ncm_nag

December 18, 2020

1 Nearest Correlation Matrices

This notebook looks at computing *nearest correlation matrices* using the NAG Library for *Java*.

All the calculations happen in the NcmNag.java file and they are presented here using *Python*. For a better understanding, it is suggested to have open NcmNag.java to look up the different functions used.

1.1 Correlation Matrices

- An n by n matrix is a correlation matrix if:
  - it is symmetric
  - it has ones on the diagonal
  - its eigenvalues are non-negative (positive semidefinite)

$$Ax = \lambda x, \quad x \neq 0$$

- The element in the ith row and jth column is the correlation between the ith and jth variables. This could be stock process, for example.

1.2 Empirical Correlation Matrices

- Empirical correlation matrices are often **not mathematically true** due to inconsistent or missing data.
- Thus we are required to find a true correlation matrix, where our input, G, is an approximate correlation matrix.
- In particular we seek the *nearest* correlation matrix, in most cases.

1.3 Computing Correlation Matrices

- The vector p_i, the ith column of a matrix, P, holds the m observations of the ith variable, of which there are n. $\bar{p}_i$ is the sample mean.

$$S_{ij} = \frac{1}{m-1}(p_i - \bar{p}_i)^T(p_j - \bar{p}_j)$$

- S is a covariance matrix, with S_{ij} the covariance between variables i and j
- R is the corresponding correlation matrix, given by:

$$D_S^{1/2} = \text{ diag}(s_{11}^{-1/2}, s_{22}^{-1/2}, \ldots, s_{nn}^{-1/2})$$

$$R = D_S^{1/2} S D_S^{1/2}$$

1.4 Approximate Correlation Matrices

- Now, what if we don't have all observations for each variable?
- We compute each covariance with observations that are available for *both* the ith and jth variable.
- For example NAG routine **G02BB**.
- We then compute the correlation matrix as before.

2 Missing Stock Price Example

- Prices for 8 stocks on the first working day of 10 consecutive months.

	Stock A	Stock B	Stock C	Stock D	Stock E	Stock F	Stock G	Stock H
Month 1	59.875	42.734	47.938	60.359	54.016	69.625	61.500	62.125
Month 2	53.188	49.000	39.500		34.750		83.000	44.500
Month 3	55.750	50.000	38.938		30.188		70.875	29.938
Month 4	65.500	51.063	45.563	69.313	48.250	62.375	85.250	
Month 5	69.938	47.000	52.313	71.016		59.359	61.188	48.219
Month 6	61.500	44.188	53.438	57.000	35.313	55.813	51.500	62.188
Month 7	59.230	48.210	62.190	61.390	54.310	70.170	61.750	91.080
Month 8	61.230	48.700	60.300	68.580	61.250	70.340		
Month 9	52.900	52.690	54.230		68.170	70.600	57.870	88.640
Month 10	57.370	59.040	59.870	62.090	61.620	66.470	65.370	85.840

- We will use NaNs where there is missing data.
- So our $P = [p_1, p_2, \ldots, p_n]$ is:

$$P = \begin{bmatrix} 59.875 & 42.734 & 47.938 & 60.359 & 54.016 & 69.625 & 61.500 & 62.125 \\ 53.188 & 49.000 & 39.500 & \text{NaN} & 34.750 & \text{NaN} & 83.000 & 44.500 \\ 55.750 & 50.000 & 38.938 & \text{NaN} & 30.188 & \text{NaN} & 70.875 & 29.938 \\ 65.500 & 51.063 & 45.563 & 69.313 & 48.250 & 62.375 & 85.250 & \text{NaN} \\ 69.938 & 47.000 & 52.313 & 71.016 & \text{NaN} & 59.359 & 61.188 & 48.219 \\ 61.500 & 44.188 & 53.438 & 57.000 & 35.313 & 55.813 & 51.500 & 62.188 \\ 59.230 & 48.210 & 62.190 & 61.390 & 54.310 & 70.170 & 61.750 & 91.080 \\ 61.230 & 48.700 & 60.300 & 68.580 & 61.250 & 70.340 & \text{NaN} & \text{NaN} \\ 52.900 & 52.690 & 54.230 & \text{NaN} & 68.170 & 70.600 & 57.870 & 88.640 \\ 57.370 & 59.040 & 59.870 & 62.090 & 61.620 & 66.470 & 65.370 & 85.840 \end{bmatrix}.$$

- And to compute the covariance between the 3rd and 4th variables:

$$\begin{aligned} v_1^T &= [47.938, 45.563, 52.313, 53.438, 62.190, 60.300, 59.870] \\ v_2^T &= [60.359, 69.313, 71.016, 57.000, 61.390, 68.580, 62.090] \\ S_{3,4} &= \frac{1}{6}(v_1 - \bar{v}_1)^T(v_2 - \bar{v}_2) \end{aligned}$$

- Let's compute this in Java.

