ncm_nag

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1 Nearest Correlation Matrices

This notebook looks at computing *nearest correlation matrices* using the NAG Library for *Python*.

1.1 Correlation Matrices

- An n by n matrix is a correlation matrix if:
  - it is symmetric
  - it has ones on the diagonal
  - its eigenvalues are non-negative (positive semidefinite)

$$Ax = \lambda x, \quad x \neq 0$$

- The element in the ith row and jth column is the correlation between the ith and jth variables. This could be stock process, for example.

1.2 Empirical Correlation Matrices

- Empirical correlation matrices are often **not mathematically true** due to inconsistent or missing data.
- Thus we are required to find a true correlation matrix, where our input, G, is an approximate correlation matrix.
- In particular we seek the *nearest* correlation matrix, in most cases.

1.3 Computing Correlation Matrices

- The vector p_i, the ith column of a matrix, P, holds the m observations of the ith variable, of which there are n. $\bar{p}_i$ is the sample mean.

$$S_{ij} = \frac{1}{m-1}(p_i - \bar{p}_i)^T(p_j - \bar{p}_j)$$

- S is a covariance matrix, with S_{ij} the covariance between variables i and j

- R is the corresponding correlation matrix, given by:

$$D_S^{1/2} = \text{diag}(s_{11}^{-1/2}, s_{22}^{-1/2}, \ldots, s_{nn}^{-1/2})$$

$$R = D_S^{1/2} S D_S^{1/2}$$

1.4 Approximate Correlation Matrices

- Now, what if we don't have all observations for each variable?
- We compute each covariance with observations that are available for *both* the ith and jth variable.
- For example NAG routine **library.correg.coeffs_pearson_miss_case**.
- We then compute the correlation matrix as before.

2 Missing Stock Price Example

- Prices for 8 stocks on the first working day of 10 consecutive months.

	Stock A	Stock B	Stock C	Stock D	Stock E	Stock F	Stock G	Stock H
Month 1	59.875	42.734	47.938	60.359	54.016	69.625	61.500	62.125
Month 2	53.188	49.000	39.500		34.750		83.000	44.500
Month 3	55.750	50.000	38.938		30.188		70.875	29.938
Month 4	65.500	51.063	45.563	69.313	48.250	62.375	85.250	
Month 5	69.938	47.000	52.313	71.016		59.359	61.188	48.219
Month 6	61.500	44.188	53.438	57.000	35.313	55.813	51.500	62.188
Month 7	59.230	48.210	62.190	61.390	54.310	70.170	61.750	91.080
Month 8	61.230	48.700	60.300	68.580	61.250	70.340		
Month 9	52.900	52.690	54.230		68.170	70.600	57.870	88.640
Month 10	57.370	59.040	59.870	62.090	61.620	66.470	65.370	85.840

- We will use NaNs where there is missing data.
- So our $P = [p_1, p_2, \ldots, p_n]$ is:

$$P = \begin{bmatrix} 59.875 & 42.734 & 47.938 & 60.359 & 54.016 & 69.625 & 61.500 & 62.125 \\ 53.188 & 49.000 & 39.500 & \text{NaN} & 34.750 & \text{NaN} & 83.000 & 44.500 \\ 55.750 & 50.000 & 38.938 & \text{NaN} & 30.188 & \text{NaN} & 70.875 & 29.938 \\ 65.500 & 51.063 & 45.563 & 69.313 & 48.250 & 62.375 & 85.250 & \text{NaN} \\ 69.938 & 47.000 & 52.313 & 71.016 & \text{NaN} & 59.359 & 61.188 & 48.219 \\ 61.500 & 44.188 & 53.438 & 57.000 & 35.313 & 55.813 & 51.500 & 62.188 \\ 59.230 & 48.210 & 62.190 & 61.390 & 54.310 & 70.170 & 61.750 & 91.080 \\ 61.230 & 48.700 & 60.300 & 68.580 & 61.250 & 70.340 & \text{NaN} & \text{NaN} \\ 52.900 & 52.690 & 54.230 & \text{NaN} & 68.170 & 70.600 & 57.870 & 88.640 \\ 57.370 & 59.040 & 59.870 & 62.090 & 61.620 & 66.470 & 65.370 & 85.840 \end{bmatrix}.$$

- And to compute the covariance between the 3rd and 4th variables:

$$\begin{aligned} v_1^T &= [47.938, 45.563, 52.313, 53.438, 62.190, 60.300, 59.870] \\ v_2^T &= [60.359, 69.313, 71.016, 57.000, 61.390, 68.580, 62.090] \\ S_{3,4} &= \frac{1}{6}(v_1 - \bar{v}_1)^T(v_2 - \bar{v}_2) \end{aligned}$$

- Let's compute this in Python.

