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Computer Algorithms
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HOMEWORK 1a

1.

$$n+10=O(n)$$

$$n+10\le c\cdot n$$

$$\frac{(n+10\le c\cdot n)}{n}$$

$$\frac{(n+10)}{n}\le c$$

$$if\ c\ge n+\frac{10}{n},$$

$$then\ N=1, c=11.$$

2.

$$f_1(n)+f_2(n)=O(n)$$

$$f_1(n)\le c_1\cdot n_1 \quad f_2(n)\le c_2\cdot n_2$$

$$f_1(n)+f_2(n)\le(c_1\cdot n)+(c_2\cdot n)$$

$$f_1(n)+f_2(n)\le n\cdot(c_1+c_2)$$

$$f_1(n)+f_2(n)\le n\cdot c_3.$$

Since **n** *doesn't change,*
$O(n)$ *for* f_1 and f_2 *is the same.*

$$N=1, c=1.$$

3.

$$2\text{n}\,!=O(n^n)$$

$$\frac{(2\text{n}\,!)}{n^n}\le\frac{(c\cdot n^n)}{n^n}$$

$$\frac{(2\text{n}\,!)}{n^n}\le c$$

$$c\ge\frac{[2\cdot(n-1)!]}{n^{(n-1)}}$$

$$c\ge\frac{[2\cdot((1)-1)!]}{(1)^{((1)-1)}}=2$$

$$N=1, c=2.$$

4.

$$\log(n!)=O(n\cdot logn)$$

$$\log(n!)\le c\cdot n\cdot logn$$

$$\log[(n-1)!]\le c\cdot n$$

$$\log[((1)-1)!]\le c\cdot(1)$$

$$if\ \log(0!)\le c,\ \ i.e.\ \ c\ge 0,$$

$$then\ N=1, c=1.$$

5.

$$\lfloor x+\frac{1}{2} \rfloor=\Theta(x), \text{where } x>0$$

$$c_1 \cdot x \leq \lfloor x \rfloor + \lfloor \frac{1}{2} \rfloor \leq c_2 \cdot x$$

$$N=x, \quad \text{so if} \quad N=1,$$

$$c_1 \cdot 1 \leq \lfloor 1 \rfloor + \lfloor \frac{1}{2} \rfloor \leq c_2 \cdot 1$$

$$\text{So}, N=1, c=2 \quad \text{for } \Omega(x) \text{ and } O(x)$$

6.

$$an^2+bn+c=\Theta(n^2),$$

$$\text{where } a>0, \quad b\geq 0, \quad c\geq 0$$

$$c_1 \cdot n^2 \leq an^2+bn+c \leq c_2 \cdot n^2$$

$$N=1, c_1 \leq a+b+c \leq c_2$$

$$a+b+c \leq 1, \text{so } c_3=1, \ N=1$$
$$(c_3 = \text{some } c \text{ that fits } c_1 \text{ and } c_2)$$
$$\text{for } \Omega(n) \text{ and } O(n)$$

7.

$$n \cdot \log n = o\left(\frac{1}{2} \cdot n^2\right)$$

$$\lim \text{ as } n \to \infty \left(\frac{(n \cdot \log n)}{[(\frac{1}{2}) \cdot n^2]}\right)=0$$

$$\lim \text{ as } n \to \infty \left(\frac{(n \cdot \log n)}{[(\frac{1}{2}) \cdot n^2]}\right) dx = \frac{[1 \cdot (\frac{1}{(n \cdot \ln b)})]}{n}$$

$$\text{where } b \text{ is some constant.}$$

$$n \to \infty \text{ faster than } \frac{1}{(n \cdot \ln b)} \to \infty,$$

$$\text{therefore:} \quad n \cdot \log n = o\left(\frac{1}{2} \cdot n^2\right)$$

8.

$$n^3 = o(1.01^n)$$

$$\lim \text{ as } n \to \infty \left(\frac{n^3}{1.01^n}\right) dx \to \left(\frac{3n^2}{(n \cdot 1.01^{(n-1)})}\right) dx$$

$$\to \left(\frac{6n}{[(n^2-n) \cdot 1.01^{(n-2)}]}\right) dx$$

$$\to \left(\frac{6}{[(n^3-3n^2+2n) \cdot 1.01^{(n-3)}]}\right) dx$$

$$(n^3-3n^2+2n) \cdot 1.01^{(n-3)} \to \infty \quad \text{faster than } 6,$$

$$\text{therefore: } n^3 = o(1.01^n)$$