2.0.1 Import required modules, set print options and define the read_file functions for *Python*

2.0.2 Initialize our *P* matrix of observations

```
// Define a 2-d array and use Double.NaN to set elements as NaNs
double[][] P = new double[][] {
        { 59.875, 42.734, 47.938, 60.359, 54.016, 69.625, 61.500, 62.125 },
        { 53.188, 49.000, 39.500, Double.NaN, 34.750, Double.NaN, 83.000, 44.500 },
        { 55.750, 50.000, 38.938, Double.NaN, 30.188, Double.NaN, 70.875, 29.938 },
        { 65.500, 51.063, 45.563, 69.313, 48.250, 62.375, 85.250, Double.NaN },
        { 69.938, 47.000, 52.313, 71.016, Double.NaN, 59.359, 61.188, 48.219 },
        { 61.500, 44.188, 53.438, 57.000, 35.313, 55.813, 51.500, 62.188 },
        { 59.230, 48.210, 62.190, 61.390, 54.310, 70.170, 61.750, 91.080 },
        { 61.230, 48.700, 60.300, 68.580, 61.250, 70.340, Double.NaN, Double.NaN },
        { 52.900, 52.690, 54.230, Double.NaN, 68.170, 70.600, 57.870, 88.640 },
        { 57.370, 59.040, 59.870, 62.090, 61.620, 66.470, 65.370, 85.840 } };
```

2.0.3 Compute the covariance, ignoring missing values

```
public static double[][] cov_bar(double[][] P) {
    double[] xi, xj;
    boolean[] xib, xjb, notp;
    int n = P[0].length;
    double[][] S = new double[n][n];
```

```
    int notpFalseCount;

    for (int i = 0; i < n; i++) {
        // Take the ith column
        xi = getMatrixColumn(P, i);

        for (int j = 0; j < i + 1; j++) {
            // Take the jth column, where j <= i
            xj = getMatrixColumn(P, j);

            // Set mask such that all NaNs are true
            xib = getNanMask(xi);
            xjb = getNanMask(xj);

            notp = addBoolArrOr(xib, xjb);

            // S[i][j] = (xi - mean(xi)) * (xj - mean(xj))
            S[i][j] = matrixMaskedDot(vectorSubScalar(xi, vectorMaskedMean(xi, notp)),
                    vectorSubScalar(xj, vectorMaskedMean(xj, notp)), notp);

            // Take the sum over !notp to normalize
            notpFalseCount = 0;
            for (boolean b : notp) {
                if (!b) {
                    notpFalseCount++;
                }
            }
            S[i][j] = 1.0 / (notpFalseCount - 1) * S[i][j];
            S[j][i] = S[i][j];
        }
    }
    return S;
}

public static double[][] cor_bar(double[][] P) {
    double[][] S, D;
    S = cov_bar(P);
    // D = 1.0 / SQRT(S)
    D = getMatrixFromDiag(vectorRightDiv(vectorSqrt(getMatrixDiag(S)), 1.0));

    // S_ = S * D
    F01CK f01ck = new F01CK();
    double[] S_ = new double[S.length * S[0].length];
    double[] S1d = convert2DTo1D(S);
    double[] D1d = convert2DTo1D(D);
    int n = S.length;
    int p = n;
    int m = n;
```

```
    double[] z = new double[0];
    int iz = 0;
    int opt = 1;
    int ifail = 0;
    f01ck.eval(S_, S1d, D1d, n, p, m, z, iz, opt, ifail);

    // D_ = D * S_
    double[] D_ = new double[n * n];
    f01ck.eval(D_, D1d, S_, n, p, m, z, iz, opt, ifail);

    return convert1DTo2D(D_, n);
}
```

2.0.4 Compute the *approximate* correlation matrix

```
double[][] G = cor_bar(P);
```

```
The approximate correlation matrix
[[ 1.     -0.325   0.1881  0.576   0.0064 -0.6111 -0.0724 -0.1589]
 [-0.325   1.      0.2048  0.2436  0.4058  0.273   0.2869  0.4241]
 [ 0.1881  0.2048  1.     -0.1325  0.7658  0.2765 -0.6172  0.9006]
 [ 0.576   0.2436 -0.1325  1.      0.3041  0.0126  0.6452 -0.321 ]
 [ 0.0064  0.4058  0.7658  0.3041  1.      0.6652 -0.3293  0.9939]
 [-0.6111  0.273   0.2765  0.0126  0.6652  1.      0.0492  0.5964]
 [-0.0724  0.2869 -0.6172  0.6452 -0.3293  0.0492  1.     -0.3983]
 [-0.1589  0.4241  0.9006 -0.321   0.9939  0.5964 -0.3983  1.    ]]
```

