

SOME GRAPH EXPLORATION

1. BASICS

Let $G = (V, E)$ be a graph with $V = \{1, \dots, n\}$. Define $\mathbb{1}$ to be the $n \times 1$ matrix (i.e., column vector) with 1 in every slot. Let A be the adjacency matrix of G .

Proposition 1. $D_{i,i}$ is the number of neighbors of i .

Proof.

□

Definition 1. $L := D - A$

Proposition 2. $L_{i,j} = \begin{cases} D_{i,i} & i = j \\ -1 & \\ 0 & \end{cases}$

Proof.

□

2. SEARCH

There might be a name for this somewhere.

Definition 2. Call a graph **local** if its adjacency matrix is block diagonal.

This is the WRONG definition.

It seems like a local graph would lend itself more to parallel BFS.

Proposition 3. Suppose a real symmetric matrix B is block-diagonal with blocks B_1, \dots, B_N . Then the B_i share a positive eigenvalue. Let λ be such a shared eigenvalue, and suppose v_1, \dots, v_N are the corresponding eigenvectors. Then $(v_1^T, \dots, v_N^T)^T$ is an eigenvector for B with eigenvalue λ .

Proof.

□