1. Prove the following for $x \in \mathbb{C}^n$:

   (a) $\|x\|_\infty \leq \|x\|_1 \leq n\|x\|_\infty$.
   Proof:

   For the first inequality, we have

$$\begin{aligned}\|x\|_\infty &= \max_{1\leq i\leq n} |x_i| \\ &\leq \sum_{i=1}^n |x_i| \\ &= \|x\|_1.\end{aligned}$$

   For the second half of our inequality chain, we have

$$\begin{aligned}\|x\|_1 &= \sum_{i=1}^n |x_i| \\ &\leq \sum_{i=1}^n \left(\max_{1\leq j\leq n} |x_j|\right) \\ &= n \max_{1\leq j\leq n} |x_j| \\ &= n\|x\|_\infty\end{aligned}$$

   Thus, $\|x\|_\infty \leq \|x\|_1 \leq n\|x\|_\infty$. □

   (b) $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$.
   Proof:

   For the first inequality, we have

$$\begin{aligned}\|x\|_\infty &= \max_{1\leq i\leq n} |x_i| \\ &= \sqrt{\left(\max_{1\leq i\leq n} |x_i|\right)^2} \\ &\leq \sqrt{\sum_{i=1}^n |x_i|^2} \\ &= \|x\|_2.\end{aligned}$$

For the second half of our inequality chain, we have

$$\begin{aligned}
\|x\|_2 &= \sqrt{\sum_{i=1}^{n} |x_i|^2} \\
&\leq \sqrt{\sum_{i=1}^{n} \left(\max_{1\leq j\leq n} |x_j| \right)^2} \\
&= \sqrt{n \left(\max_{1\leq j\leq n} |x_j| \right)^2} \\
&= \sqrt{n} \max_{1\leq j\leq n} |x_j| \\
&= n\|x\|_\infty.
\end{aligned}$$

Thus, $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$. □

(c) $\|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2$.
Proof:

□