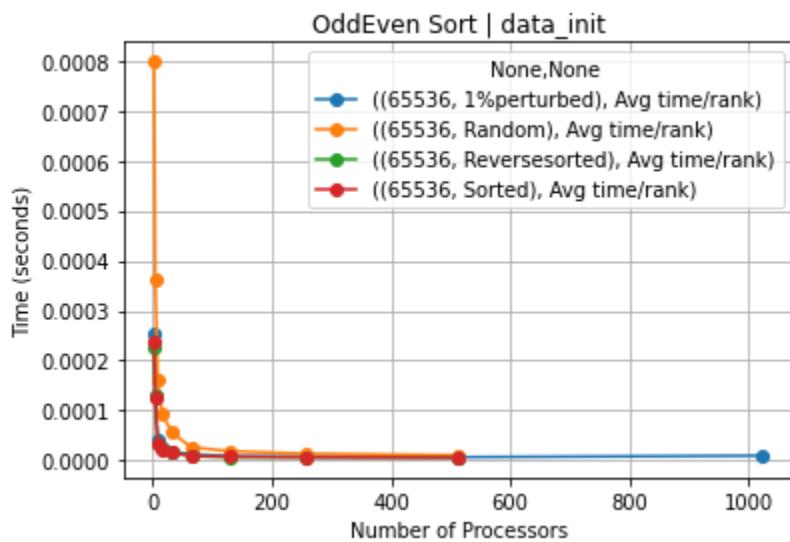
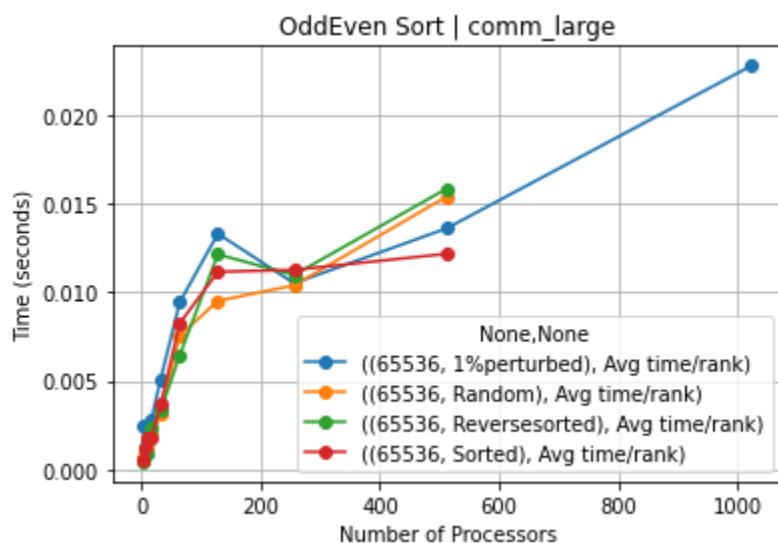
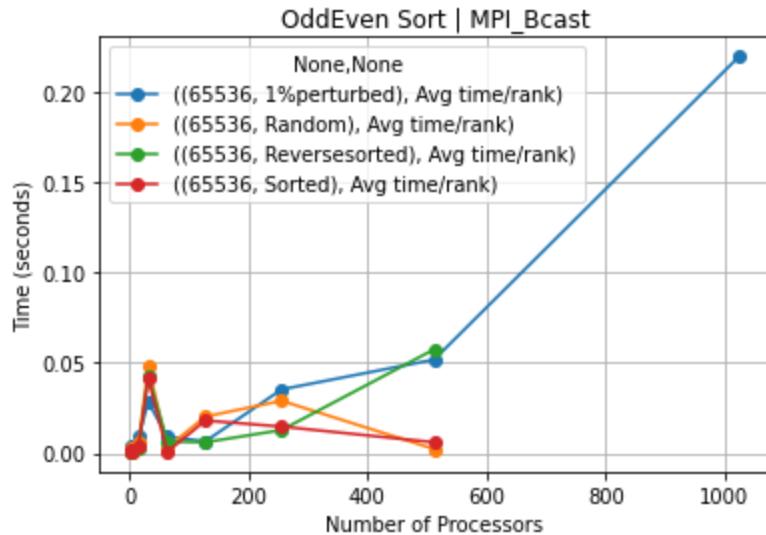


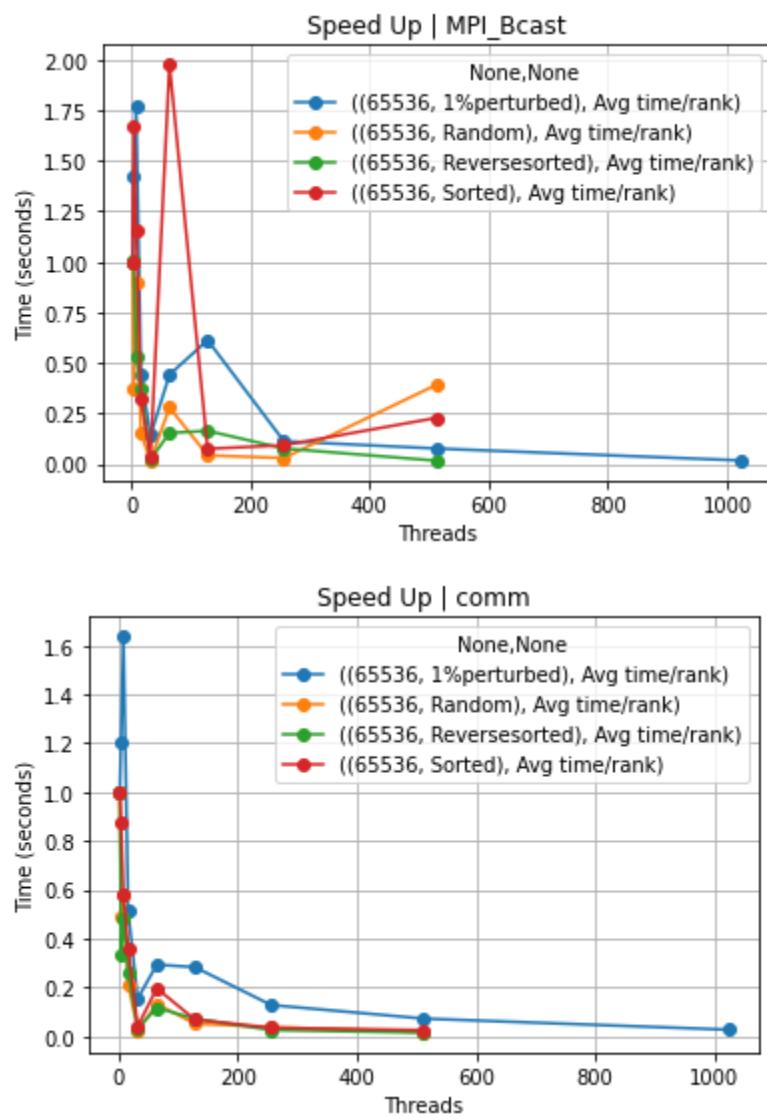


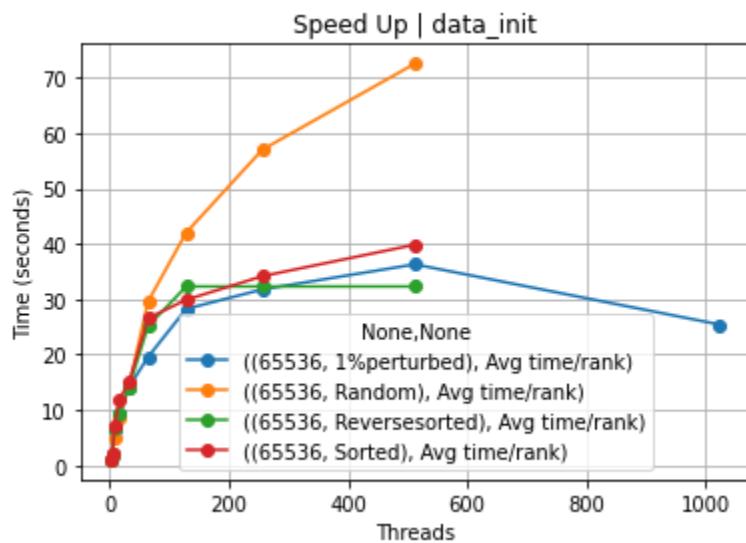
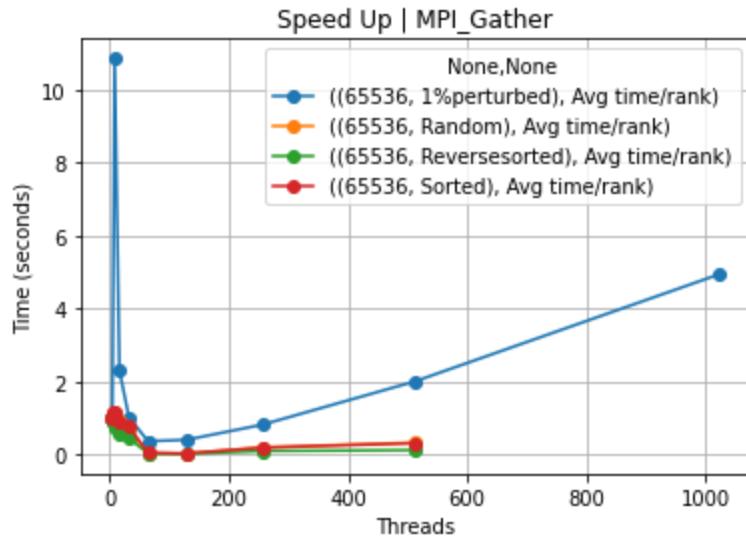


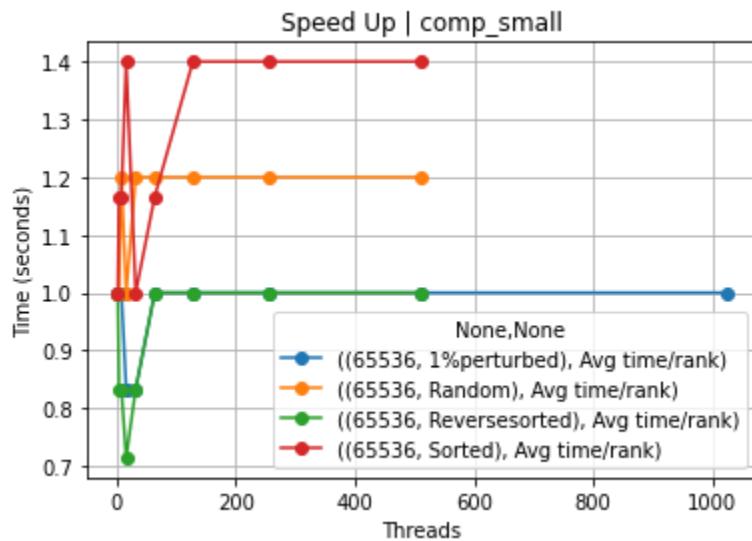
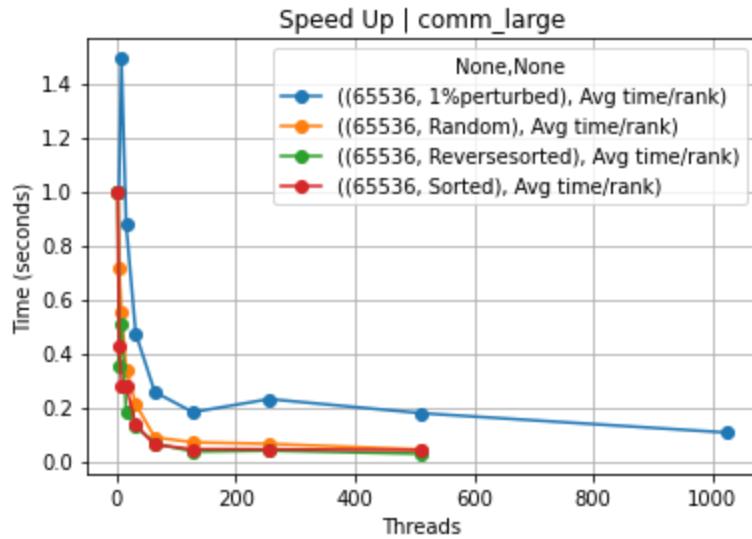






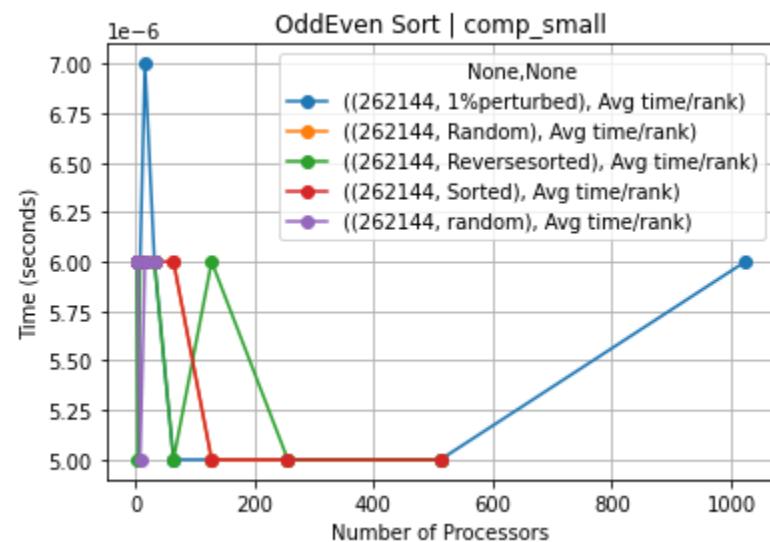
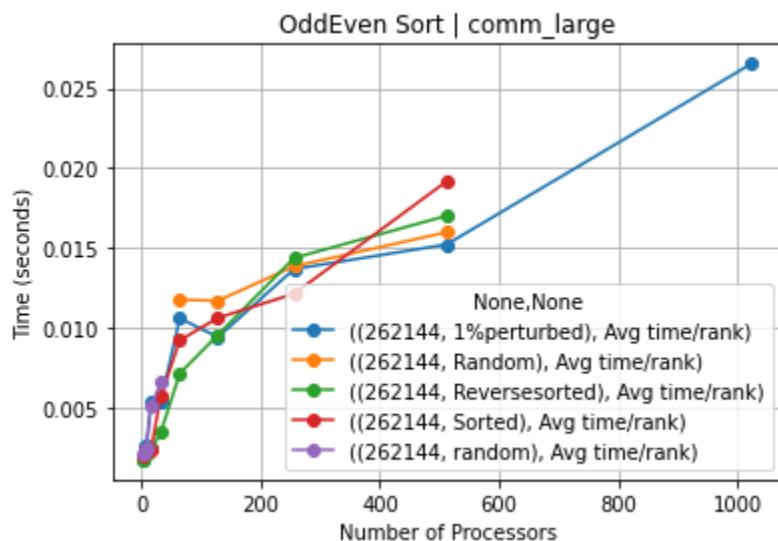
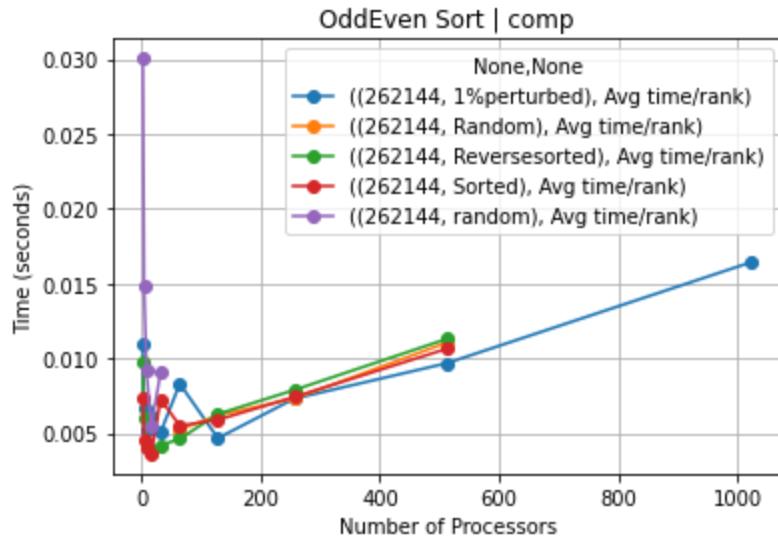


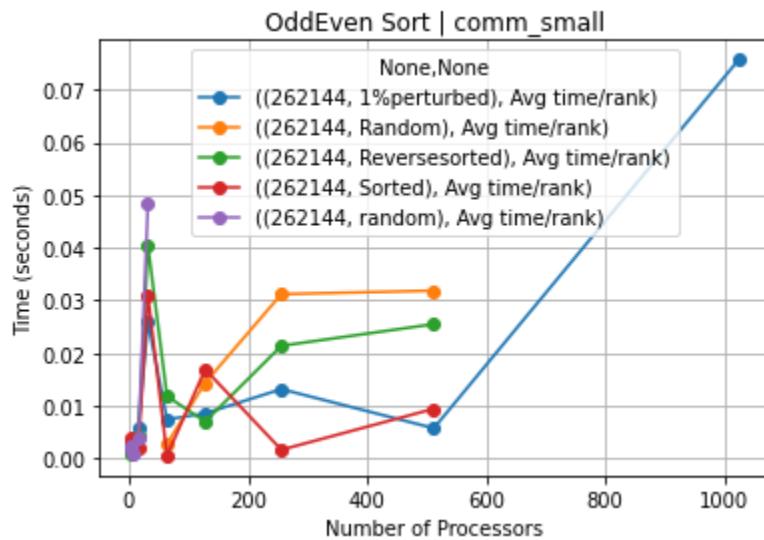
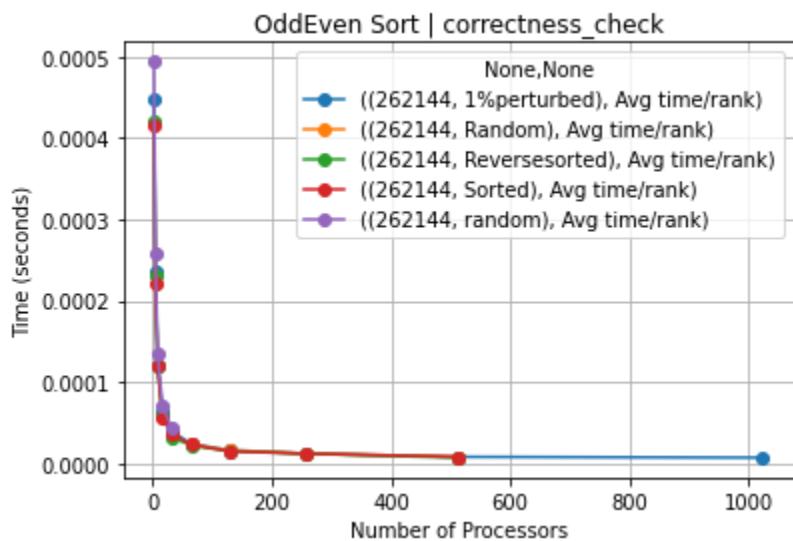
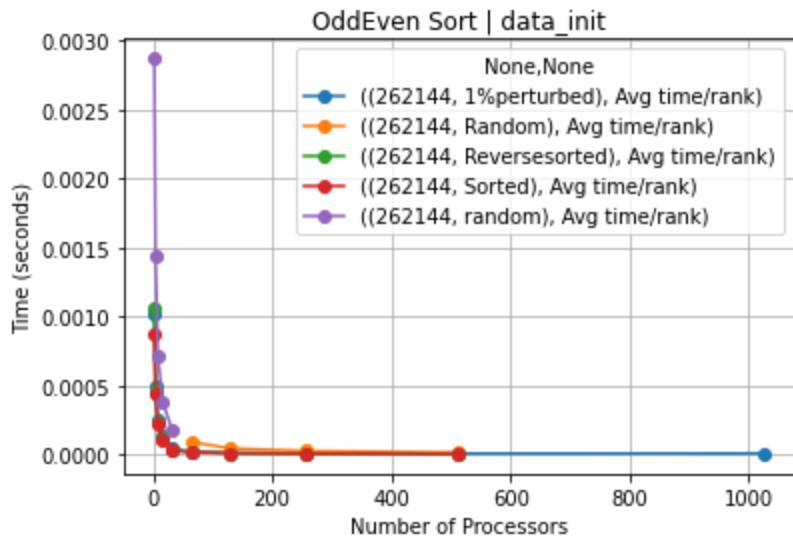


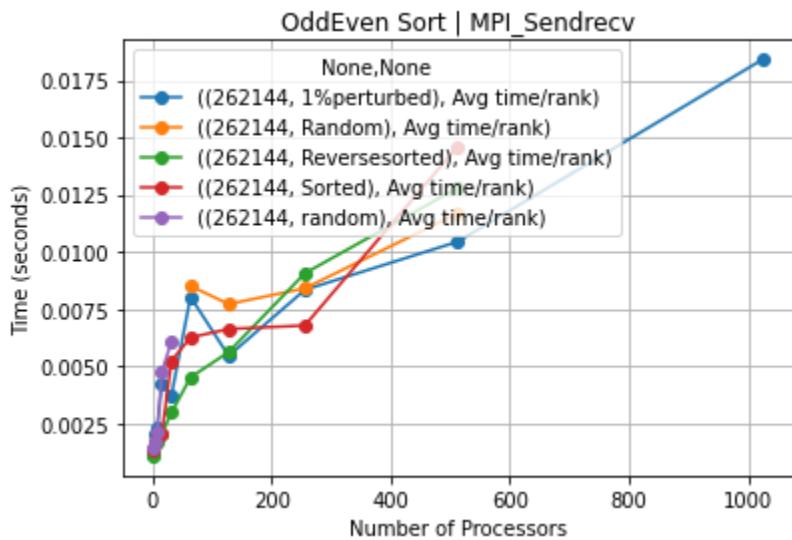
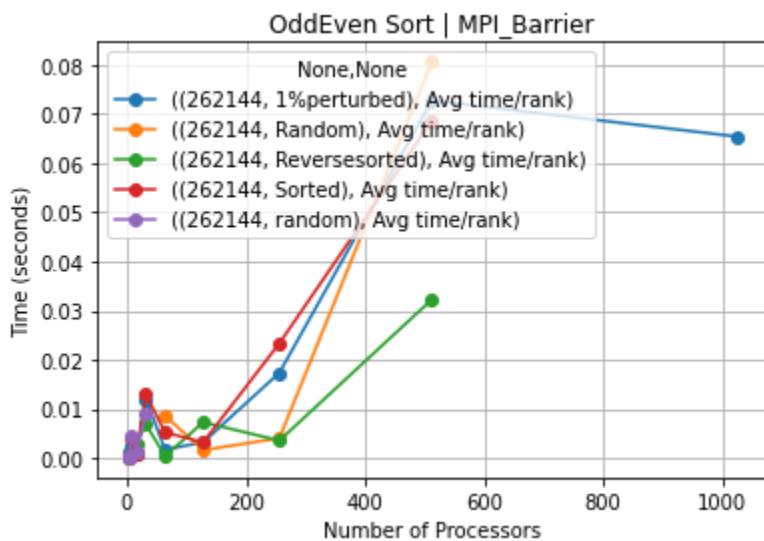
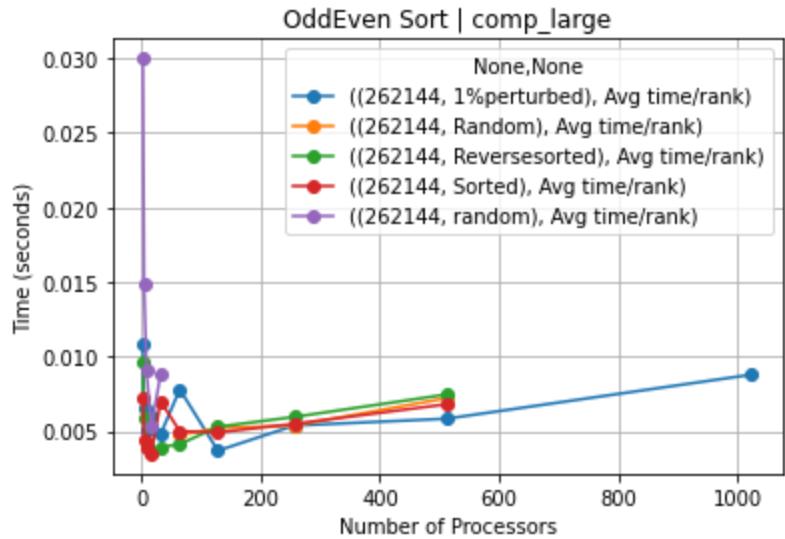


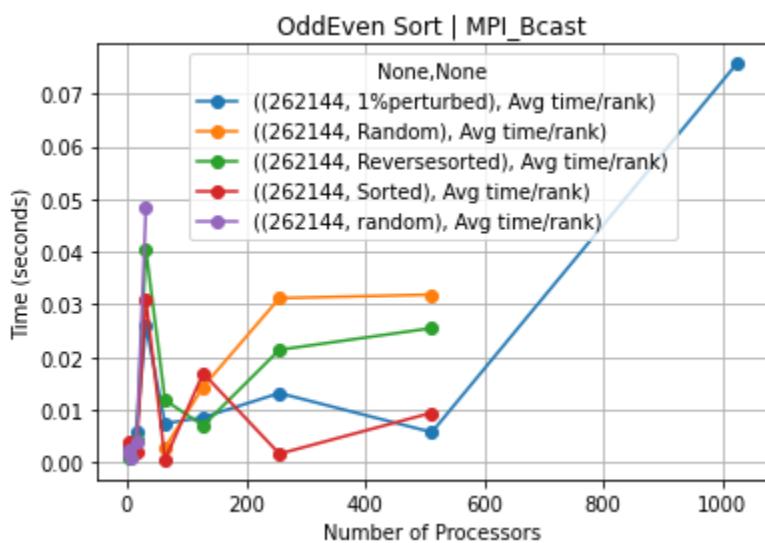
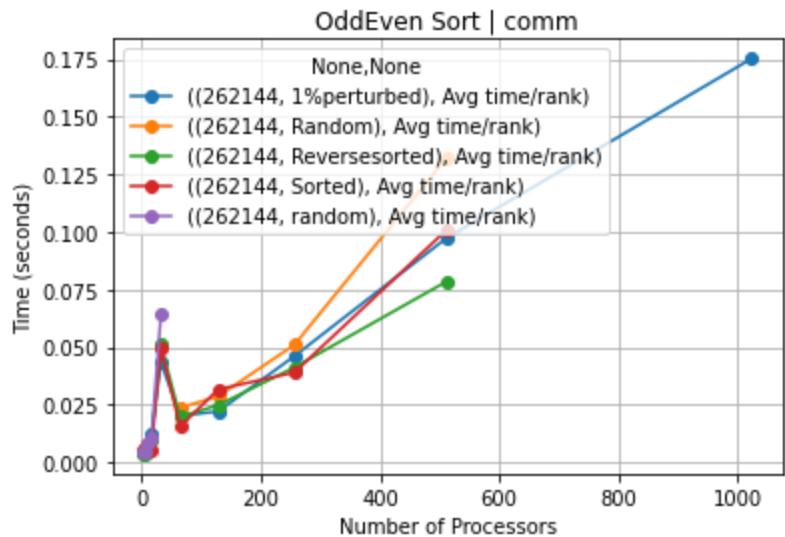
We see a linear increase in the time for large computation as the number of processes increases. This is most likely due to the fact that we are operating with a relatively small number of processes, increasing the overhead. We also observe a high communication time relative to everything else. This is because my implementation of the algorithm requires processes to transmit at most half of the entire array during each iteration of the algorithm.

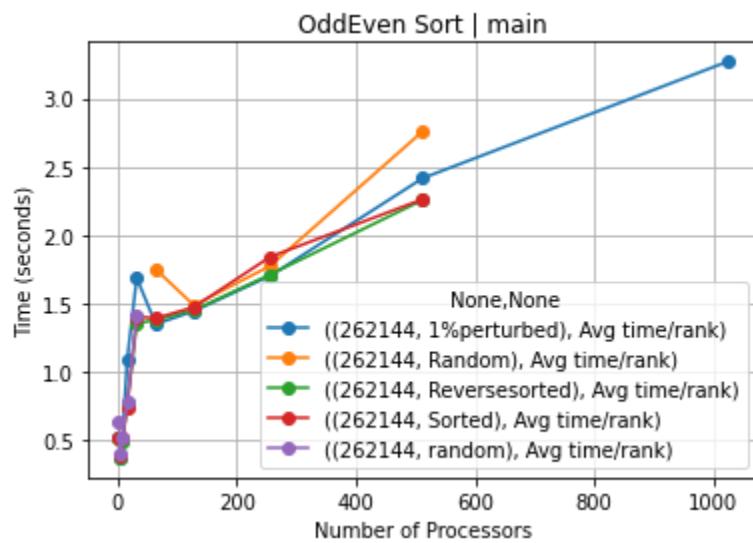
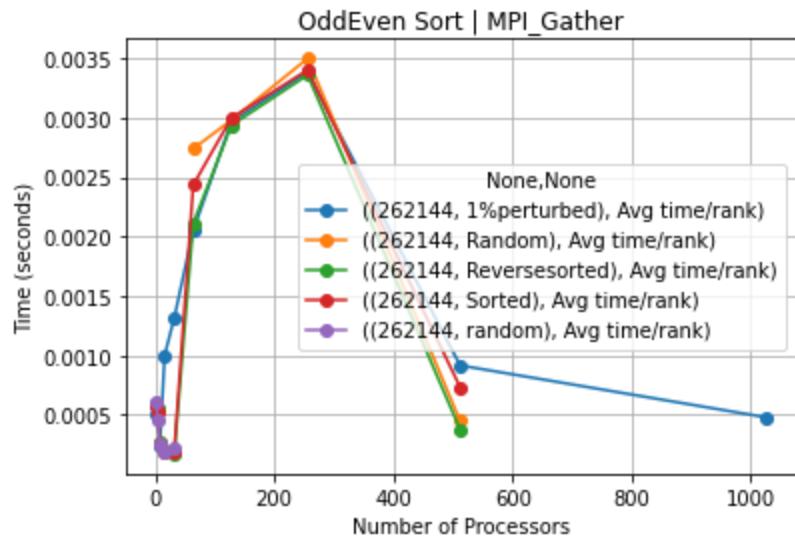
## Strong Scaling 262144

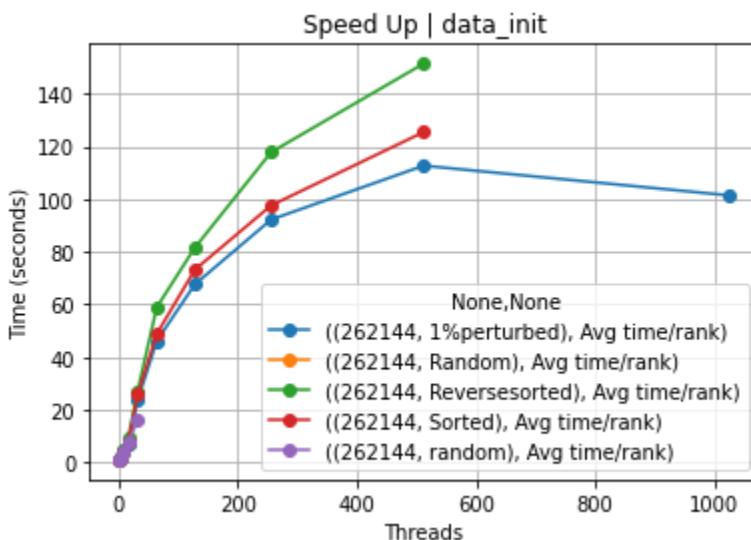
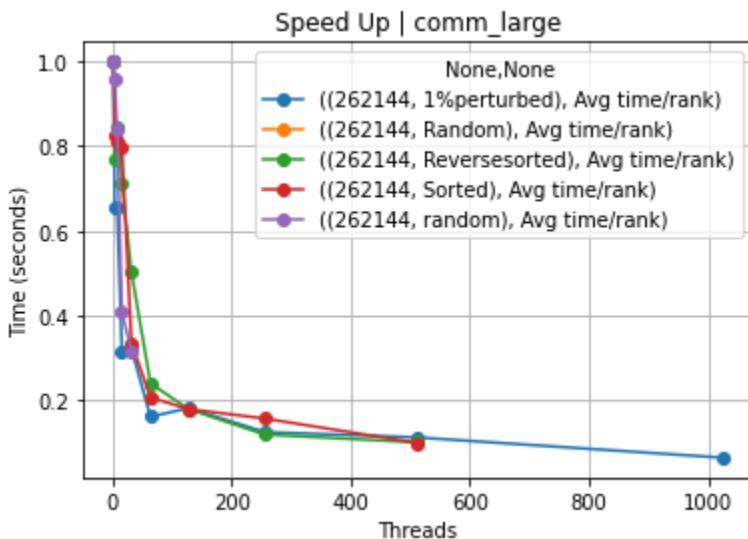
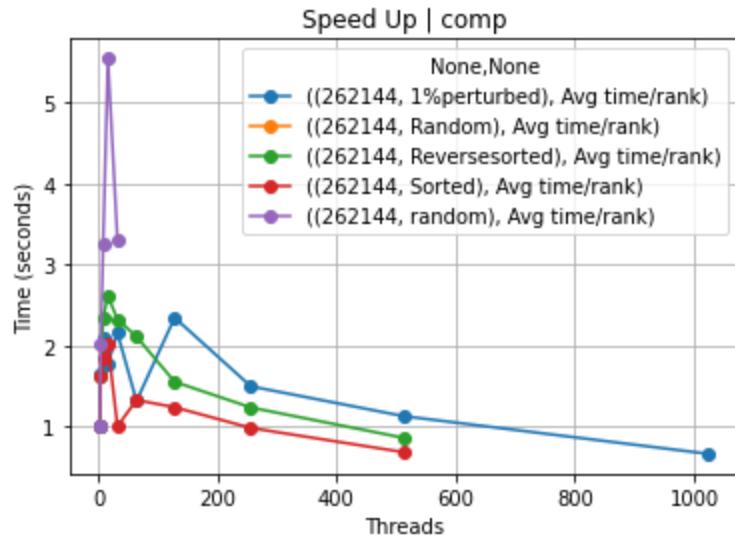