2.0.5 Compute the eigenvalues of our (indefinite) G.

- We see below that our matrix G is not a mathematically true correlation matrix.

```
F08NA f08na = new F08NA();
String jobvl = "N";
String jobvr = "N";
int n = G[0].length;
double[] G1d = convert2DTo1D(G);
int lda = G.length;
double[] wr = new double[n];
double[] wi = new double[n];
int ldvl = 1;
double[] vl = new double[ldvl];
int ldvr = 1;
double[] vr = new double[ldvr];
int lwork = 3 * n;
double[] work = new double[lwork];
int info = 0;
```

```
f08na.eval(jobvl, jobvr, n, G1d, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info);
Arrays.sort(wr);
```

```
Sorted eigenvalues of G [-0.2498 -0.016   0.0895  0.2192  0.7072  1.7534  1.9611
3.5355]
```

3 Nearest Correlation Matrices

- Our problem now is to solve:

$$\min \frac{1}{2}\|G - X\|_F^2 = \min \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n}|G(i,j) - X(i,j)|^2$$

- In order to find X, a true correlation matrix, where G is an approximate correlation matrix.
- An algorithm by Qi and Sun (2006), applies an inexact Newton method to a dual (unconstrained) formulation of this problem.
- Improvements were suggested by Borsdorf and Higham (2010 MSc).
- It is globally and quadratically (fast!) convergent.
- This is implemented in NAG routine **G02AA**.

3.1 Using G02AA to compute the nearest correlation matrix in the Frobenius norm

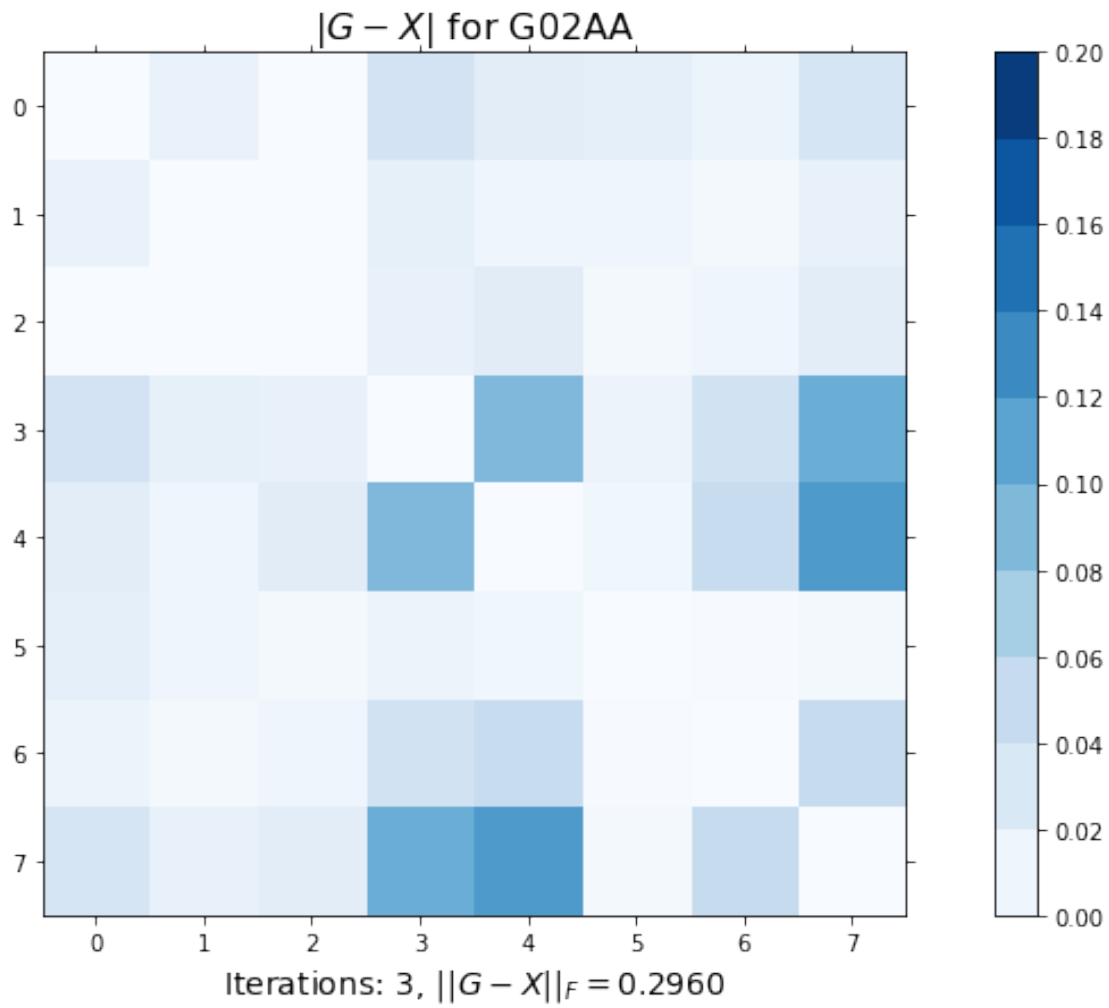
```
// Call NAG routine G02AA and print the result
G02AA g02aa = new G02AA();
G1d = convert2DTo1D(G);
n = G.length;
int ldg = n;
int ldx = n;
double errtol = 0.0;
int maxits = 0;
int maxit = 0;
double[] X1d = new double[ldx * n];
int iter = 0;
int feval = 0;
double nrmgrd = 0.0;
int ifail = 0;
g02aa.eval(G1d, ldg, n, errtol, maxits, maxit, X1d, ldx, iter, feval, nrmgrd, ifail);

double[][] X = convert1DTo2D(X1d, ldx);
iter = g02aa.getITER();
```

```
Nearest correlation matrix
[[ 1.     -0.3112  0.1889  0.5396  0.0268 -0.5925 -0.0621 -0.1921]
```

```
 [-0.3112  1.      0.205   0.2265  0.4148  0.2822  0.2915  0.4088]
 [ 0.1889  0.205   1.     -0.1468  0.788   0.2727 -0.6085  0.8802]
 [ 0.5396  0.2265 -0.1468  1.      0.2137  0.0015  0.6069 -0.2208]
 [ 0.0268  0.4148  0.788   0.2137  1.      0.658  -0.2812  0.8762]
 [-0.5925  0.2822  0.2727  0.0015  0.658   1.      0.0479  0.5932]
 [-0.0621  0.2915 -0.6085  0.6069 -0.2812  0.0479  1.     -0.447 ]
 [-0.1921  0.4088  0.8802 -0.2208  0.8762  0.5932 -0.447   1.    ]]
```

```
jobvl = "N";
jobvr = "N";
n = X[0].length;
lda = X.length;
wr = new double[n];
wi = new double[n];
ldvl = 1;
vl = new double[ldvl];
ldvr = 1;
vr = new double[ldvr];
lwork = 3 * n;
work = new double[lwork];
info = 0;
f08na.eval(jobvl, jobvr, n, X1d, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info);
Arrays.sort(wr);
```

```
Sorted eigenvalues of X [-0.      0.      0.038   0.1731  0.6894  1.7117  1.9217
3.4661]
```