2.0.1 Import required modules and set print options

[1]:
```
import numpy as np
from naginterfaces.library import correg as nl_correg
import matplotlib.pyplot as plt
# Set the print precision
np.set_printoptions(precision=4, suppress=True)
```

[2]:
```
# Select the display backend for Jupyter:
%matplotlib inline
```

2.0.2 Initialize our P matrix of observations

[3]:
```
# Define a 2-d array and use np.nan to set elements as NaNs
P = np.array([[59.875, 42.734, 47.938, 60.359, 54.016, 69.625, 61.500, 62.125],
              [53.188, 49.000, 39.500, np.nan, 34.750, np.nan, 83.000, 44.500],
              [55.750, 50.000, 38.938, np.nan, 30.188, np.nan, 70.875, 29.938],
              [65.500, 51.063, 45.563, 69.313, 48.250, 62.375, 85.250, np.nan],
              [69.938, 47.000, 52.313, 71.016, np.nan, 59.359, 61.188, 48.219],
              [61.500, 44.188, 53.438, 57.000, 35.313, 55.813, 51.500, 62.188],
              [59.230, 48.210, 62.190, 61.390, 54.310, 70.170, 61.750, 91.080],
              [61.230, 48.700, 60.300, 68.580, 61.250, 70.340, np.nan, np.nan],
              [52.900, 52.690, 54.230, np.nan, 68.170, 70.600, 57.870, 88.640],
              [57.370, 59.040, 59.870, 62.090, 61.620, 66.470, 65.370, 85.840]])
```

```
m, n = P.shape
```

2.0.3 Compute the covariance, ignoring missing values

[4]:
```
def cov_bar(P):
    """Returns an approximate sample covariance matrix"""
    # P.shape returns a tuple (m, n) that we unpack to _m and n
    _m, n = P.shape # pylint: disable=unused-variable
    # Initialize an n-by-n zero matrix
    S = np.zeros((n, n))
    for i in range(n):
        # Take the ith column
        xi = P[:, i]
        for j in range(i+1):
            # Take the jth column, where j <= i
            xj = P[:, j]
            # Set mask such that all NaNs are True
            notp = np.isnan(xi) | np.isnan(xj)
            # Apply the mask to xi
            xim = np.ma.masked_array(xi, mask=notp)
            # Apply the mask to xj
            xjm = np.ma.masked_array(xj, mask=notp)
            S[i, j] = np.ma.dot(xim - np.mean(xim), xjm - np.mean(xjm))
            # Take the sum over ~notp to normalize
            S[i, j] = 1.0 / (sum(~notp) - 1) * S[i, j]
            S[j, i] = S[i, j]
    return S
```

[5]:
```
def cor_bar(P):
    """Returns an approximate sample correlation matrix"""
    S = cov_bar(P)
    D = np.diag(1.0 / np.sqrt(np.diag(S)))
    return D @ S @ D
```

2.0.4 Compute the *approximate* correlation matrix

[6]:
```
G = cor_bar(P)
print("The approximate correlation matrix \n{}".format(G))
```

```
The approximate correlation matrix
[[ 1.     -0.325   0.1881  0.576   0.0064 -0.6111 -0.0724 -0.1589]
 [-0.325   1.      0.2048  0.2436  0.4058  0.273   0.2869  0.4241]
 [ 0.1881  0.2048  1.     -0.1325  0.7658  0.2765 -0.6172  0.9006]
 [ 0.576   0.2436 -0.1325  1.      0.3041  0.0126  0.6452 -0.321 ]
 [ 0.0064  0.4058  0.7658  0.3041  1.      0.6652 -0.3293  0.9939]
```

```
[-0.6111  0.273   0.2765  0.0126  0.6652  1.      0.0492  0.5964]
[-0.0724  0.2869 -0.6172  0.6452 -0.3293  0.0492  1.     -0.3983]
[-0.1589  0.4241  0.9006 -0.321   0.9939  0.5964 -0.3983  1.    ]]
```

2.0.5 Compute the eigenvalues of our (indefinite) G.

- We see below that our matrix G is not a mathematically true correlation matrix.

[7]:
```
print("Sorted eigenvalues of G {}".format(np.sort(np.linalg.eig(G)[0])))
```

```
Sorted eigenvalues of G [-0.2498 -0.016   0.0895  0.2192  0.7072  1.7534  1.9611
3.5355]
```

3 Nearest Correlation Matrices

- Our problem now is to solve:

$$\min \frac{1}{2}\|G - X\|_F^2 = \min \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n}|G(i,j) - X(i,j)|^2$$

- In order to find X, a true correlation matrix, where G is an approximate correlation matrix.
- An algorithm by Qi and Sun (2006), applies an inexact Newton method to a dual (unconstrained) formulation of this problem.
- Improvements were suggested by Borsdorf and Higham (2010 MSc).
- It is globally and quadratically (fast!) convergent.
- This is implemented in NAG routine **library.correg.corrmat_nearest**.