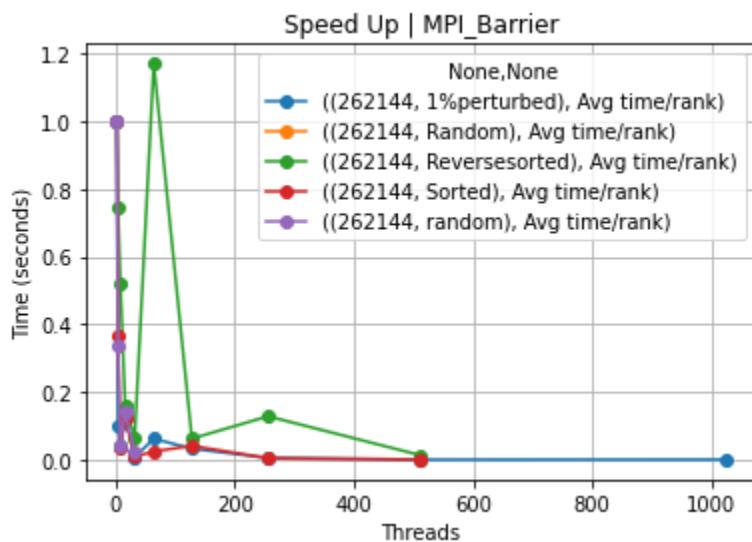
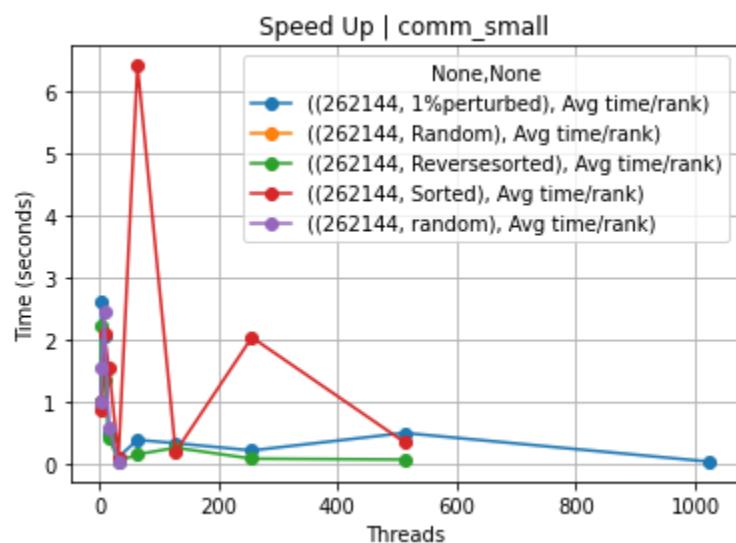
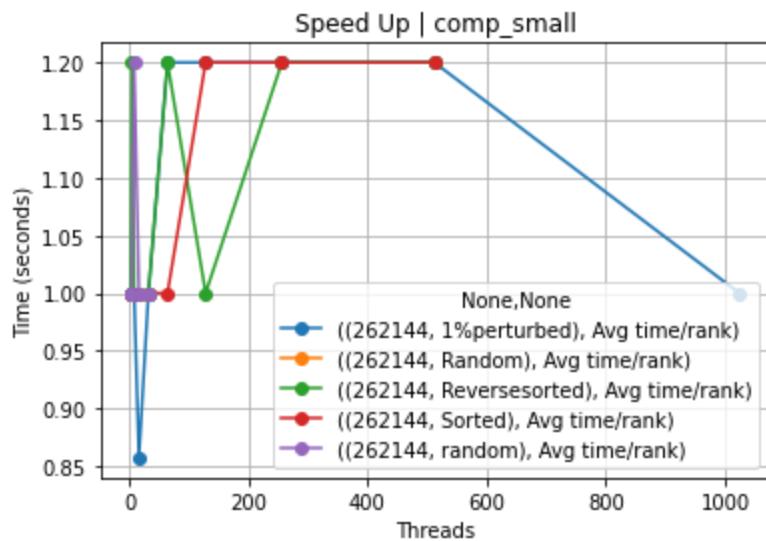


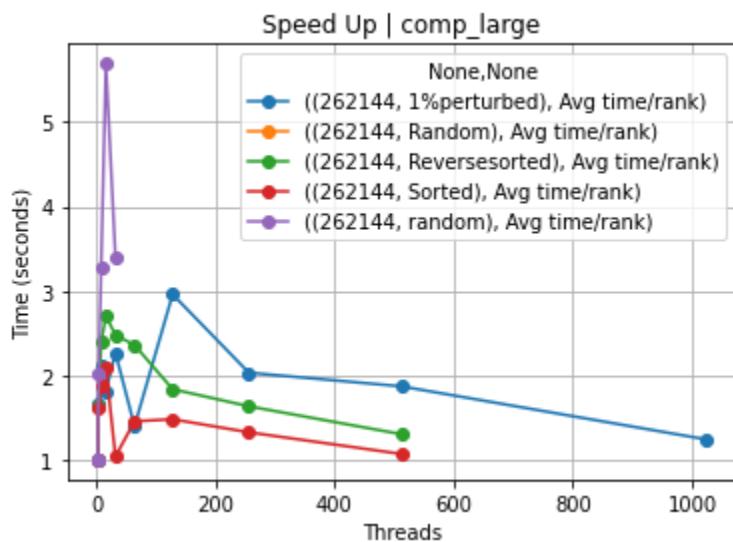
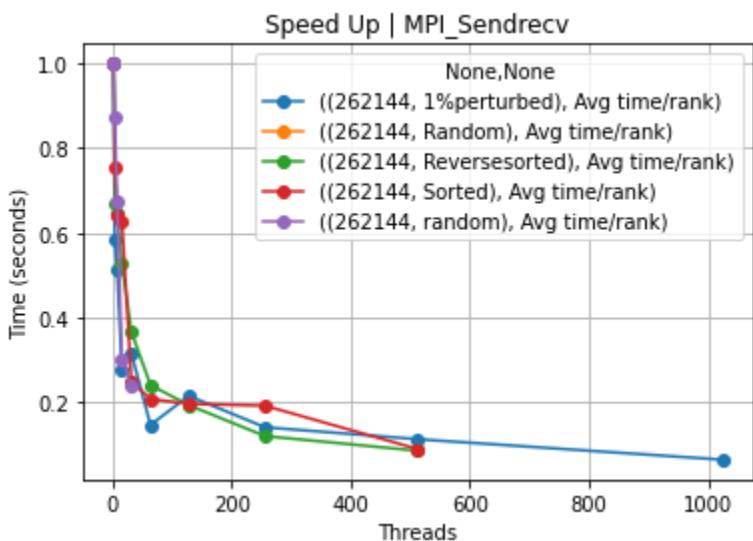
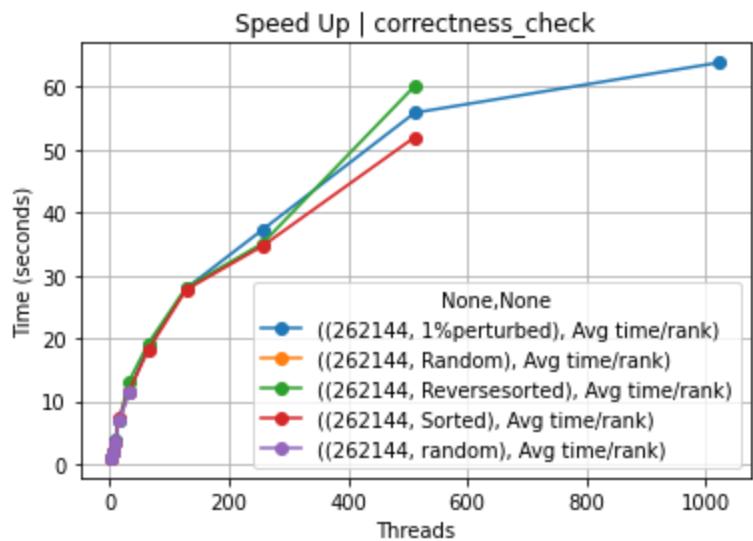


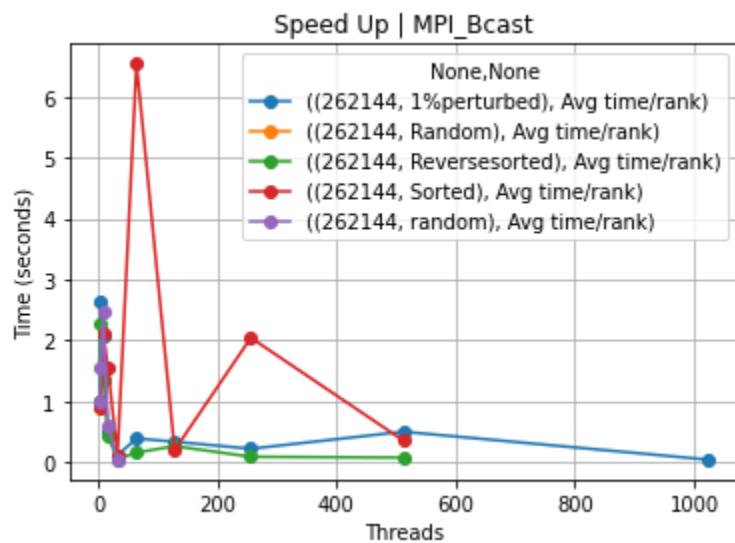
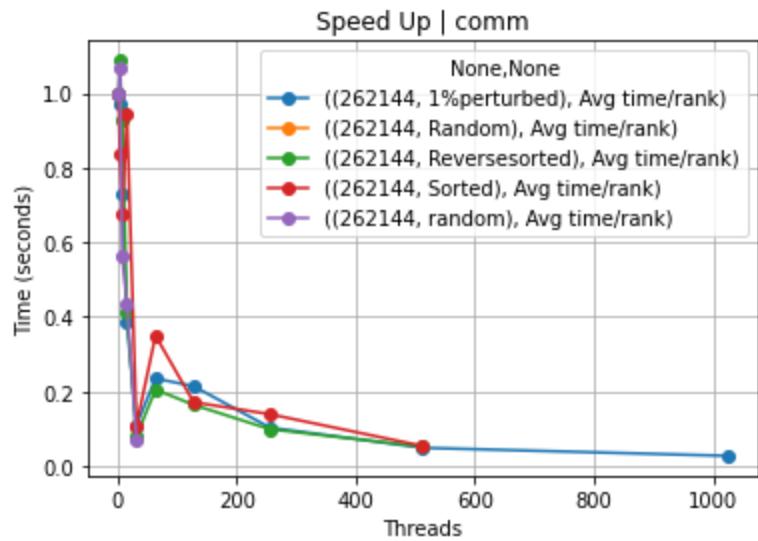


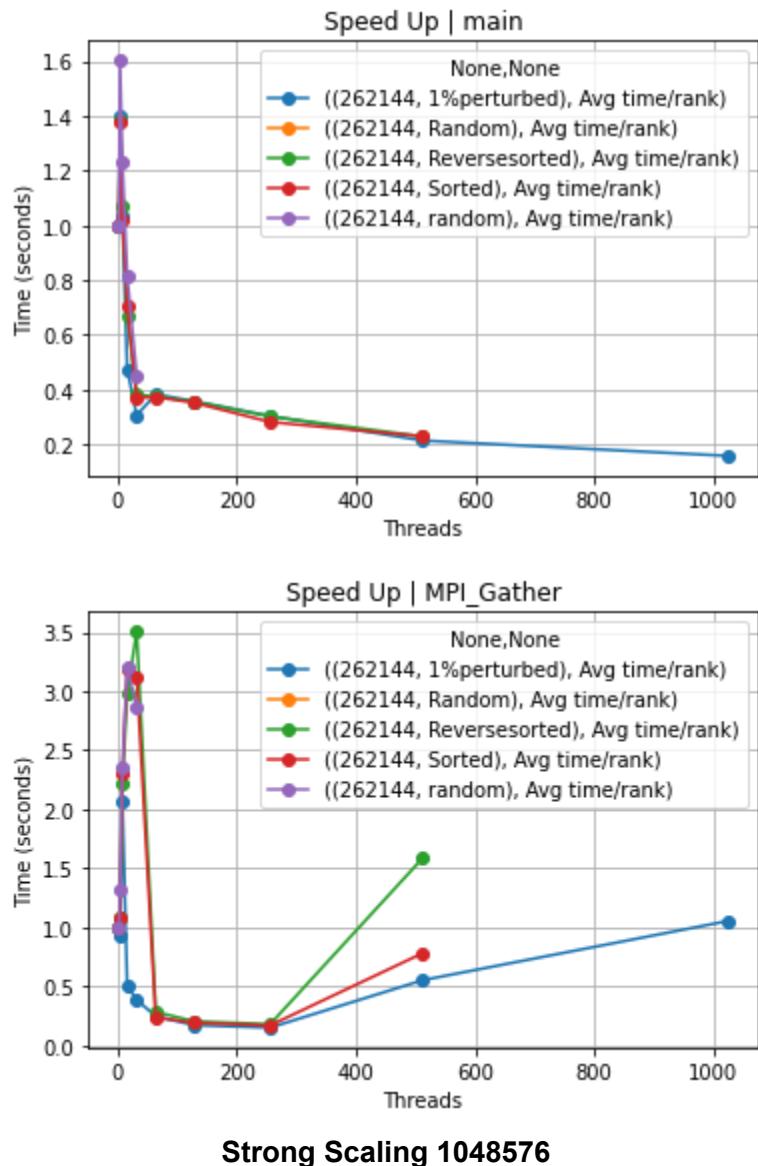


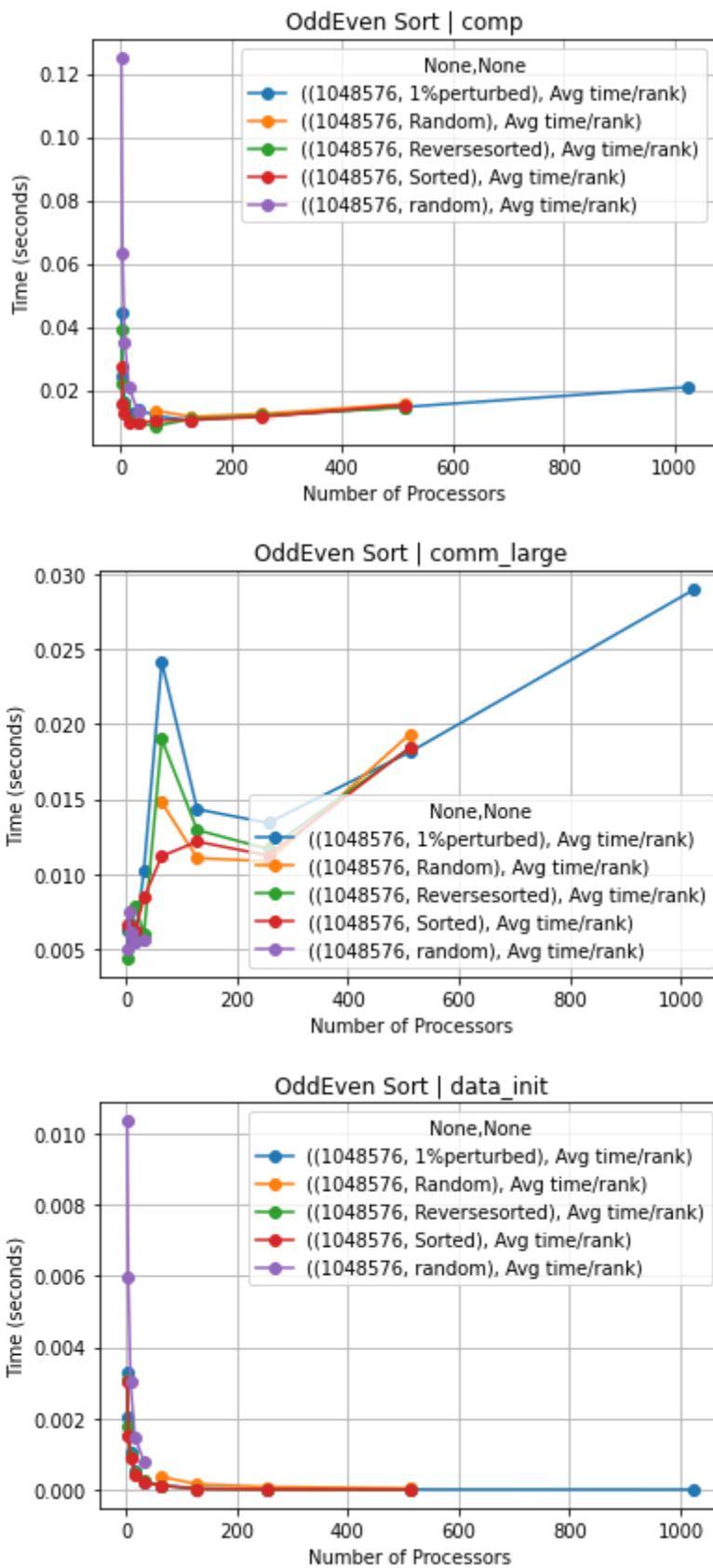


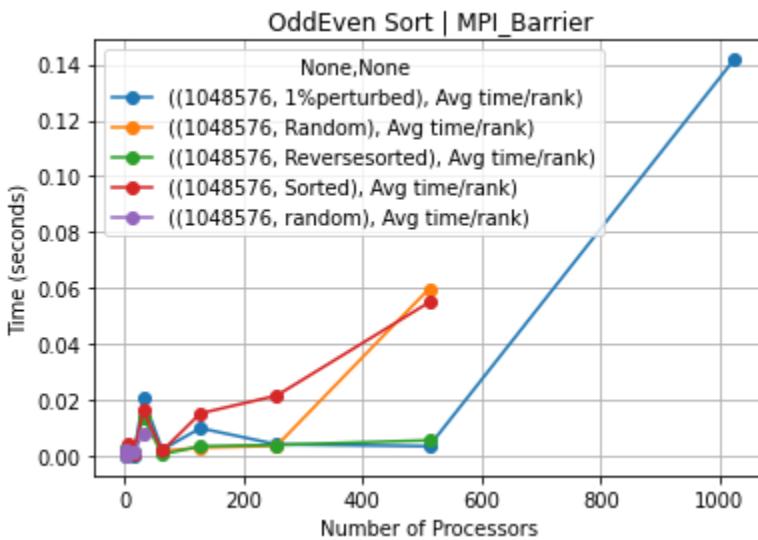
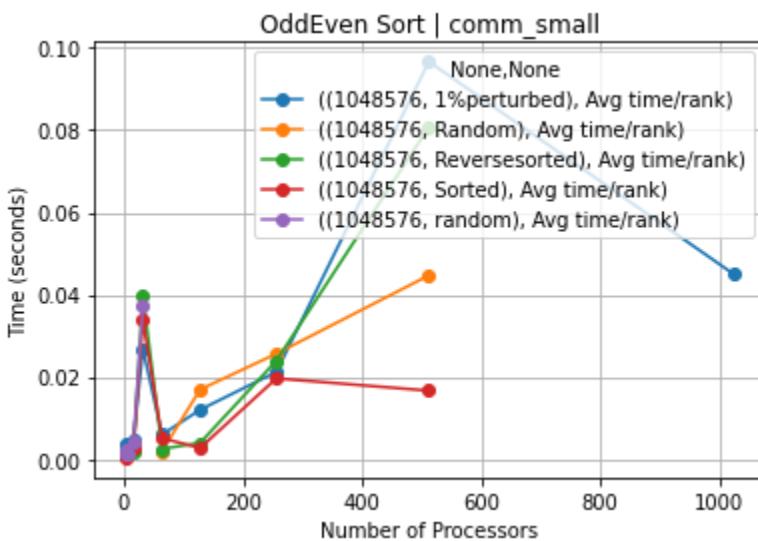
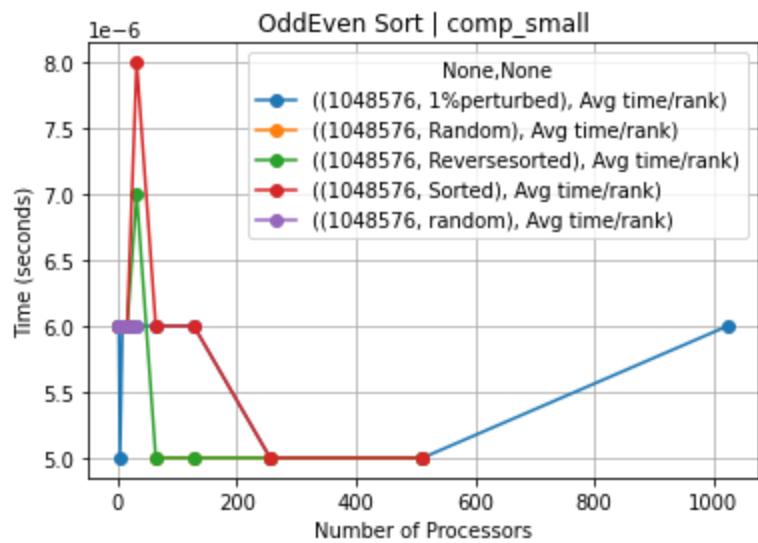


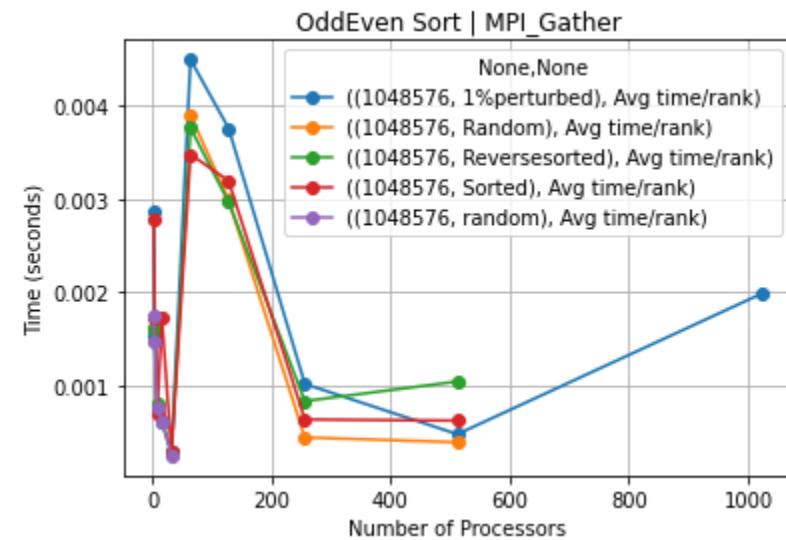
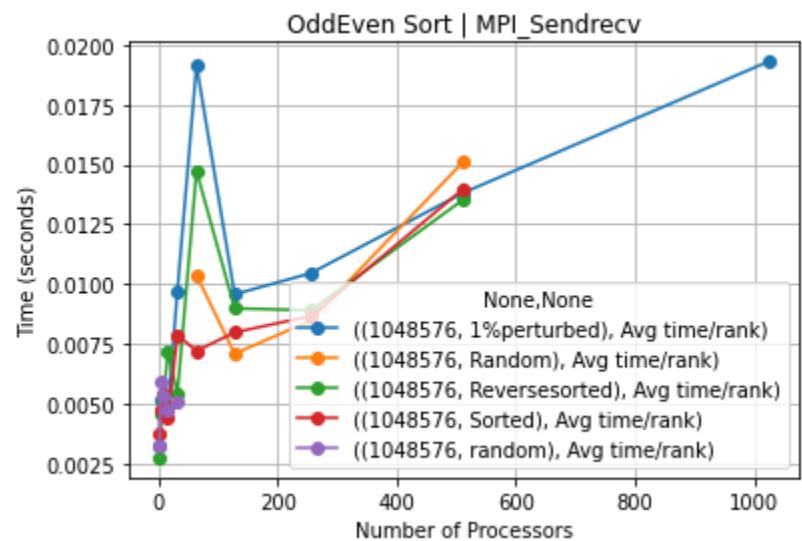
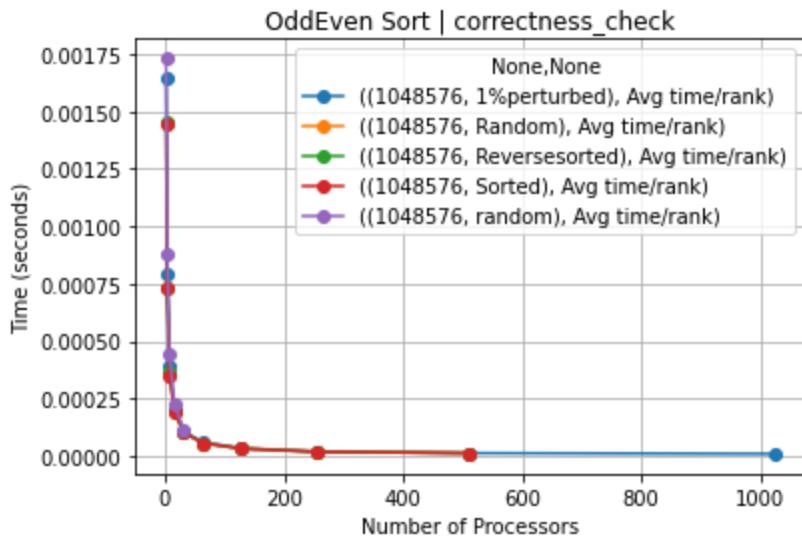


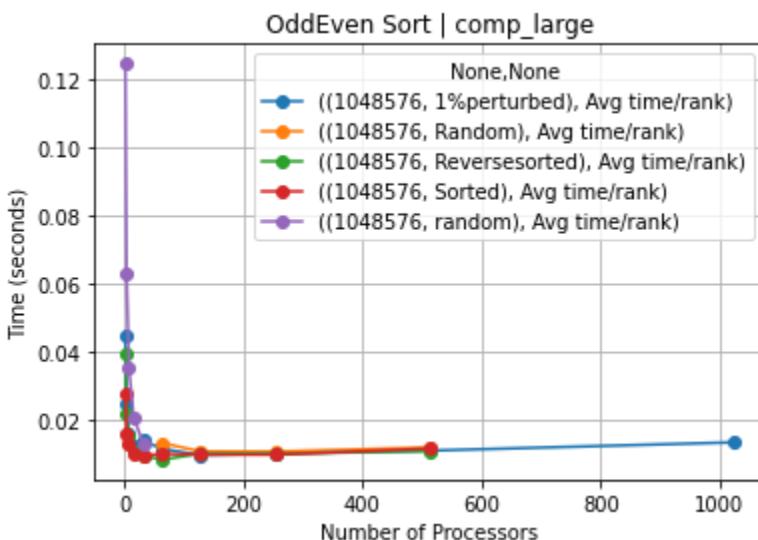
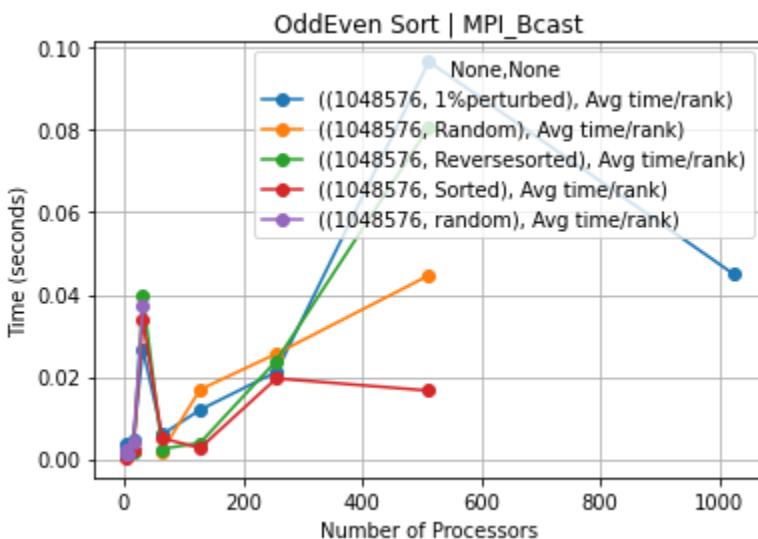
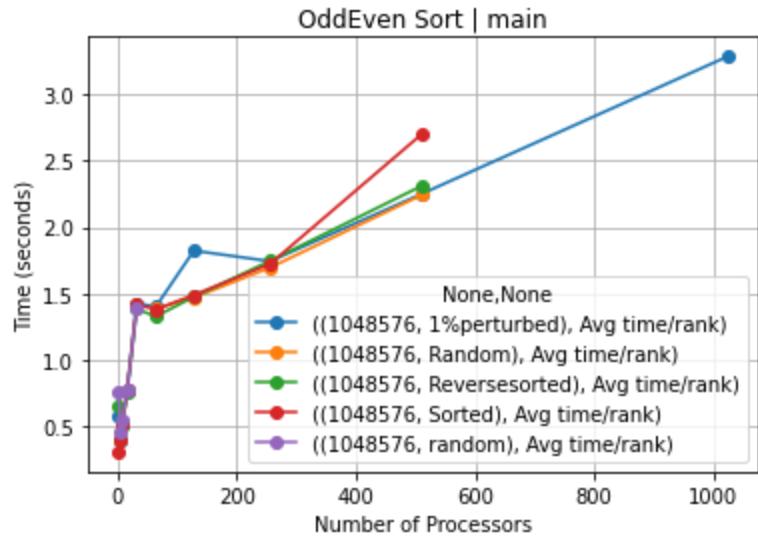


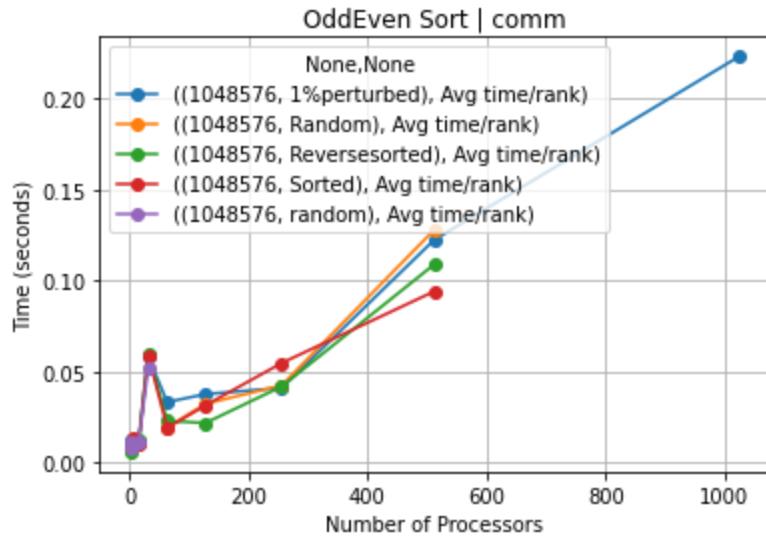


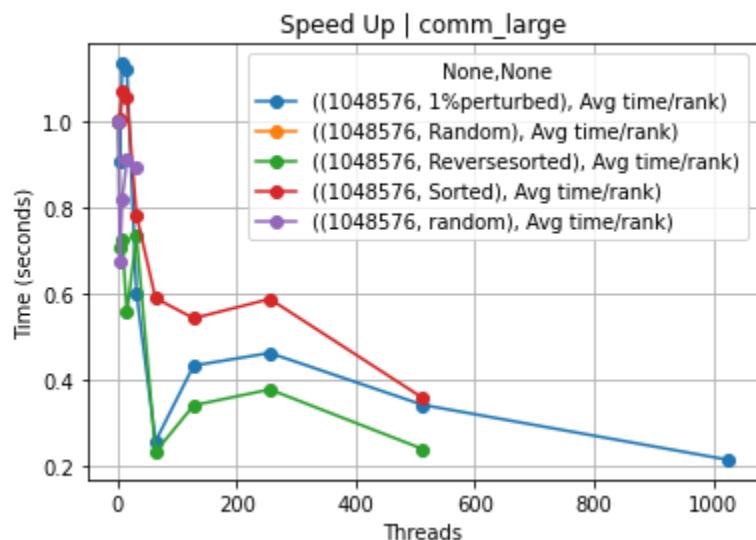
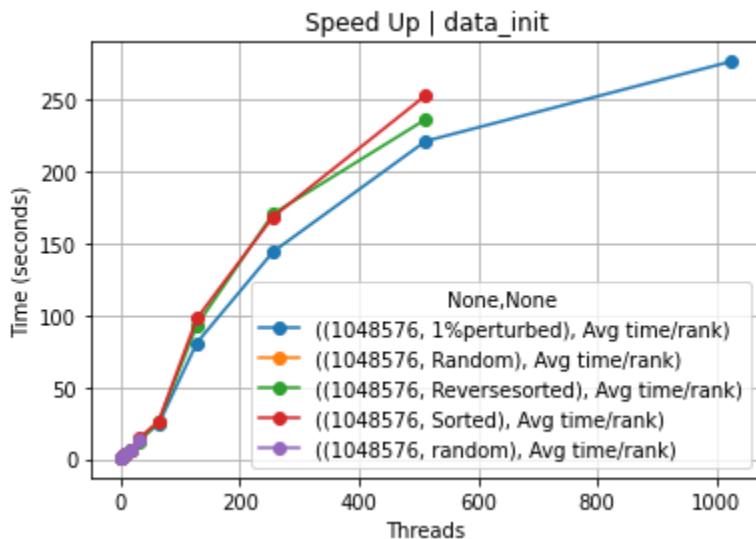
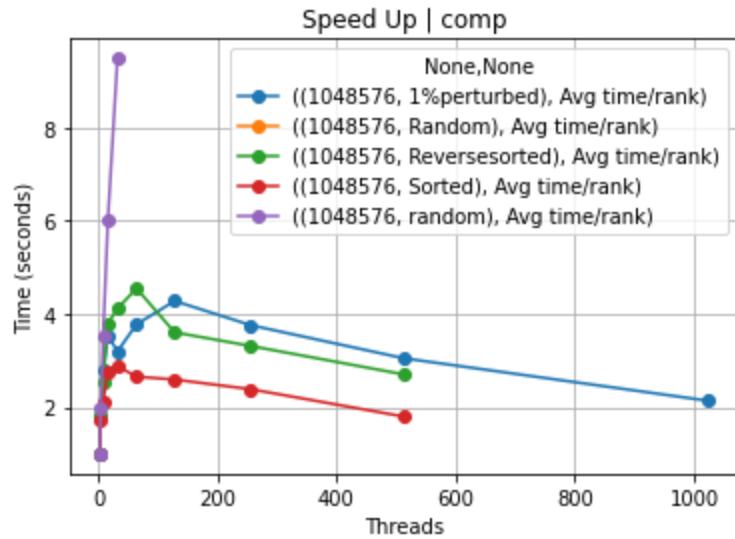


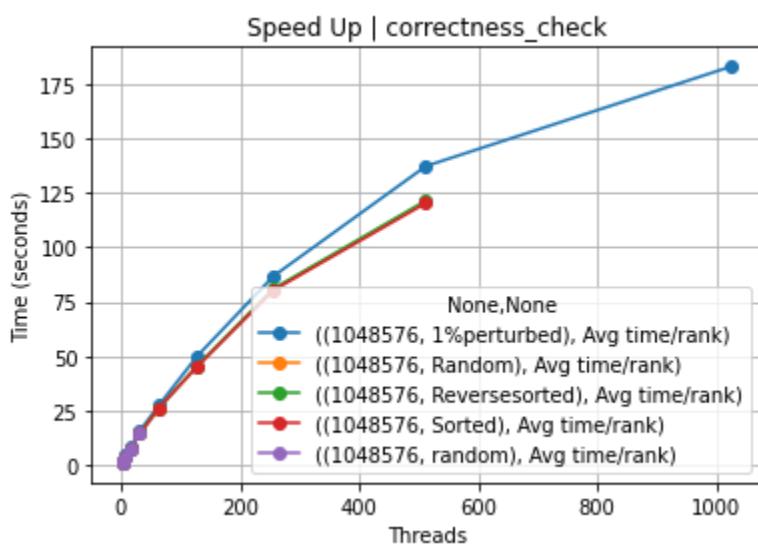
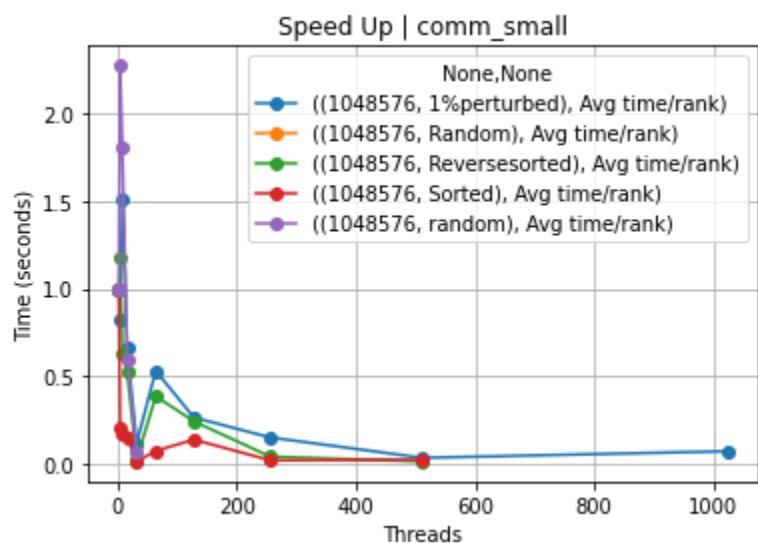
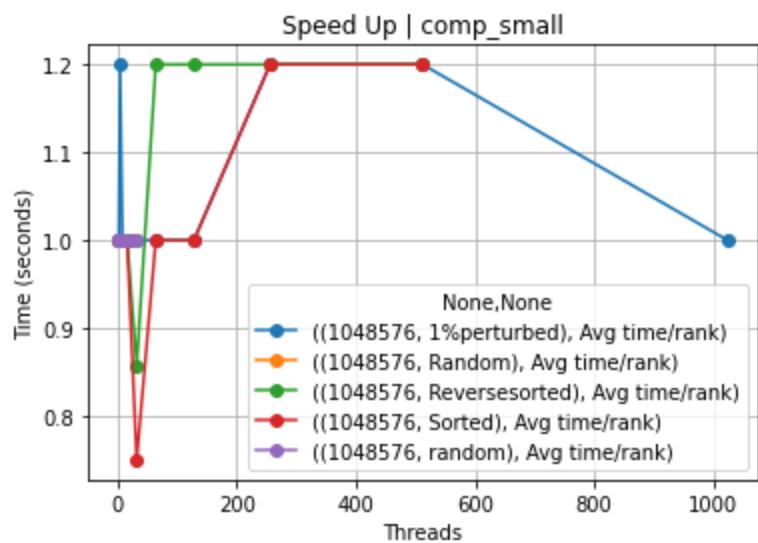