4 Weighting rows and columns of elements

- Now, we note that for Stocks A to C we have a complete set of observations.

$$P = \begin{bmatrix} 59.875 & 42.734 & 47.938 & 60.359 & 54.016 & 69.625 & 61.500 & 62.125 \\ 53.188 & 49.000 & 39.500 & \text{NaN} & 34.750 & \text{NaN} & 83.000 & 44.500 \\ 55.750 & 50.000 & 38.938 & \text{NaN} & 30.188 & \text{NaN} & 70.875 & 29.938 \\ 65.500 & 51.063 & 45.563 & 69.313 & 48.250 & 62.375 & 85.250 & \text{NaN} \\ 69.938 & 47.000 & 52.313 & 71.016 & \text{NaN} & 59.359 & 61.188 & 48.219 \\ 61.500 & 44.188 & 53.438 & 57.000 & 35.313 & 55.813 & 51.500 & 62.188 \\ 59.230 & 48.210 & 62.190 & 61.390 & 54.310 & 70.170 & 61.750 & 91.080 \\ 61.230 & 48.700 & 60.300 & 68.580 & 61.250 & 70.340 & \text{NaN} & \text{NaN} \\ 52.900 & 52.690 & 54.230 & \text{NaN} & 68.170 & 70.600 & 57.870 & 88.640 \\ 57.370 & 59.040 & 59.870 & 62.090 & 61.620 & 66.470 & 65.370 & 85.840 \end{bmatrix}.$$

- Perhaps we wish to preserve part of the correlation matrix?
- We could solve the *weighted* problem, NAG routine **G02AB**

$$\|W^{\frac{1}{2}}(G - X)W^{\frac{1}{2}}\|_F$$

- Here W is a diagonal matrix.
- We can also force the resulting matrix to be positive definite.