3.1 Using corrmat_nearest to compute the nearest correlation matrix in the Frobenius norm

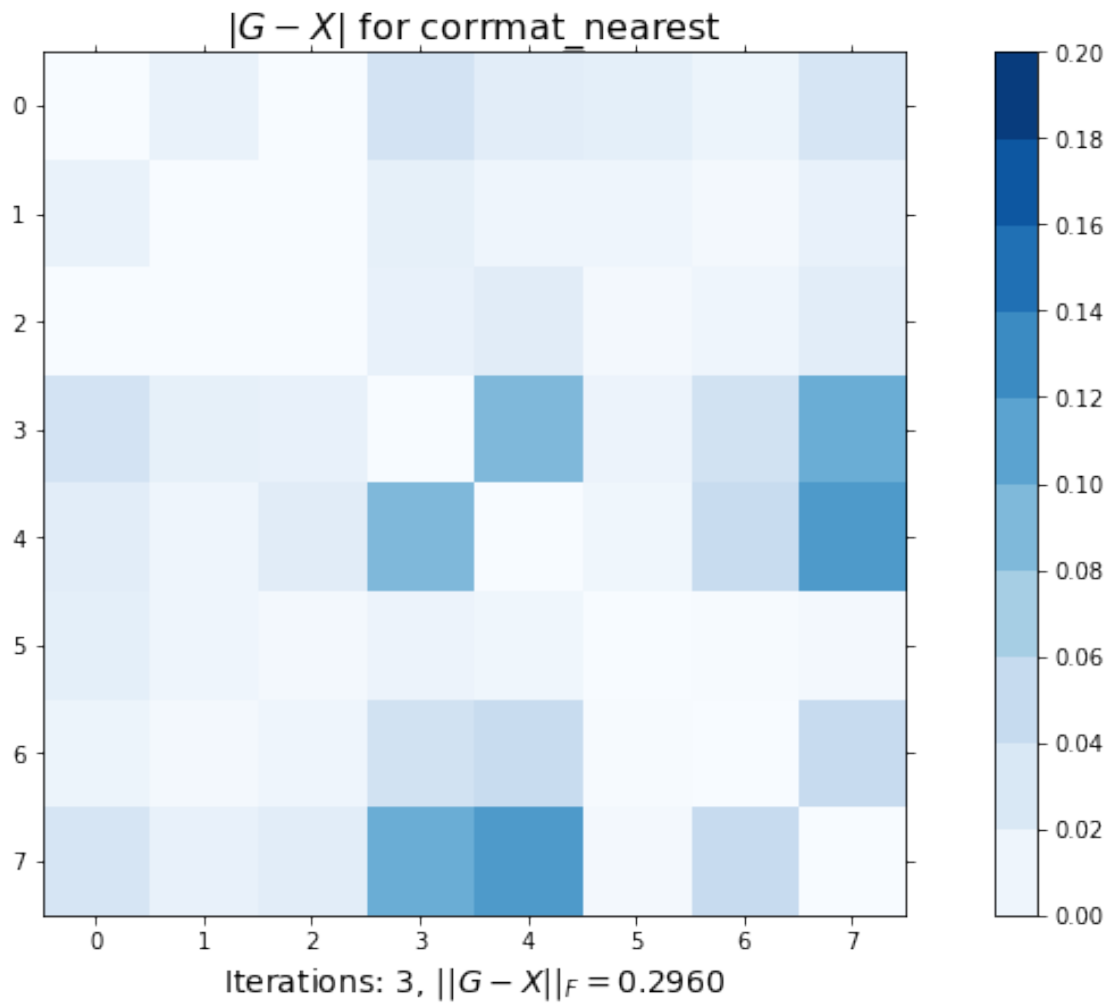
[8]:
```
# "Call NAG routine library.correg.corrmat_nearest and print the result
X, itr, _, _ = nl_correg.corrmat_nearest(G)
print("Nearest correlation matrix\n{}".format(X))
```

```
Nearest correlation matrix
[[ 1.     -0.3112  0.1889  0.5396  0.0268 -0.5925 -0.0621 -0.1921]
 [-0.3112  1.      0.205   0.2265  0.4148  0.2822  0.2915  0.4088]
 [ 0.1889  0.205   1.     -0.1468  0.788   0.2727 -0.6085  0.8802]
 [ 0.5396  0.2265 -0.1468  1.      0.2137  0.0015  0.6069 -0.2208]
 [ 0.0268  0.4148  0.788   0.2137  1.      0.658  -0.2812  0.8762]
 [-0.5925  0.2822  0.2727  0.0015  0.658   1.      0.0479  0.5932]
 [-0.0621  0.2915 -0.6085  0.6069 -0.2812  0.0479  1.     -0.447 ]
 [-0.1921  0.4088  0.8802 -0.2208  0.8762  0.5932 -0.447   1.    ]]
```

[9]:
```
print("Sorted eigenvalues of X [{0}]".format(
    ''.join(
        ['{:.4f} '.format(x) for x in np.sort(np.linalg.eig(X)[0])]
    )
))
```

```
Sorted eigenvalues of X [-0.0000 0.0000 0.0380 0.1731 0.6894 1.7117 1.9217
3.4661 ]
```

