Chapter 4

calebren

2025-10-14

Exercises

Exercise 4.1

Consider a branching process with offspring distribution $\vec{a} = (a, b, c)$, where $a + b + c = 1$. Let P be the Markov transition matrix. Exhibit the first three rows of P. That is, find P_{ij} for $i = 0, 1, 2$ and $j = 0, 1, \ldots$.

Solution: Since 0 is an absorbing state, it can only transition to 0. For $i = 0$, $P_{0j} = 0$ for all $j \neq 0$ and $P_{00} = 1$.

For $i = 1$, the only individual in the generation can transition to $j = 0, 1, 2$ with distribution $\vec{a}$, so

$$P_{1j} = \begin{cases} a & \text{for } j = 0 \\ b & \text{for } j = 1 \\ c & \text{for } j = 2 \\ 0 & \text{otherwise} \end{cases}$$

For $i = 2$, we can condition on the outcomes of each individual.

$$P_{2j} = \begin{cases} a^2 & \text{for } j = 0 \\ 2ab & \text{for } j = 1 \\ 2ab + c^2 & \text{for } j = 2 \\ 2bc+ & \text{for } j = 3 \end{cases}$$