Optimal tests of the composite null hypothesis arising in mediation analysis

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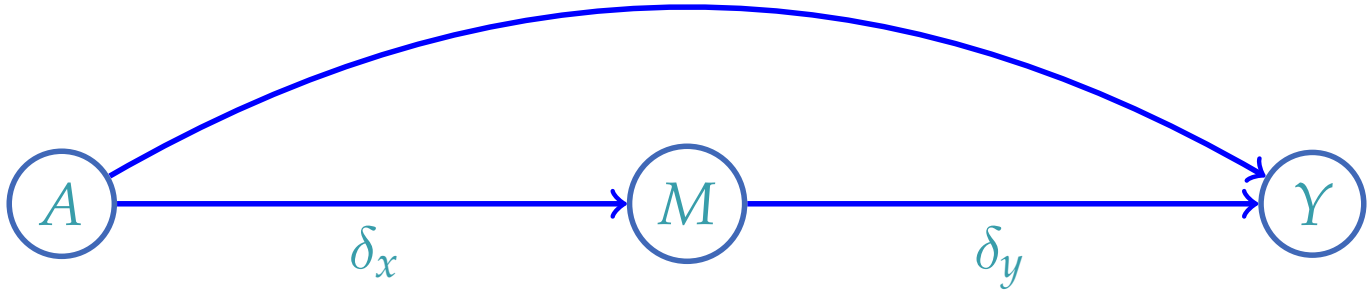
Mediation testing setting

