

## SOME GRAPH EXPLORATION

### 1. BASICS

Let  $G = (V, E)$  be a graph with  $V = \{1, \dots, n\}$ . Define  $\mathbb{1}$  to be the  $n \times 1$  matrix (i.e., column vector) with 1 in every slot. Let  $A$  be the adjacency matrix of  $G$ .

**Proposition 1.**  $D_{i,i}$  is the number of neighbors of  $i$ .

*Proof.* □

**Definition 1.**  $L := D - A$

$$\text{Proposition 2. } L_{i,j} = \begin{cases} D_{i,i} & i = j \\ -1 & \\ 0 & \end{cases}$$

*Proof.* □

### 2. SEARCH

There might be a name for this somewhere.

**Definition 2.** Call a graph **local** if its adjacency matrix is block diagonal.

*This is the WRONG definition.*

It seems like a local graph would lend itself more to parallel BFS.

**Proposition 3.** Suppose a real symmetric matrix  $B$  is block-diagonal with blocks  $B_1, \dots, B_N$ . Then the  $B_i$  share a positive eigenvalue. Let  $\lambda$  be such a shared eigenvalue, and suppose  $v_1, \dots, v_N$  are the corresponding eigenvectors. Then  $(v_1^T, \dots, v_N^T)^T$  is an eigenvector for  $B$  with eigenvalue  $\lambda$ .

*Proof.* □