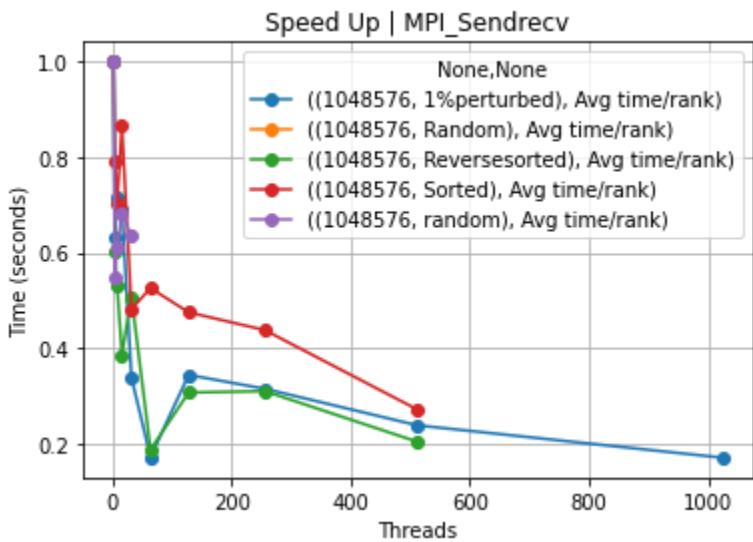
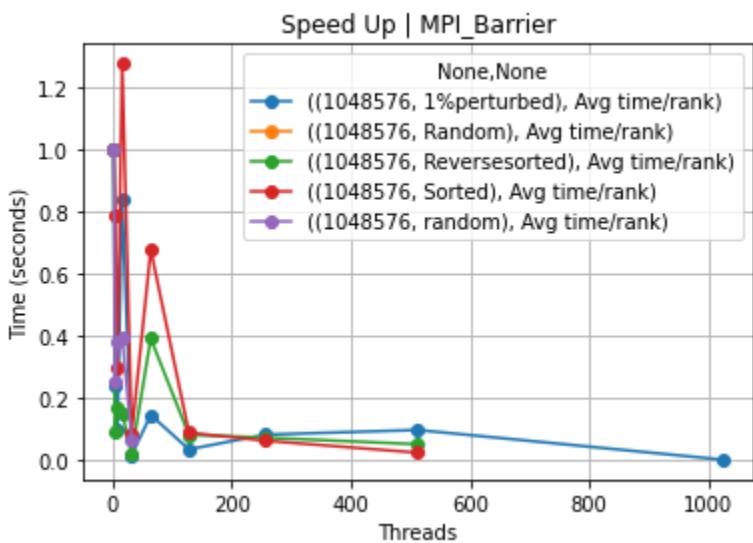
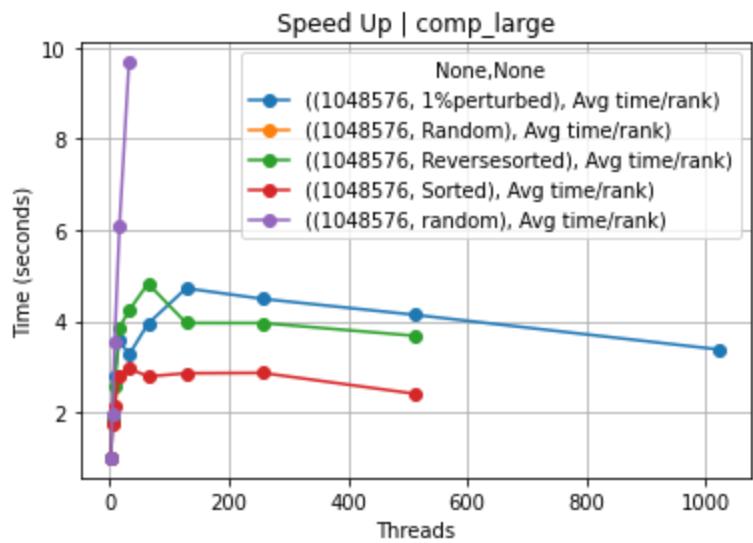


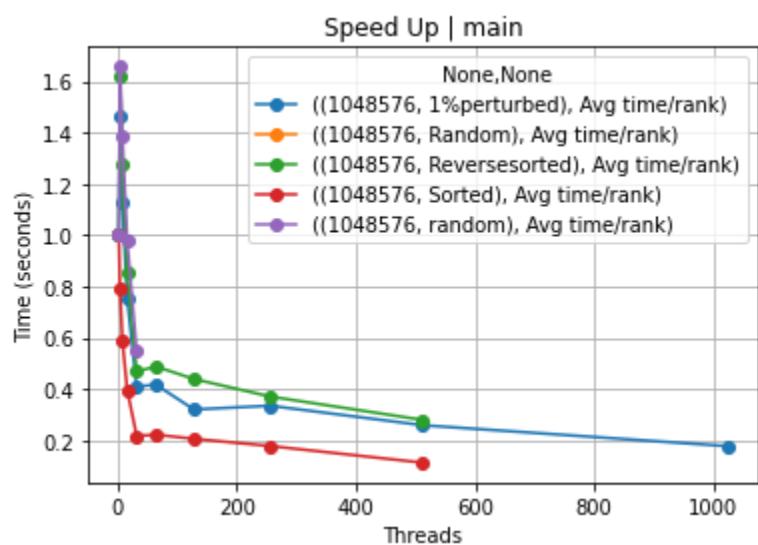
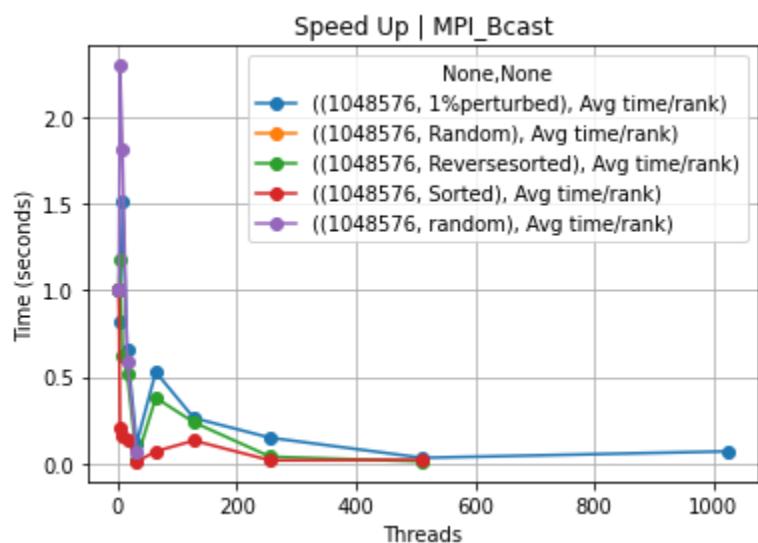
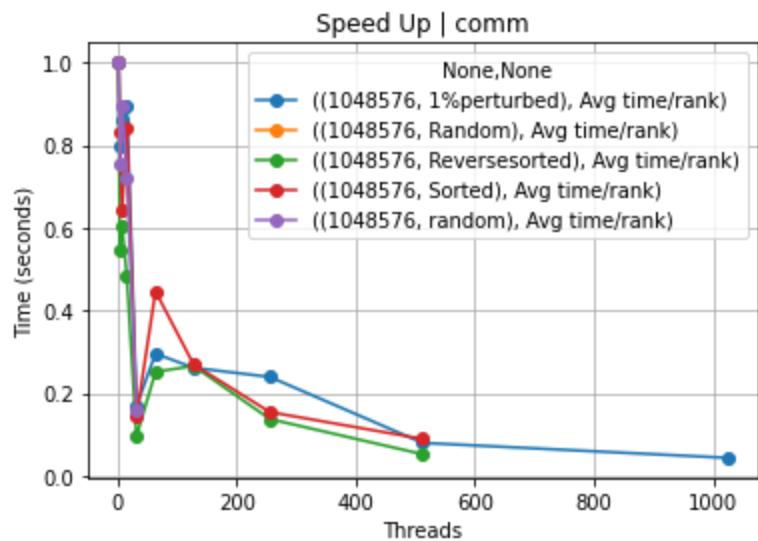


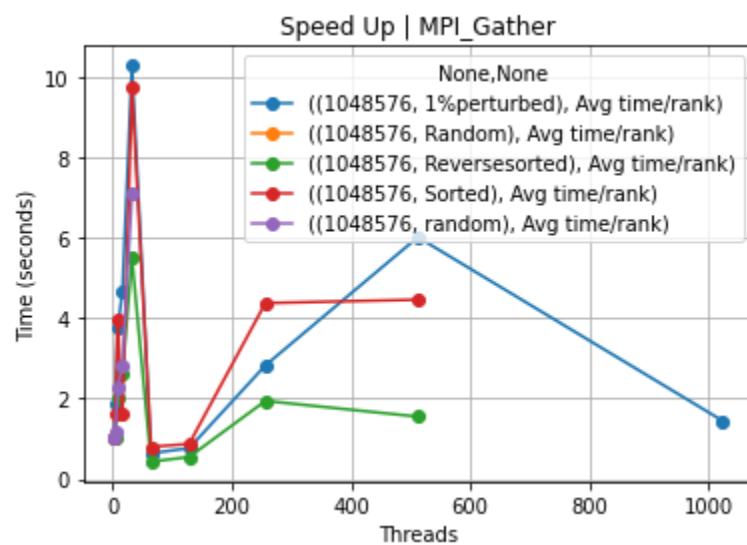
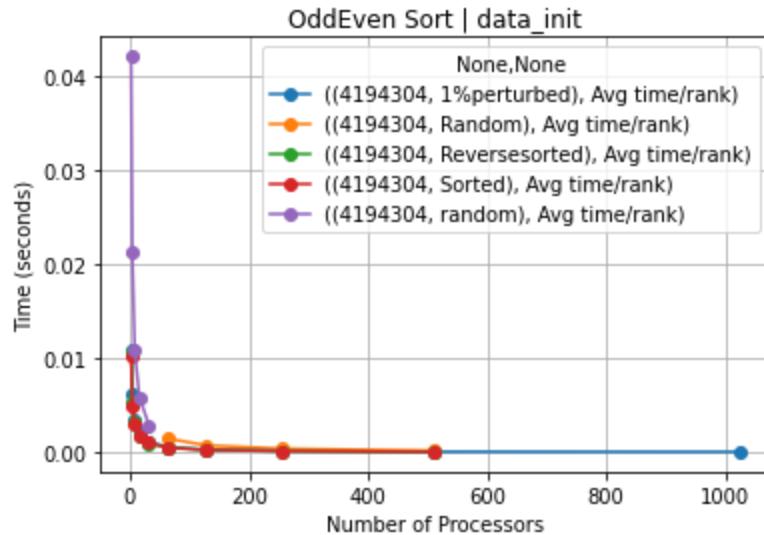




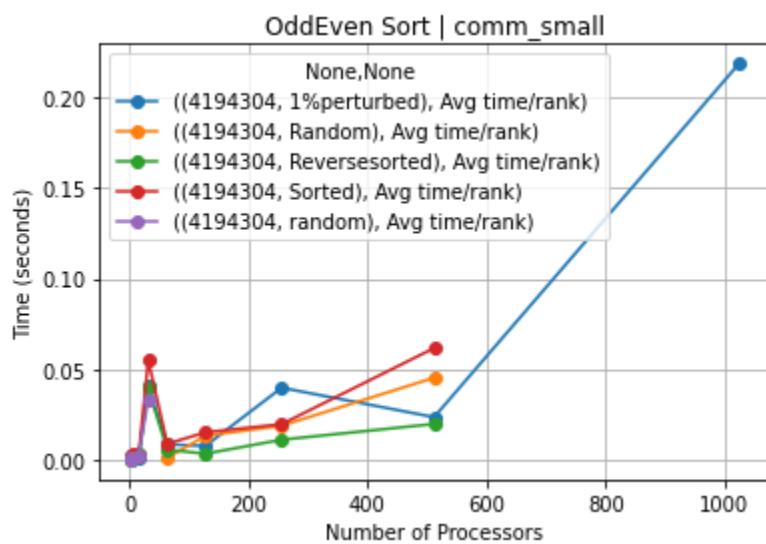
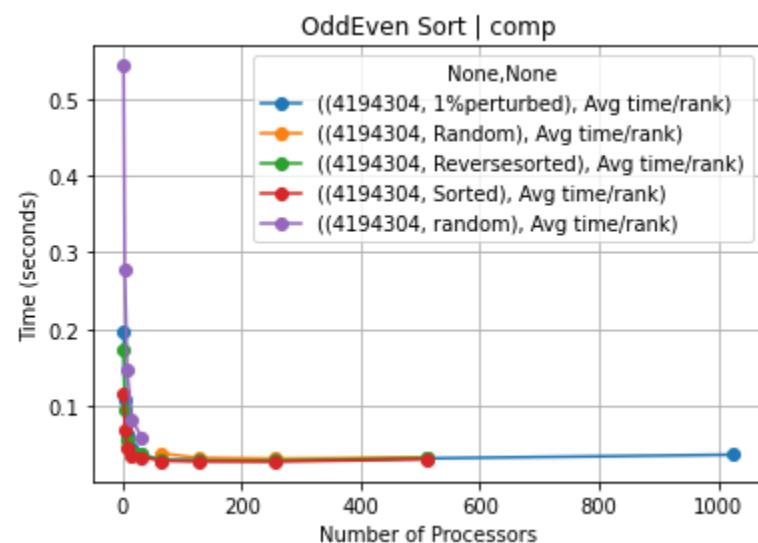
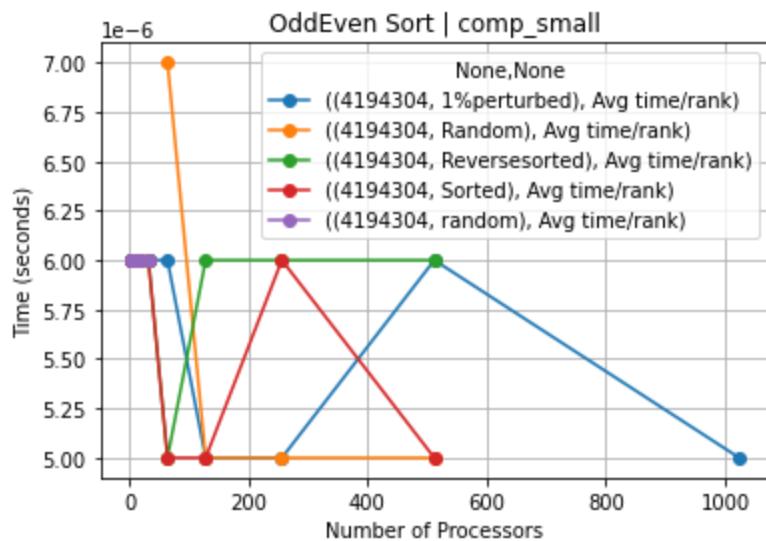


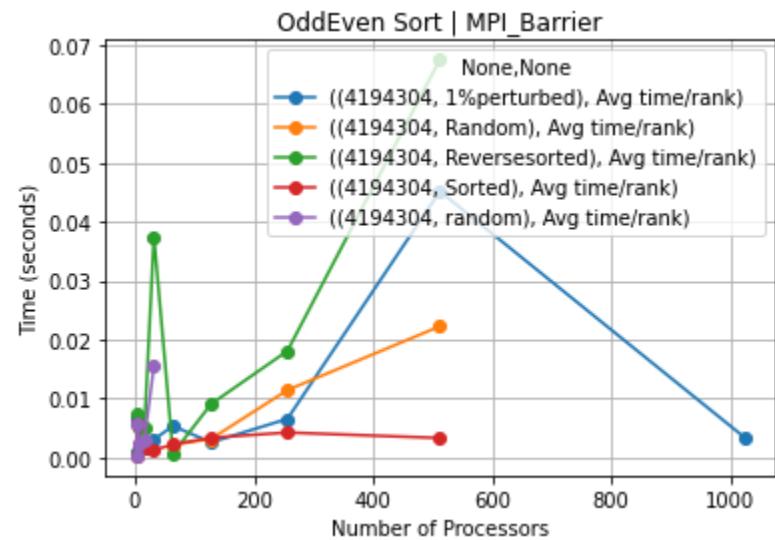
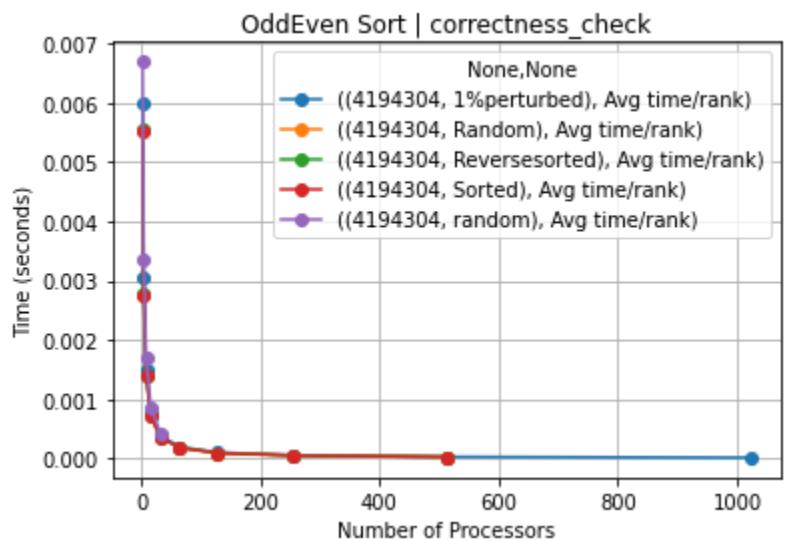
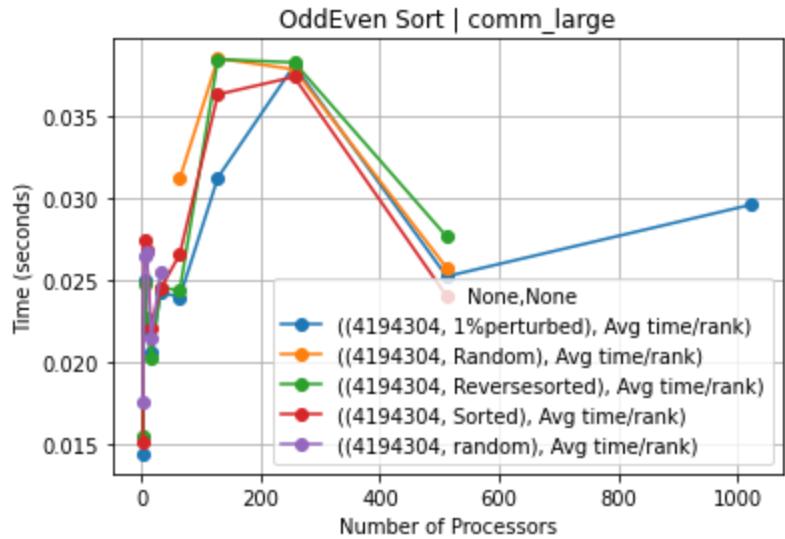


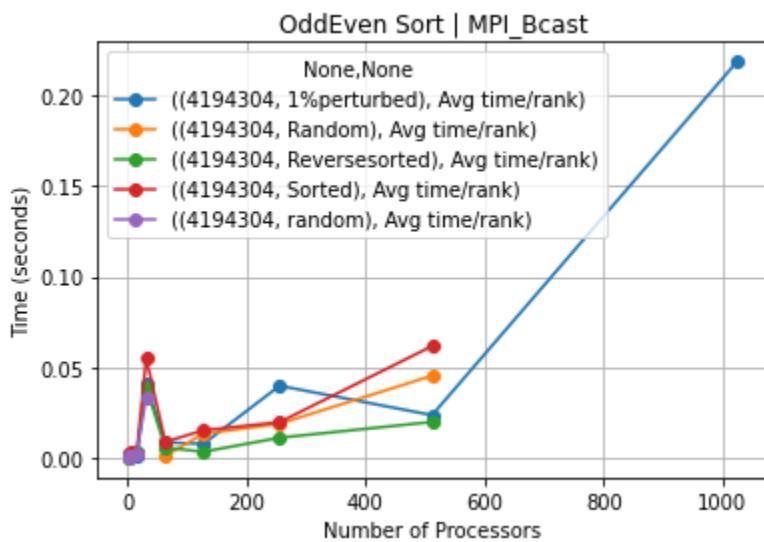
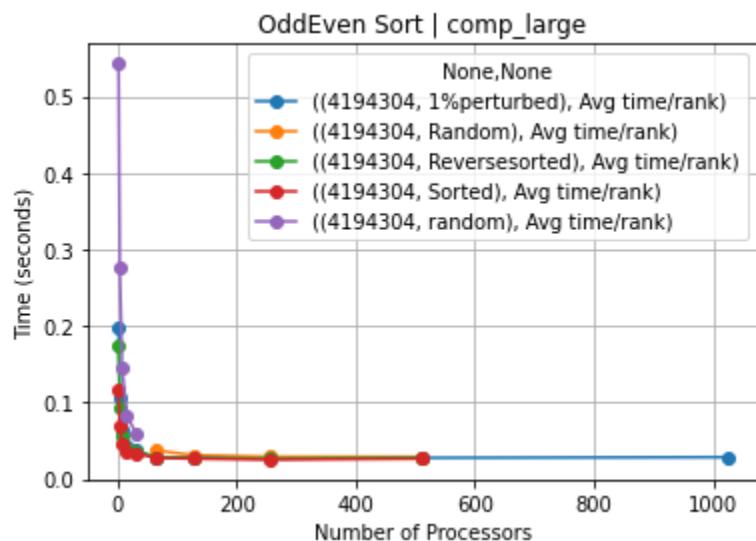
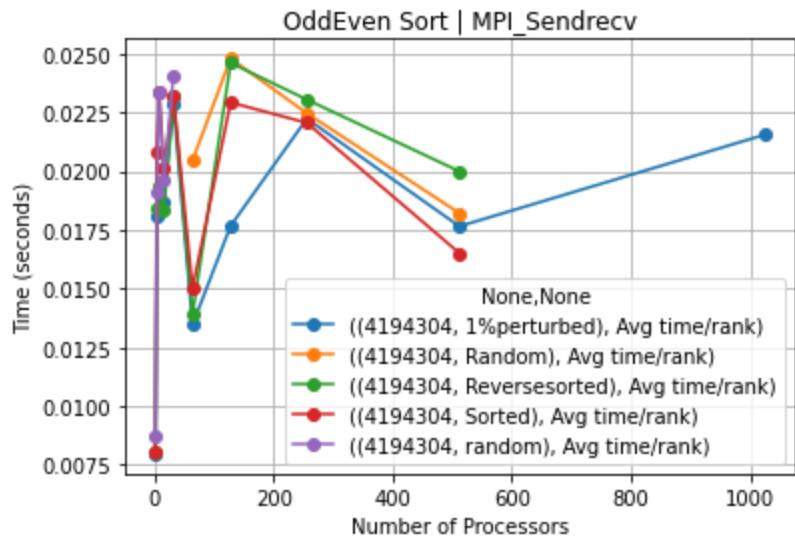


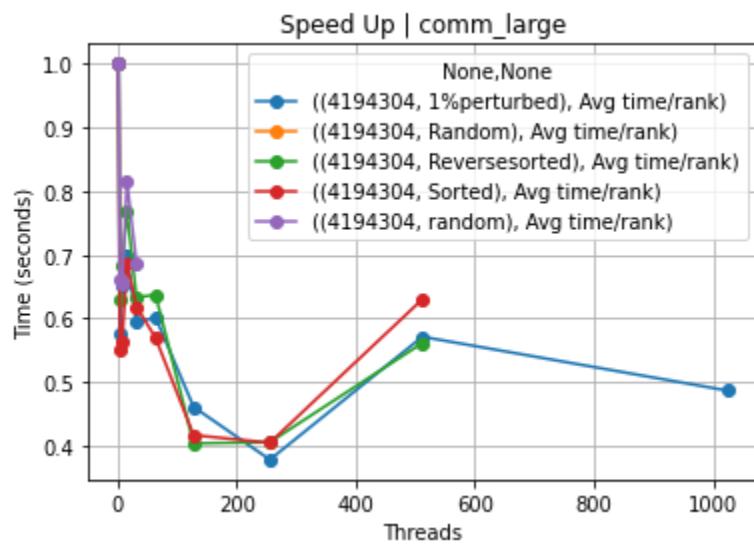
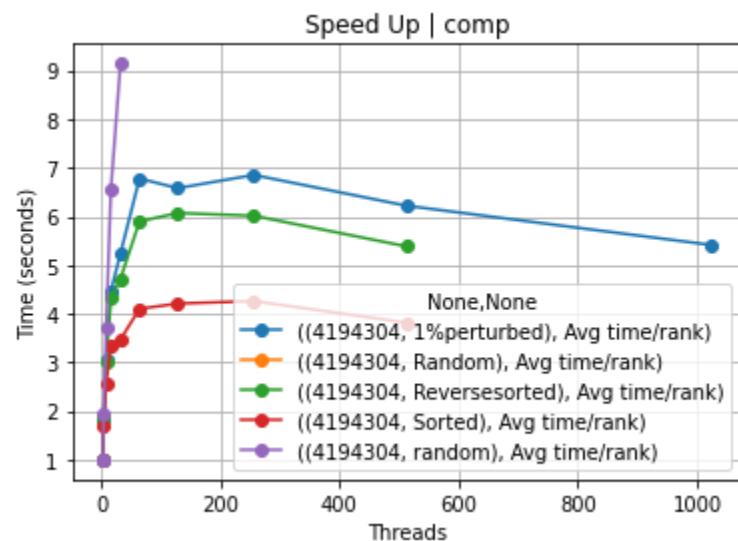
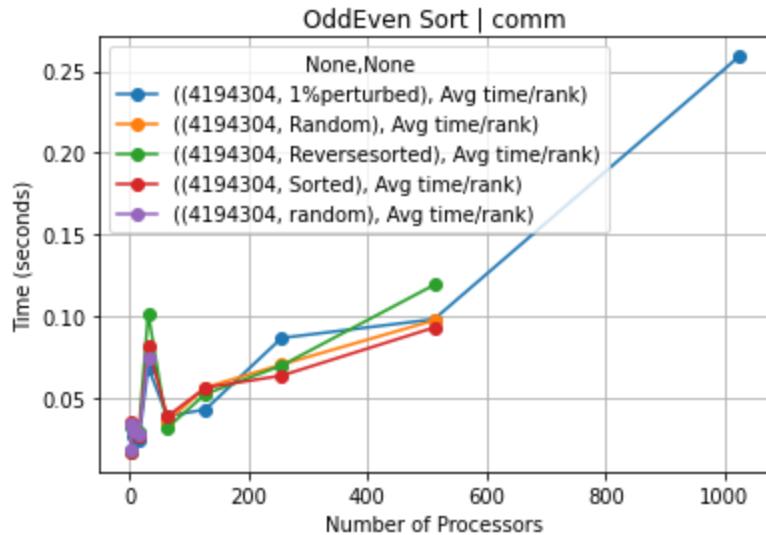


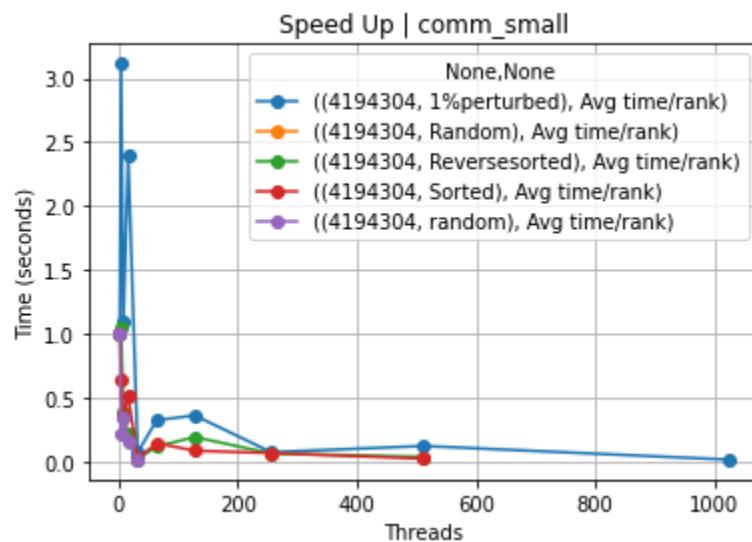
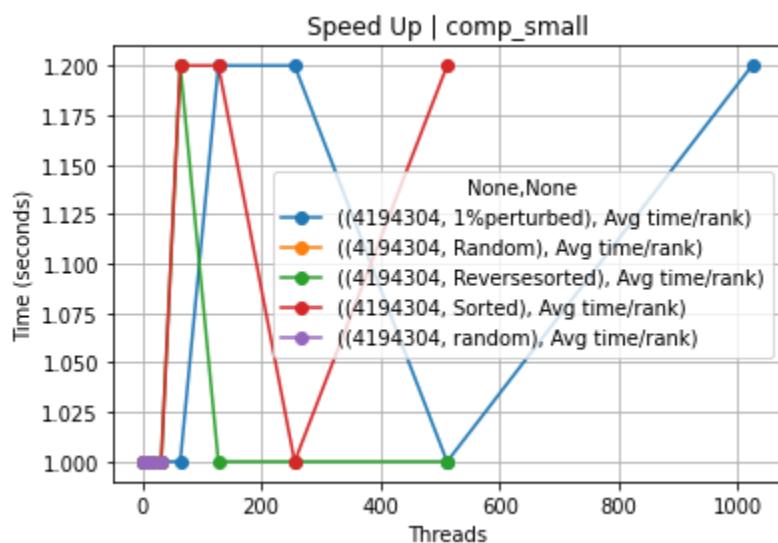
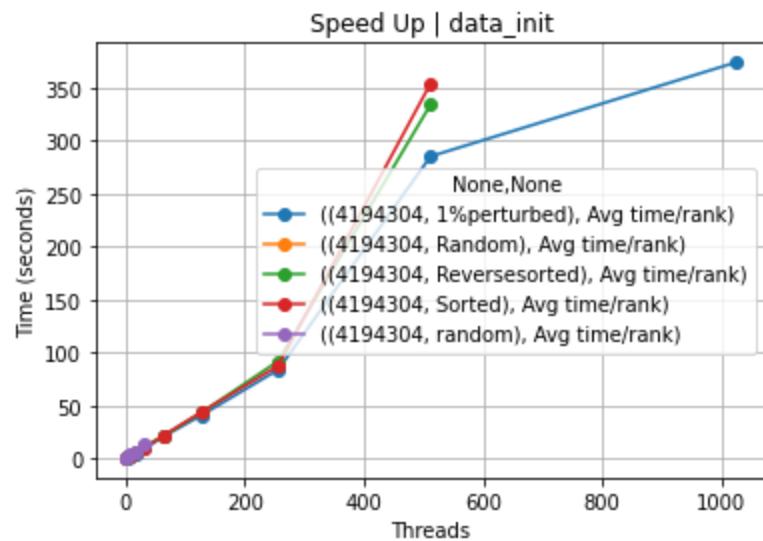
**Strong Scaling 4194304**

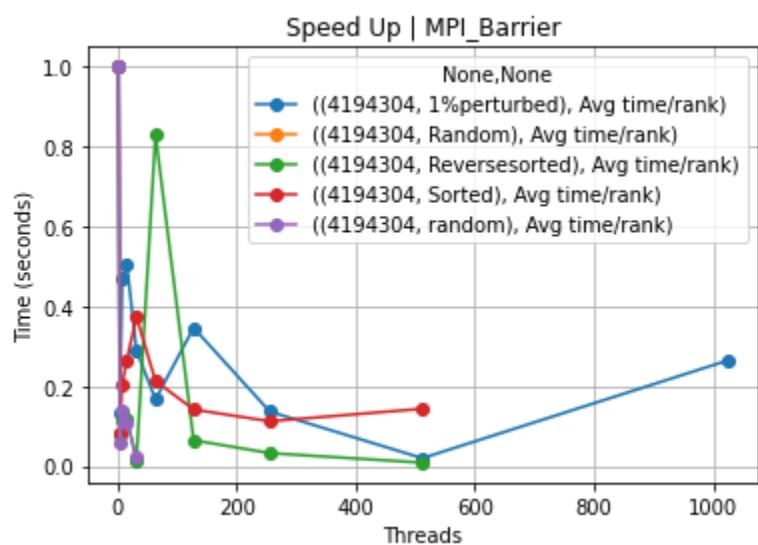
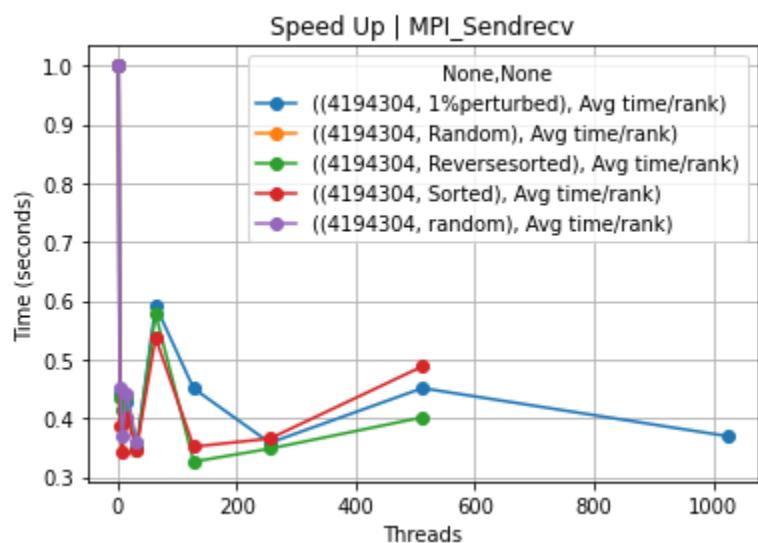
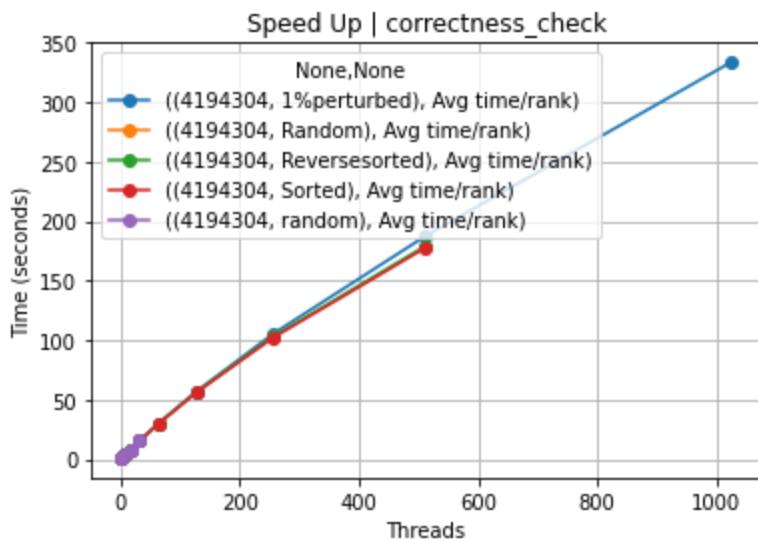