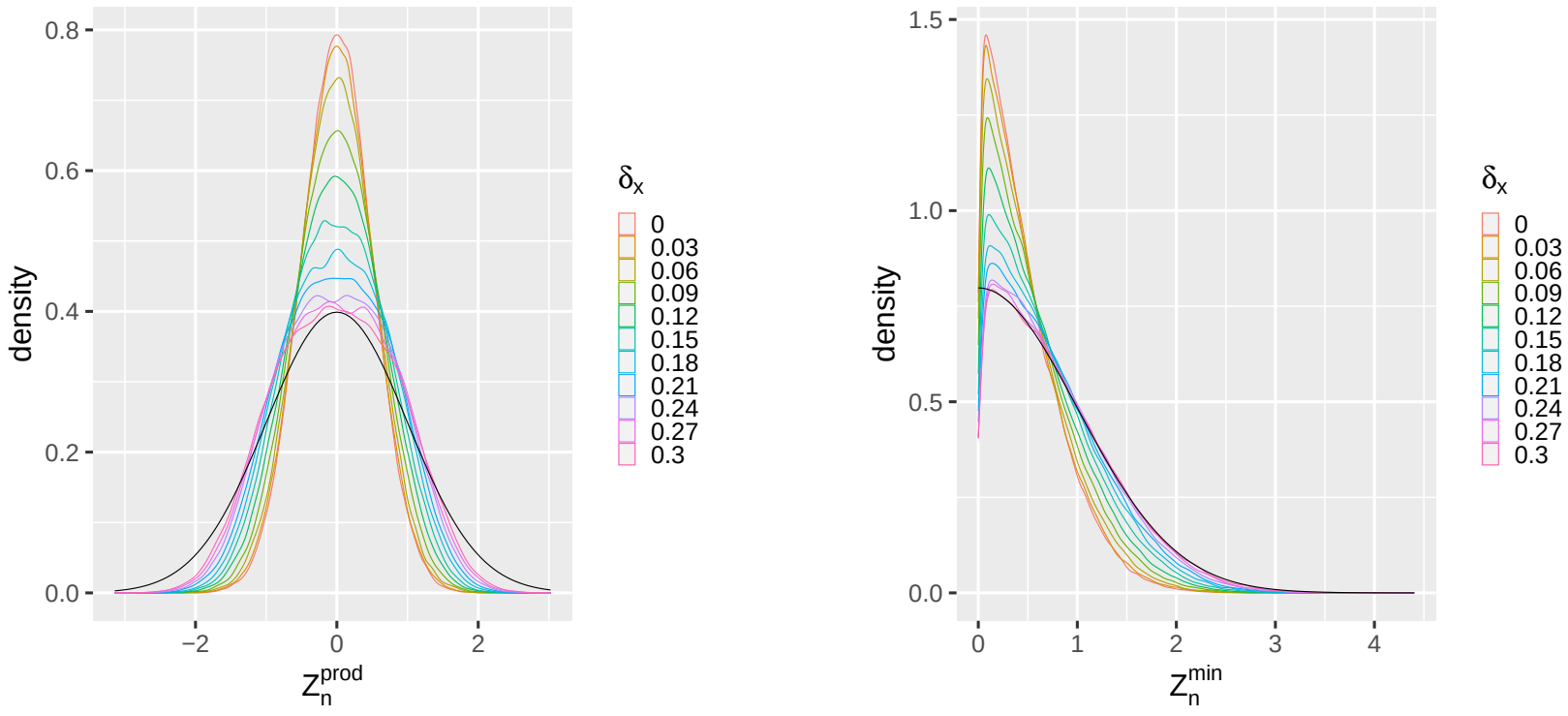


- **Under certain causal and regression modeling assumptions,** $NIE = \delta_x \delta_y$**.**
- **We want to test** $H_0 : \delta_x \delta_y = 0$
  1. **with at most** α **type 1 error for each possible** (δ_x, δ_y) **satisfying** $\delta_x \delta_y = 0$
  2. **maximizing power (in some sense) everywhere else.**
- **Coefficient estimators are asymptotically normal:**

$$\sqrt{n}\Sigma_n^{-1/2}\left\{(\hat{\delta}_x, \hat{\delta}_y)^\top - (\delta_x, \delta_y)^\top\right\} \rightsquigarrow \mathcal{N}\left\{(0,0)^\top, I_2\right\}$$

uniformly in (δ_x, δ_y)**.**

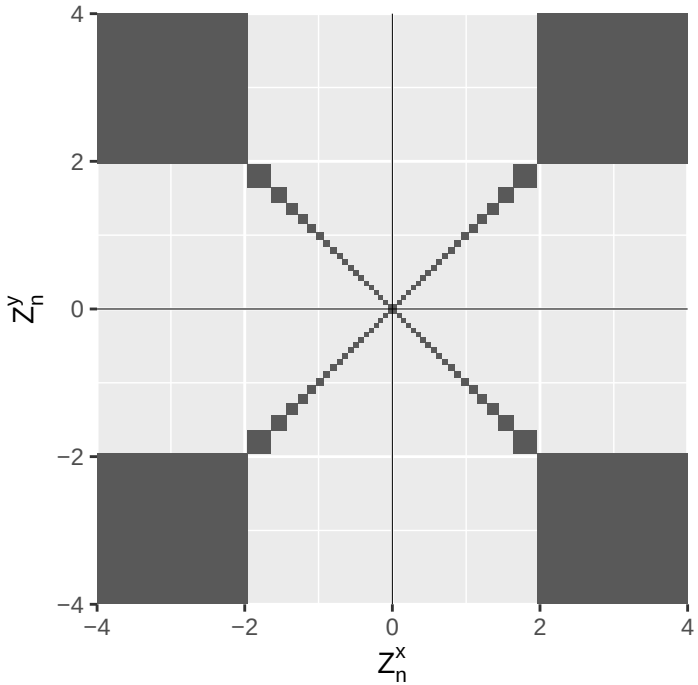
Traditional tests are overly conservative

