Ch21

ASEN 3113 HW#8

#21, 39, 59, 113

21.39| Given: 40 cm x 40 cm window
Blackbody assumption
$T = 1200\,K$
$\tau_1 = 0.7$ for $\lambda < 3\,\mu m$
$\tau_2 = \emptyset$ for $\lambda > 3\,\mu m$

Find: τ and G_{tr}

Solve: $\lambda T = (3\mu m)(1200\,K) = 3600\,\mu m K$

table 21-2 $\Rightarrow f_\lambda = 0.403607$

$$\tau_{total} = \tau_1 f_\lambda + \tau_2 (1 - f_\lambda)$$

$$= 0.7(0.403607) + \emptyset(0.596393)$$

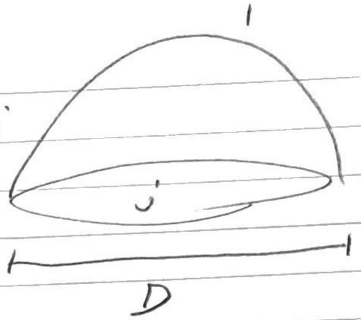
$$\boxed{\tau_{total} = 0.2825}$$

$$G_{tr} = \tau_{total}\, G = \tau_{total}(A \sigma T^4)$$

$$= (0.2825)(0.4 m^2)(5.67e\text{-}08\ W/m^2K^4)(1200\,K)^4$$

$$\boxed{G_{tr} = 531.5\ W}$$

21.59// Given:



Find: $F_{i \to j}$

Solve: Surface j cannot see itself, therefore, $F_{jj} = \emptyset$ (1)

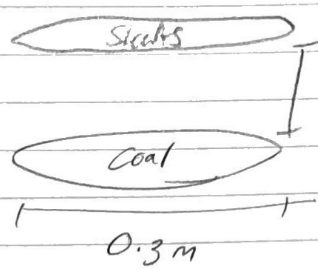
By Summation Rule: $F_{jj} + F_{ji} = 1$ (2)

Combine (1) & (2): $F_{ji} = 1$

Reciprocity theorem: $A_j F_{ji} = A_i F_{ij}$

$$F_{ij} = F_{ji}\left(\frac{A_j}{A_i}\right) = 1\left(\frac{\pi\left(\frac{d}{2}\right)^2}{\frac{1}{2}\left(4\pi\left(\frac{d}{2}\right)^2\right)}\right) = \boxed{\frac{1}{2} = F_{ij}}$$

21.113// Given:



Assume: Coal & Steak ≈ blackbody

$T_c = 950\,K$
$T_s = -5°C = 278\,K$

Find: $\dot{Q}_{c\to s}$ w/ open side & reradiating side

Solve: Table 21-3 $\Rightarrow F_{cs} = 1 + \dfrac{1-\sqrt{4R^2+1}}{2R^2} = 1 + \dfrac{1-\sqrt{4(0.75)^2+1}}{2(0.75)^2} = 0.28642$

$$\dot{Q}_{cs} = A_c F_{cs}\sigma\left(T_c^{\,4} - T_s^{\,4}\right) \qquad \to R = \frac{r}{L} = \frac{0.15\,m}{0.2\,m} = 0.75$$

$$\dot{Q}_{cs} = \left(\pi(0.15\,m)^2\right)(0.28642)\left(5.67e\text{-}08\,\tfrac{W}{m^2K^4}\right)\left(950\,K^4 - 278\,K^4\right)$$

$$= \boxed{928.2\,W}$$

For a reradiating surface, $\dot{Q}_f = \varnothing = \dot{Q}_{cf} + \dot{Q}_{sf} = A_c F_{cf}(E_{bc} - E_{bf}) + A_s F_{sf}(E_{bs} - E_{bf})$

$$E_{bf} = \frac{A_c F_{cf} E_{bc} + A_s F_{sf} E_{bs}}{A_c F_{cf} + A_s F_{sf}}$$

$F_{cf} + F_{cs} = 1 \Rightarrow F_{cf} = 1 - F_{cs} = 0.71358$, by symmetry $F_{cf} = F_{sf} = 0.71358$

$E_{bc} = \sigma T_c^{\,4} = \left(5.67e\text{-}08\,\tfrac{W}{m^2K^4}\right)(950\,K)^4 = 46{,}182\,\tfrac{W}{m^2}$

$E_{bs} = \sigma T_s^{\,4} = \left(5.67e\text{-}08\,\tfrac{W}{m^2K^4}\right)(278\,K)^4 = 338.7\,\tfrac{W}{m^2}$

$$E_{bf} = \frac{\left(\pi(0.15\,m)^2\right)(0.7136)\left(46{,}182\,\tfrac{W}{m^2}\right) + \left(\pi(0.15\,m)^2\right)(0.7136)\left(338.7\,\tfrac{W}{m^2}\right)}{\left(\pi(0.15\,m)^2\right)(0.7136) + \left(\pi(0.15\,m^2)\right)(0.7136)} = 23{,}260\,\tfrac{W}{m^2}$$

$\dot{Q}_s = \dot{Q}_{cs} + \dot{Q}_{fs} = \dot{Q}_{cs} + A_f F_{fs}(E_{bf} - E_{bs}) = \dot{Q}_{cs} + A_s F_{sf}(E_{bf} - E_{bs})$

$$= 928.2\,W + \left(\pi(0.15\,m)^2\right)(0.7136)\left(23{,}260\,\tfrac{W}{m^2} - 338.7\,\tfrac{W}{m^2}\right) = \boxed{2084.4\,W}$$