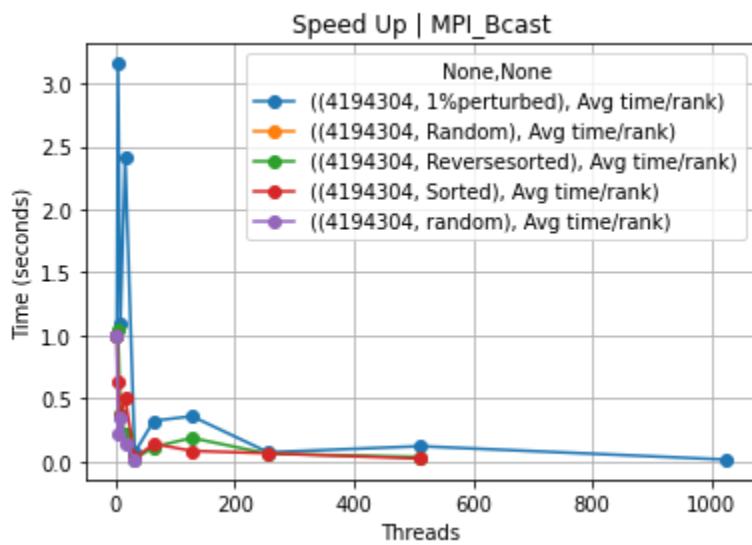
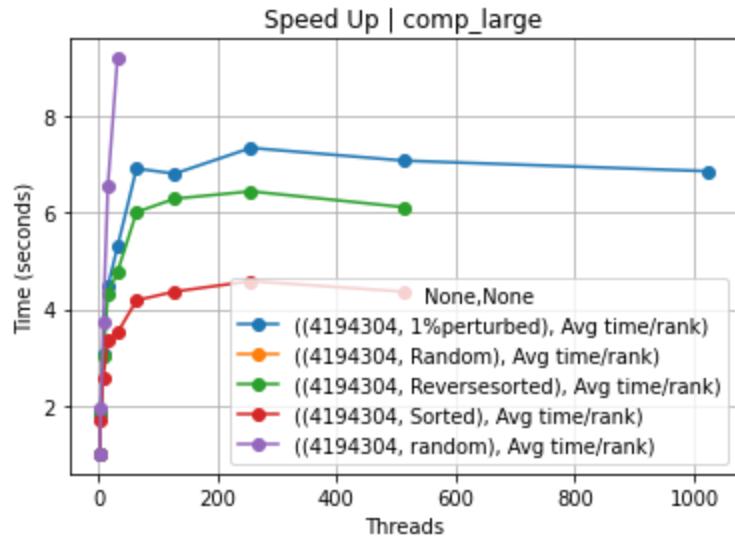


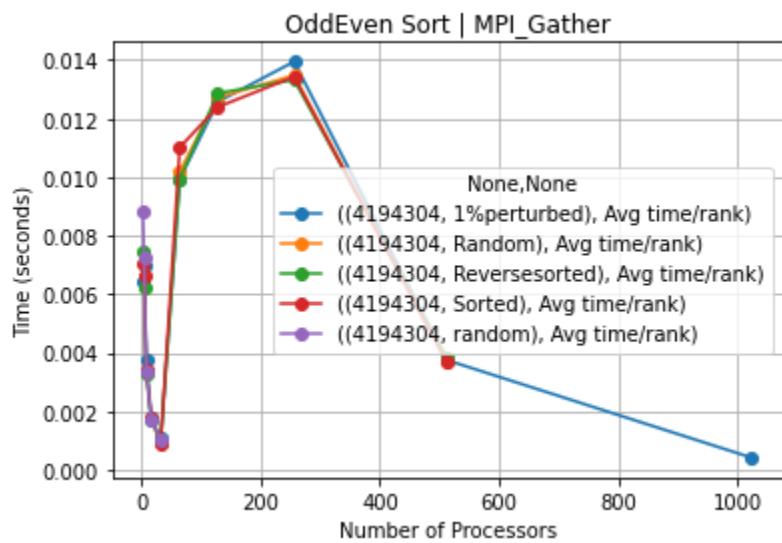
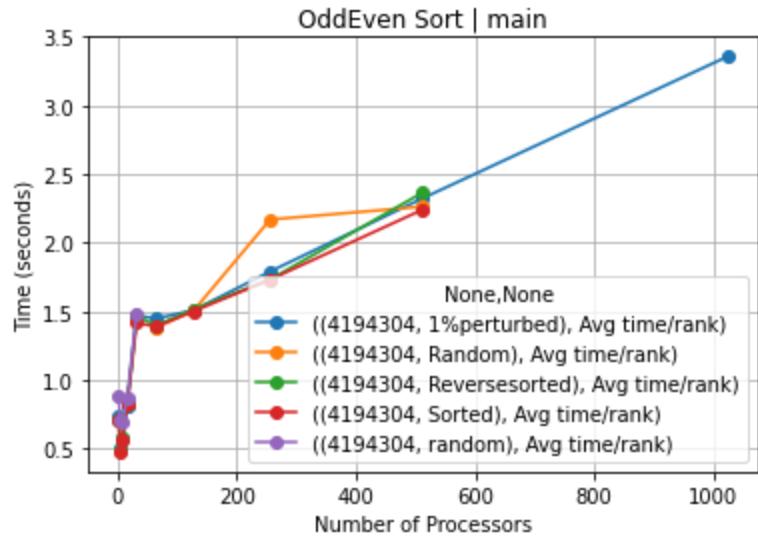


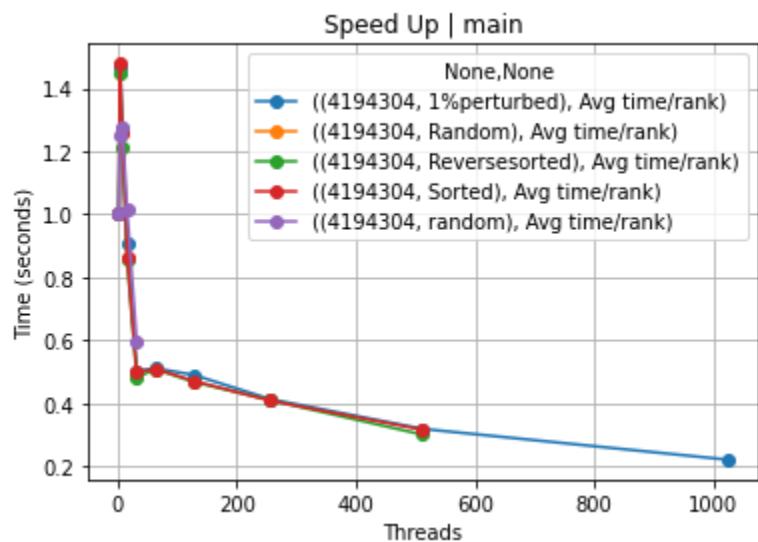
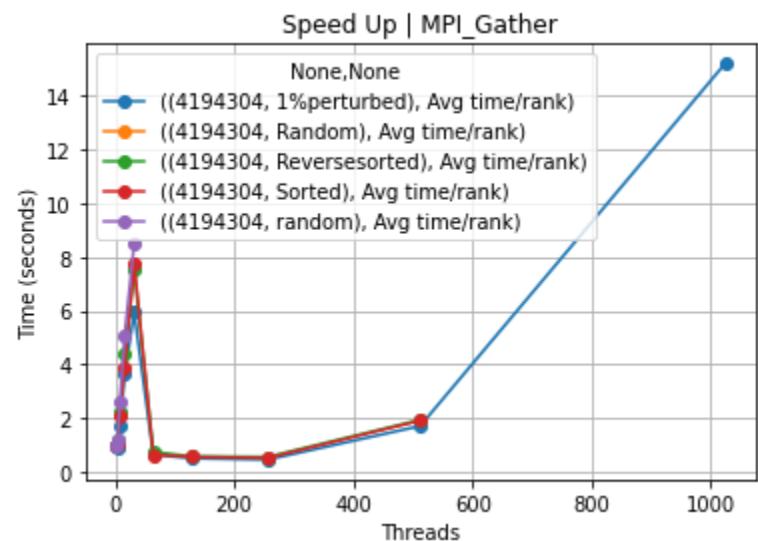
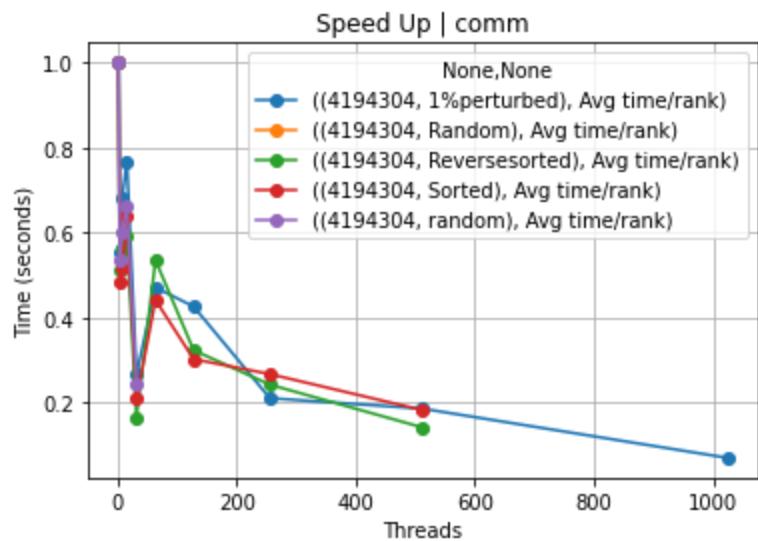




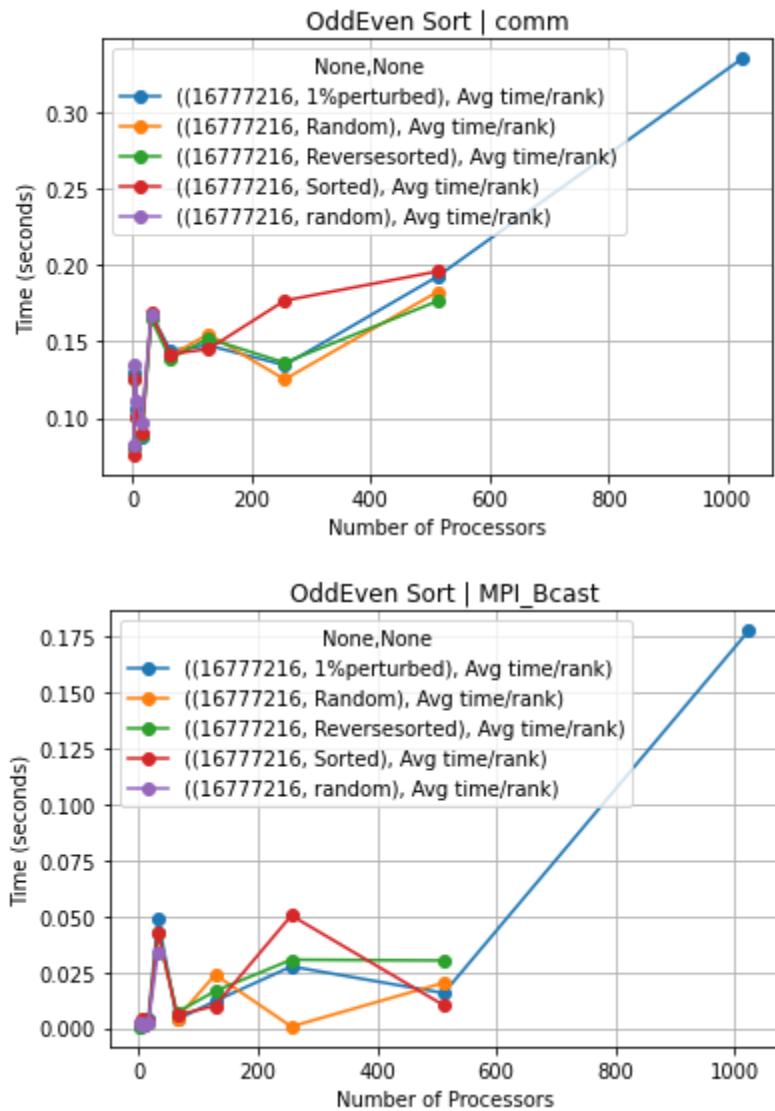


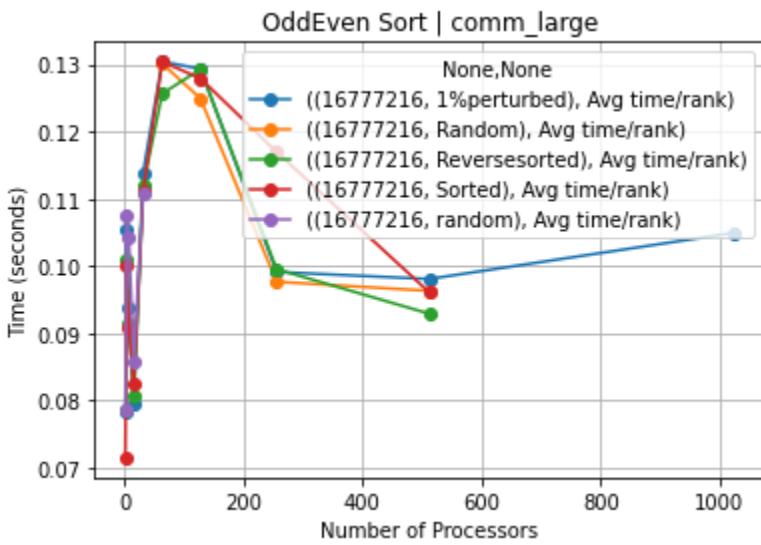
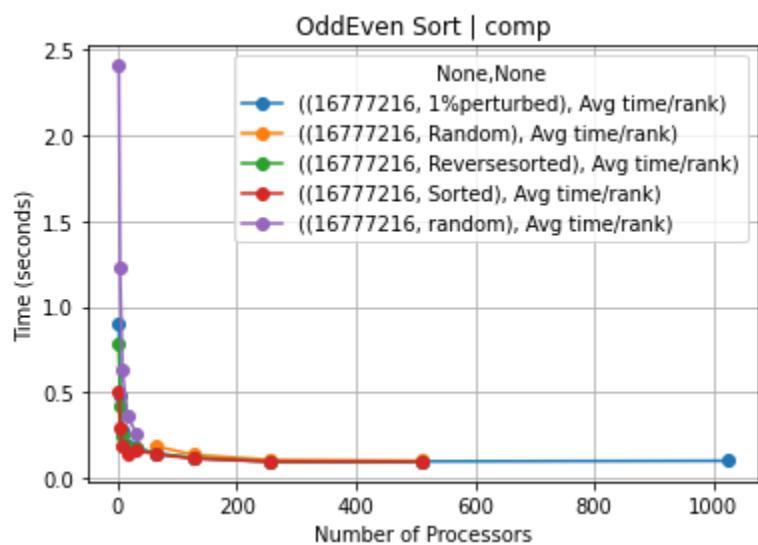
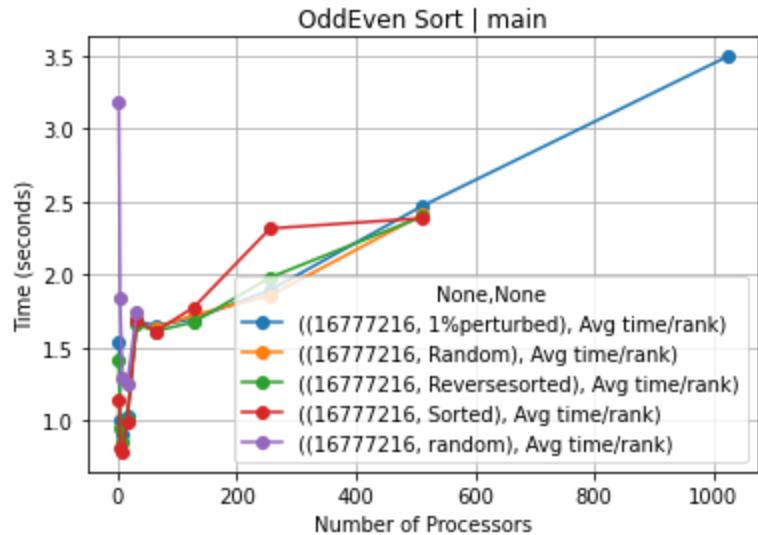


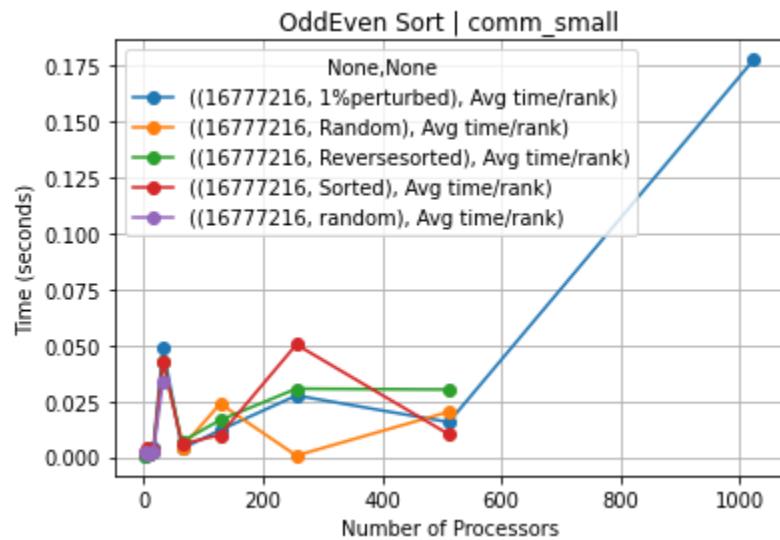
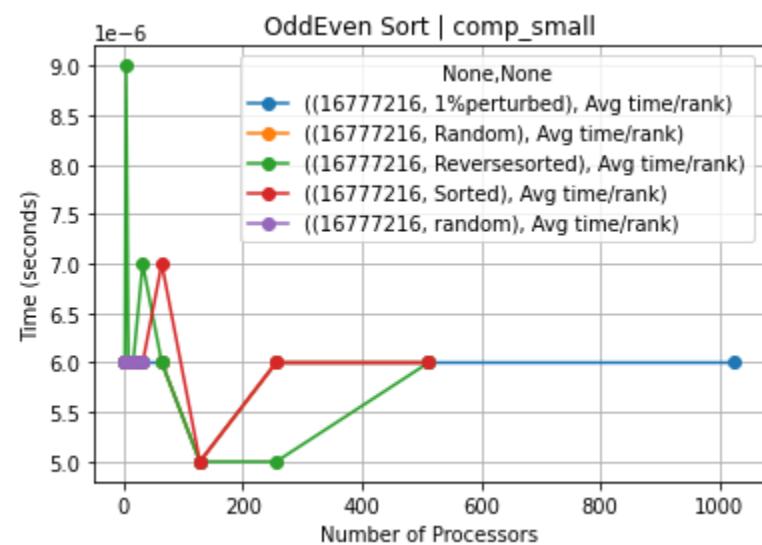
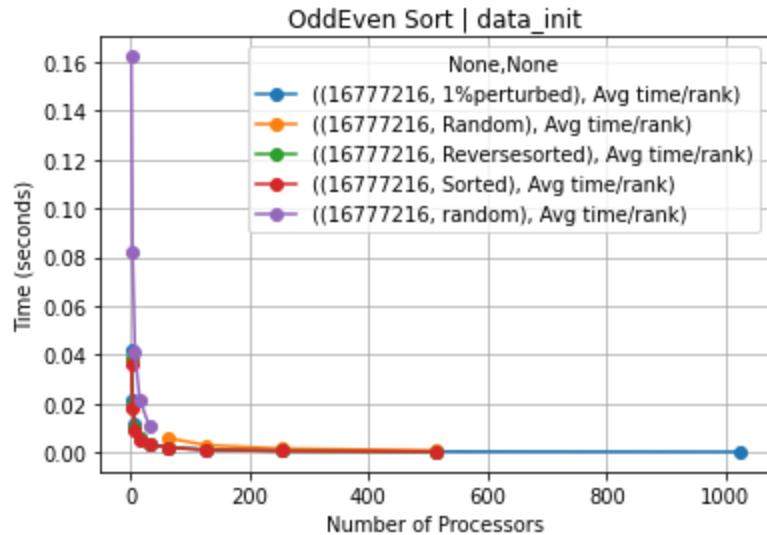


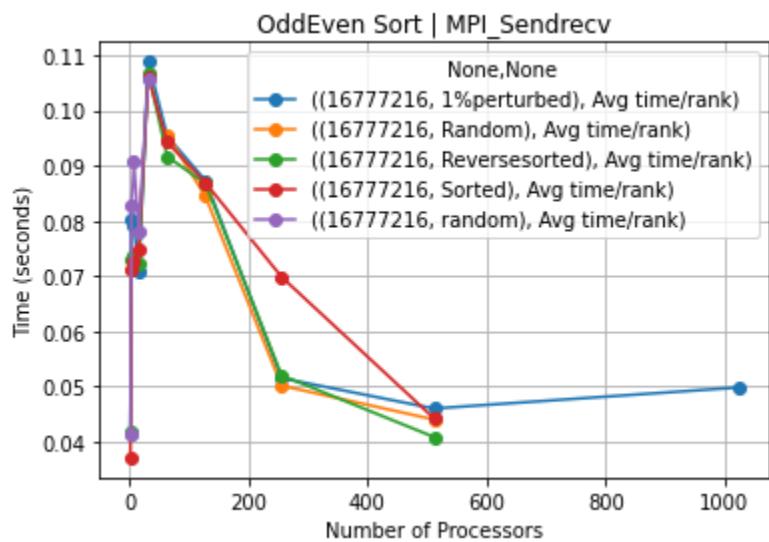
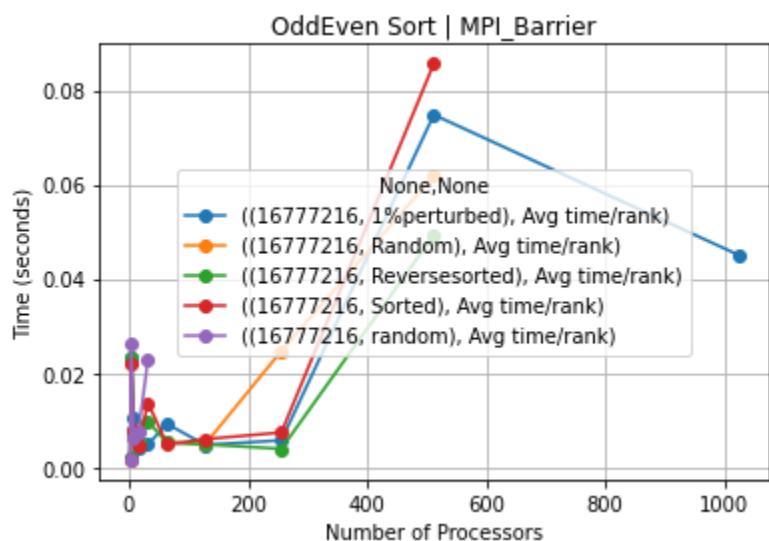
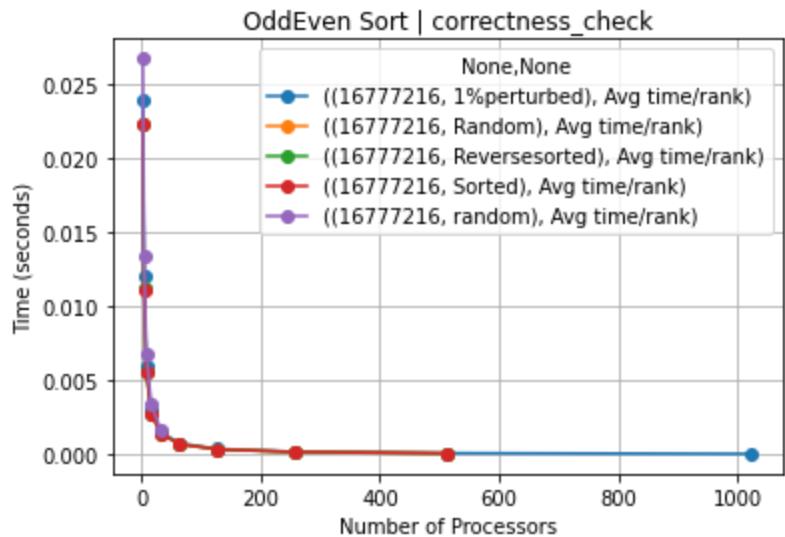


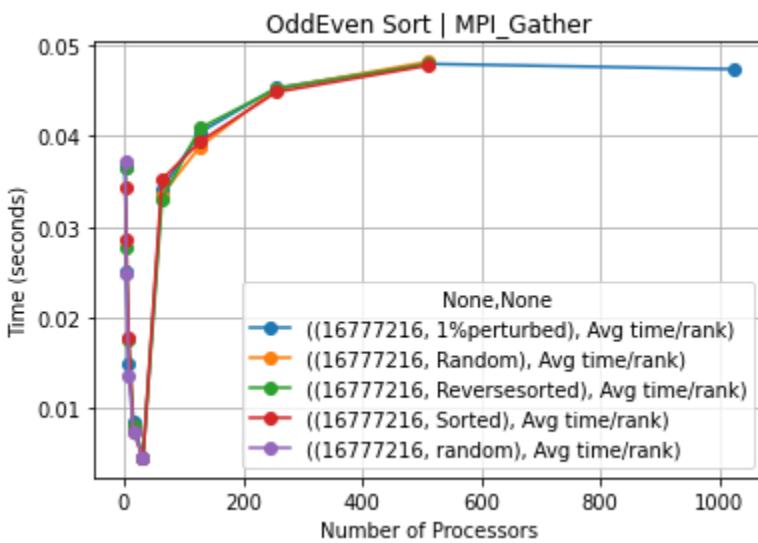
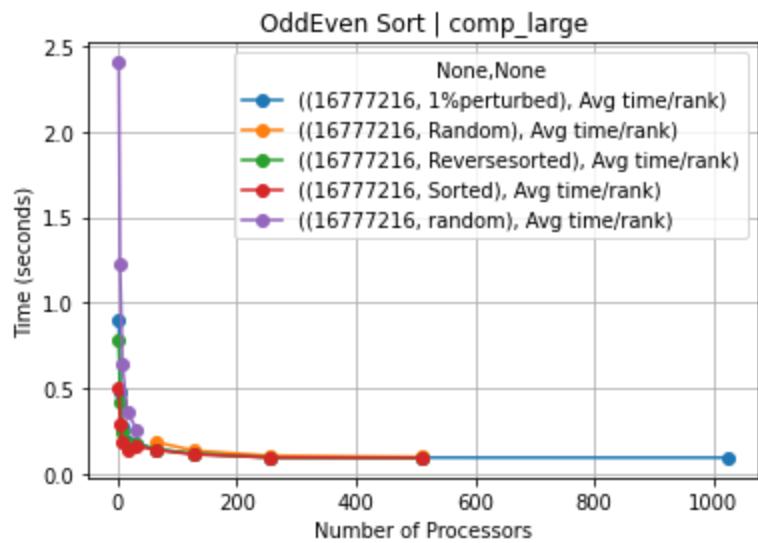
### Strong Scaling 16777216

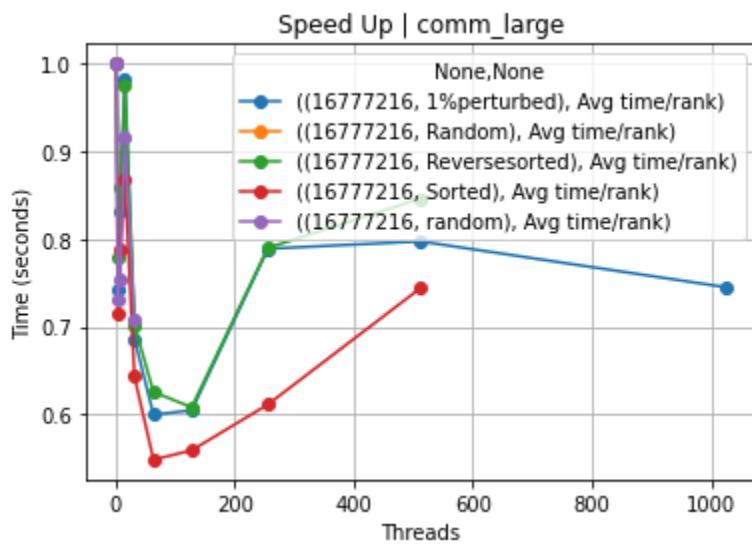
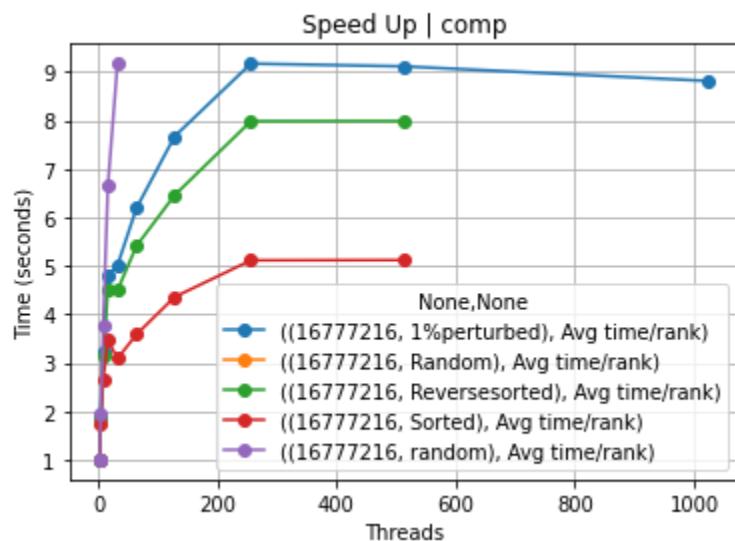
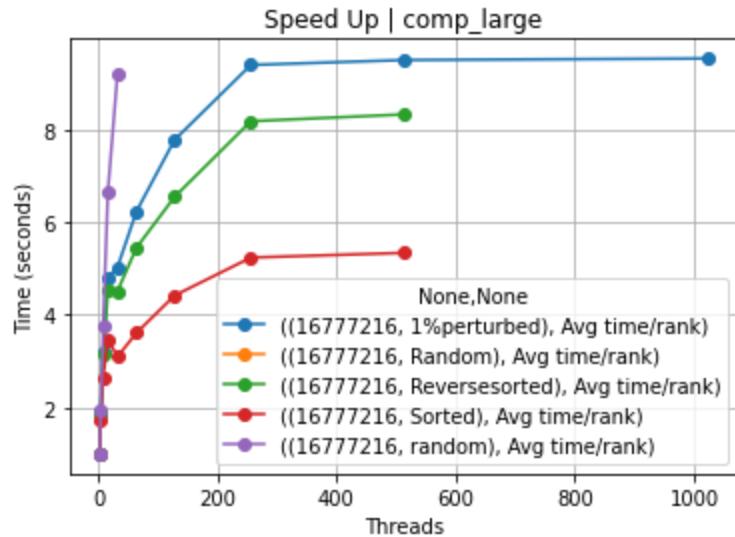


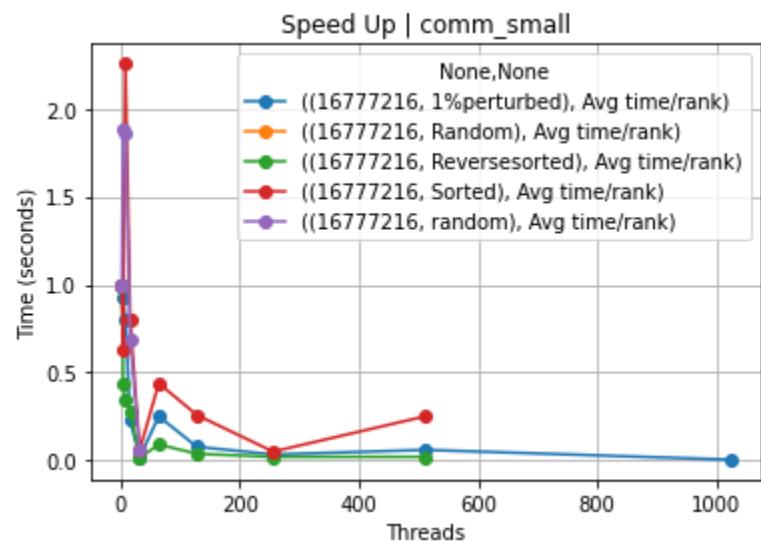
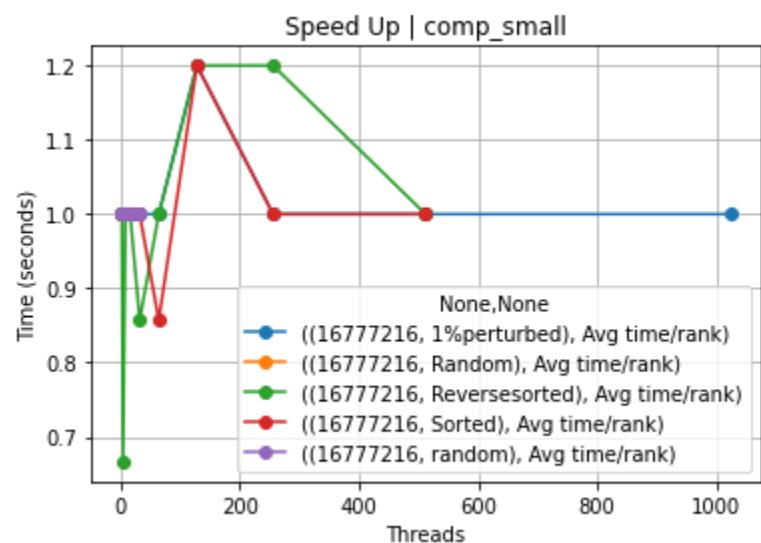
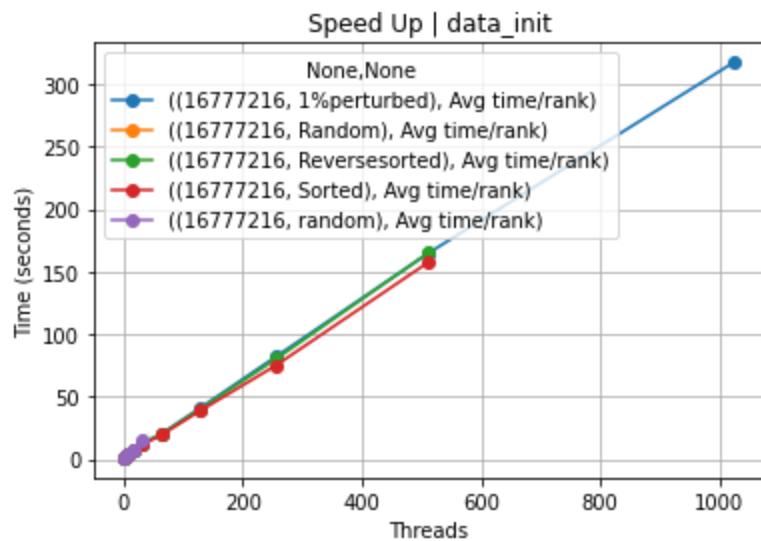