4.0.1 Use G02AB to compute the nearest correlation matrix with row and column weighting

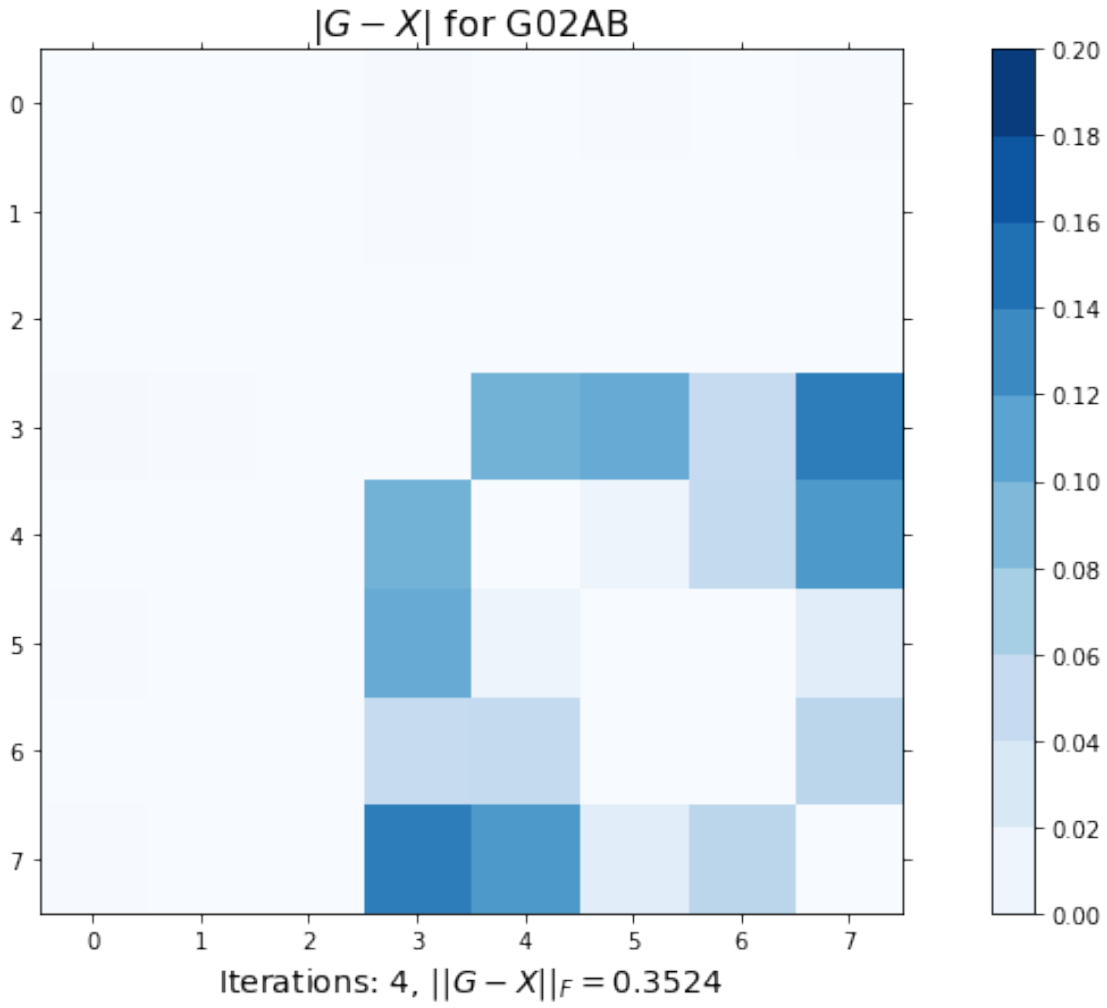
```
// Define an arrray of weights
double[] W = new double[] { 10, 10, 10, 1, 1, 1, 1, 1 };

// Set up and call the NAG routine using weights and a minimum eigenvalue
G02AB g02ab = new G02AB();
G1d = convert2DTo1D(G);
ldg = G.length;
n = G[0].length;
String opt = "B";
double alpha = 0.001;
errtol = 0.0;
maxits = 0;
maxit = 0;
ldx = n;
X1d = new double[ldx * n];
iter = 0;
feval = 0;
nrmgrd = 0;
ifail = 0;
g02ab.eval(G1d, ldg, n, opt, alpha, W, errtol, maxits, maxit, X1d, ldx, iter, feval, nrmgrd, i

X = convert1DTo2D(X1d, ldx);
iter = g02ab.getITER();
```

```
Nearest correlation matrix using row and column weighting
[[ 1.     -0.325   0.1881  0.5739  0.0067 -0.6097 -0.0722 -0.1598]
 [-0.325   1.      0.2048  0.2426  0.406   0.2737  0.287   0.4236]
 [ 0.1881  0.2048  1.     -0.1322  0.7661  0.2759 -0.6171  0.9004]
 [ 0.5739  0.2426 -0.1322  1.      0.2085 -0.089   0.5954 -0.1805]
 [ 0.0067  0.406   0.7661  0.2085  1.      0.6556 -0.278   0.8757]
 [-0.6097  0.2737  0.2759 -0.089   0.6556  1.      0.049   0.5746]
 [-0.0722  0.287  -0.6171  0.5954 -0.278   0.049   1.     -0.455 ]
 [-0.1598  0.4236  0.9004 -0.1805  0.8757  0.5746 -0.455   1.    ]]
```

```
jobvl = "N";
jobvr = "N";
```

```
n = X[0].length;
lda = X.length;
wr = new double[n];
wi = new double[n];
ldvl = 1;
vl = new double[ldvl];
ldvr = 1;
vr = new double[ldvr];
lwork = 3 * n;
work = new double[lwork];
info = 0;
f08na.eval(jobvl, jobvr, n, X1d, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info);
Arrays.sort(wr);
```

```
Sorted eigenvalues of X [0.001  0.001  0.0305 0.1646 0.6764 1.7716 1.891
3.4639]
```



5 Weighting Individual Elements

- Would it be better to be able to *weight individual elements* in our approximate matrix?
- In our example the top left 3 by 3 block of exact correlations, perhaps.
- Element-wise weighting means we wish to find the minimum of

$$\|H \circ (G - X)\|_F$$

- So individually $h_{ij} \times (g_{ij}–x_{ij})$.
- However, this is a more "difficult" problem, and more computationally expensive.
- This is implemented in the NAG routine **G02AJ**.

