Prove the following statements (10 points)

- 1. If $a|b$ and $a|c$, then $a|(mb+nc)$, where $m,n \in \mathbb{Z}$

  $a|b$ means $\exists k_1 \in \mathbb{Z}$ such that $b = k_1 a$
  $a|c$ means $\exists k_2 \in \mathbb{Z}$ such that $c = k_2 a$

  We want to know if $mb+nc$ can be factorized as the product of a and an integer.

  Obviously, $mb = mk_1 a$, $nc = nk_2 c$. Adding them together, we have

  $$mb + nc = mk_1 a + nk_2 a = (mk_1 + nk_2)a$$

  Obviously, $(mk_1 + nk_2)$ is an integer, that is $mb + nc$ can be factorized as the product of a and an integer!
  According to the definition of division, we have $a|(mb+nc)$.