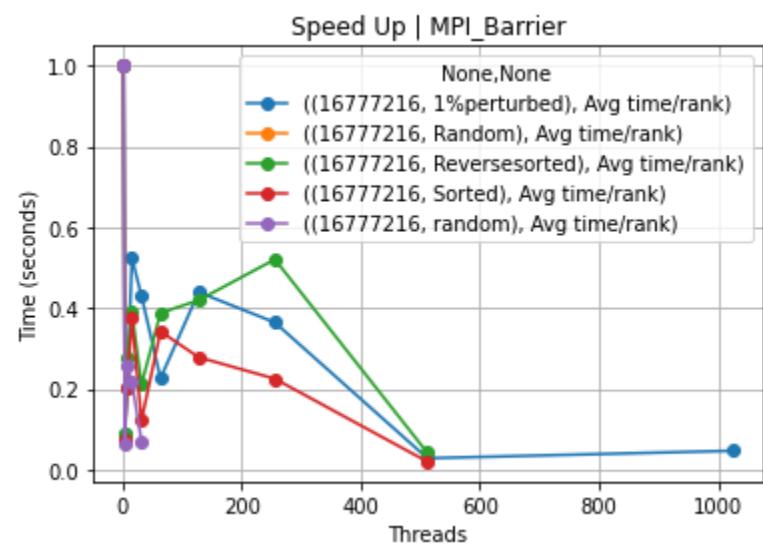
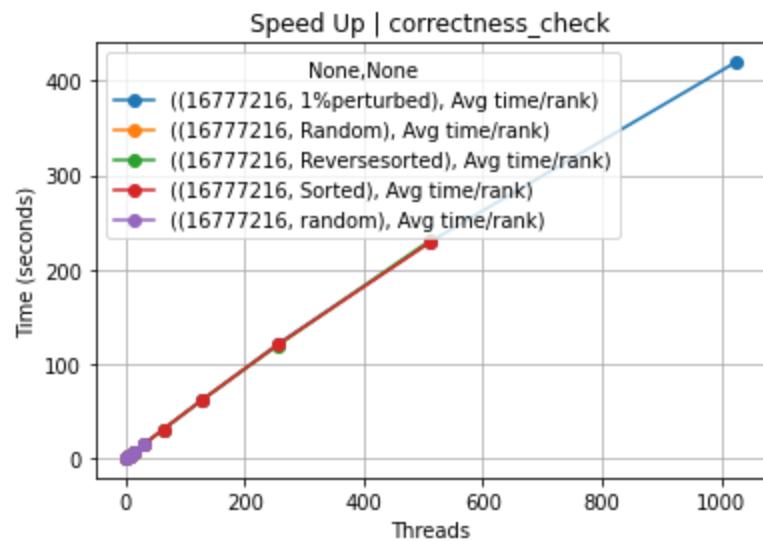


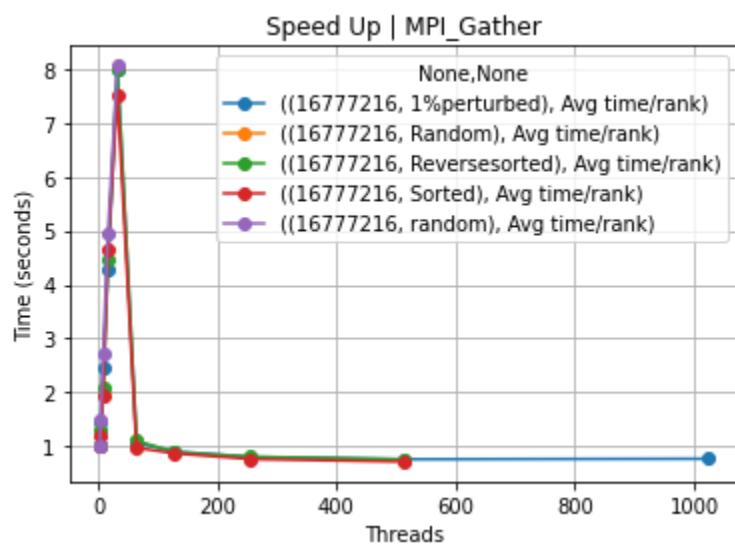
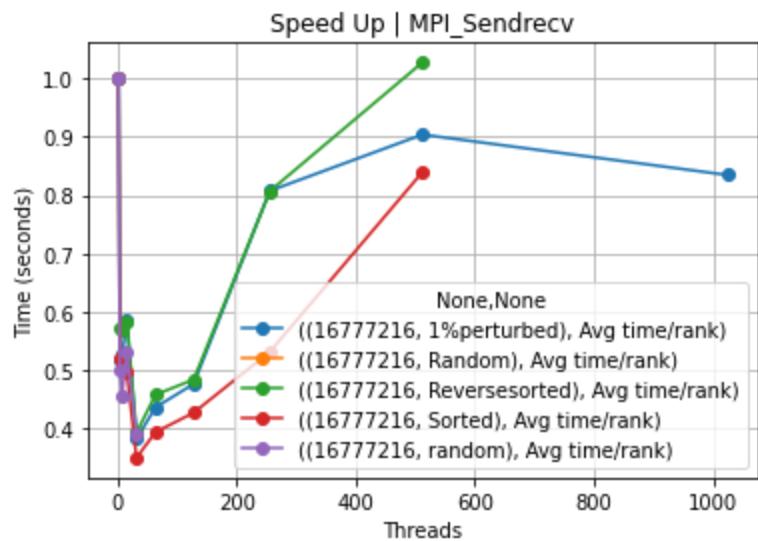


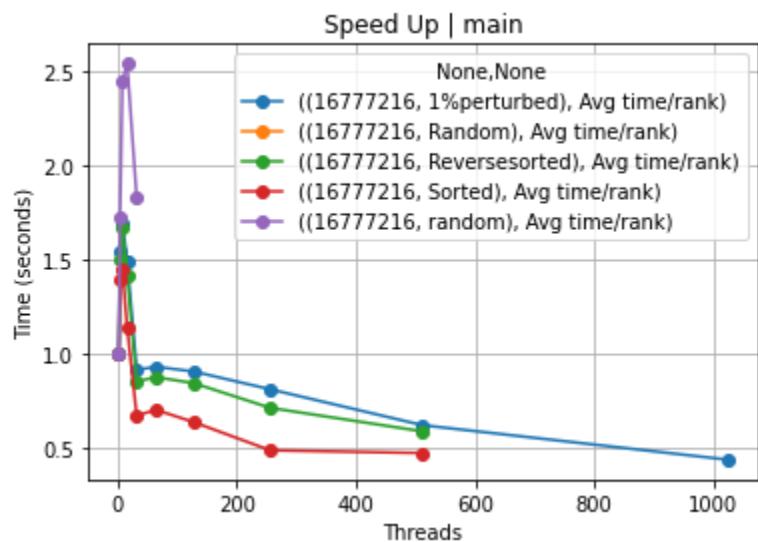
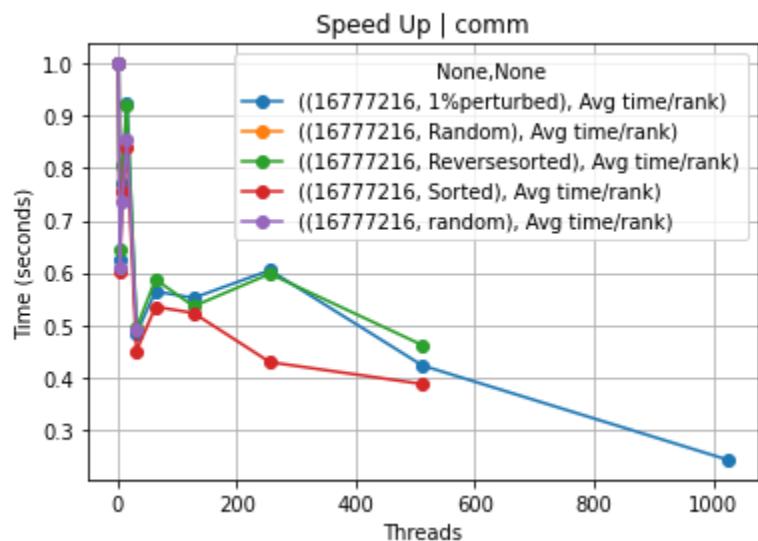
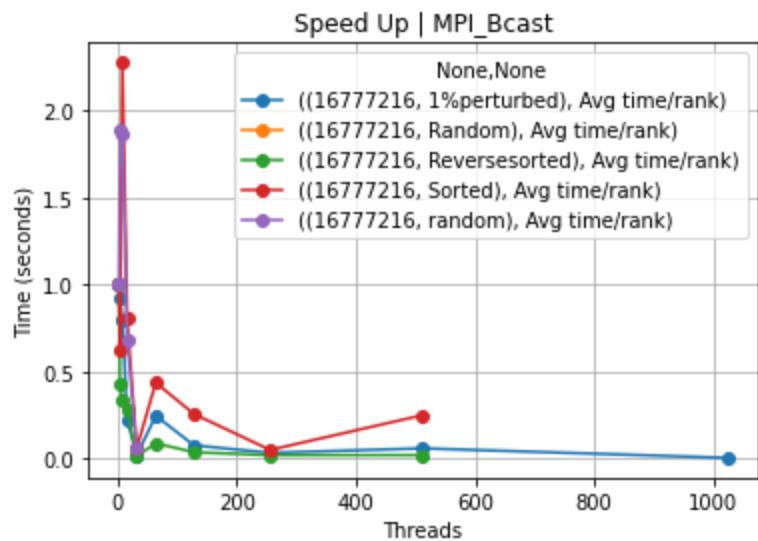




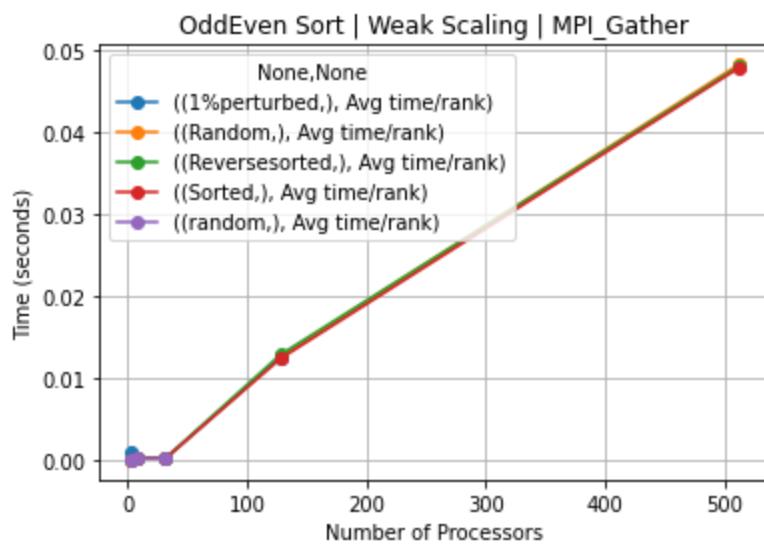
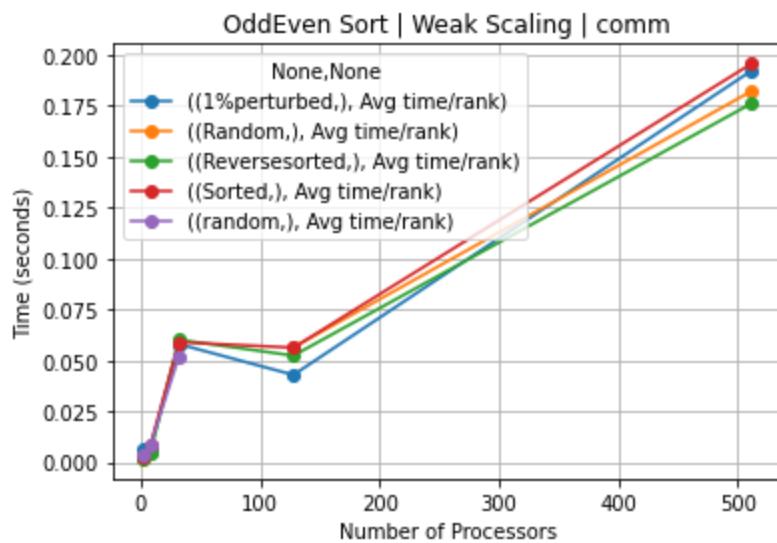


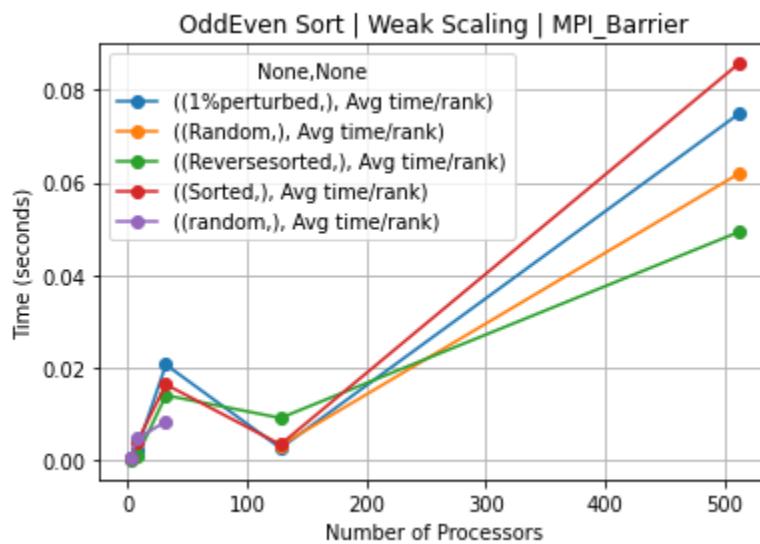
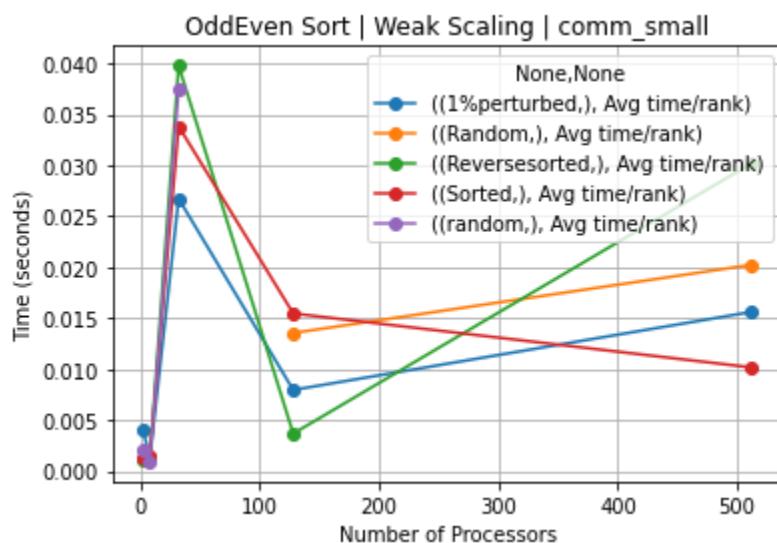
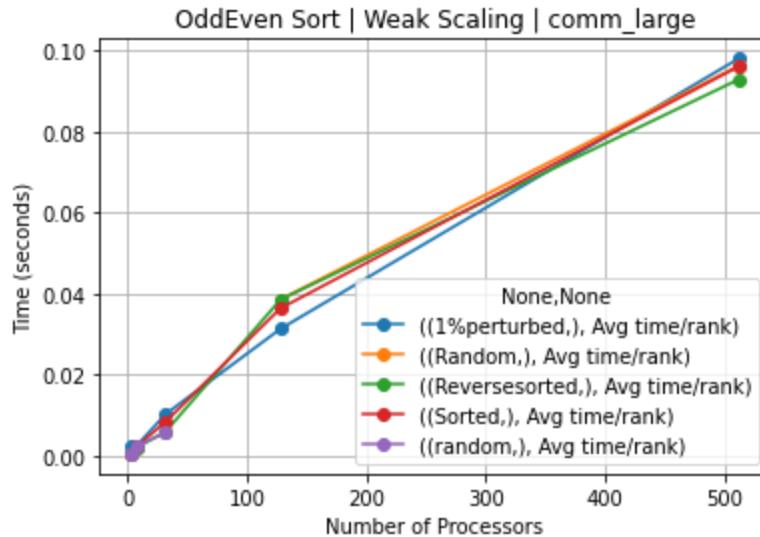


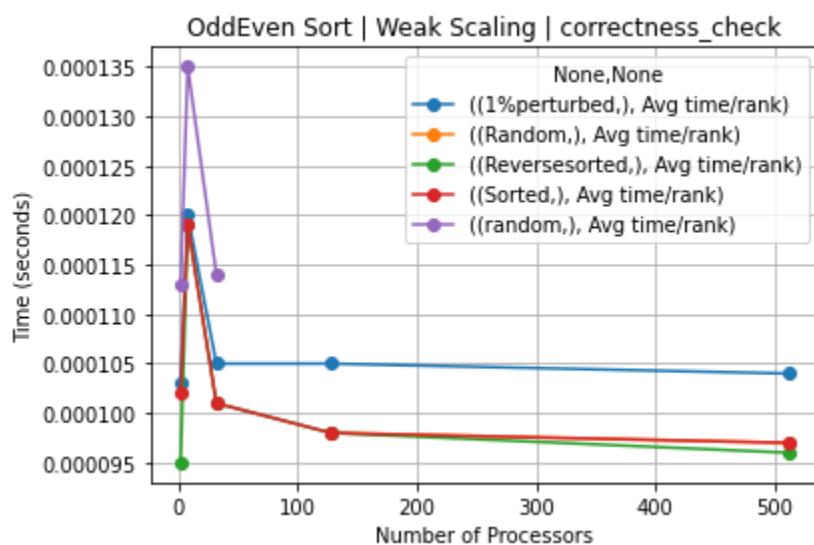
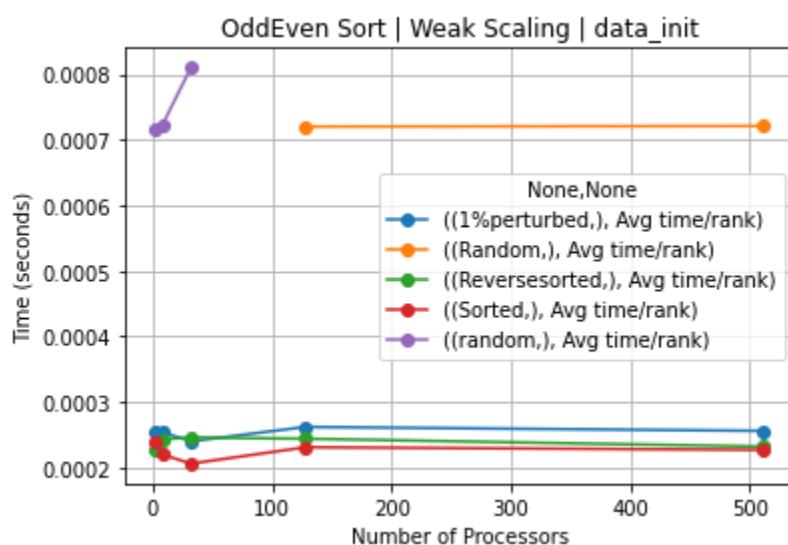
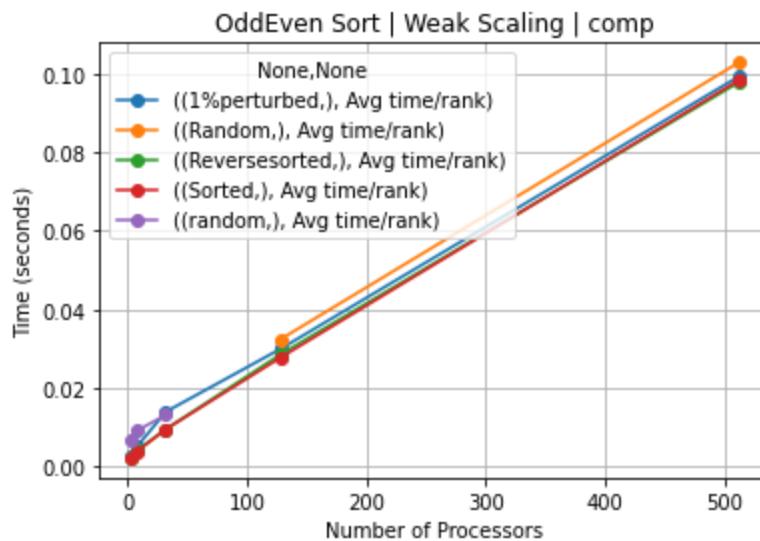


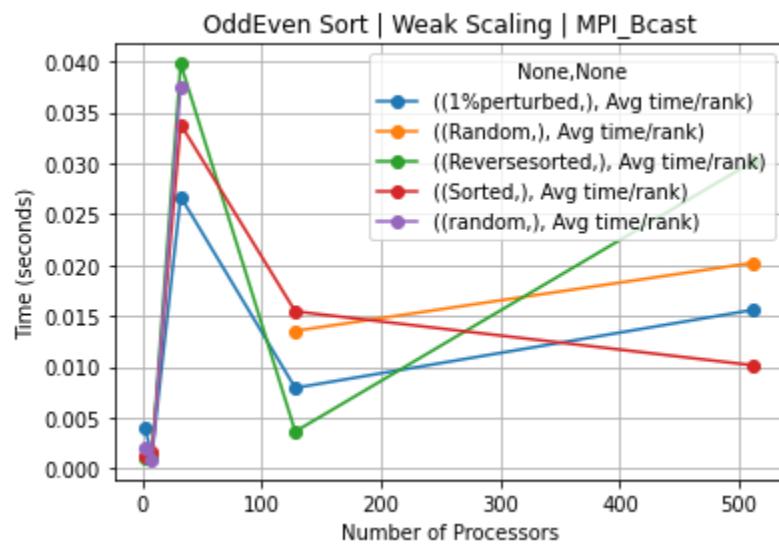
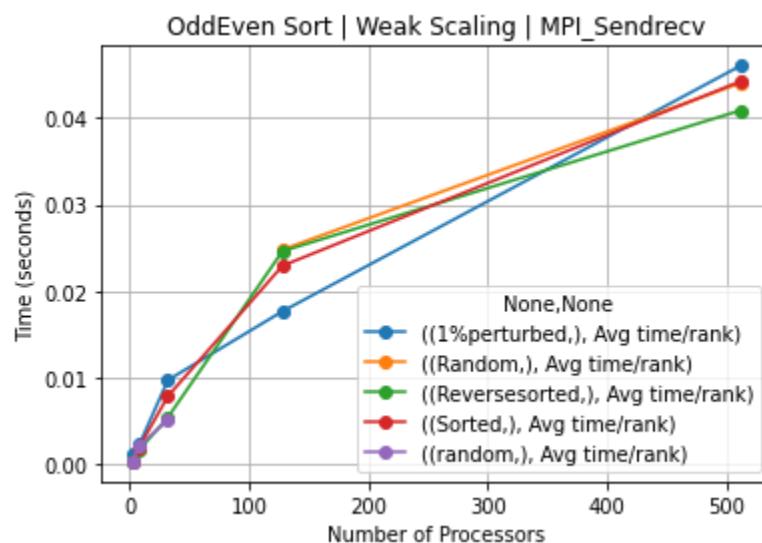
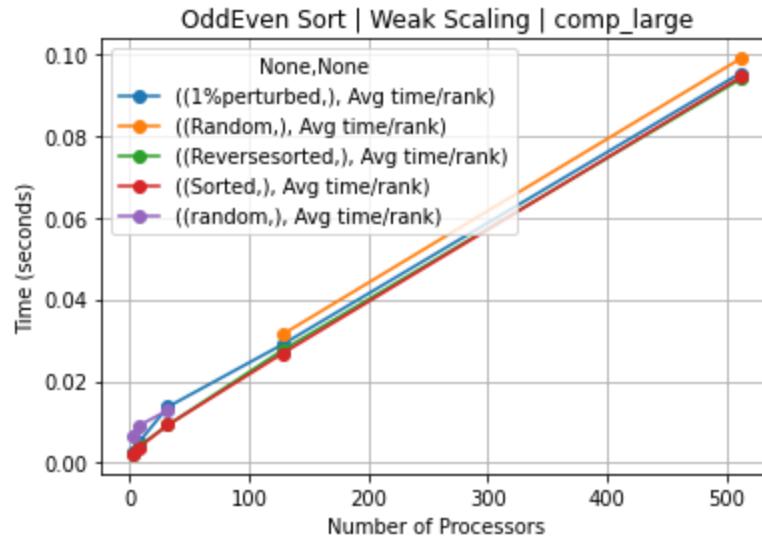


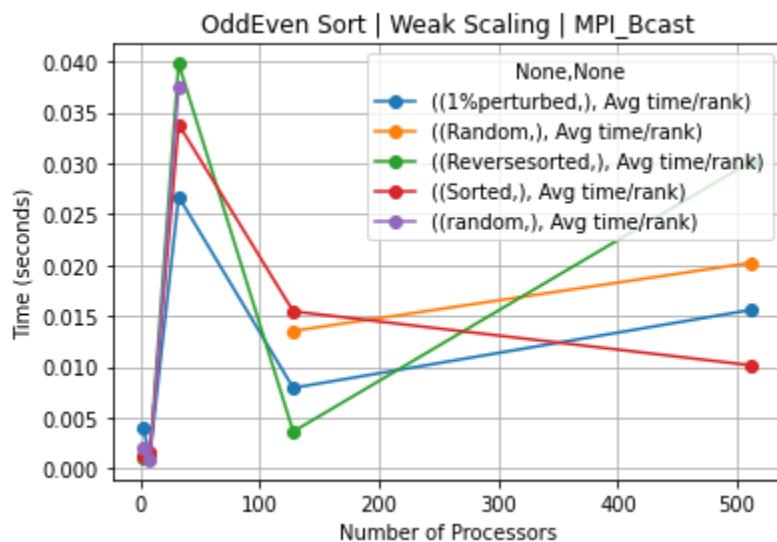
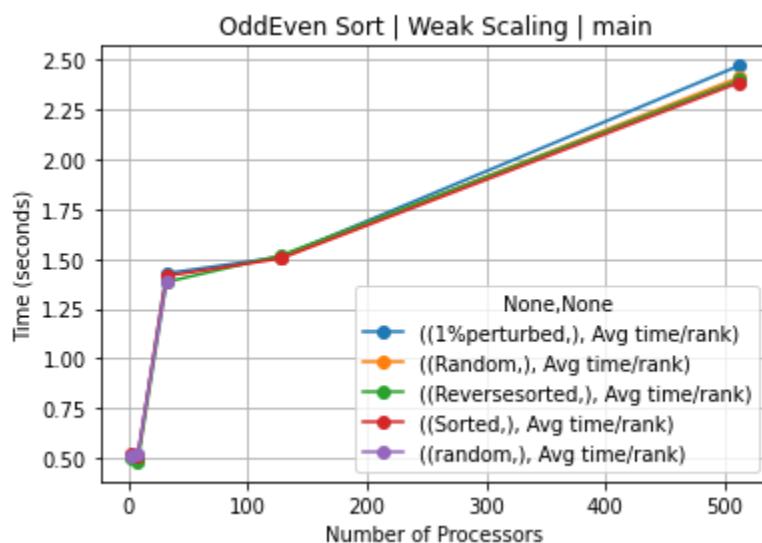
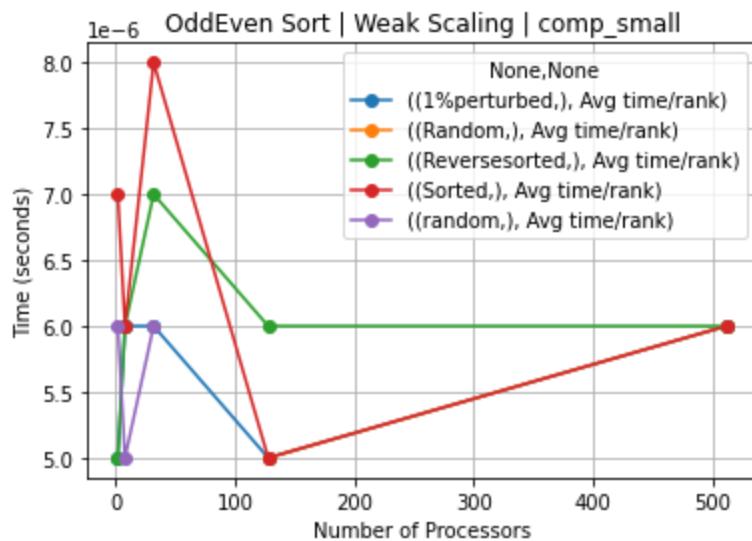
## **Weak Scaling**





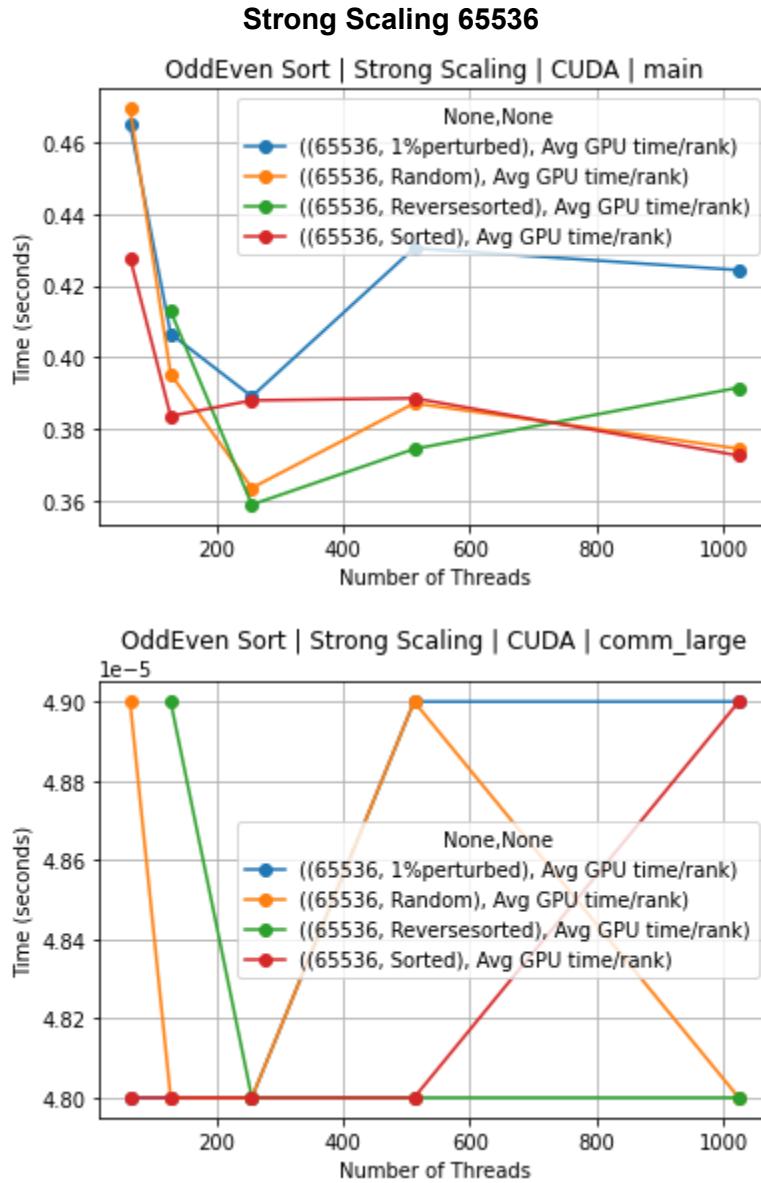


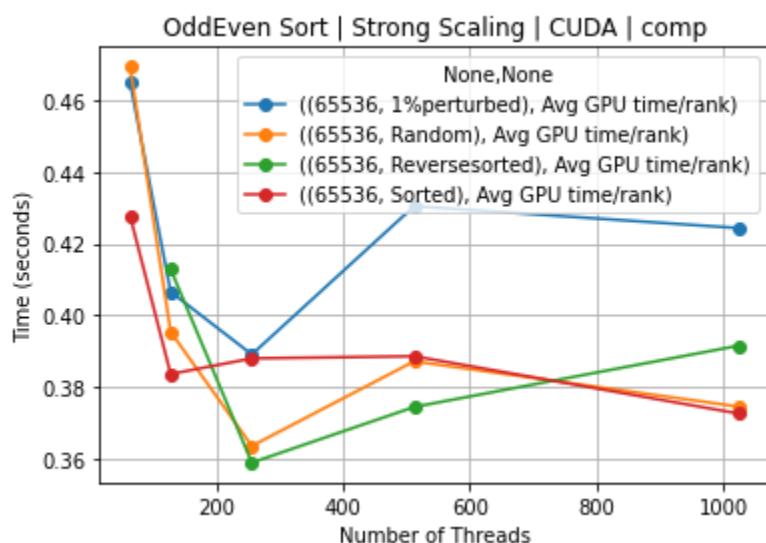
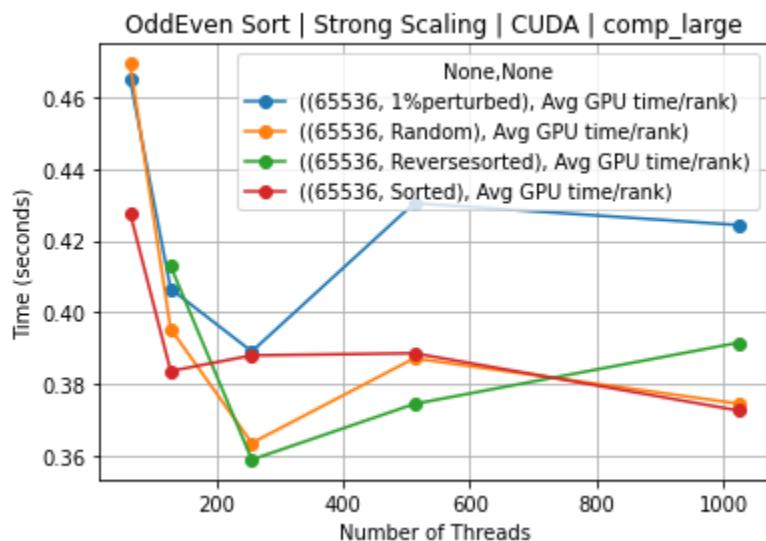
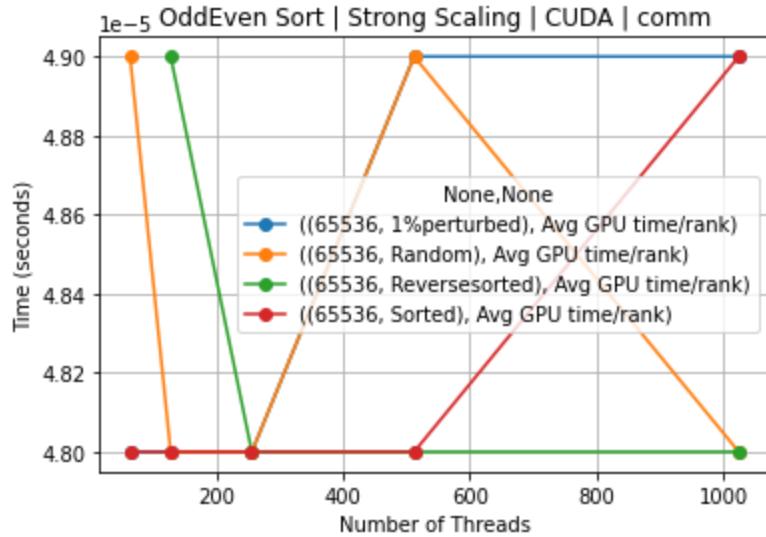