[10]:
```
# Plot the difference between G and X as a small shaded square for each element
fig1, ax1 = plt.subplots(figsize=(14, 7))
cax1 = ax1.imshow(abs(X-G), interpolation='none', cmap=plt.cm.Blues,
                  vmin=0, vmax=0.2)
cbar = fig1.colorbar(cax1, ticks = np.linspace(0.0, 0.2, 11, endpoint=True),
                     boundaries=np.linspace(0.0, 0.2, 11, endpoint=True))
cbar.mappable.set_clim([0, 0.2])
ax1.tick_params(axis='both', which='both',
                bottom='off', top='off', left='off', right='off',
                labelbottom='off', labelleft='off')
ax1.set_title(r'$|G-X|$ for corrmat_nearest', fontsize=16)
plt.xlabel(
    r'Iterations: {0}, $||G-X||_F = {1:.4f}$'.format(itr, np.linalg.norm(X-G)),
    fontsize=14,
)
plt.show()
```



4 Weighting rows and columns of elements

- Now, we note that for Stocks A to C we have a complete set of observations.

$$P = \begin{bmatrix} 59.875 & 42.734 & 47.938 & 60.359 & 54.016 & 69.625 & 61.500 & 62.125 \\ 53.188 & 49.000 & 39.500 & \text{NaN} & 34.750 & \text{NaN} & 83.000 & 44.500 \\ 55.750 & 50.000 & 38.938 & \text{NaN} & 30.188 & \text{NaN} & 70.875 & 29.938 \\ 65.500 & 51.063 & 45.563 & 69.313 & 48.250 & 62.375 & 85.250 & \text{NaN} \\ 69.938 & 47.000 & 52.313 & 71.016 & \text{NaN} & 59.359 & 61.188 & 48.219 \\ 61.500 & 44.188 & 53.438 & 57.000 & 35.313 & 55.813 & 51.500 & 62.188 \\ 59.230 & 48.210 & 62.190 & 61.390 & 54.310 & 70.170 & 61.750 & 91.080 \\ 61.230 & 48.700 & 60.300 & 68.580 & 61.250 & 70.340 & \text{NaN} & \text{NaN} \\ 52.900 & 52.690 & 54.230 & \text{NaN} & 68.170 & 70.600 & 57.870 & 88.640 \\ 57.370 & 59.040 & 59.870 & 62.090 & 61.620 & 66.470 & 65.370 & 85.840 \end{bmatrix}.$$

- Perhaps we wish to preserve part of the correlation matrix?
- We could solve the *weighted* problem, NAG routine **library.correg.corrmat_nearest_bounded**

$$\|W^{\frac{1}{2}}(G - X)W^{\frac{1}{2}}\|_F$$

- Here W is a diagonal matrix.
- We can also force the resulting matrix to be positive definite.

4.0.1 Use corrmat_nearest_bounded to compute the nearest correlation matrix with row and column weighting

[11]:
```
# Define an array of weights
W = np.array([10, 10, 10, 1, 1, 1, 1, 1], dtype = np.float64)
```

[12]:
```
# Set up and call the NAG routine using weights and a minimum eigenvalue
opt = 'B'
alpha = 0.001
X, itr, _, _ = nl_correg.corrmat_nearest_bounded(G, opt, alpha, W)
print("Nearest correlation matrix using row and column weighting\n{}".format(X))
```

```
Nearest correlation matrix using row and column weighting
[[ 1.     -0.325   0.1881  0.5739  0.0067 -0.6097 -0.0722 -0.1598]
 [-0.325   1.      0.2048  0.2426  0.406   0.2737  0.287   0.4236]
 [ 0.1881  0.2048  1.     -0.1322  0.7661  0.2759 -0.6171  0.9004]
 [ 0.5739  0.2426 -0.1322  1.      0.2085 -0.089   0.5954 -0.1805]
 [ 0.0067  0.406   0.7661  0.2085  1.      0.6556 -0.278   0.8757]
 [-0.6097  0.2737  0.2759 -0.089   0.6556  1.      0.049   0.5746]
 [-0.0722  0.287  -0.6171  0.5954 -0.278   0.049   1.     -0.455 ]
 [-0.1598  0.4236  0.9004 -0.1805  0.8757  0.5746 -0.455   1.    ]]
```

