1. Let $A \in \mathbb{R}^{n\times n}$ be a tridiagonal matrix where the diagonal entries are given by a_j for $j = 1, \ldots, n$, the lower diagonal entries are b_j for $j = 2, \ldots, n$ and the upper diagonal entries are c_j for $j = 1, \ldots, n-1$.

   (a) For $n = 3$, derive the LU factorization of the matrix A.

2. Consider the linear system

$$\begin{aligned} 6x + 2y + 2z &= -2 \\ 2x + \frac{2}{3}y + \frac{1}{3}z &= 1 \\ x + 2y - z &= 0 \end{aligned}$$

   (a) Verify that $(x, y, z) = (2.6, -3.8, -5)$ is the exact solution.
   To verify the solution, let's first rewrite the LHS of the system and multiply by our vector to get

$$\begin{pmatrix} 6 & 2 & 2 \\ 2 & \frac{2}{3} & \frac{1}{3} \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2.6 \\ -3.8 \\ -5 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

   which shows the exact solution is given by $(x, y, z) = (2.6, -3.8, -5)$.

   (b) Let's create our augmented matrix and begin Gaussian elimination

$$\left(\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 2 & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0.3333 & 0.3333 & -0.3333 \\ 2 & 0.6667 & 0.3333 & 1 \\ 1 & 2 & -1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0.3333 & 0.3333 & -2 \\ 0 & 0.0001 & -0.3333 & 1.666 \\ 0 & 0 & -1.333 & 0.3333 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0.3333 & 0.3333 & -2 \\ 0 & 1 & -3333 & 16660 \\ 0 & 0 & -1.333 & 0.3333 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1111 & -5554 \\ 0 & 1 & -3333 & 16660 \\ 0 & 0 & -1.333 & 0.3333 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1111 & -5554 \\ 0 & 1 & -3333 & 16660 \\ 0 & 0 & 1 & -0.2500 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -5276 \\ 0 & 1 & 0 & 15820 \\ 0 & 0 & 1 & -0.2500 \end{array}\right)$$