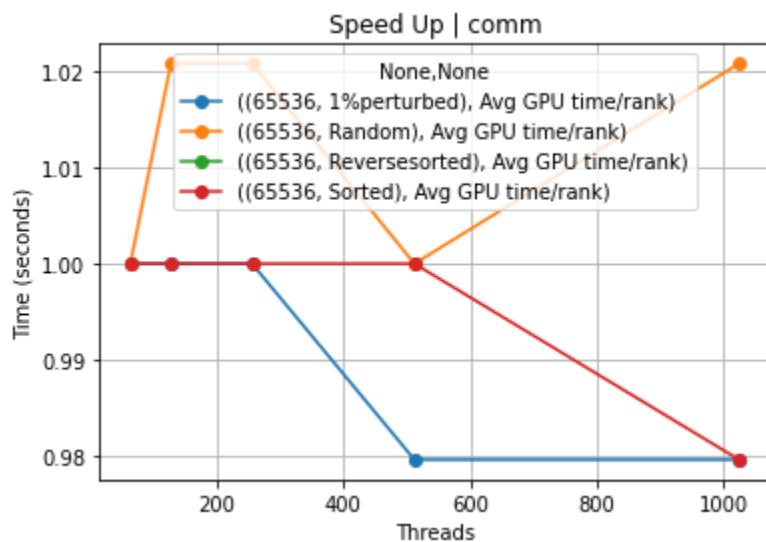
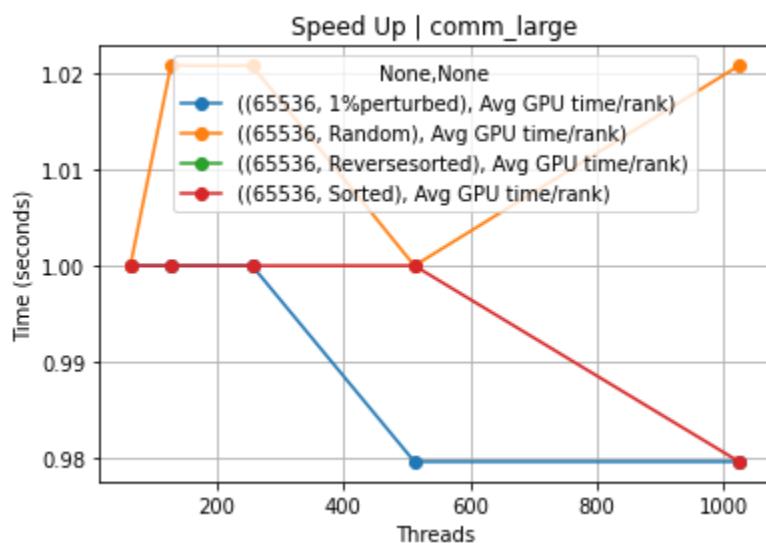
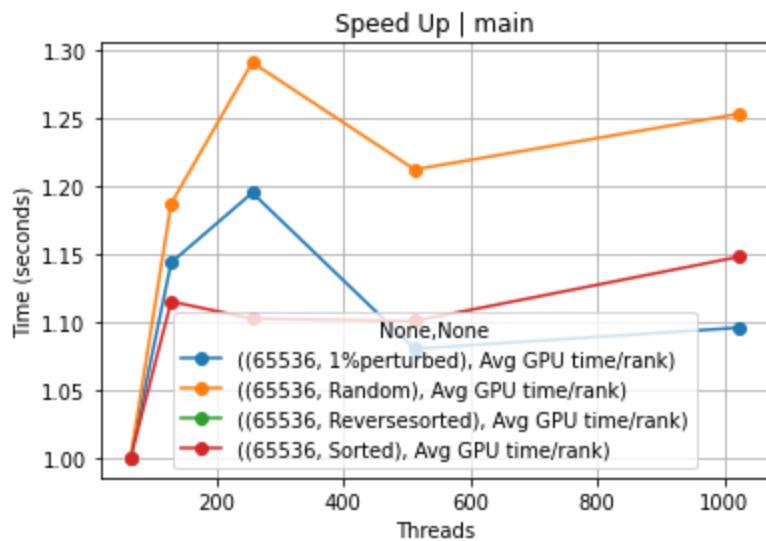


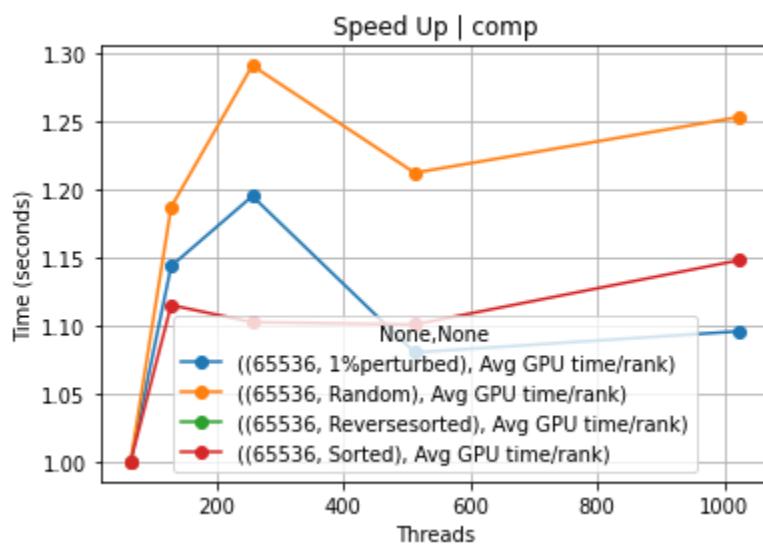
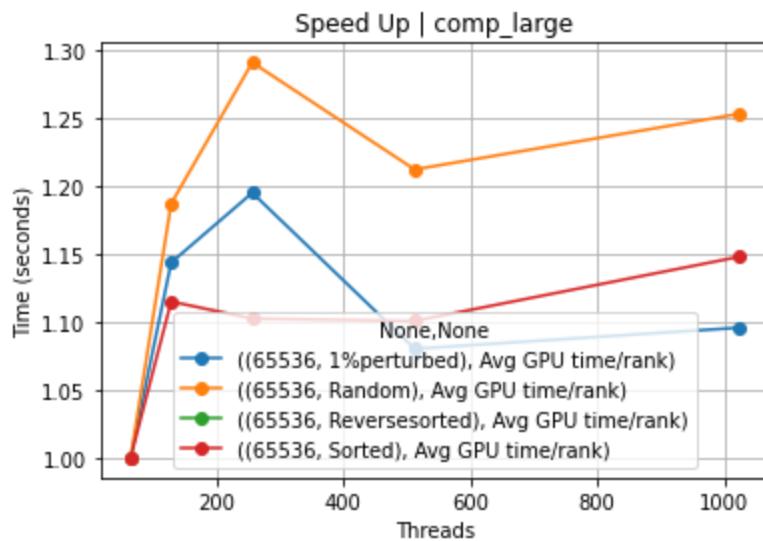
For weak scaling, we see a few trends. For comp\_large, comm and main, we see a linear increase with nearly perfect straight lines. This scaling is unideal, and is most likely due to the inefficient nature of the algorithm. In fact, my implementation can have at most half of all data being transferred between processes in any given iteration of the algorithm. This shoots up the communication time, increasing the overhead of the algorithm.

OddEven Sort CUDA:

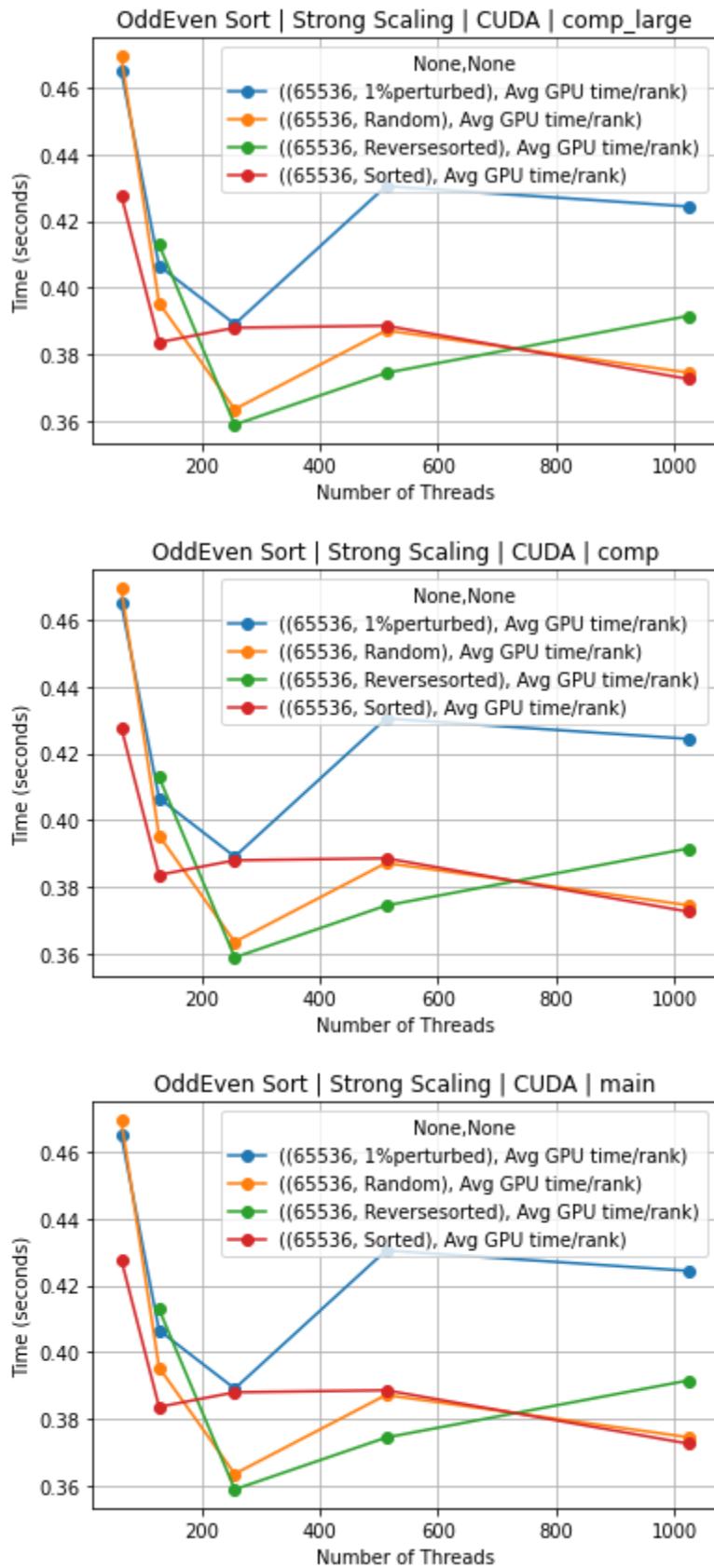


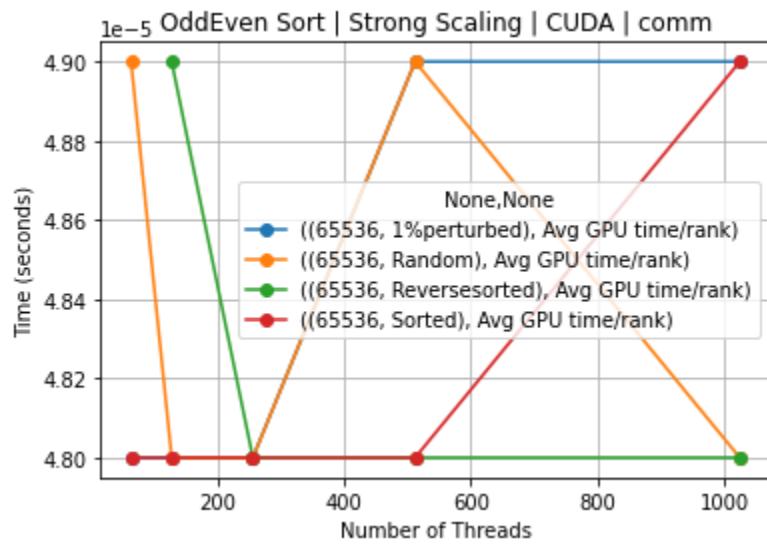
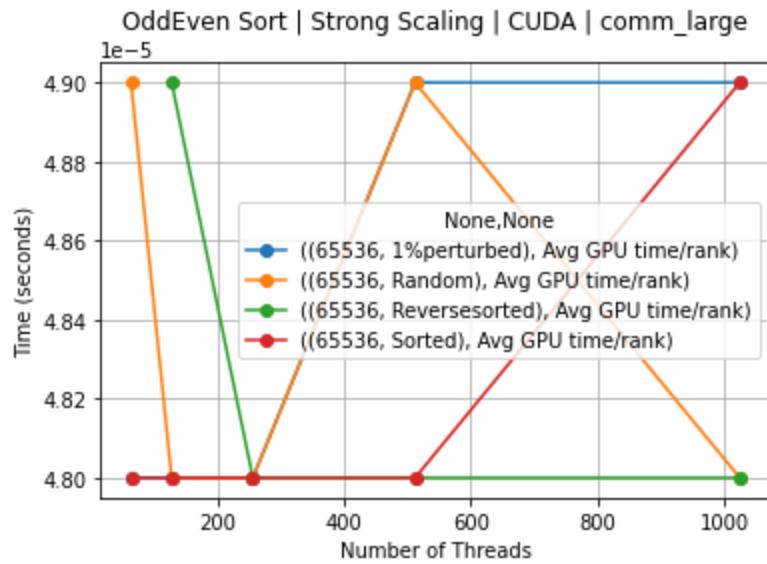




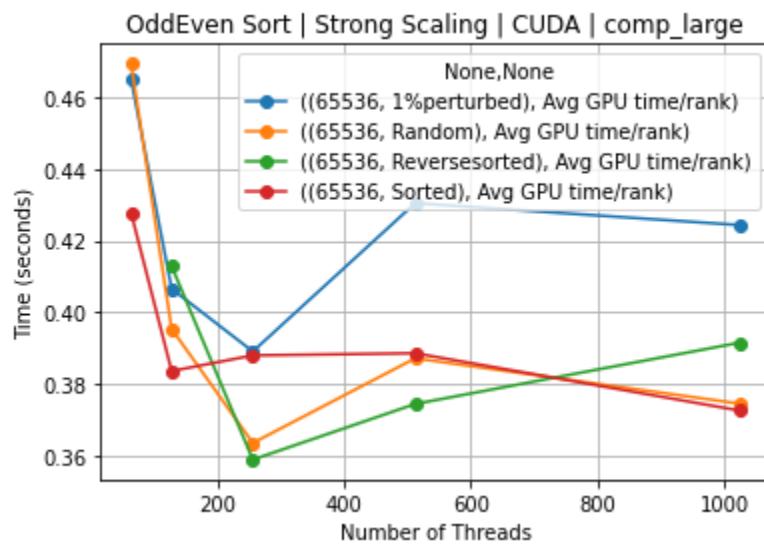
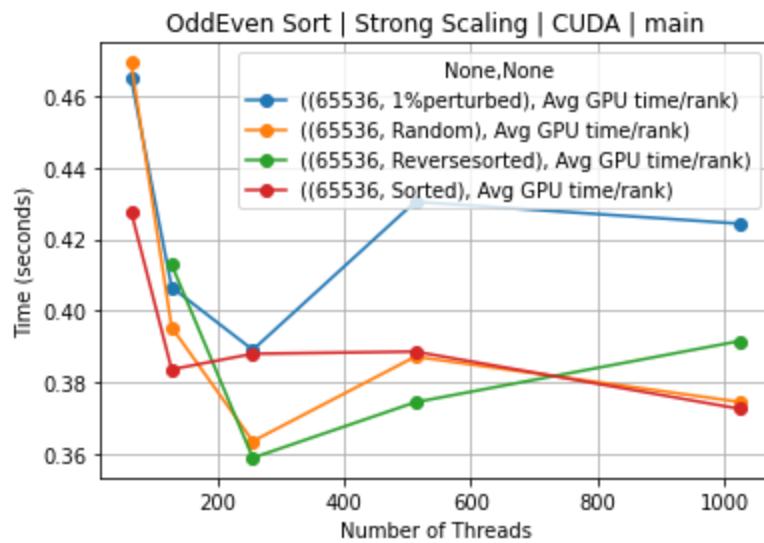


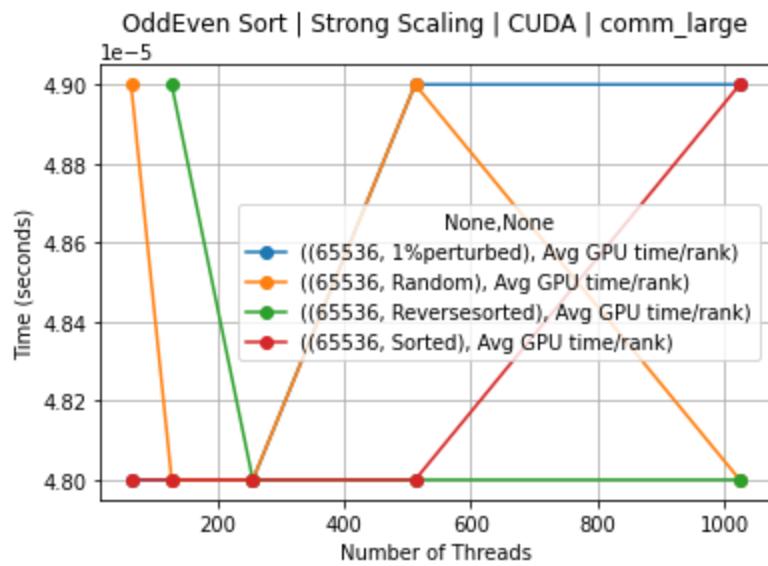
**Strong Scaling 262144**

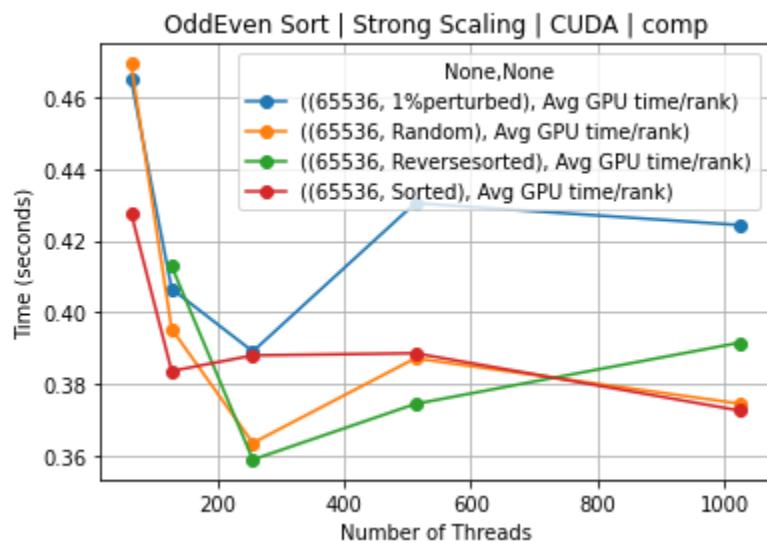
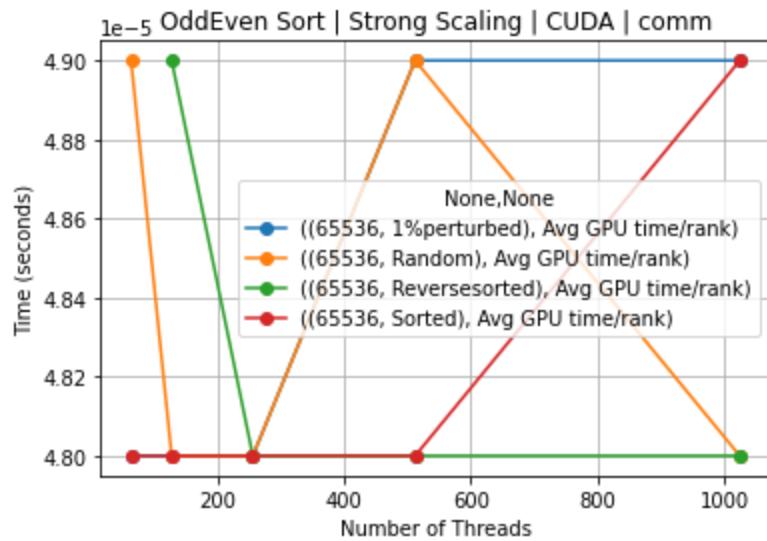




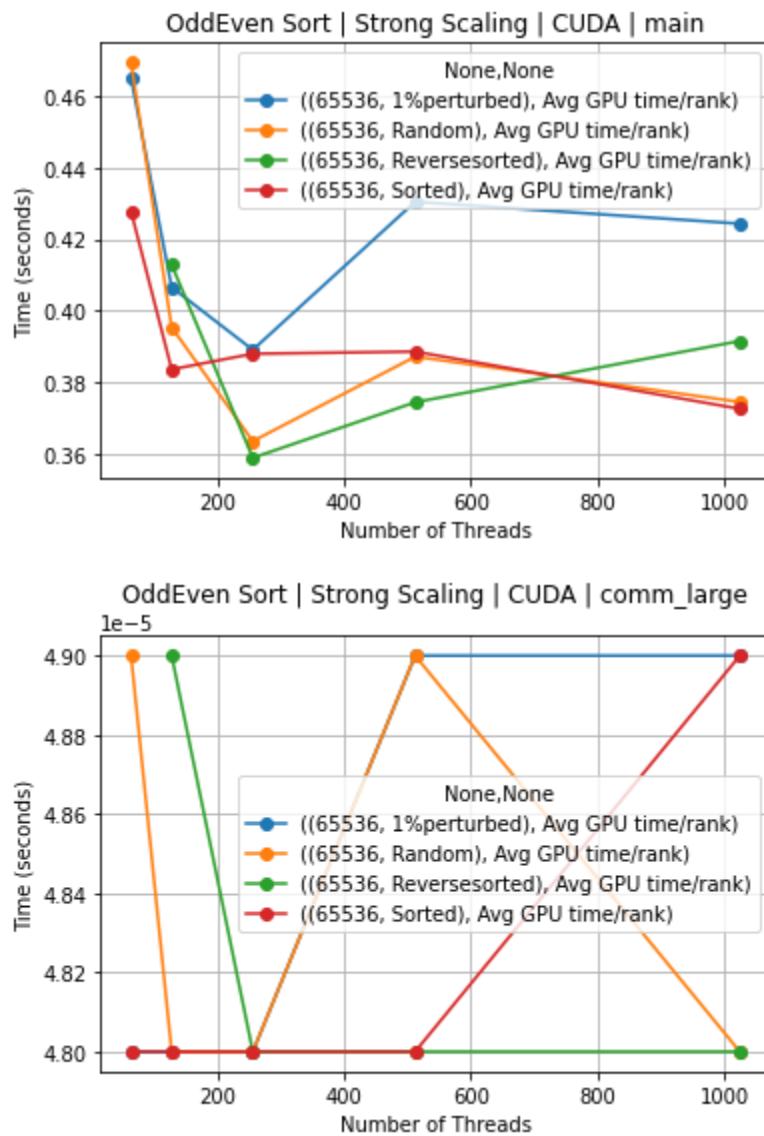
**Strong Scaling 1048576**

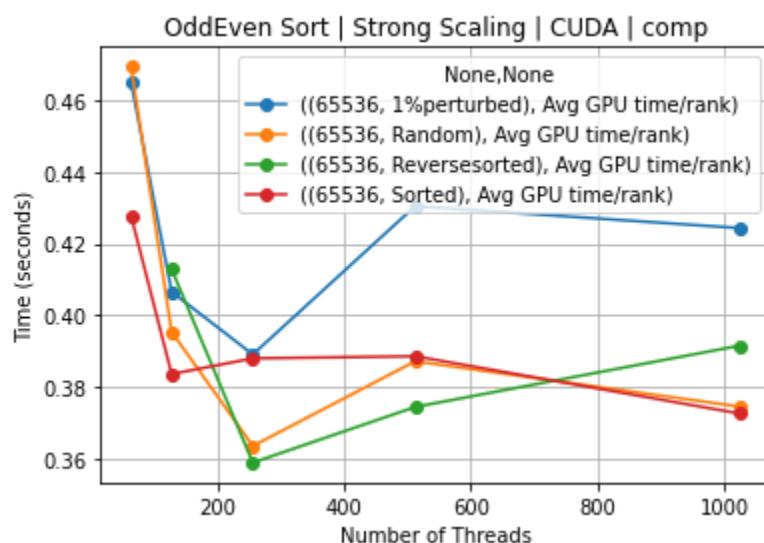
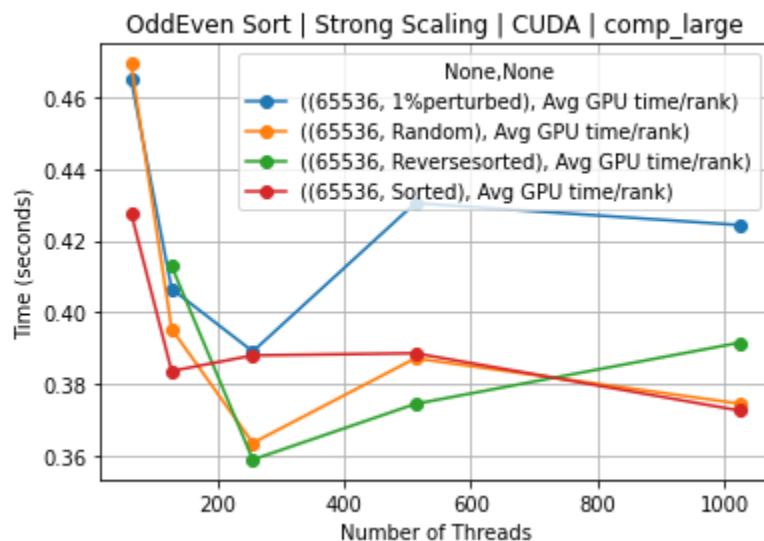
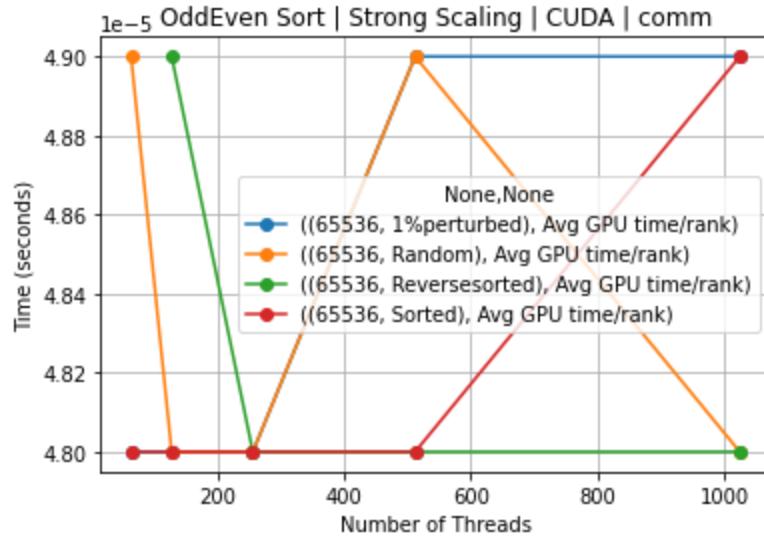




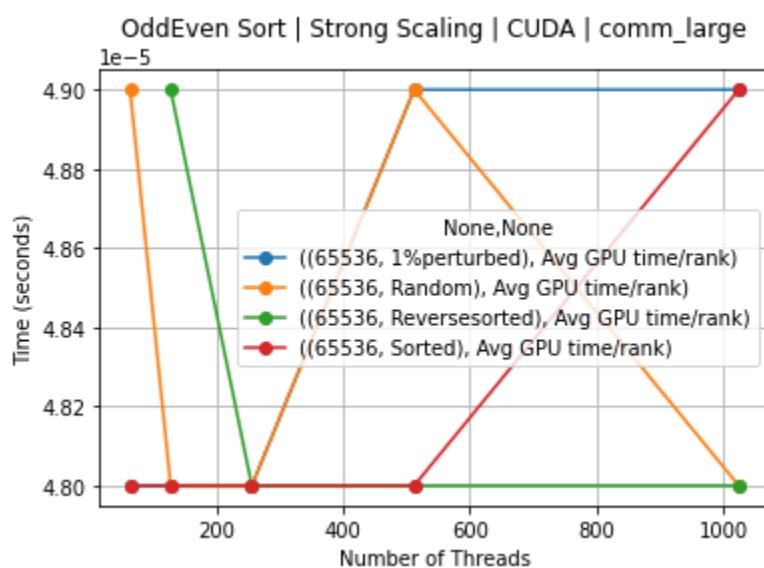
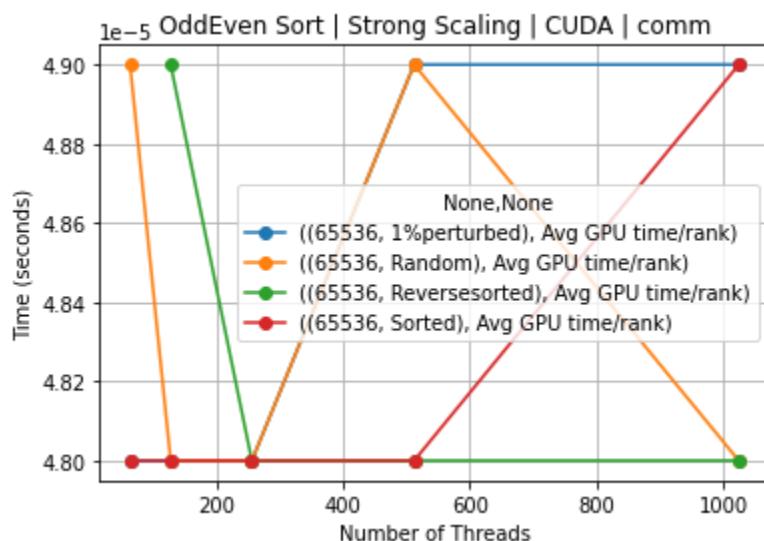


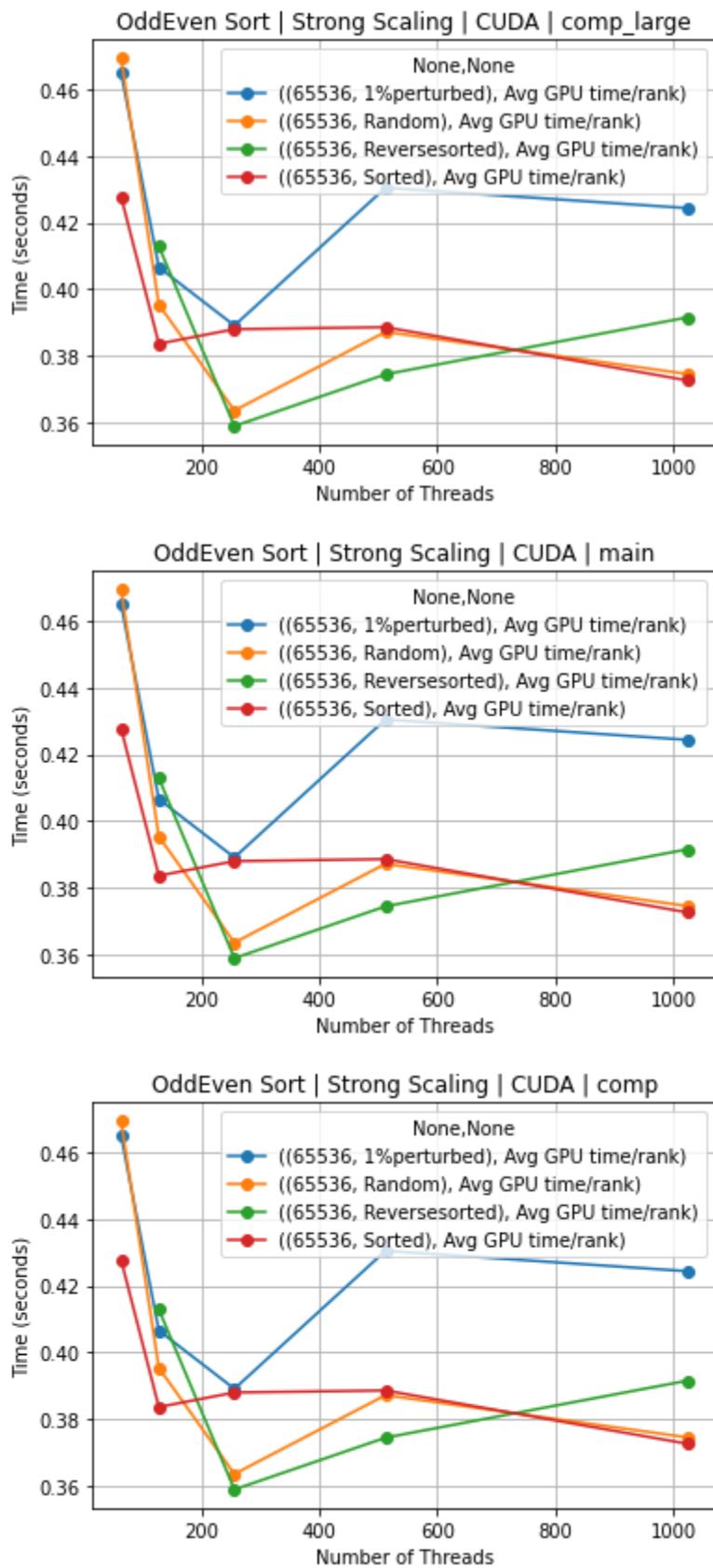
**Strong Scaling 4194304**





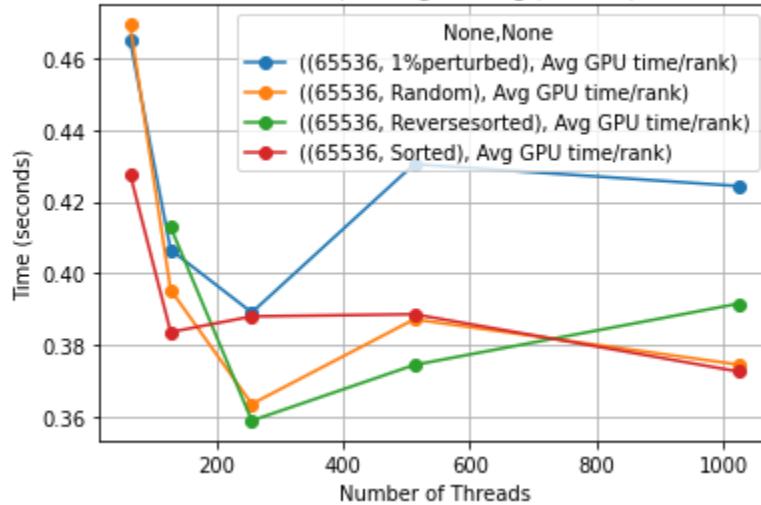
## Strong Scaling 1048576



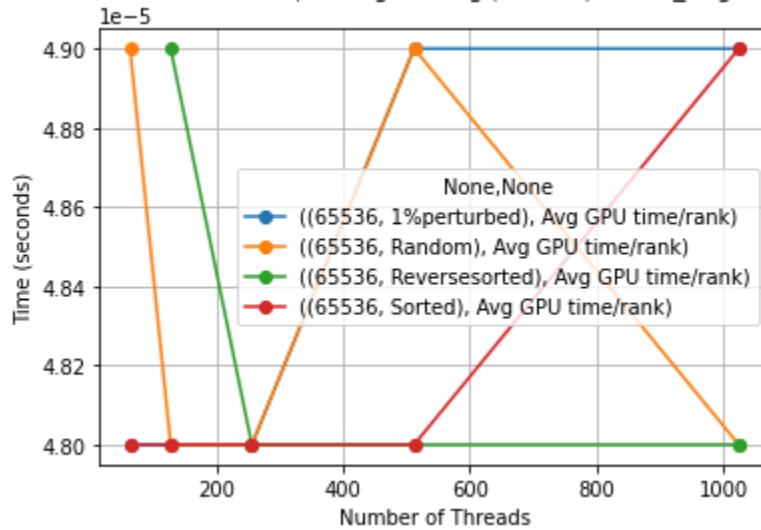


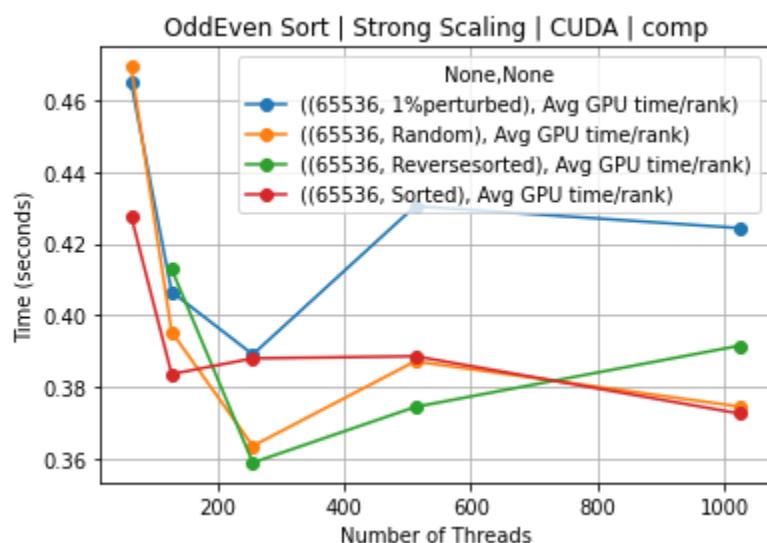
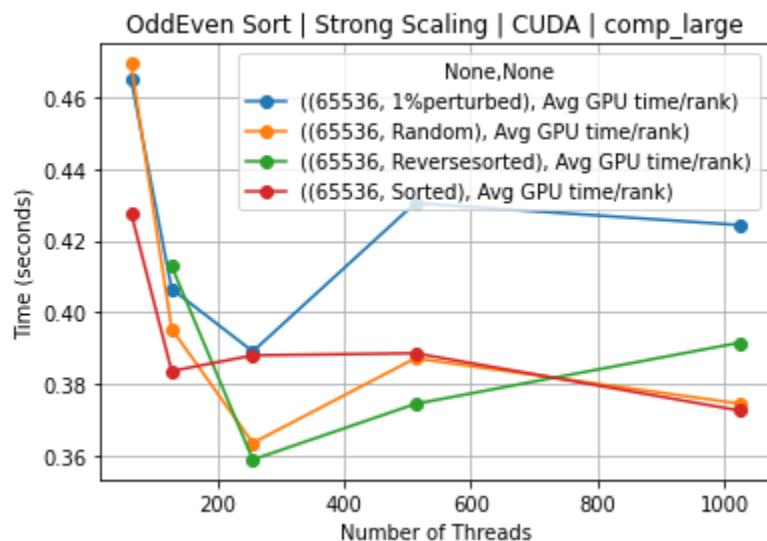
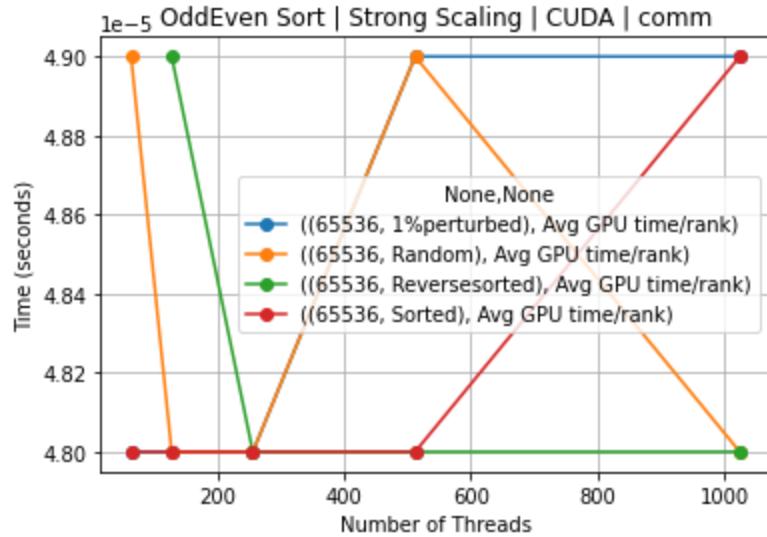
### Strong Scaling 16777216

