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Homework 1b: Questions 1 & 2

1.

$$[f_1(n)=O(g_1(n)) \ \wedge \ f_2(n)=O(g_2(n))] \Rightarrow f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$$

$$(f_1 \le c_1 \cdot g_1 \ \wedge \ f_2 \le c_2 \cdot g_2) \Rightarrow (f_1 \cdot f_2 \ \le \ c_1 \cdot g_1 \cdot c_2 \cdot g_2),$$

$$Let\, c = c_1 \cdot c_2 \ \ , \ f = f_1 \cdot f_2 \ \ , \ g = g_1 \cdot g_2, \ so \ f \le c \cdot g$$

This matches our definition of Big-O. Therefore, the product of f1 and f2 is upwardly bounded by the product of g1 and g2.

2. $3^n = \omega(n^k), where\, constant\, k > 1 \Rightarrow \underset{n \to \infty}{limit} \dfrac{f(n)}{g(n)} \ = \ \dfrac{3^n}{n^k} \Rightarrow \infty$

This means that ***f(n)*** must grow at a faster rate than ***g(n)***. If ***k*** is a constant, that means for some input size, ***n*** will exceed ***k***. We know that $\mathbf{2^n}$ grows faster that $\mathbf{n^2}$. Therefore, we may induce that there exists an ***N*** where: $3^N \ge N^k, \ where \ N > k$