ASEN 3113 HW #7

Ch 18

#15, 19, 42, 43

18.15 // Given: 1000 W

$L = 0.5\,cm$

Al = 2024-T6

$\rho = 2770\ kg/m^3$

$C_p = 875\ J/kg\,K$

$\alpha = 7.3e\text{-}05\ m^2/s$

$T_a = 22°C = 295\,K$

$h = 12\ W/m^2K$

eff = 85%

$A = 0.03\,m^2$

Assume:

- thermal properties constant
- h is uniform & constant

Find: Valid for lumped analysis?

t @ plate = 140°C = 413 K

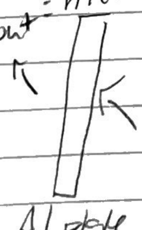
Solve: $k = \alpha \rho C_p = (7.3\cdot10^{-5}\ m^2/s)(2770\ kg/m^3)(875\ J/kg\,K) = 177\ W/mK$

$L_c = \frac{L}{2} = \frac{0.5\,cm}{2} = 0.0025\,m$

$Bi = \frac{hL_c}{k} = \frac{(12\ W/m^2K)(0.0025\,m)}{177\ W/mK} = 1.70e\text{-}04 < 0.1$

Therefore, it is valid to assume lumped analysis.

$\dot{Q}_{out} = hA\Delta T$



$\dot{Q}_{in} = P(eff) = 1000\,W\,(0.85) = 850\,W$

Al plate

$m = \rho A L$

$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE}{dt} \Rightarrow 850\,W - hA(T - T_a) = mC_p\frac{dT}{dt}$

$850\,W - (12\ W/m^2K)(0.03\,m^2)(T - 295\,K) = (2770\ kg/m^3)(0.03\,m^2)(0.005\,m)(875\ J/kg\,K)\frac{dT}{dt}$

$\int_0^t \frac{dt}{363.5\ J/K} = \int_{295}^{413} \frac{dT}{956.2 - 0.36T} \Rightarrow \frac{t}{363.5} = -\frac{1}{0.36}\ln(956.2 - 0.36T)\Big|_{295}^{413}$

$t = -1009.7\ln(956.2 - 0.36T)\Big|_{295}^{413} = \boxed{518\,s}$

18.11 Given: $L = 2\,cm$

$k = 21\ W/mK$

$\rho = 8000\ kg/m^3$

$C_p = 570\ J/kg K$

$T_i = 18°C$

$T_a = 950°C$

$h = 150\ W/m^2K$

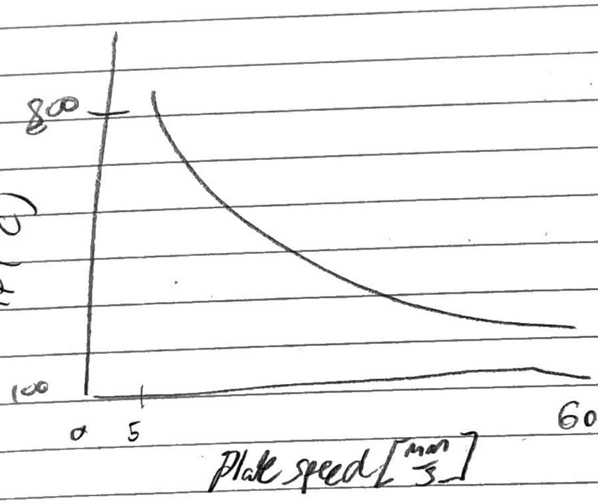
$d = 3\,m$

$V = 5\ mm/s$ to $60\ mm/s$

Assume:
Thermal Props Const
Lumped Analysis

Find: Variation in final temperature by velocity

Solve: See MATLAB Code *



18.42// Given: $L = \frac{10cm}{2} = 5cm$

$\rho = 2702\ kg/m^3$

$C_p = 903\ J/kg\,K$

$k = 237\ W/mK$

$\alpha = 97.1e{-06}\ m^2/s$

$T_\infty = 500°C$

$T_i = 25°C$

Assume:

- Surface temp $T_s \approx T_\infty$

Find: T_f @ $t = 15s$ using 1 term approx

Solve: Because we assume $T_s \approx T_\infty$ it implies $h \approx \infty$

$$B_i = \frac{hL}{k} = \infty$$

Table 18-2 -> $\lambda_1 = 1.5708$

$A_1 = 1.2732$

$$\theta_{wall} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right)$$

$$\tau = \frac{\alpha t}{L^2} = \frac{97.1e{-06}\ m^2/s\ t}{(0.05m)^2} = 0.03885^{-1}$$

$$T(\emptyset, 15) = T_\infty + (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} \cos(\emptyset)$$

$$= 500°C + (25°C - 500°C) 1.2732 e^{-(1.5708)^2 (0.03885^{-1})(15s)} (1)$$

$$= 500°C + (-143.9°C)$$

$$\boxed{T_f = 356.1°C}$$

18.43// Given: $L = \frac{3cm}{2} = 1.5cm$

$k = 110\ W/mK$

$\rho = 8530\ kg/m^3$

$c_p = 380\ J/kgK$

$\alpha = 33.9e{-06}\ m^2/s$

$T_i = 25°C$

$T_\infty = 700°C$

$t = 10\ min = 600\ s$

$h = 80\ W/m^2K$

Find: T_s @ t

Is lumped analysis valid

Solve: $Bi = \frac{hL}{k} = \frac{80\ W/m^2K\,(0.015\ m)}{110\ W/mK} = 0.0109$

$\tau = \frac{\alpha t}{L^2} = \frac{33.9e{-06}\ m^2/s\,(600\ s)}{(0.015\ m)^2} = 90.4 > 0.2$

Table 10-2 $\xrightarrow{Bi=0.01}$ $\begin{cases} \lambda_1 = 0.0998 \\ A_1 = 1.0017 \end{cases}$

$$\theta_{wall} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\lambda_1 \frac{x}{L}\right)$$

$$T(L, 600) = T_\infty + (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

$$= 700°C + (25°C - 700°C)(1.0017) e^{-0.0998^2 (90.4)} \cos(0.0998)$$

$$\boxed{T_s = 426.6\ °C}$$

$Bi = 0.0109 \leq 0.1$

therefore, a lumped analysis would also be accurate.