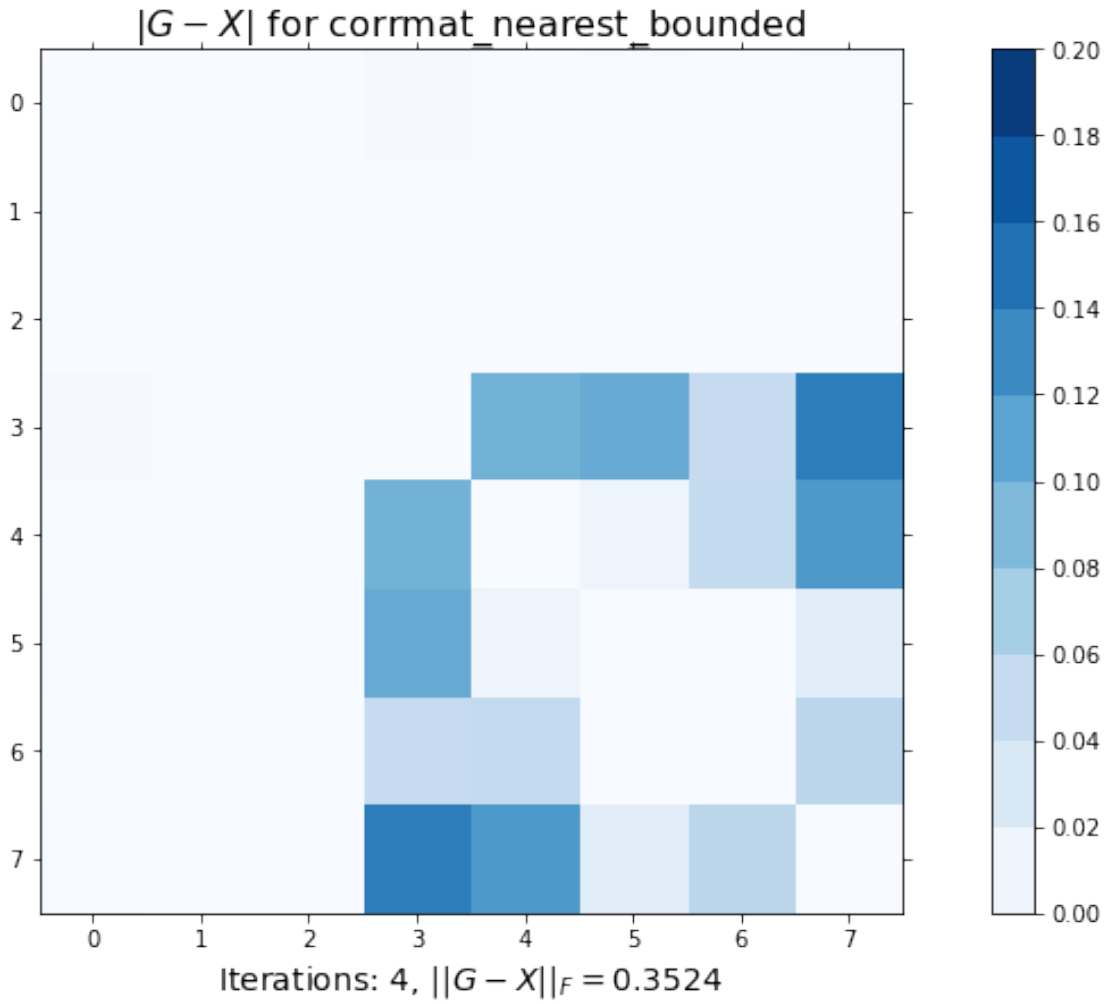
[13]:
```
print("Sorted eigenvalues of X [{0}]".format(
    ''.join(
        ['{:.4f} '.format(x) for x in np.sort(np.linalg.eig(X)[0])]
    )
))
```

```
Sorted eigenvalues of X [0.0010 0.0010 0.0305 0.1646 0.6764 1.7716 1.8910 3.4639
]
```

[14]:
```
fig1, ax1 = plt.subplots(figsize=(14, 7))
cax1 = ax1.imshow(abs(X-G), interpolation='none', cmap=plt.cm.Blues, vmin=0,
                  vmax=0.2)
cbar = fig1.colorbar(cax1, ticks = np.linspace(0.0, 0.2, 11, endpoint=True),
                     boundaries=np.linspace(0.0, 0.2, 11, endpoint=True))
```

```
cbar.mappable.set_clim([0, 0.2])
ax1.tick_params(axis='both', which='both',
                bottom='off', top='off', left='off', right='off',
                labelbottom='off', labelleft='off')

ax1.set_title(r'$|G-X|$ for corrmat_nearest_bounded', fontsize=16)
plt.xlabel(
    r'Iterations: {0}, $||G-X||_F = {1:.4f}$'.format(itr, np.linalg.norm(X-G)),
    fontsize=14,
)
plt.show()
```



5 Weighting Individual Elements

- Would it be better to be able to *weight individual elements* in our approximate matrix?
- In our example the top left 3 by 3 block of exact correlations, perhaps.
- Element-wise weighting means we wish to find the minimum of

$$\|H \circ (G - X)\|_F$$

- So individually $h_{ij} \times (g_{ij}–x_{ij})$.
- However, this is a more "difficult" problem, and more computationally expensive.
- This is implemented in the NAG routine **library.correg.corrmat_h_weight**.