5.0.1 Use G02AJ to compute the nearest correlation matrix with element-wise weighting

```
// Set up a matrix of weights
n = P[0].length;
double[][] H = new double[n][n];
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        if ((i < 3) && (j < 3)) {
            H[i][j] = 100.0;
        } else {
            H[i][j] = 1;
        }
    }
}
```

```
 array([[100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
        [100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
        [100., 100., 100.,   1.,   1.,   1.,   1.,   1.],
        [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
        [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
        [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
        [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.],
        [  1.,   1.,   1.,   1.,   1.,   1.,   1.,   1.]])
```

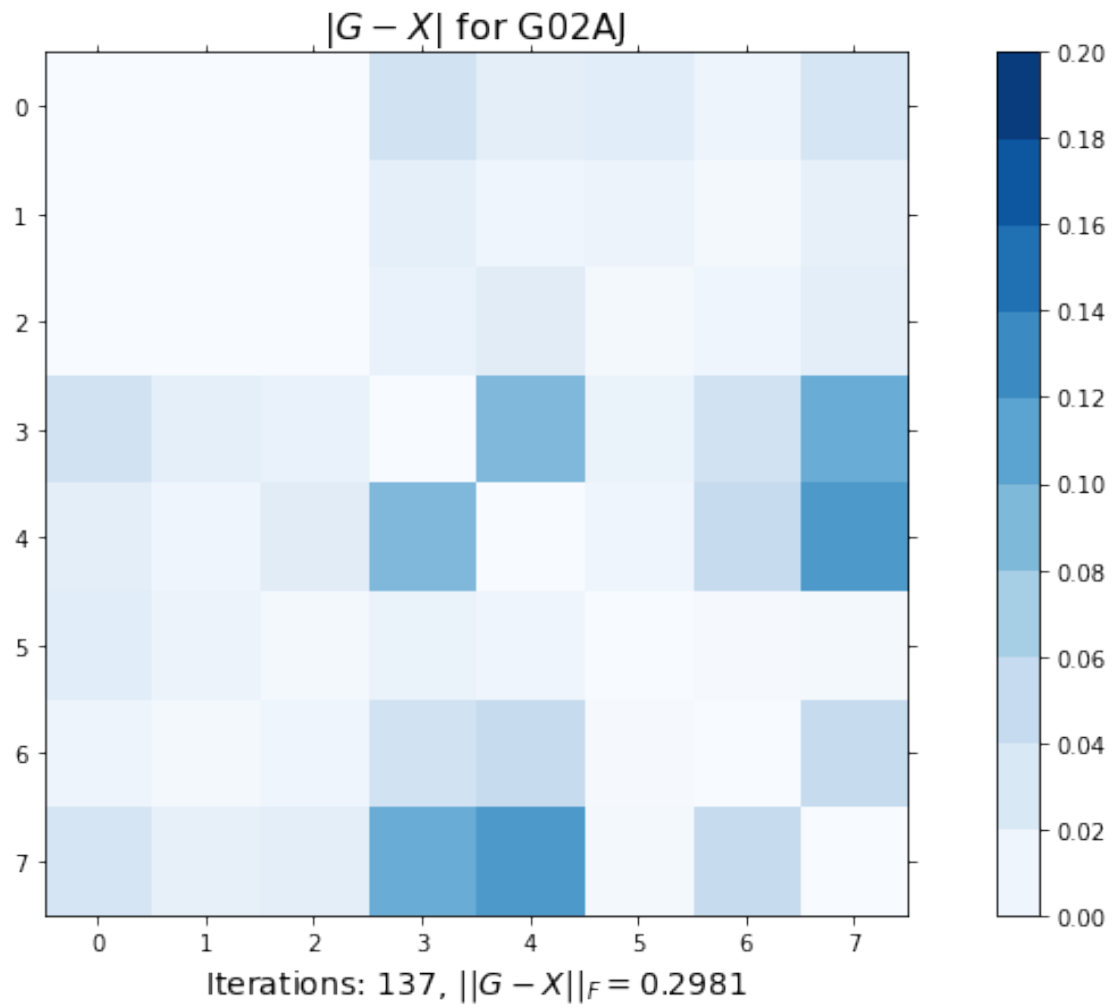
```
// Call the NAG routine specifying a minimum eigenvalue
G02AJ g02aj = new G02AJ();
G1d = convert2DTo1D(G);
ldg = G.length;
n = G[0].length;
alpha = 0.001;
double[] H1d = convert2DTo1D(H);
int ldh = H.length;
```

```
errtol = 0;
maxit = 0;
ldx = n;
X1d = new double[ldx * n];
iter = 0;
double norm2 = 0;
ifail = 0;
g02aj.eval(G1d, ldg, n, alpha, H1d, ldh, errtol, maxit, X1d, ldx, iter, norm2, ifail);

X = convert1DTo2D(X1d, ldx);
iter = g02aj.getITER();
```

```
Nearest correlation matrix using element-wise weighting
[[ 1.     -0.3251  0.1881  0.5371  0.0255 -0.5893 -0.0625 -0.1929]
 [-0.3251  1.      0.2048  0.2249  0.4144  0.2841  0.2914  0.4081]
 [ 0.1881  0.2048  1.     -0.1462  0.7883  0.2718 -0.6084  0.8804]
 [ 0.5371  0.2249 -0.1462  1.      0.2138 -0.0002  0.607  -0.2199]
 [ 0.0255  0.4144  0.7883  0.2138  1.      0.6566 -0.2807  0.8756]
 [-0.5893  0.2841  0.2718 -0.0002  0.6566  1.      0.0474  0.593 ]
 [-0.0625  0.2914 -0.6084  0.607  -0.2807  0.0474  1.     -0.4471]
 [-0.1929  0.4081  0.8804 -0.2199  0.8756  0.593  -0.4471  1.    ]]
```

```
jobvl = "N";
jobvr = "N";
n = X[0].length;
lda = X.length;
wr = new double[n];
wi = new double[n];
ldvl = 1;
vl = new double[ldvl];
ldvr = 1;
vr = new double[ldvr];
lwork = 3 * n;
work = new double[lwork];
info = 0;
f08na.eval(jobvl, jobvr, n, X1d, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info);
Arrays.sort(wr);
Arrays.sort(wi);
```

```
Sorted eigenvalues of X [0.001  0.001  0.0375 0.1734 0.6882 1.7106 1.9224 3.466
]
```