OddEven Sort | Strong Scaling | CUDA | main

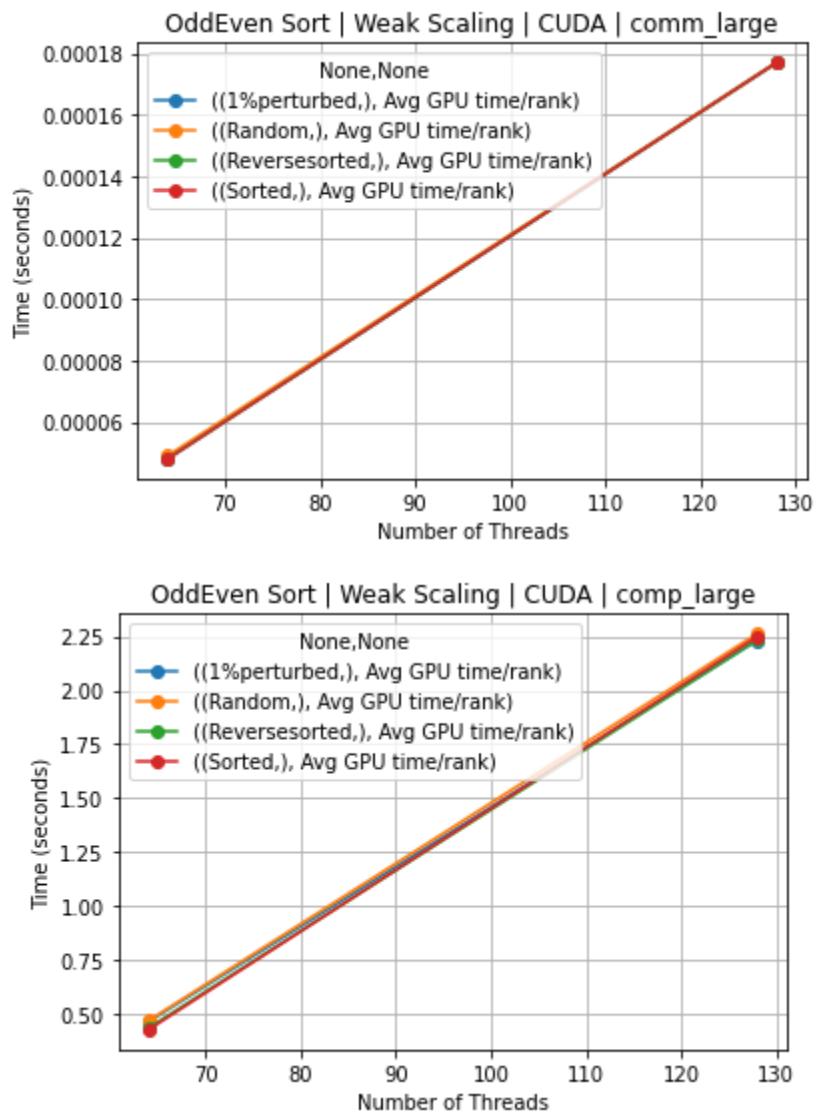


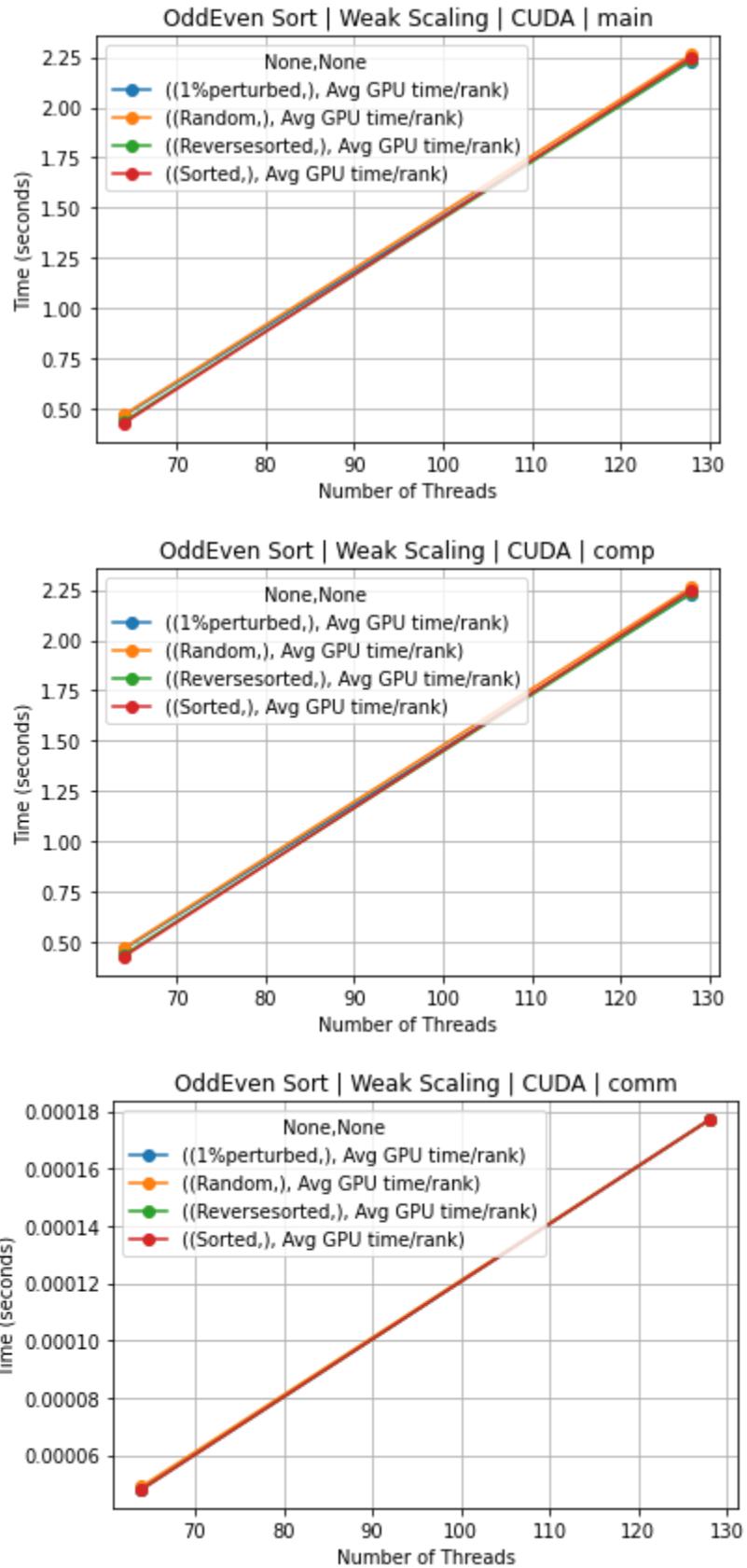
OddEven Sort | Strong Scaling | CUDA | comm\_large





## Weak Scaling

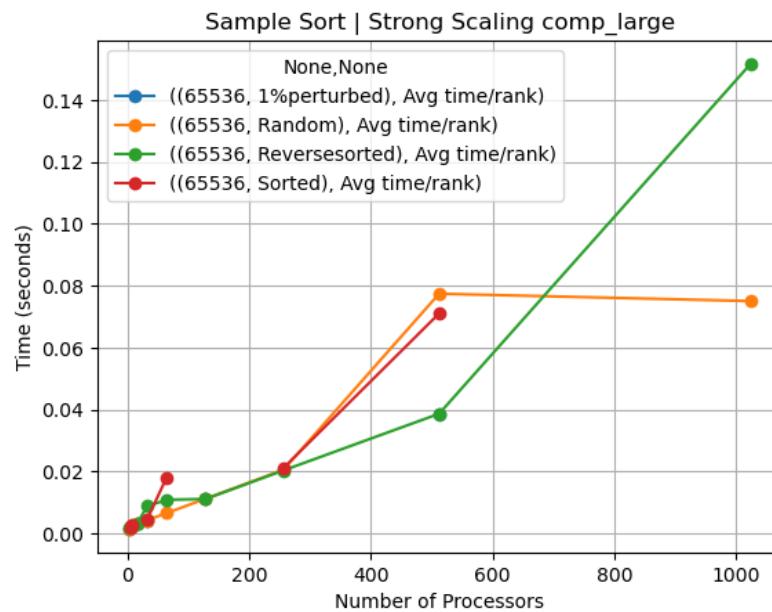
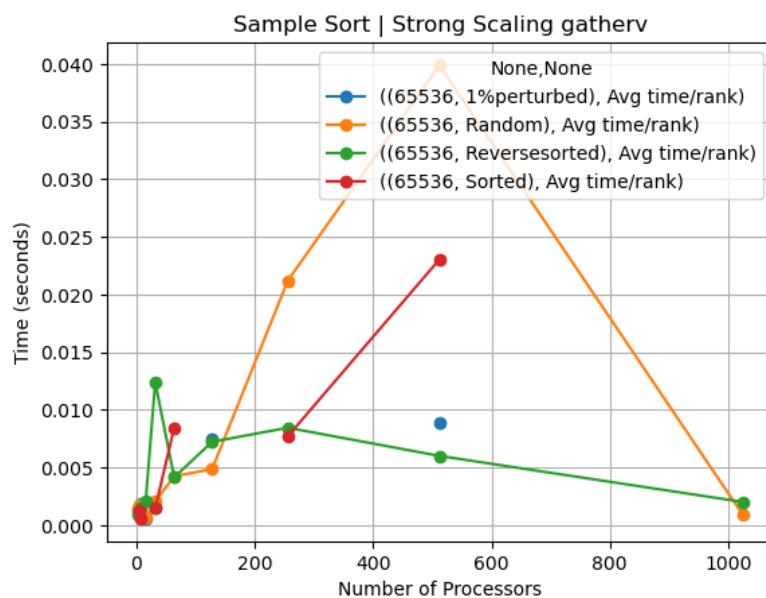


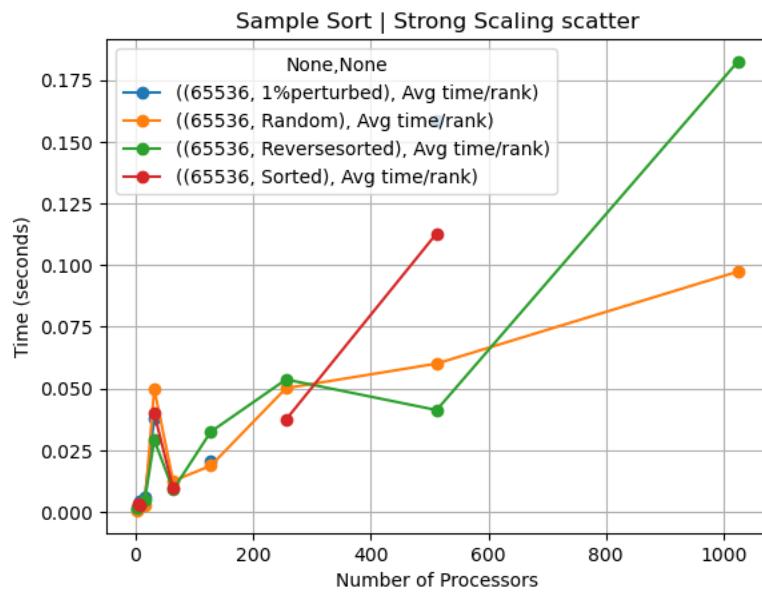
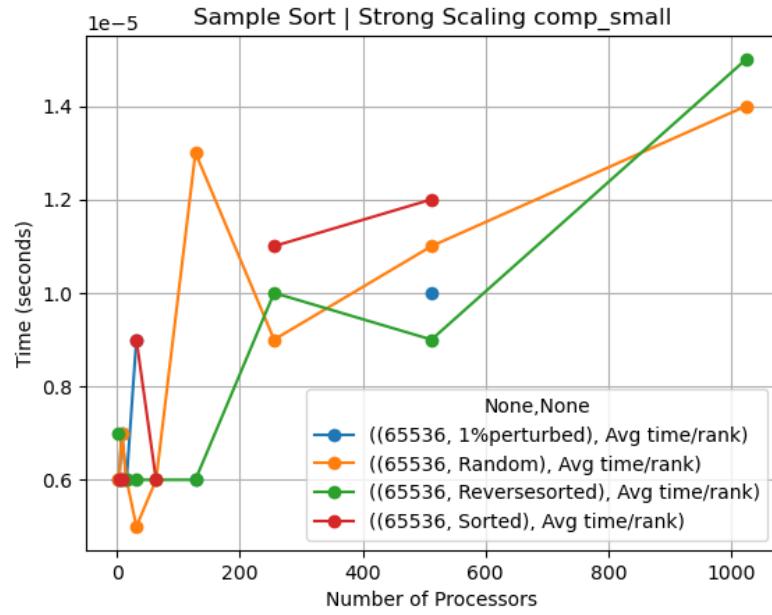


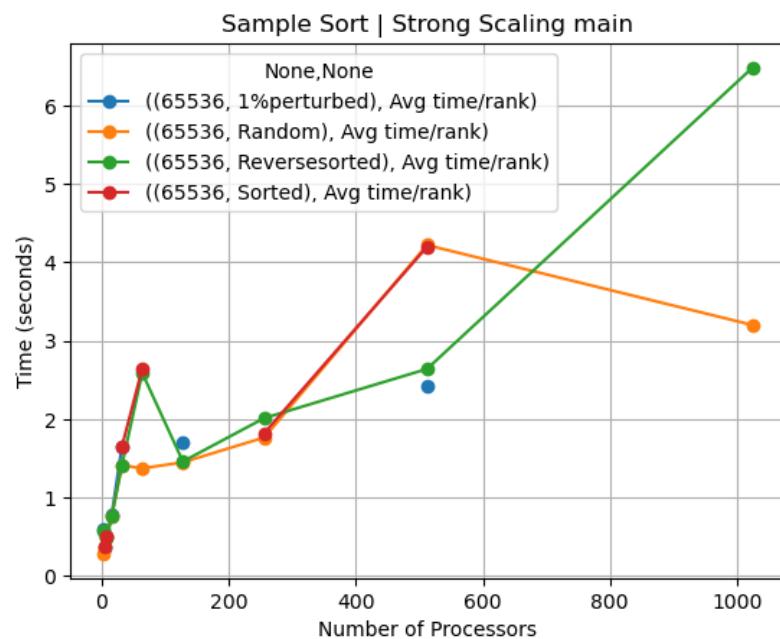
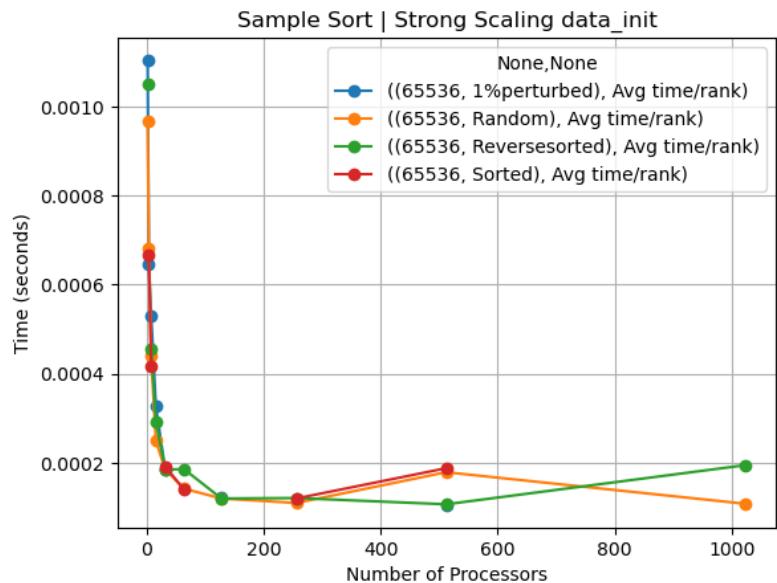
We see a linear decrease time for number of threads, until a point where things increase up.

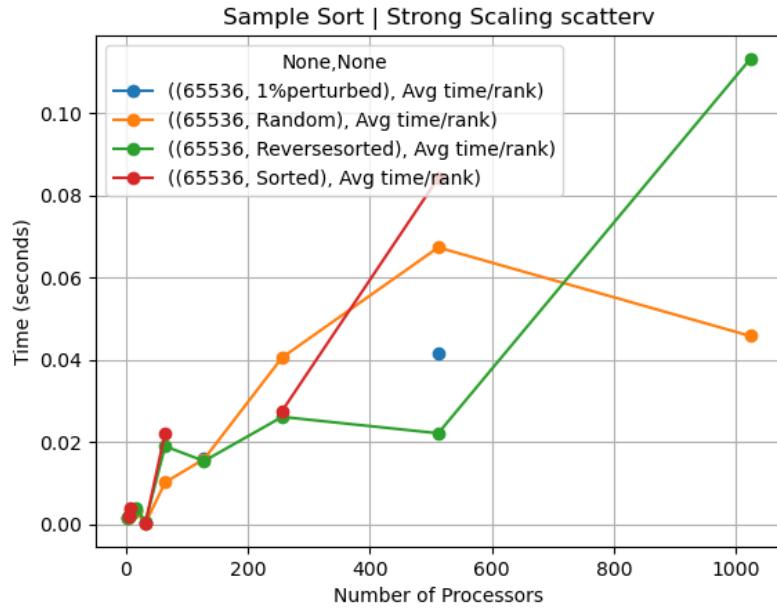
### Sample Sort MPI:

#### Strong Scaling:



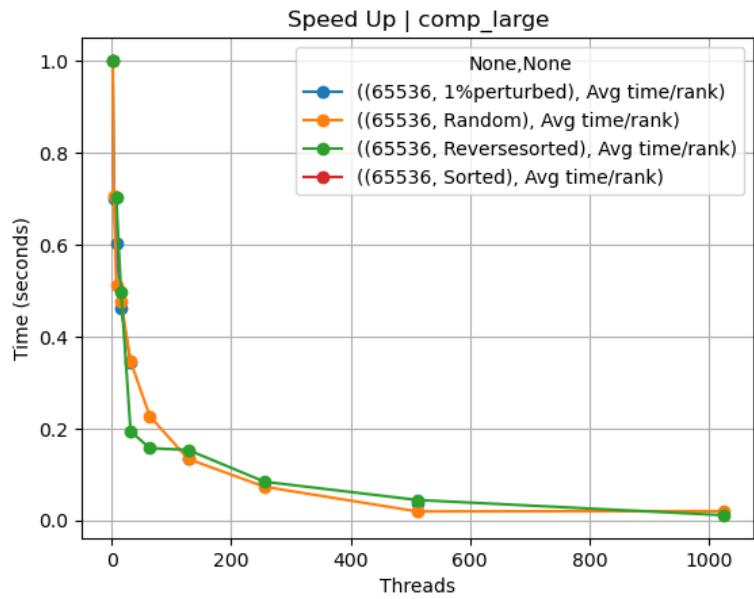
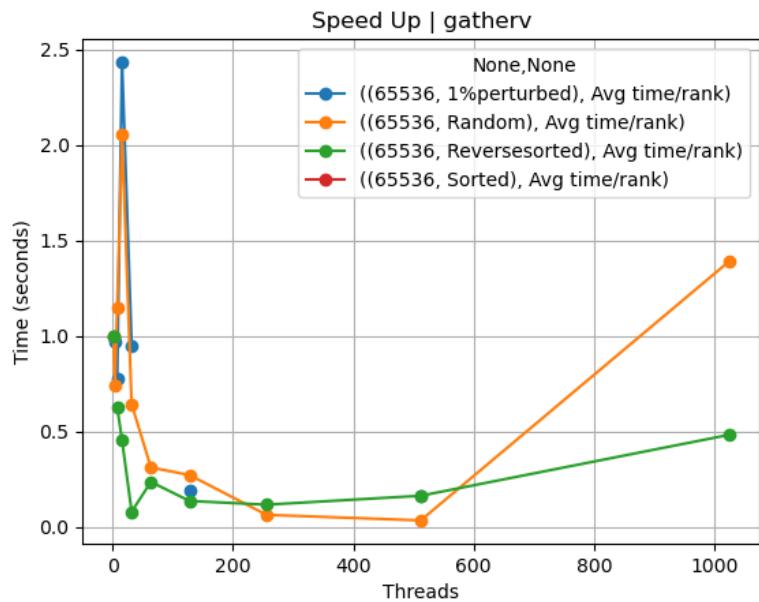




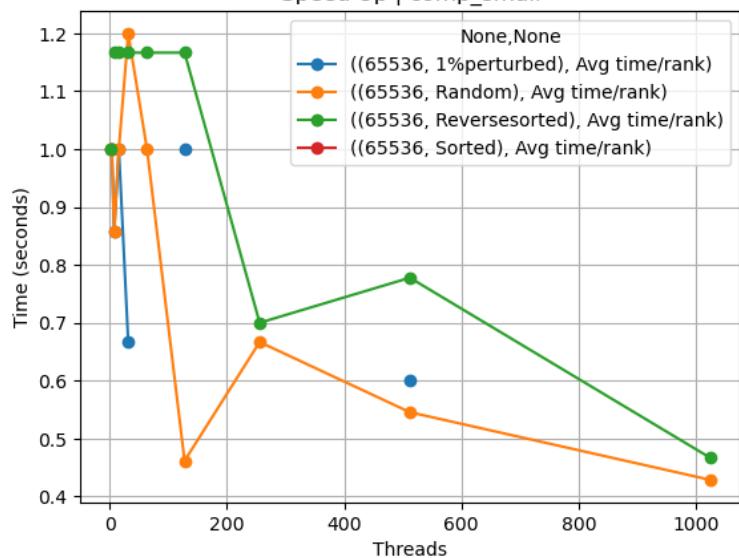


The time complexity for these processors are very linear. There may be an inefficiency in the algorithm such that the extra processors do not improve the final time. I assume that when I increase the amount of processors that my program is using the inefficiencies in the communication between processors are exacerbated. This result in the performance to linearly decrease as I increase the number of processors that my program is using.

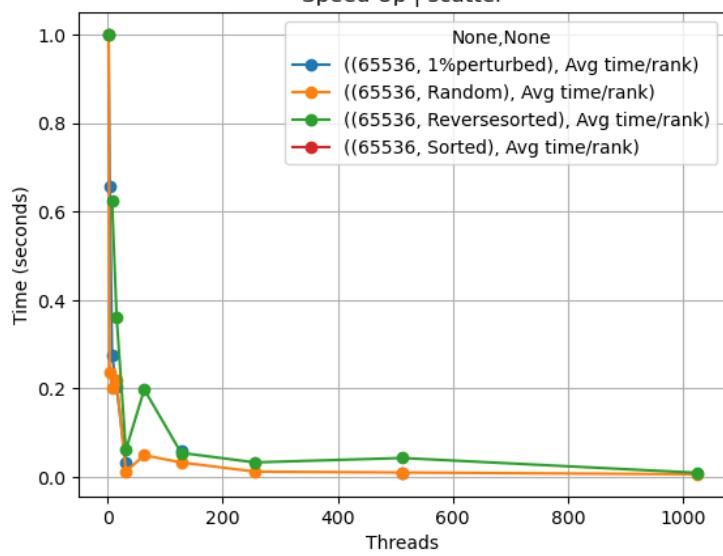
Speedup:

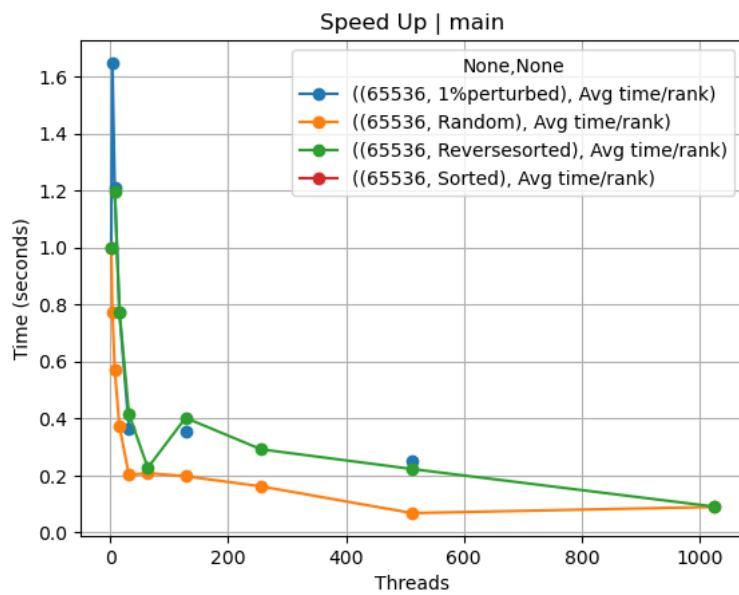
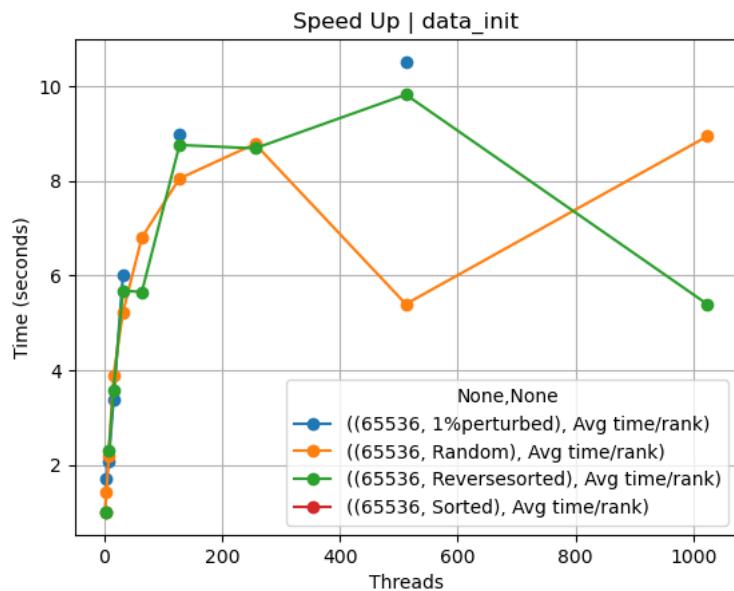


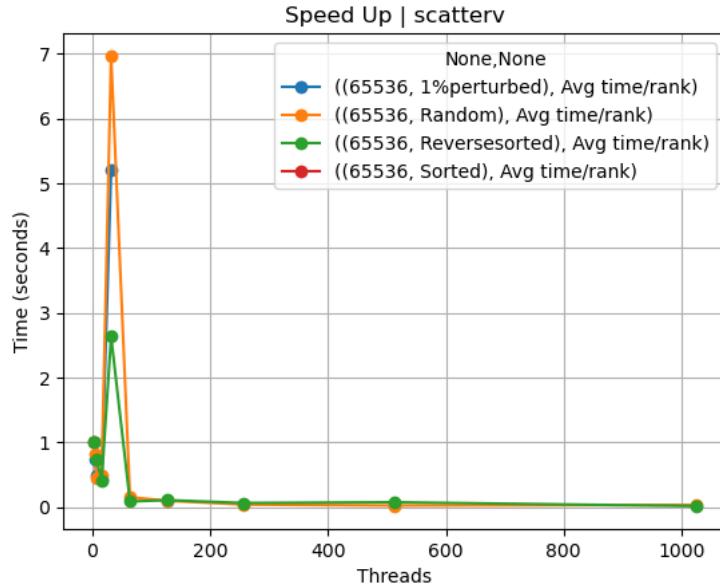
Speed Up | comp\_small



Speed Up | scatter

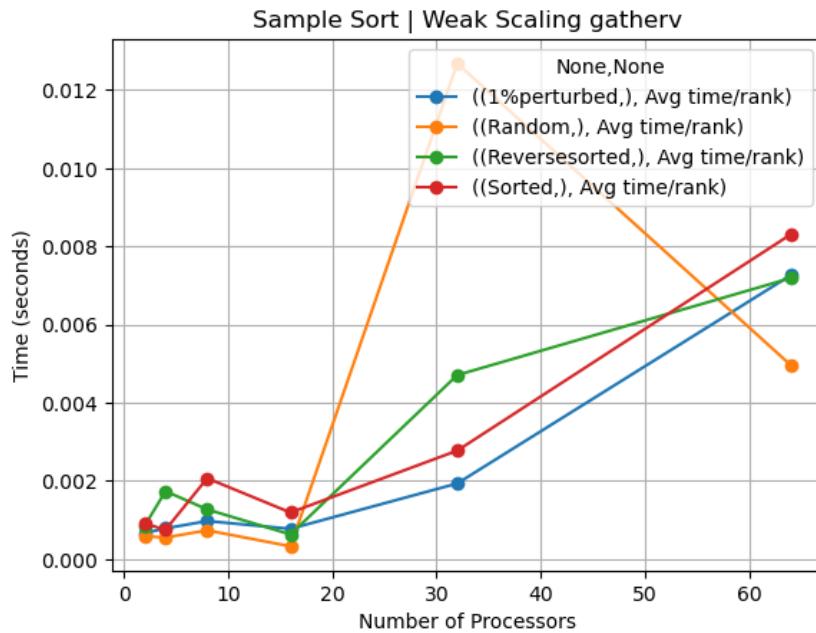




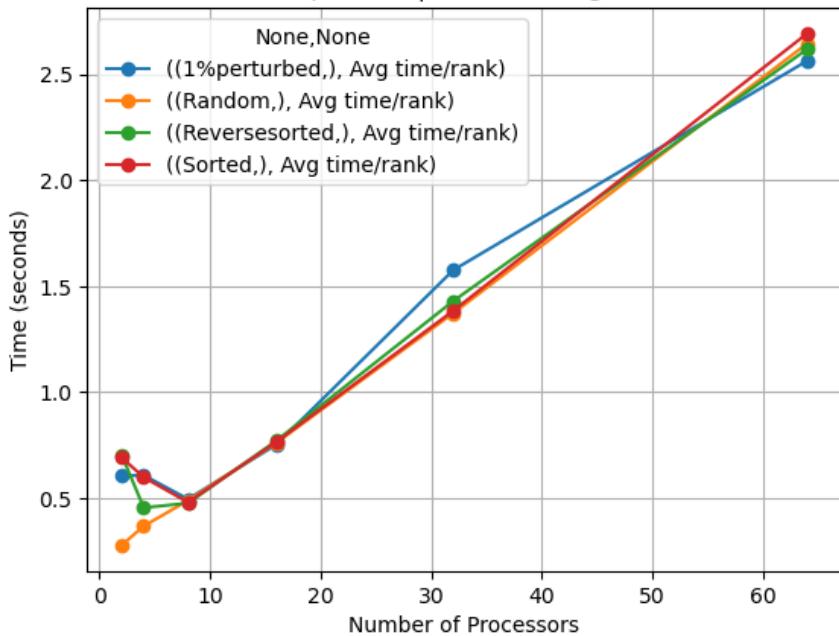


The performance graphs of the speedup further show the inefficiency of communications between nodes. One interesting observation is that for 16 nodes the communication performance is vastly improved. With that being said, I believe that this shows that the communication functions could be improved to have that same communication speedup for all numbers of processors to better show the benefits of parallelism.