5.0.1 Use corrmat_h_weight to compute the nearest correlation matrix with element-wise weighting

[15]:
```
# Set up a matrix of weights
H = np.ones([n, n])
H[:3, :3] = 100
H
```

[15]:
```
array([[100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
       [100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
       [100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
       [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
       [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
       [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
       [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
       [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.]])
```

[16]:
```
# "Call the NAG routine specifying a minimum eigenvalue
alpha = 0.001
X, itr, _ = nl_correg.corrmat_h_weight(G, alpha, H, maxit=200)
print("Nearest correlation matrix using element-wise weighting\n{}".format(X))
```

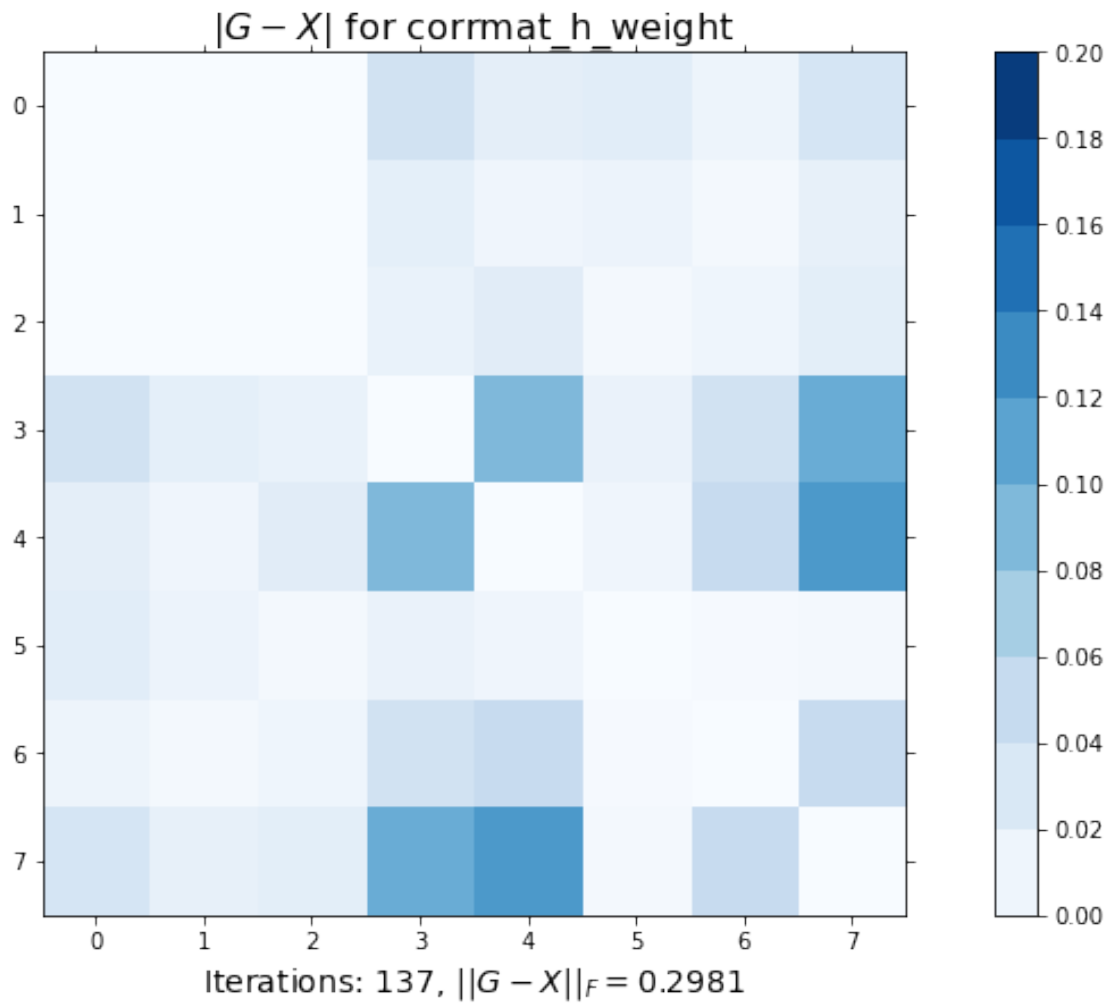
```
Nearest correlation matrix using element-wise weighting
[[ 1.     -0.3251  0.1881  0.5371  0.0255 -0.5893 -0.0625 -0.1929]
 [-0.3251  1.      0.2048  0.2249  0.4144  0.2841  0.2914  0.4081]
 [ 0.1881  0.2048  1.     -0.1462  0.7883  0.2718 -0.6084  0.8804]
 [ 0.5371  0.2249 -0.1462  1.      0.2138 -0.0002  0.607  -0.2199]
 [ 0.0255  0.4144  0.7883  0.2138  1.      0.6566 -0.2807  0.8756]
 [-0.5893  0.2841  0.2718 -0.0002  0.6566  1.      0.0474  0.593 ]
 [-0.0625  0.2914 -0.6084  0.607  -0.2807  0.0474  1.     -0.4471]
 [-0.1929  0.4081  0.8804 -0.2199  0.8756  0.593  -0.4471  1.    ]]
```