6 Fixing a Block of Elements

- We probably really wish to *fix* our leading block of true correlations, so it does not change at all.
- We have the NAG routine **G02AN**.
- This routine fixes a leading block, which we require to be positive definite.
- We apply the *shrinking algorithm* of Higham, Strabić and Šego. The approach is **not** computationally expensive.
- What we find is the smallest , such that X is a true correlation matrix:

$$X = \alpha \begin{pmatrix} G_{11} & 0 \\ 0 & I \end{pmatrix} + (1-\alpha)G, \qquad G = \begin{pmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{pmatrix}$$

- G_{11} is the leading k by k block of the approximate correlation matrix that we wish to fix.
- α is in the interval $[0, 1]$.

6.0.1 Use G02AN to compute the nearest correlation matrix with fixed leading block

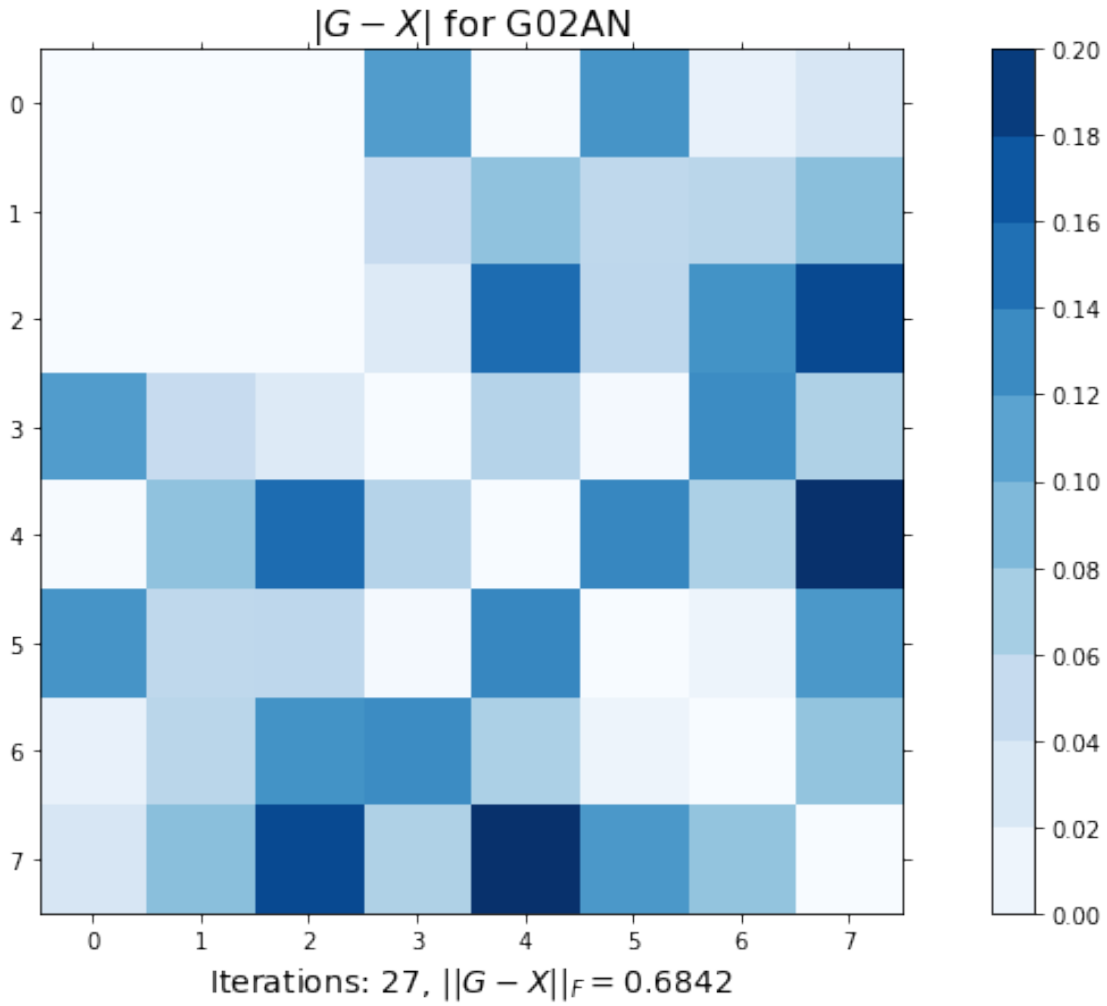
```
// Call the NAG routine fixing the top 3-by-3 block
G02AN g02an = new G02AN();
G1d = convert2DTo1D(G);
ldg = G.length;
n = G[0].length;
int k = 3;
errtol = 0;
double eigtol = 0;
ldx = n;
X1d = new double[ldx * n];
alpha = 0.001;
iter = 0;
double eigmin = 0;
norm2 = 0;
ifail = 0;
g02an.eval(G1d, ldg, n, k, errtol, eigtol, X1d, ldx, alpha, iter, eigmin, norm2, ifail);

X = convert1DTo2D(X1d, ldx);
iter = g02an.getITER();
alpha = g02an.getALPHA();
```

```
Nearest correlation matrix with fixed leading block
[[ 1.     -0.325   0.1881  0.4606  0.0051 -0.4887 -0.0579 -0.1271]
 [-0.325   1.      0.2048  0.1948  0.3245  0.2183  0.2294  0.3391]
 [ 0.1881  0.2048  1.     -0.106   0.6124  0.2211 -0.4936  0.7202]
 [ 0.4606  0.1948 -0.106   1.      0.2432  0.0101  0.516  -0.2567]
 [ 0.0051  0.3245  0.6124  0.2432  1.      0.532  -0.2634  0.7949]
 [-0.4887  0.2183  0.2211  0.0101  0.532   1.      0.0393  0.4769]
 [-0.0579  0.2294 -0.4936  0.516  -0.2634  0.0393  1.     -0.3185]