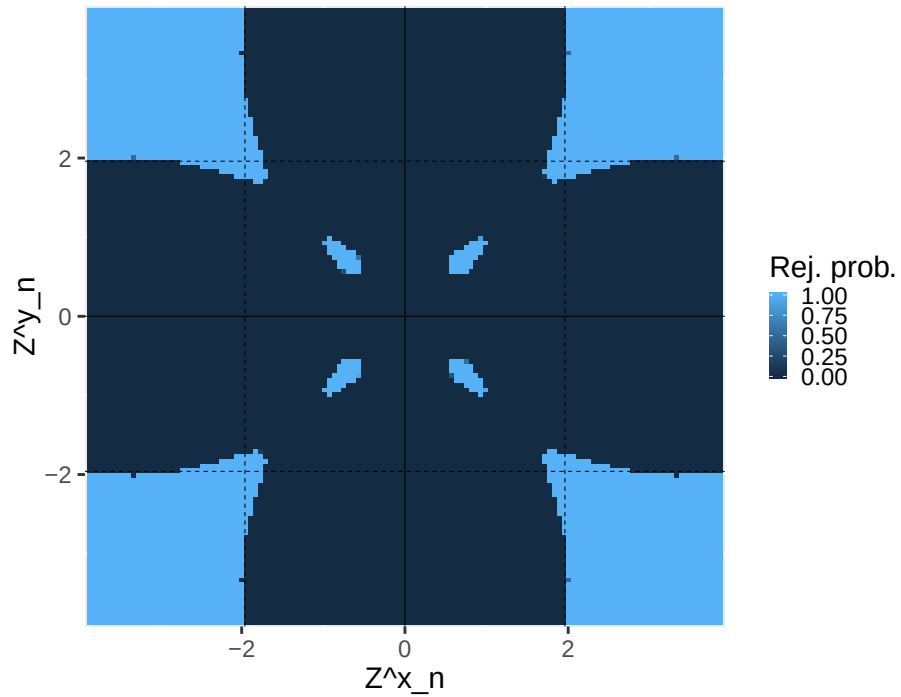


(a) **(b)**

Density plots of (a) the delta method and (b) joint significance test statistics under $\delta_y = 0$ and varying δ_x with $n = 100$.

Novel tests of H_0 with optimal power

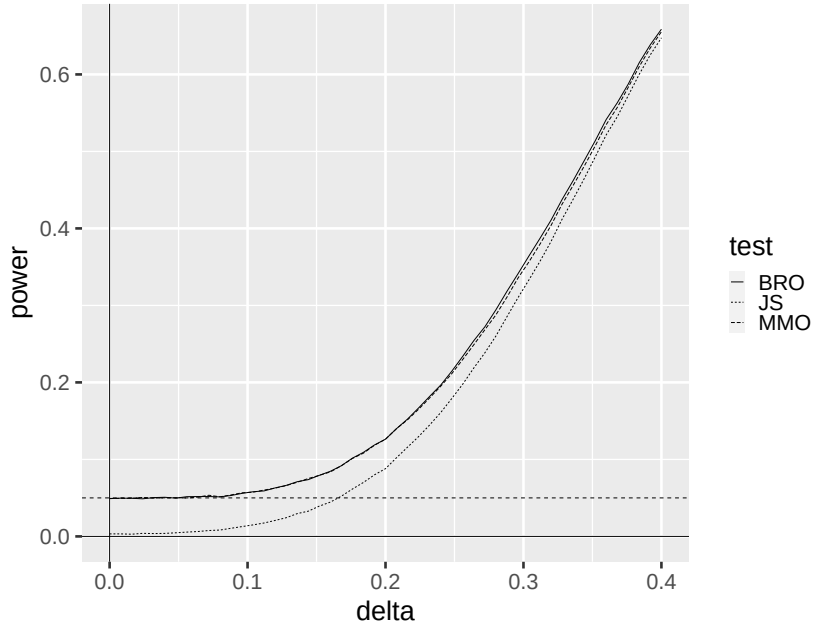




(a) **(b)**

Rejection regions of (a) the minimax optimal test and (b) the Bayes risk optimal test

Our tests dominate traditional tests in terms of power



Rejection probabilities of the minimax optimal (MMO), Bayes risk optimal (BRO), and joint significance (JS) tests.

Paper: `https://arxiv.org/abs/2107.07575`

R package: `https://github.com/achambaz/mediation.test`

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