[17]:
```
print("Sorted eigenvalues of X [{0}]".format(
    ''.join(
        ['{:.4f} '.format(x) for x in np.sort(np.linalg.eig(X)[0])]
    )
))
```

```
Sorted eigenvalues of X [0.0010-0.0000j 0.0010+0.0000j 0.0375+0.0000j
0.1734+0.0000j 0.6882+0.0000j 1.7106+0.0000j 1.9224+0.0000j 3.4660+0.0000j ]
```

[18]:
```
fig1, ax1 = plt.subplots(figsize=(14, 7))
cax1 = ax1.imshow(abs(X-G), interpolation='none', cmap=plt.cm.Blues, vmin=0,
                  vmax=0.2)
cbar = fig1.colorbar(cax1, ticks = np.linspace(0.0, 0.2, 11, endpoint=True),
                     boundaries=np.linspace(0.0, 0.2, 11, endpoint=True))
cbar.mappable.set_clim([0, 0.2])
ax1.tick_params(axis='both', which='both',
                bottom='off', top='off', left='off', right='off',
                labelbottom='off', labelleft='off')

ax1.set_title(r'$|G-X|$ for corrmat_h_weight', fontsize=16)
plt.xlabel(
    r'Iterations: {0}, $||G-X||_F = {1:.4f}$'.format(itr, np.linalg.norm(X-G)),
    fontsize=14,
)
plt.show()
```



6 Fixing a Block of Elements

- We probably really wish to *fix* our leading block of true correlations, so it does not change at all.
- We have the NAG routine **library.correg.corrmat_shrinking**.
- This routine fixes a leading block, which we require to be positive definite.
- We apply the *shrinking algorithm* of Higham, Strabić and Šego. The approach is **not** computationally expensive.
- What we find is the smallest , such that X is a true correlation matrix:

$$X = \alpha \begin{pmatrix} G_{11} & 0 \\ 0 & I \end{pmatrix} + (1-\alpha)G, \qquad G = \begin{pmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{pmatrix}$$

- G_{11} is the leading k by k block of the approximate correlation matrix that we wish to fix.
- α is in the interval $[0, 1]$.

6.0.1 Use corrmat_shrinking to compute the nearest correlation matrix with fixed leading block

[19]:
```
# Call the NAG routine fixing the top 3-by-3 block
k = 3
X, alpha, itr, _, _ = nl_correg.corrmat_shrinking(G, k)
print("Nearest correlation matrix with fixed leading block \n{}".format(X))
```

```
Nearest correlation matrix with fixed leading block
[[ 1.     -0.325   0.1881  0.4606  0.0051 -0.4887 -0.0579 -0.1271]
 [-0.325   1.      0.2048  0.1948  0.3245  0.2183  0.2294  0.3391]
 [ 0.1881  0.2048  1.     -0.106   0.6124  0.2211 -0.4936  0.7202]
 [ 0.4606  0.1948 -0.106   1.      0.2432  0.0101  0.516  -0.2567]
 [ 0.0051  0.3245  0.6124  0.2432  1.      0.532  -0.2634  0.7949]
 [-0.4887  0.2183  0.2211  0.0101  0.532   1.      0.0393  0.4769]
 [-0.0579  0.2294 -0.4936  0.516  -0.2634  0.0393  1.     -0.3185]
 [-0.1271  0.3391  0.7202 -0.2567  0.7949  0.4769 -0.3185  1.    ]]
```