 [-0.1271  0.3391  0.7202 -0.2567  0.7949  0.4769 -0.3185  1.    ]]
```

```
jobvl = "N";
jobvr = "N";
n = X[0].length;
lda = X.length;
wr = new double[n];
wi = new double[n];
ldvl = 1;
vl = new double[ldvl];
ldvr = 1;
vr = new double[ldvr];
lwork = 3 * n;
```

```
work = new double[lwork];
info = 0;
f08na.eval(jobvl, jobvr, n, X1d, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info);
Arrays.sort(wr);
```

```
Sorted eigenvalues of X [0.     0.1375 0.2744 0.3804 0.7768 1.6263 1.7689
3.0356]
Value of alpha returned: [0.2003]
```



7 Fixing Arbitrary Elements

- The routine **G02AP** fixes arbitrary elements by finding the smallest , such that X is a true correlation matrix in:

$$X = \alpha T + (1 - \alpha)G, \quad T = H \circ G, \quad h_{ij} \in [0, 1]$$

- A "1" in H fixes corresponding elements in G.
- $0 < h_{ij} < 1$ weights corresponding element in G.
- α is again in the interval $[0, 1]$.

7.1 Alternating Projections

- First method proposed to solve our original problem, however, it is very slow.
- The idea is we alternate projecting onto two sets, which are:
  - the set of smeidefinite matrices (S1), and
  - matrices with unit diagonal (s2)
- We do this until we converge on a matrix with both properties.

7.2 Alternating Projections with Anderson Acceleration

- A new approach by Higham and Strabić uses *Anderson Acceleration*, and makes the method worthwhile.
- In particular, we will be able to fix elements whilst finding the nearest true correlation matrix in the Frobenius norm.
- Our projections are now:
  - the set of (semi)definite matrices with some minimum eigenvalue, and
  - matrix with elements $G_{i,j}$ for some given indices i and j
- To appear in a future NAG Library.

8 More on using the NAG Library for *Java*:

https://www.nag.com/content/nag-library-for-java