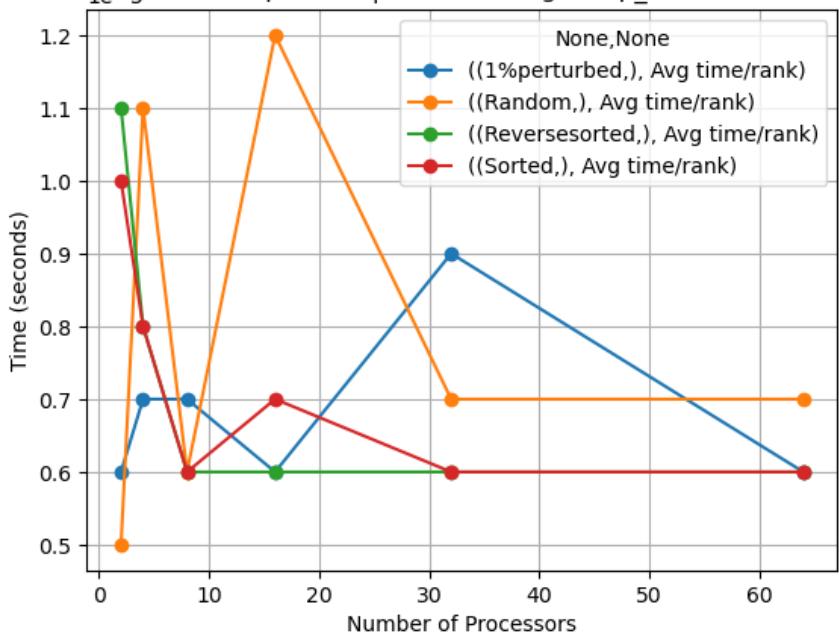
### Weak Scaling:

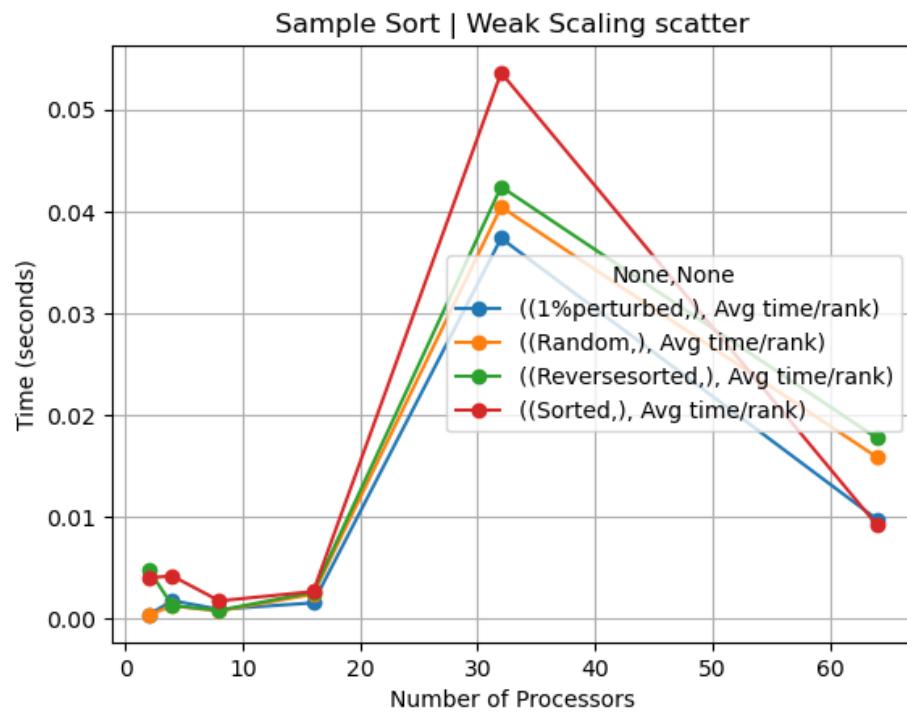
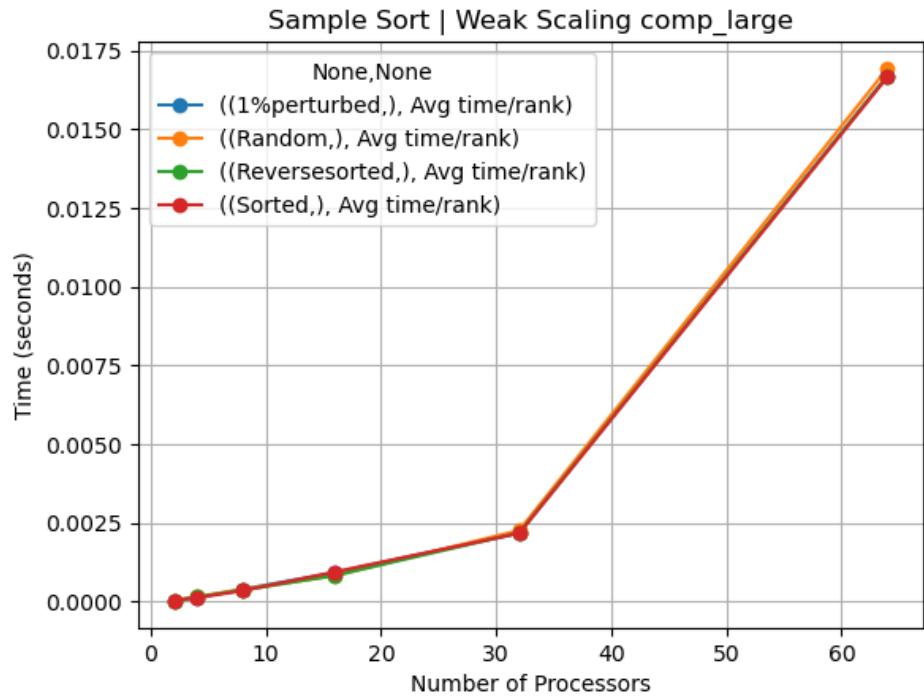


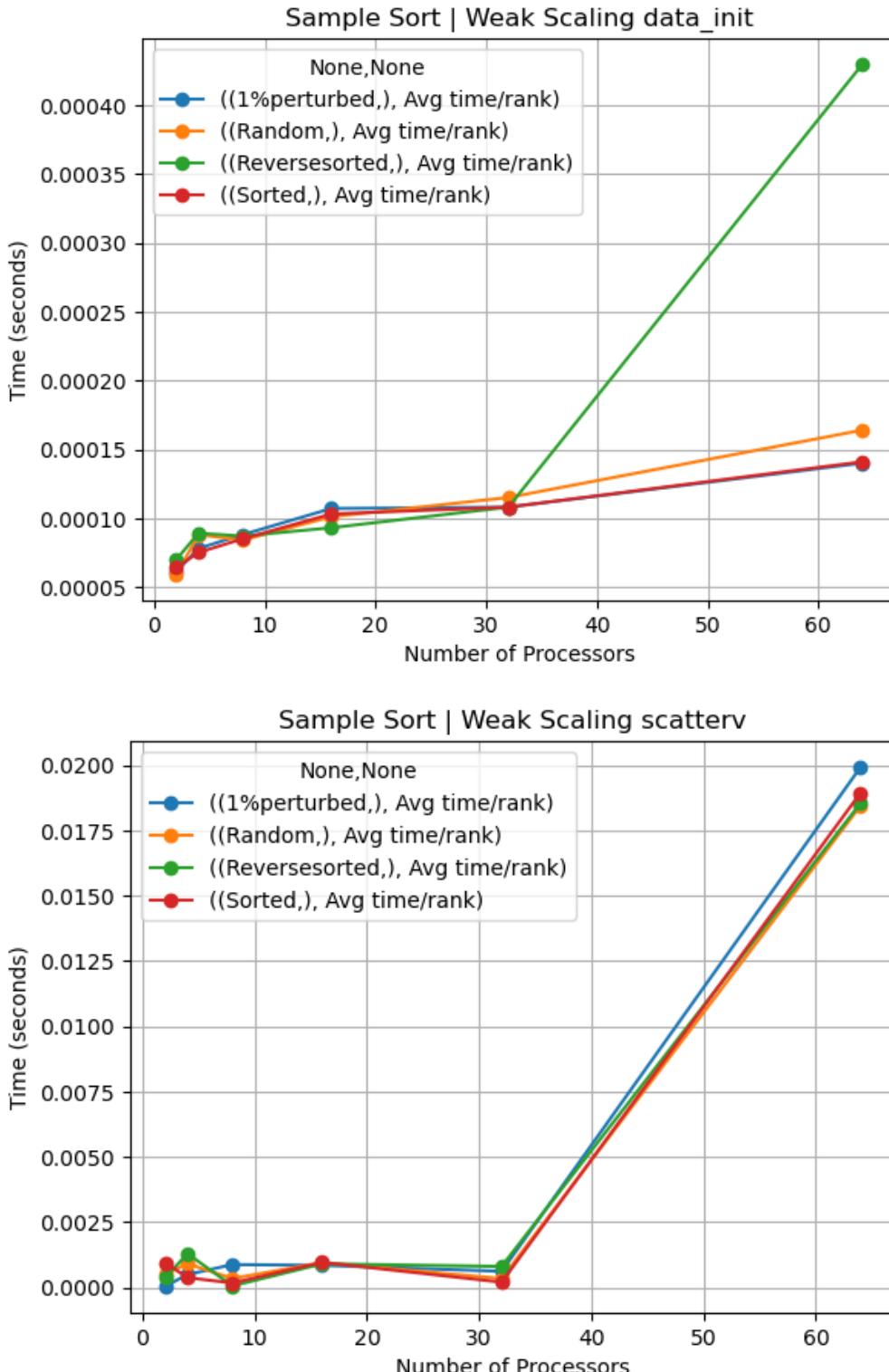
Sample Sort | Weak Scaling main



1e-5 Sample Sort | Weak Scaling comp\_small





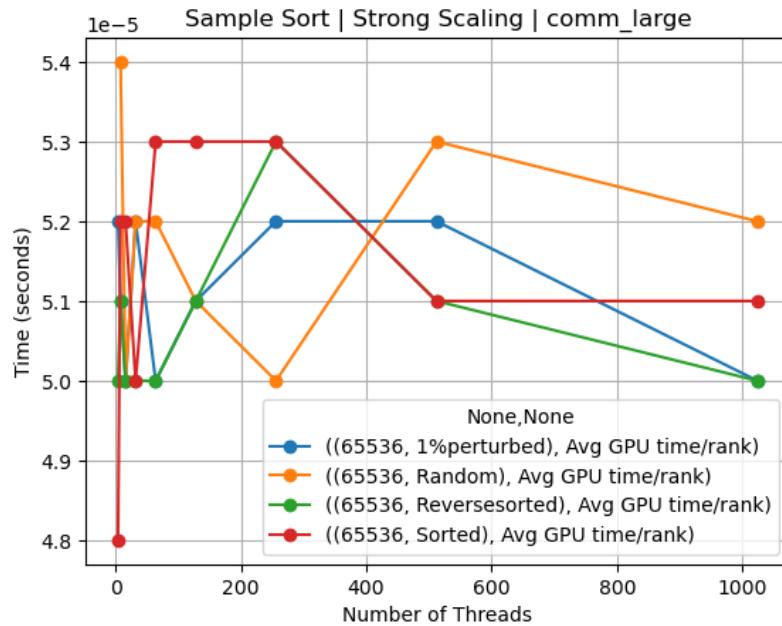
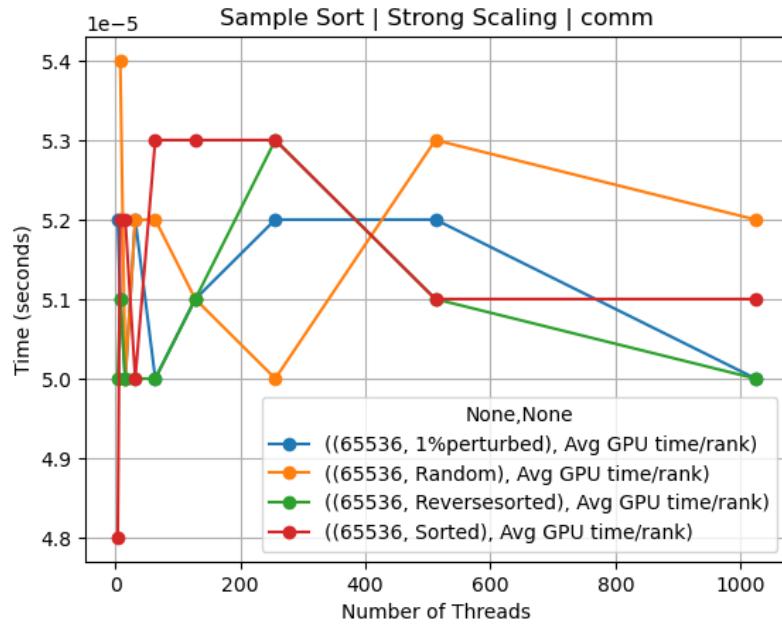


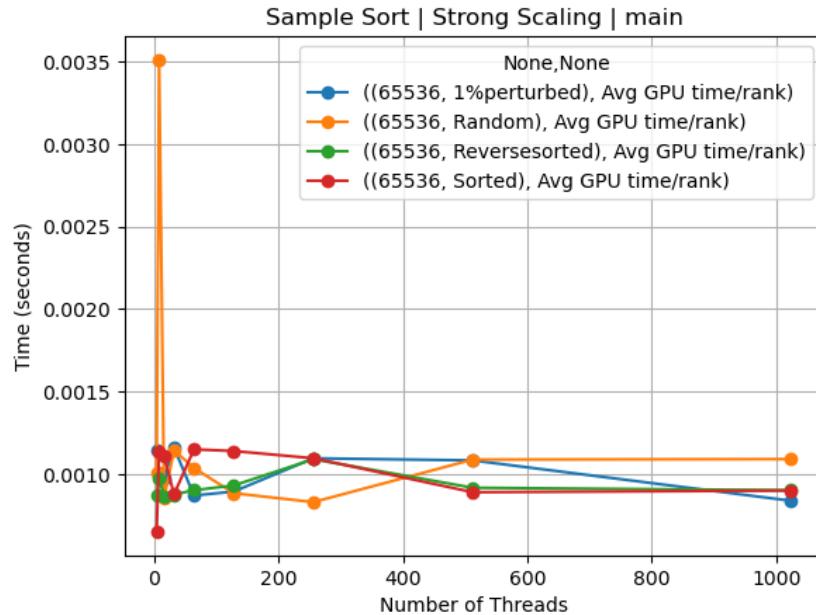
The weak scaling graph shows that as soon as the amount of node being using goes above 32 the communication performance suffers greatly. This may be due to the aforementioned issue in communication. It is important to note that the main graph does not reflect this because it

aggregates all of the performance within the program rather than just the communication performance and include some linear computation in that section.

### **Sample Sort CUDA:**

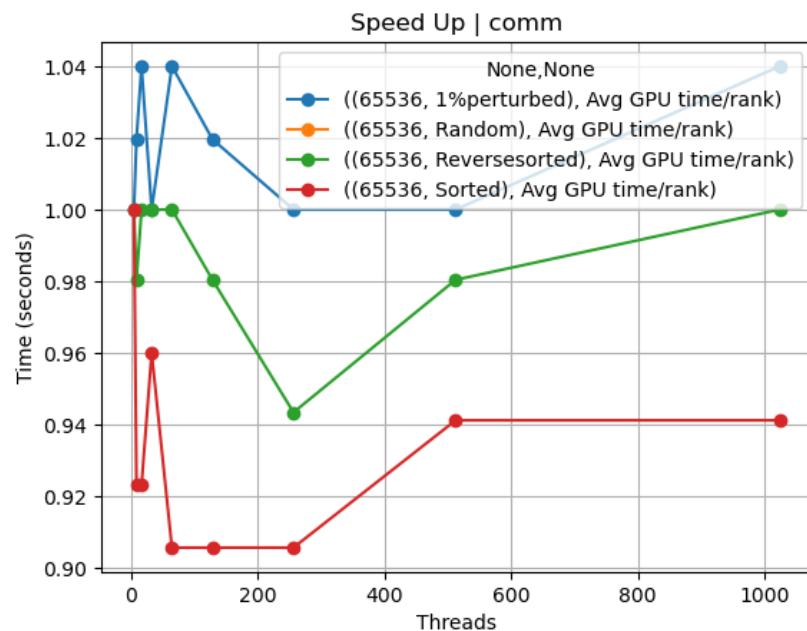
#### Strong Scaling:

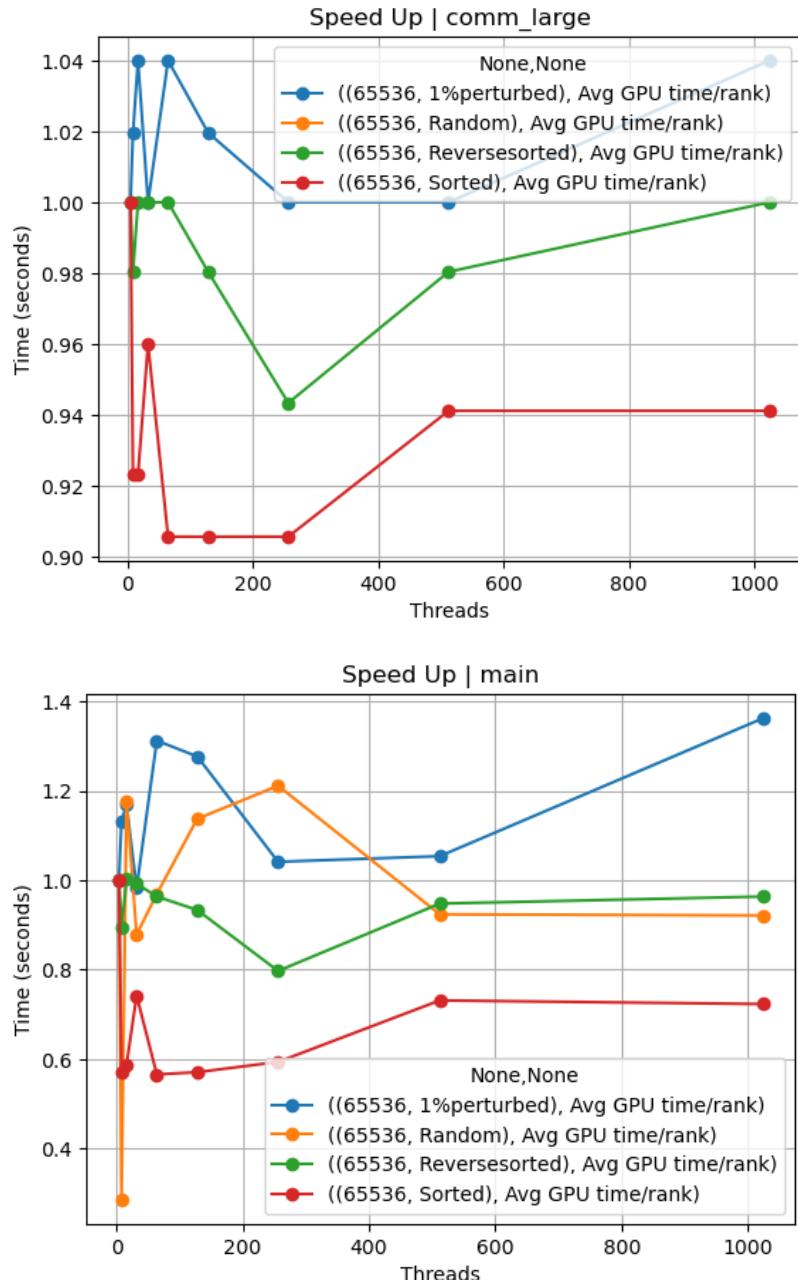




The cuda implementation shows a lot better performance than the mpi implementation. As the number of threads increases the comm and main performance shows slight improvement. Comm shows the largest improvement out of all of the different metrics.

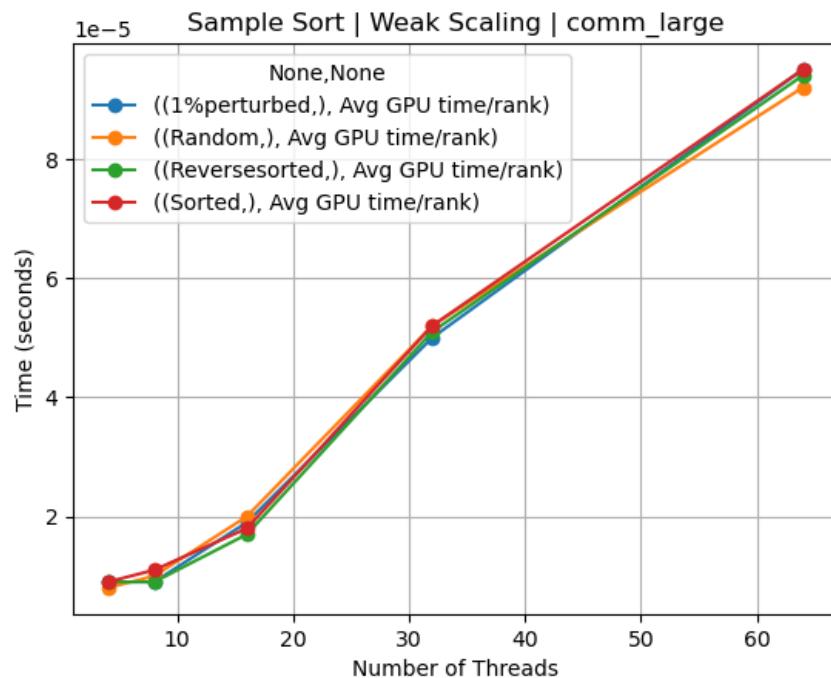
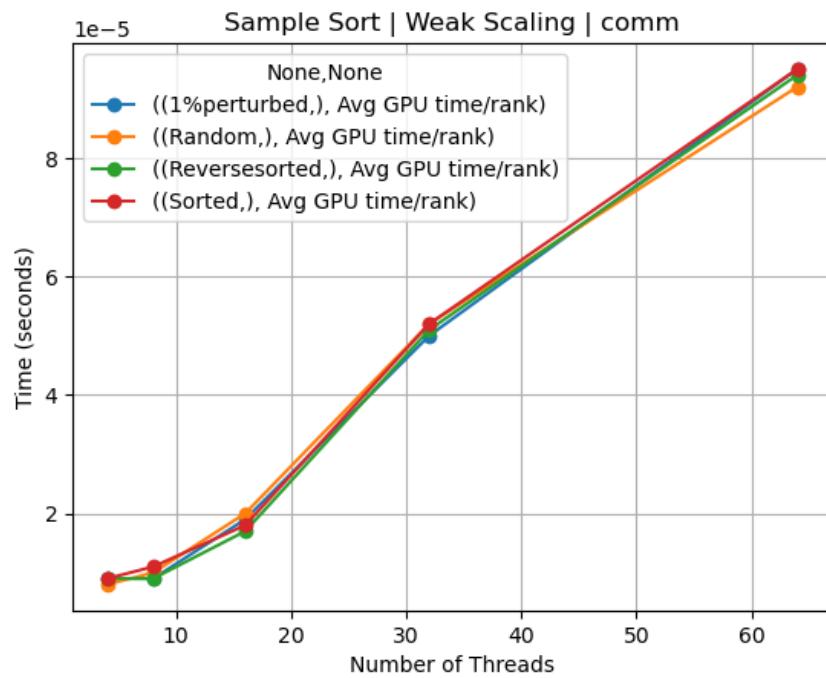
### Speedup:

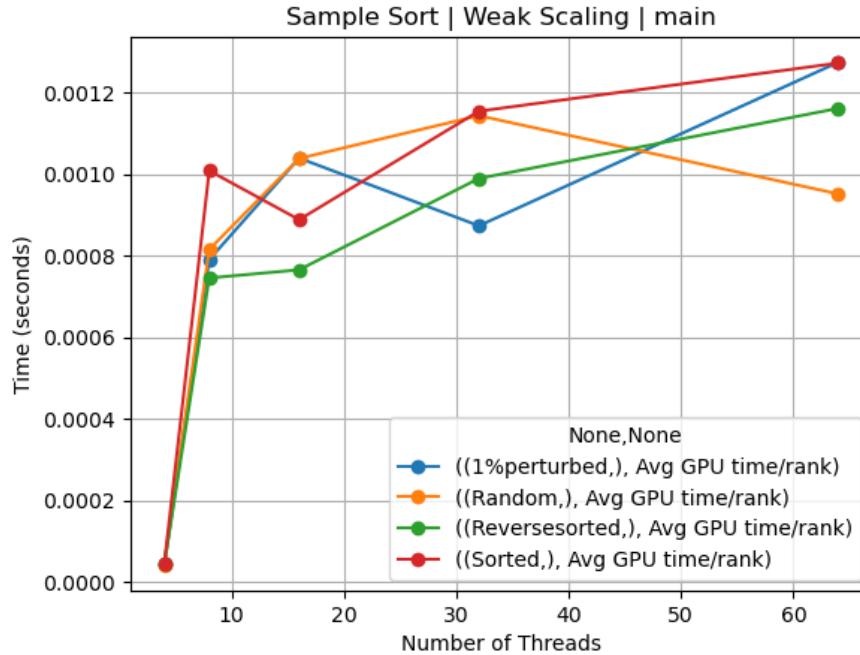




The speedup shows the performance improvement gained from increasing the number of processors. I noticed an outlier with a dip in performance in the random sort. I do not believe that this is representative of an average run, rather it is just an instance of varying performance.

## Weak Scaling:

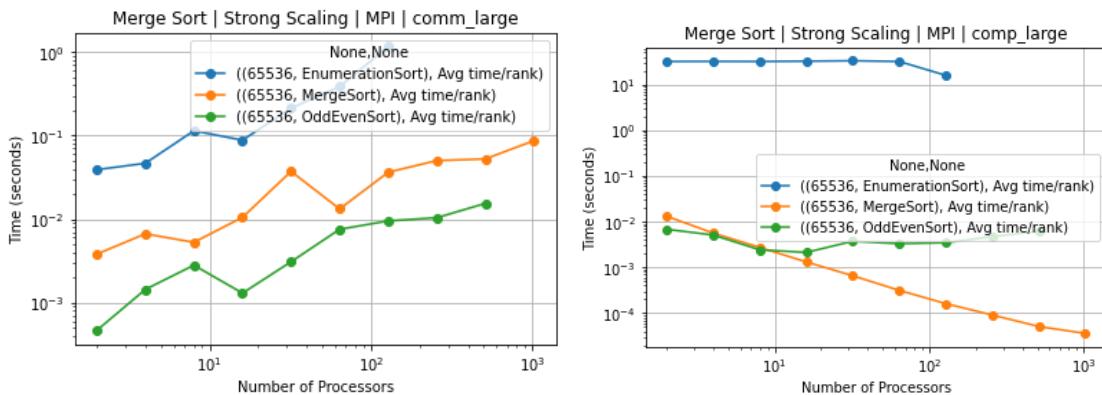


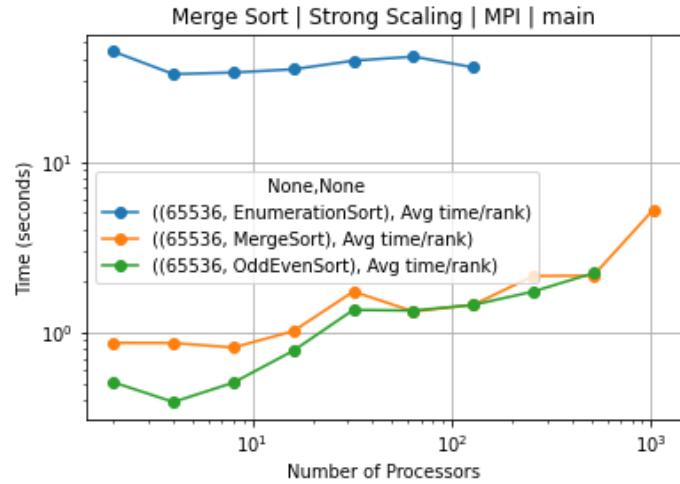


The result from the weak scaling are roughly what I would expect. The comm performance is mainly linear. I notice that the main scaling is less linear. I assume that this is because data initialization is linear and therefore as we increase the number of N, it takes a bit longer to generate those values.

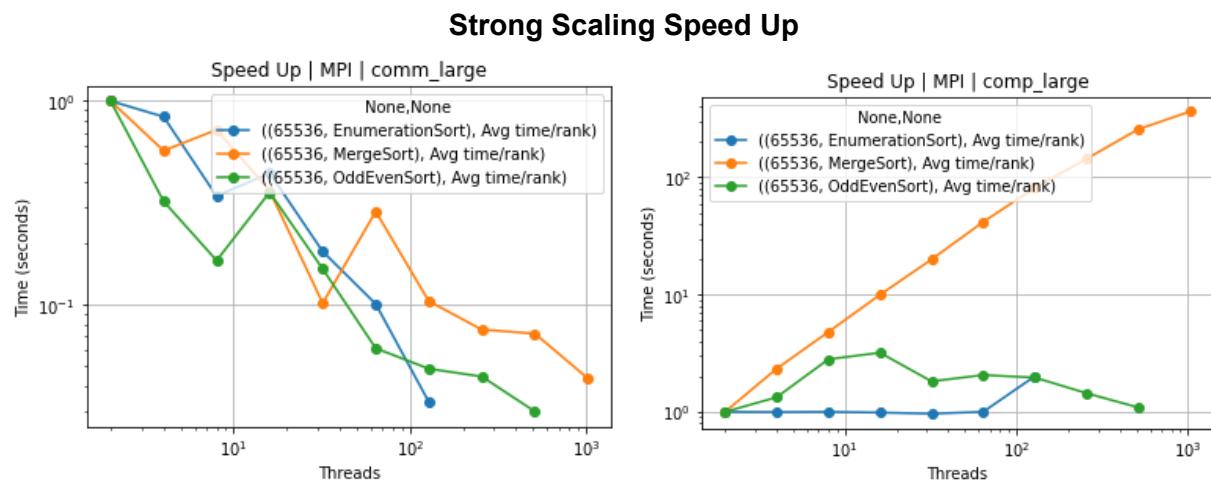
#### MPI Comparison:

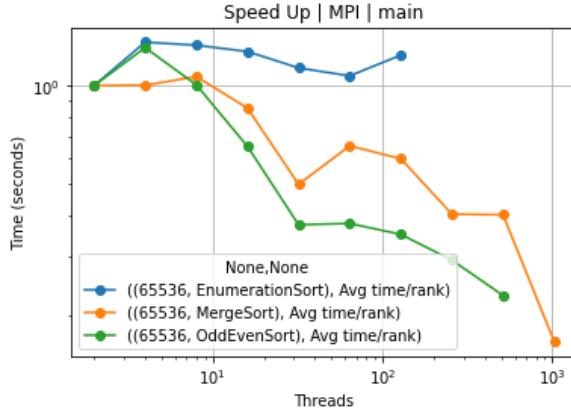
#### Strong Scaling (No SampleSort due to issues in metadata)





For the Strong Scaling between all the algorithms there seems to be a clear increase in the time it takes for the comm<sub>\_large</sub> to run as the number of processors increase, where as, the time it takes to run comp<sub>\_large</sub> decreases over time, and while Enumeration Sort doesn't have as much data it shows that it begins to decrease and may go even further down.

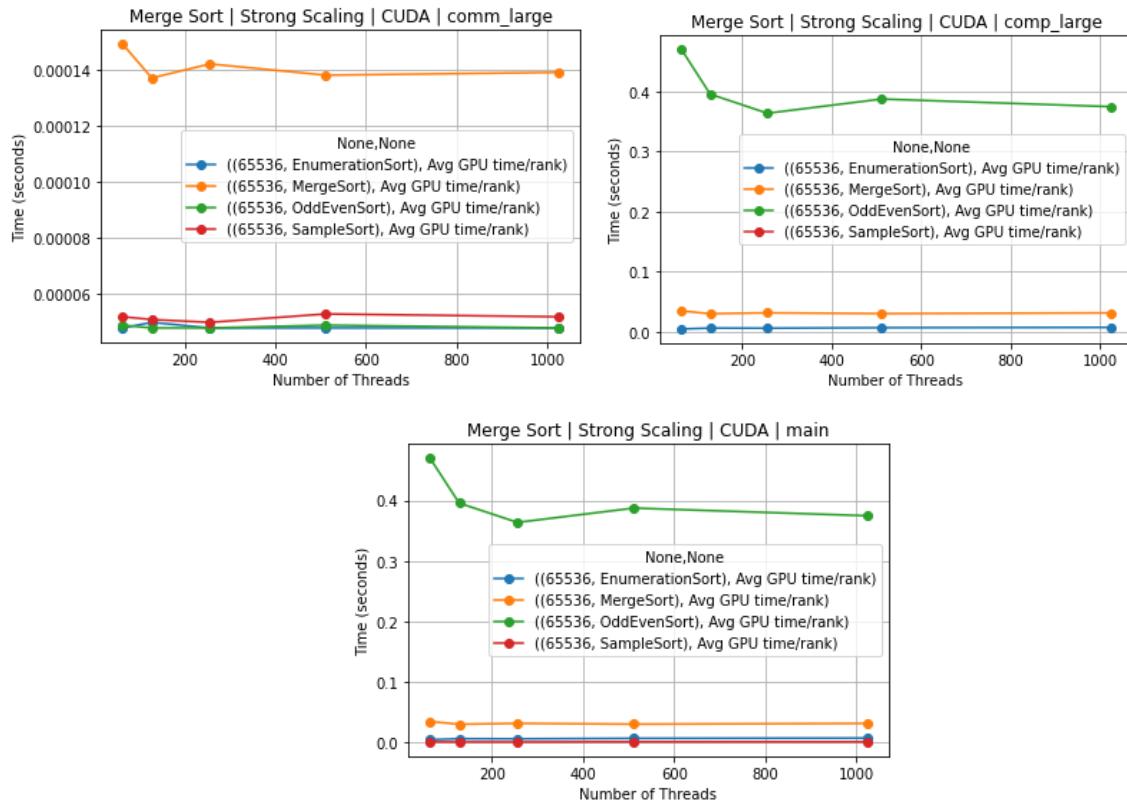




The speed up has a similar showing in that there is little to no speed up for the Algorithms in the comm<sub>\_large</sub>, but comp<sub>\_large</sub> does show speed up for each of the present Algorithms.

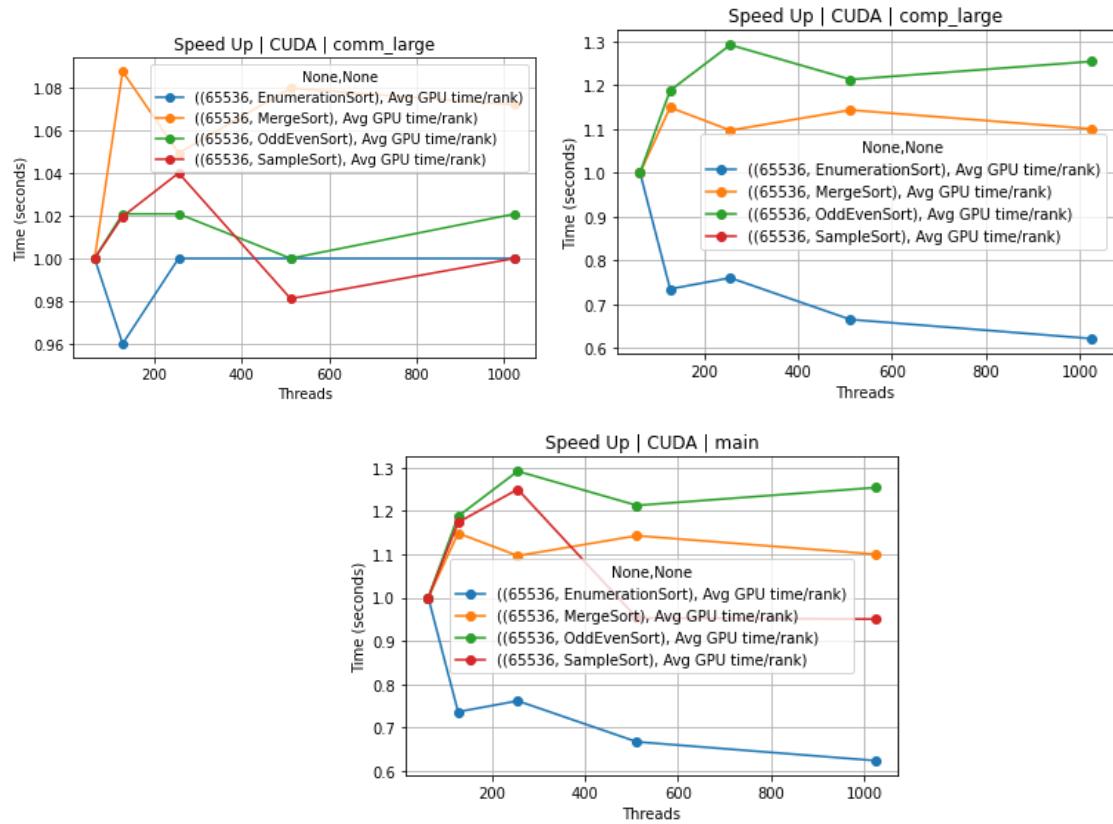
### CUDA Comparison:

#### Strong Scaling



Here it is a bit difficult to see due to the differences in the Algorithms, but it is clear that for comm<sub>\_large</sub>, the MergeSort takes quite a bit of time, whereas for comp<sub>\_large</sub>, OddEven Sort seems to take the most time. And further analysis is a little difficult given the scale of the algorithms when put together.

## Strong Scaling Speed Up



Finally the speed up here shows a bit of a mixture for both comm\_large and comp\_large, though while never substantial, there are slight increases over time as the processors increase.

Seemingly more so in OddEven and MergeSort.