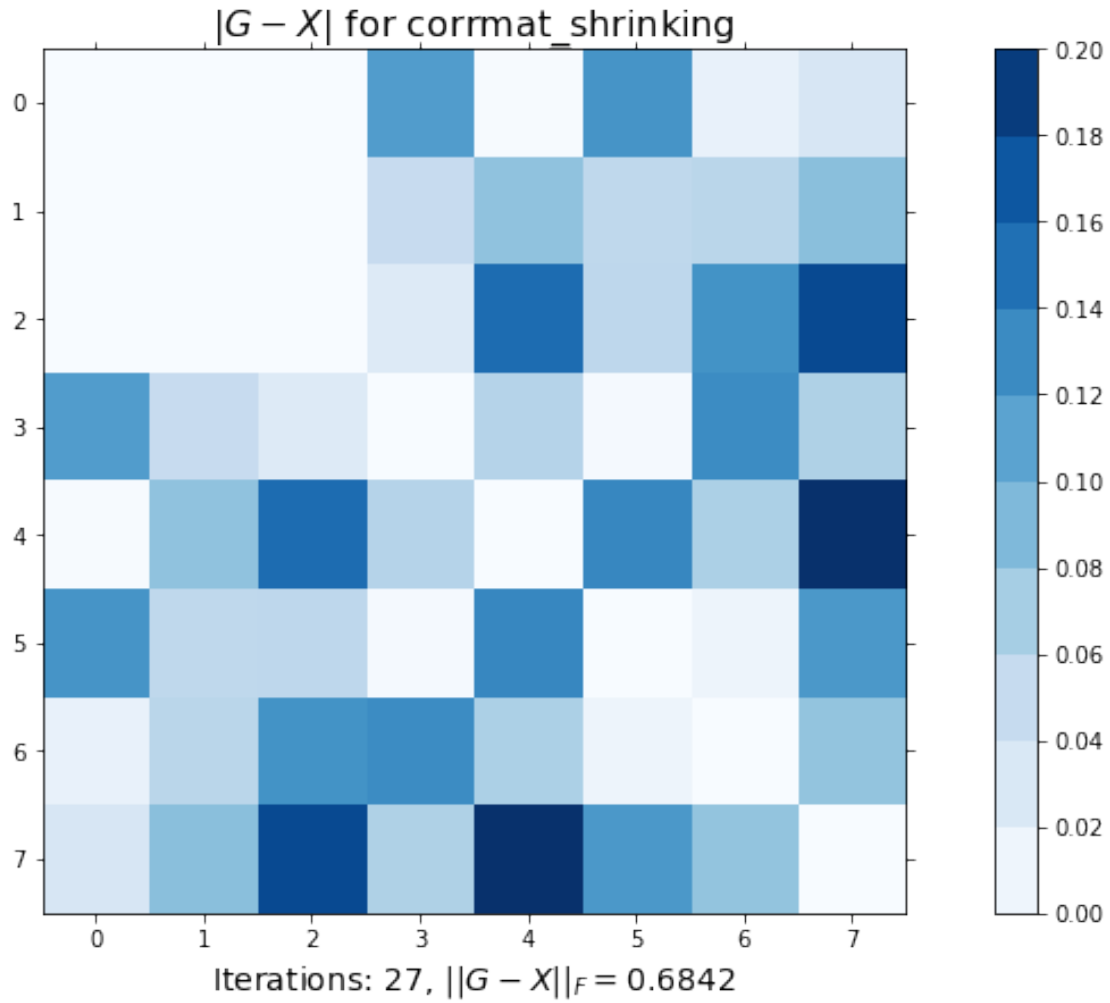
[20]:
```
print("Sorted eigenvalues of X [{0}]".format(
    ''.join(
        ['{:.4f} '.format(x) for x in np.sort(np.linalg.eig(X)[0])]
    )
))
print("Value of alpha returned: {:.4f}".format(alpha))
```

```
Sorted eigenvalues of X [0.0000 0.1375 0.2744 0.3804 0.7768 1.6263 1.7689 3.0356
]
Value of alpha returned: 0.2003
```

[21]:
```
fig1, ax1 = plt.subplots(figsize=(14, 7))
cax1 = ax1.imshow(abs(X-G), interpolation='none', cmap=plt.cm.Blues, vmin=0,
                  vmax=0.2)
cbar = fig1.colorbar(cax1, ticks = np.linspace(0.0, 0.2, 11, endpoint=True),
                     boundaries=np.linspace(0.0, 0.2, 11, endpoint=True))
cbar.mappable.set_clim([0, 0.2])
ax1.tick_params(axis='both', which='both',
                bottom='off', top='off', left='off', right='off',
                labelbottom='off', labelleft='off')

ax1.set_title(r'$|G-X|$ for corrmat_shrinking', fontsize=16)
plt.xlabel(
    r'Iterations: {0}, $||G-X||_F = {1:.4f}$'.format(itr, np.linalg.norm(X-G)),
    fontsize=14,
```

```
)
plt.show()
```



7 Fixing Arbitrary Elements

- The routine **library.correg.corrmat_target** fixes arbitrary elements by finding the smallest , such that X is a true correlation matrix in:

$$X = \alpha T + (1 - \alpha)G, \quad T = H \circ G, \quad h_{ij} \in [0, 1]$$

- A "1" in H fixes corresponding elements in G.
- $0 < h_{ij} < 1$ weights corresponding element in G.
- α is again in the interval $[0, 1]$.

7.1 Alternating Projections

- First method proposed to solve our original problem, however, it is very slow.
- The idea is we alternate projecting onto two sets, which are:
  - the set of smeidefinite matrices (S1), and
  - matrices with unit diagonal (s2)
- We do this until we converge on a matrix with both properties.

7.2 Alternating Projections with Anderson Acceleration

- A new approach by Higham and Strabić uses *Anderson Acceleration*, and makes the method worthwhile.
- In particular, we will be able to fix elements whilst finding the nearest true correlation matrix in the Frobenius norm.
- Our projections are now:
  - the set of (semi)definite matrices with some minimum eigenvalue, and
  - matrix with elements $G_{i,j}$ for some given indices i and j
- To appear in a future NAG Library.

8 More on using the NAG Library for *Python*:

https://www.nag.com/nag-library-python